

MODERN COMPUTER MATHEMATICS

featuring The ARKAY COMPUTER TRAINER

> Model CT-601

ARKAY INTERNATIONAL, INC.



The ARKAY COMPUTER TRAINER Model CT-601

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ing the Computer Trainer in the Classroom

Below is a suggested course of study in computer thematics. Each topic may require several lessons bending upon the maturity and preparation of the dents. It should be emphasized that the purpose of trainer is not to arrive at answers quickly but to estrate and permit analysis of each step that a inputer follows in arriving at a solution. The sugsted activities are by no means all inclusive. The enious teacher can devise many others.

TOPIC I READING AND WRITING BINARY NUMBERS

a. 4 bit numbers (bit is short for binary digit) - iltrate with computer trainer. See Introductory sson Plan.

b. Larger bit numbers - powers of two - use in ogramming.

c. Binary coded decimal (BCD) numbers.

TOPIC II ADDITION OF 4 BIT NUMBERS

a. Addition table:

 $\begin{array}{cccccccc} & 0 & & 0 & & 1 \\ & + & & + & & + \\ \hline & 0 & & 1 & & 1 \\ \hline & 0 & & 1 & & 10 \end{array}$

b. Using the computer trainer add function - operation of switches to display answer.

TOPIC III COMPLEMENTS OF 4 BIT NUMBERS (Shown as negative numbers on computer trainer.)

In modern courses zero as identity element for addition may be introduced or reviewed. (Negative numbers are then additive inverses, e.g. The fifth bit is a one and therefore the sign lights on the trainer.)

5	0101
- 5	1011
0	10000

Changing a positive binary number to a negative binary number by changing all 1's to 0's; all 0's to 1's and then adding 1 to the least significant bit:

Exercise:

Positive	Negative	Check
1001	0110+1	1001
0110	(0111)	0111
		10000

Use computer trainer: Encode positive number in A, negative in B. Use add function (sign only should light).

TOPIC IV SUBTRACTION OF 4 BIT NUMBERS (Minuend larger than subtrahend.)

No problems should arise in this case - with pencil and paper use the additive method of subtraction using the addition table.

0101	5	1011	11
-0010	-3	-0111	- 7
0010	2	0100	4

Use the trainer to check answers with the subtraction function - changing switches to display answer.

TOPIC VI SHIFTING OF 4 BIT NUMBERS ON TRAINER

a. Shifting Left: Radix moves right. (Multiplication by powers of two.)

Relate to multiplication of decimal numbers by powers of ten - moving decimal point right.

b. Shifting Right: Radix moves left. (Division by powers of two.)

Relate to division of decimal numbers by powers of ten - moving decimal point left.

TOPIC V SUBTRACTION OF 4 BIT NUMBERS (Subtrahend larger than minuend.)

Answers will be negative:

3	0011
-5	0101
-2	1.1110 (negative of 0010)

NOTE: The 1 to the left of the radix point indicates a negative number. Use trainer to illustrate lighting of sign bit. (See Triangle Problem Lesson Plan.)

TOPIC VII BASE EIGHT NUMBERS (OCTAL)

More convenient representation of binary numbers used in programming. One octal place = three binary places.

Exercises:

Binary	Octal	Decima	
010 101	25	21	
000 010	0002	0002	

TOPIC VIII INTRODUCING 10 BASIC INSTRUCTIONS OF COMPUTER TRAINER (Coded in binary and octal.)

Students may punch-out and use practice code sheets - thereby becoming very familiar with instructions and their meanings. Although no program is involved, the student develops an insight into necessary and possible steps which may occur in the solution of a problem. (As a teaching tool, the computer trainer shows its superiority over a regular computer in providing this detailed kind of analysis of the basic steps performed.)

TOPIC IX CODING ADDRESSES AND COMBINING THEM WITH THE BASIC INSTRUCTIONS

TOPIC X THE PROGRAM FOR ADDITION OF 2 DIGITS

a. Necessity for storing required constants in core memory: 0001 at one address in case the sum is more than 10 (in binary coded decimals this becomes the tens digit). 0101 at another address for testing the sum (see Introductory Lesson Plan for developing understanding of this step).

b. Practicing add routine with various digits - explaining each step.

TOPIC XI⁴ MULTIPLICATION OF 4 BIT NUMBERS

a. Simplicity of table:

1	0	1	
0	0	0	
1	0	1	

b. Develops understanding of importance of shifting: e.g.

 $\frac{x \begin{array}{c} 0111 \\ 0011 \\ \hline 0111 \\ \hline 0111 \\ 101011 \end{array}}$ (original number shifted left)

TOPIC XII PROGRAM FOR MULTIPLICATION

TOPIC XIII PROGRAM FOR DIVISION

TOPIC XIV OTHER PROGRAMS

TOPIC XV WRITING ORIGINAL PROGRAMS

TOPIC XVI RELATING COMPUTER TRAINER TO REGULAR COMPUTERS WITH LARGER BIT INPUTS

Introductory Lesson Plan

AIMS:

To establish interest in learning about computers.

