# VITERBI DECODER PROCESSING

#### Viterbi References

#### • Suggested references:

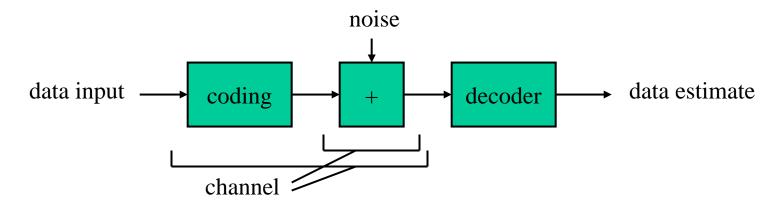
- J. G. Proakis and M. Salehi, *Fundamentals of Communica-tion Systems*, Prentice Hall, 2005. [coding and viterbi]
- A. J. Viterbi, "Error Bounds for Convolutional Codes and an Asymptotically Optimum Decoding Algorithm," *IEEE Trans.* on Information Theory, April 1967.
- G. D. Forney, "Maximum-Likelihood Sequence Estimation of Digital Sequences in the Presence of Intersymbol Interference," *IEEE Trans. on Information Theory*, May 1972.
- H.-L. Lou, "Implementing the Viterbi Algorithm," *IEEE Signal Processing Magazine*, Sept. 1995.
- P. J. Black, "Algorithms and Architectures for High Speed Viterbi Decoding," Ph.D. dissertation, March 1993.

# Viterbi Algorithm

- Widely used in communication systems to decode a data sequence that has been encoded by a "finitestate" process
  - Ex: ethernet receiver
  - Ex: hard disk read electronics
- A very common demanding DSP task
  - Ex: viterbi building blocks are in every serious DSP benchmarking suite
  - Ex: The Texas Instruments C64x DSP processor has a dedicated on-chip viterbi decoder. The viterbi decoder and a "turbo decoder" occupy about 8% of the chip's area

# Viterbi Algorithm

- Involves coding data, adding noise, and decoding
  - Deliberate encoding: convolutional or trellis codes
  - Unintentional encoding: intersymbol interference
- Output is an *estimate* of the original data
- Viterbi algorithm is optimal in the maximum likelihood sense—it finds the input that is most likely, given the observed channel output



- Encoding is a process which adds redundancy to data to reduce the probability of errors or increase the level of acceptable noise in the channel
- The constraint length (commonly called *k*) is a measure of the complexity of the code
  - Ex: k = 7  $\rightarrow$   $2^{k-1}$  states =  $2^6$  states = 64 states (in Wi-Fi)
  - Ex: k = 9 → 256 states (a challenging design)
- Ex: a 4-state, Rate =  $\frac{1}{2}$  convolutional encoder
  - input data  $x_n$
  - encoded symbols  $y_1$  and  $y_2$
  - $y_{1,n} = x_n \text{ XOR } x_{n-1} \text{ XOR } x_{n-2}$
  - $y_{2,n} = x_n \text{ XOR } x_{n-2}$
  - Two data bits output by decoder for each input bit (rate =  $\frac{1}{2}$ )
  - "Double rate" output bits typically sent serially into one "channel"

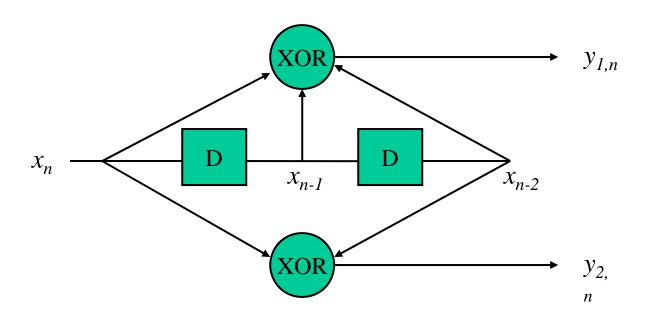
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• Present state:

$$x_{n-2}$$
  $x_{n-1}$ 

• Next state:

$$x_{n-1}$$
  $x_n$  (input)



• 8-entry truth table

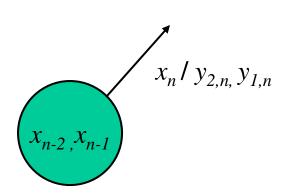
- inputs: three *x*'s

- outputs: two *y*'s

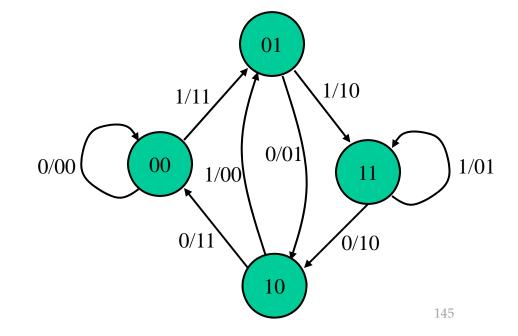
$y_{1,n}$	$y_{2,n}$
0	0
1	1
1	0
0	1
1	1
0	0
0	1
1	0
	0 1 1 0 1 0 0

- Each state can transition to only 2 other states
  - These two states are reached by either  $x_n = 0$  or 1
- Next state found by:

$$x_{n-1}$$
  $x_n$  (input)

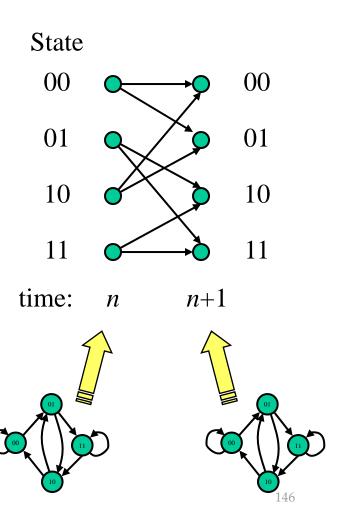


Notation used in State Graph



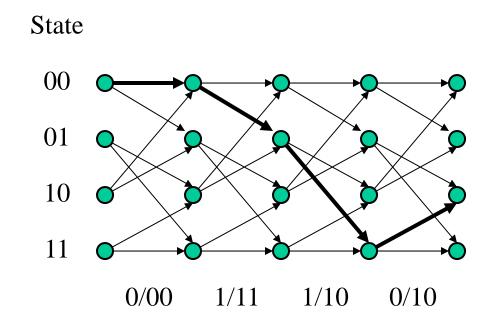
#### **Trellis**

- The *trellis* is a time indexed version of the state diagram
- Each state transition (input bit into the encoder) corresponds to a forward step in the trellis



#### **Trellis**

- Given a starting state, every possible input sequence corresponds to a unique path through the trellis
- Example:
  - Assume start in state 00
  - Inputs  $x_n = 0, 1, 1, 0$
  - Outputs =
    {0,0},
    {1,1},
    {1,0},
    {1,0}



## Viterbi Decoding

- The Viterbi decoder calculates a semi-brute-force estimate of the likelihood for each path through the trellis
- Key point: Once the estimates for all states in a step/iteration of the trellis have been calculated, the probabilities for all previous steps/iterations can be discarded; only the most likely entry to a state must be remembered
- It comprises four basic steps:
  - 1. Calculate the trellis
    - a) Weight the trellis branches by calculating branch metrics
    - b) Compute the minimum weight path to time *n*+1 in terms of the minimum weight path to time *n*. Retain step decisions. Uses *add-compare-select (ACS)* algorithm.
  - 2. Find the last state of the minimum weight path.
  - 3. Find the entire minimum weight path. Also called survivor path decode or *traceback*.
  - 4. Reorder bits into correct forward ordering.