To learn what a program is.

To introduce the binary system as a necessary part of any computer; relating it to decimal notation.

To become familiar with four bit binary numbers.

(For enrichment in brighter classes: To become familiar with the binary-decimal code of representing decimal numbers.)

BACKROUND:

Decimal notation system. Writing decimal numbers in exponential form:

e.g. 1346 as

$$(10)^3$$
 + $3(10)^2$ + $4(10)^1$ + $6(10)^0$

PROCEDURES:

(The language and examples may be modified for more mature students.)

The presence of the CT-601 on the desk or demonstration table should provide motivation for the lesson. The students will certainly be impressed by its appearance and want to find out what it is and how it works. The teacher might have the addition program on the drum and go through some of the steps so that the students can see the lights flashing and that something is happening. Surely one of them will suggest the word "computer". Question the students about previous contacts with regular computers. Who has seen one in operation? What did you see? (The answers will most probably be that noises were heard, flashing lights were seen and perhaps that the answer to a question was flashed on a screen.

Explain that this computer (the CT-601), while it has flashing lights, is a little different from a regular computer in that they (the students) will be part of it, performing functions accomplished automatically in regular computers and that this computer will thus, in effect, work in slow motion. (A regular computer uses electrical impulses traveling at the speed of light to perform the orders of the program.) The students will perform each order of the program manually so that each step can be analyzed and understood. (We have used the word "program". The next question - from a student or the teacher - might well be: What is a program?)

Write an arithmetic problem on the board such as:

57 +89 Ask the class to write out the steps involved in the solution of this problem, i.e.:

- 1. Add 9 + 7 (Add)
- 2. Write 6 under the 9 (Read in)
- 3. Carry 1 to the tens column. (Store in core memory)
- 4. Add 5 + 8 (Add)
- 5. Add the 1 that was carried. (Read in the stored number and add)
- 6. Write a 4 under the 8. (Read in)
- 7. Write a 1 in the hundreds place. (Read out the answer)

¹Tell the students that they have just written a program in which the human brain is the computer. In the language of electrical computers the steps above might correspond to the words (in parenthesis) at the right of each step.

Demonstrate the memory and control sections of the CT-601: The Core Memory which stores numbers; the Drum which holds the program and the Control Panel which displays the orders. Let the students come up to take a closer look. They will notice that 2^3 , 2^2 , 2^1 and 2^0 are marked on the panels, leading to the next question: What are binary numbers and how and why are they used?

Refer to the arithmetic problem on the board and to the program derived for its solution by the human computer. The human computer has a stored knowledge of answers to many addition combinations in decimal notation but the electrical computer knows only two digits: OFF and ON. We must therefore use a number system with only two digits: 1 for ON and 0 for OFF. At this point review the decimal system in exponential form:

Thousands	Hundreds	Tens	Ones	
10 ³	10^{2}	10 ¹	100	

In the binary system, the places would be:

Eights	Fours	Twos	Ones
2 ³	2 ²	2^1	2^{0}

Demonstrate the Encoder, which changes decimal digits to binary. Have pupils write numbers from 1 to 9 in binary form.

¹. For more mature students, the teacher might refer to "Mathematics and Computers" by George R. Slibitz and Jules A. Larrivee. p. 130-134. A more sophisticated problem is used to show the analogy between the human computer and the electrical computer.

Have students come to the machine and read out the binary equivalent of a given number, then have him check himself on the machine.

While students are practicing the conversion of numbers from decimal to binary and binary to decimal, pairs of students could take turns using the trainer in a contest situation. One of the pair displays a binary number on the A input display, covering the switch. The other student must match the number on the B input display.

The home assignment would be to convert numbers from decimal to binary and from binary to decimal.

In a brighter or faster class, the binary coded decimal system (BCD) of notation would be introduced and a home assignment given to code 3 and 4 place decimal numbers into BCD.

Triangle Problem Lesson Plan

AIMS:

To illustrate how a computer might solve a simple problem.

To provide practice in using the add function of the computer trainer.

To give practice in recognizing the significance of the sign bit on computer trainer.

PROCEDURES:

Review the theorem, from geometry, that in any triangle the sum of any two sides must be larger than the third.

Introduce problem: Can a triangle have sides 9, 3, 5?

How would we solve this?

1. 9 + 3 = 12 12 > 5 O.K. 2. 9 + 5 = 14 14 > 3 O.K. 3. 5 + 3 = 88 < 9 No triangle

Note that <u>any</u> two sides must have a sum greater than the third side.