# 1. Calculating the Trellis

- Each branch is assigned a weight, called a *branch metric*
- The branch metric is a measure of the likelihood of the transition given the noisy observations
- Likelihood of a transition is given by an appropriate measure of the "distance" between an ideal encoder output and the actual received signal
- The overall goal is to find the minimum weight path (often called *shortest path*) through the trellis

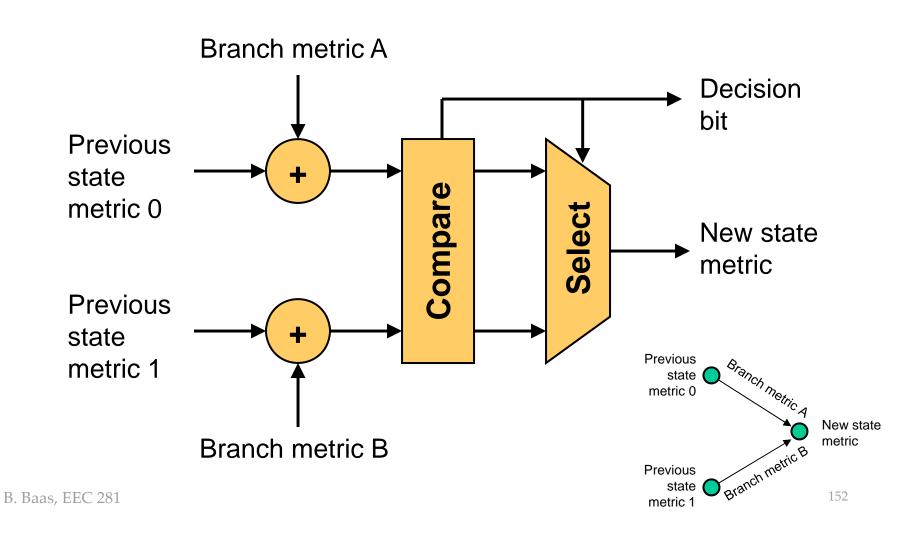
# 1a) Branch Metric Calculations

- More likely transitions receive *lower* weights
- Example:
  - Receive: {0.13, 0.89}
  - Branch with {0,1} outputs receives a lower weight (because it was the more likely transmitted pair) than the {1,0} branch
- A very simple implementation would add just the differences between received values, for example:
  - Starting state: 0,1 Notice from State Graph 0/01 and 1/10 are only options
  - Receive: {0.13, 0.89}
  - Branch {0,0} Impossible, not present on State Graph
  - Branch  $\{0,1\}$  Branch metric = |0-0.13| + |1-0.89| = 0.24
  - Branch  $\{1,0\}$  Branch metric = |1-0.13| + |0-0.89| = 1.76
  - Branch {1,1}
     Impossible, not present on State Graph
- Higher performance decoders would use more complex mapping functions between received values and the branch metric weights

# 1b) ACS

- Goal is to find the most likely entry path into a state
- ADD previous state metric to current branch metric
  - for first branch
  - for second branch
- COMPARE which incoming branch produces the lower metric (higher probability)
- SELECT the minimum and save the branch
- Fully parallel viterbi decoders use many ACS datapaths to calculate an entire trellis state update in one cycle

#### **ACS** Hardware

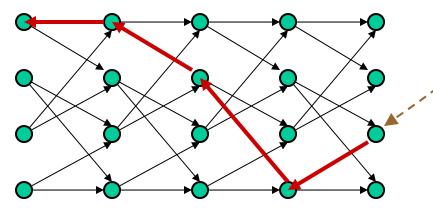


# 2. Finding The Most Likely Path

- After all data have been processed, we must find the most likely path through the trellis
- Done by searching through the final "column" of states and selecting the most likely (lowest weight)
- Operation done infrequently compared to other calculations

#### 3. Traceback

- The goal is to read out the best estimate for all decoded bits
- The process begins at the most likely state and it follows this procedure for each state in the trellis
  - 1) Read stored decision bit from ACS
  - 2) Output this "decoded bit"
  - 3) Use the decision bit to calculate the previous state in the trellis
  - 4) <repeat>



## 4. Reorder Output Bits

- Bits output from the traceback operation are in reverse order (last bit out corresponds to first received)
- Need logic and memory to buffer and reverse order
- Fundamental to the algorithm, cannot be avoided
- Summary
  - Viterbi is data and memory intensive
    - Forward ACS
    - Reverse traceback
    - Forward re-ordering