The human can make a judgement about whether the sum is greater or not. The computer, however, must have a way to make this same judgement. The computer subtracts the third side from the sum. If the sum is greater, the remainder shown will be positive and the sign bit will not light.

If the third side is greater than the sum, the remainder shown is negative and the sign bit lights, showing that no triangle is possible.

Demonstrate steps with computer trainer: Given 3 digits (a, b and c), can a triangle be formed with sides a, b and c?

- 1. Encode a into register A.
- 2. Store contents of register A in core memory 1.
- 3. Encode b into register B.
- 4. Store contents of register B in core memory 2.
- 5. Encode c into register A.
- 6. Store contents of register A in core memory 3.
- 7. Read in contents of core memory 1 to accumulator.
- 8. Transfer contents of accumulator to X-register.
- 9. Transfer contents of core memory 2 to accumulator.

TRIANGLE PROBLEM LESSON PLAN

- 10. Add (Change accumulator switches to (a + b) corresponding with answer); turn all X-register switches to zero, display answer.
- 11. Store contents of accumulator in core memory 4.
- 12. Read in contents of core memory 3 (c) to accumulator.
- 13. Transfer contents of accumulator to X-register.
- 14. Read in contents of core memory 4 to accumulator.
- 15. Subtract (a + b) c.Note sign bit. If sign bit lights, no triangle is possible. If sign bit does not light, continue.
- 16. Read in contents of core memory 1 (a again) to accumulator.
- 17. Transfer contents of accumulator to X-register.
- 18. Read in contents of core memory 3 (c) to accumulator.
- 19. Add (a + c). (See step 10.)
- 20. Store sum in core memory 4.

- 21. Read in contents of core memory 2 (b) to accumulator.
- 22. Transfer contents of accumulator to X-register.
- 23. Read in contents of core memory 4(a + c) to accumulator.
- 24. Subtract [(a + c) b]Note sign - if it is not lit continue repeating steps using [(c + b) - a].

NOTE: At this point the student has no understanding of shifting. Practice digits must therefore be chosen so that their sum is not greater than 15.

Have the students take turns operating the machine using various combinations of numbers. EMPHASIZE CONTINUITY OF STEPS. Note that the regular computer would arrive at the answers very quickly but that we are analyzing the steps needed.

Assignment: Convert to binary and show addition and subtraction needed to test whether a triangle may be constructed with sides:

a = 3, b = 2, c = 1
a = 5, b = 6, c = 7
a = 4, b = 2, c = 7

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Add Routine Lesson Plan

AIMS:

To show how the computer determines whether a sum is larger than 10.

To provide an understanding of the add function.

To introduce the student to shifting operations.

PROCEDURES:

The add function on the Arithmetic unit operates automatically. After the accumulator switches have been set to correspond to the sum and the X-register switches have been set to zero, the sum can be displayed in the accumulator. Before the sum can be displayed as a decimal number in the Decoder, the computer must have some way of resolving the sum into binary coded decimals. To determine whether the sum is greater than 10, the first thought might be to subtract 10 from the sum. A positive remainder would indicate that the 101 place should be 1 which is stored in the core memory. There is a limitation here due to the design of the trainer. If the sum is 16, 17 or 18, the 2^4 bit is used but is not connected in the subtraction function. This kind of limitation could occur in any regular computer no matter how many bits are available in the accumulator. The shifting feature now shows its usefulness. Shifting the sum one place to the right divides it by 2. Of course we lose the 2⁰ bit from the accumulator but this is either zero for an even number or one for an odd number and can be retrieved later on. Since we have divided the sum by 2, we now subtract 5 instead of 10.

Relate this operation to decimal numbers:

e.g.	$\frac{\text{Decimal}}{6+7=13}$	Binary 0110+0111 =	= 1101	
Divide by 2:	6.5	Shift right:	0110	1
Subtract 5:	5		0101	Extension
	1.5		0001	register
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Positive remainder indicates sum greater than 1. 10^1 bit on decoder is 0001.

Multiply by 2:	3.00	Shift left:	0011	
10^0 digit =	3		10 ⁰ k	oit is 0011

Students should analyze several examples in this way to thoroughly understand these steps of the program.

It might be well to analyze the case which makes shifting necessary:

e.g.	Decimal	Binary
	8+9=17	$\overline{1000 + 1001} = 10001$
		24
Divide by 2:	8.5	Shift left: 1000 1
		Now the 2^4 - 101 Extension
Subtract 5:	5	bit is in the register
		accumulator
	3.5	0011

Positive remainder indicates sum greater than 10. 10^1 bit on decoder is 0001.

Multiply	by	2:	7	Shift right: 0111	
$10^0 = 7$				10^0 bit = 0111	

