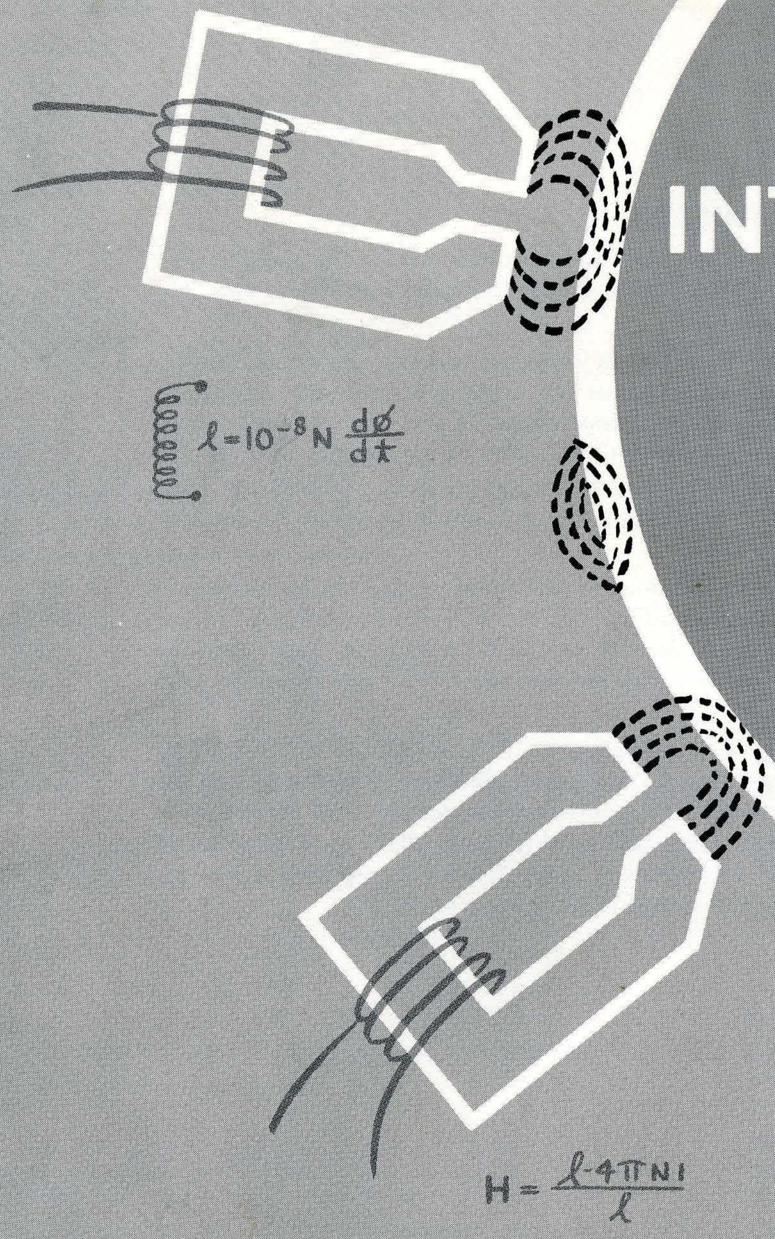

INTRODUCTION TO PULSE MAGNETIC RECORDING

by **FRANK J. LOHAN**
SENIOR DEVELOPMENT ENGINEER

BRYANT MAGNETIC STORAGE DRUMS

BRYANT COMPUTER PRODUCTS DIVISION
P.O. BOX 620, SPRINGFIELD, VERMONT, U.S.A. • TEL. TURNER 5-2141

INTRODUCTION

This article on pulse magnetic recording deals directly with the theories involved in recording on magnetic memory drums. Normal tolerances to which drums, magnetic heads, amplifiers, etc., are manufactured are not taken into consideration.

Present day accomplishments in the field of recording on magnetic memory drums are briefly stated. Bit densities of approximately 150 per inch or lower are reasonable from the standpoint of pulse spreading and general resolution of the recorded information. In the case of applications wherein the head is permitted to be very close to the rotating surface, a bit density of up to 225 bits per inch is practical. Bryant has supplied drum systems for use at up to 750 bits per inch. However, these applications were special and were designed to use playback about 95 per cent down from that used in the case of the low bit density.

One of the most important determining factors in deciding on operation at a given bit density is how sophisticated the electronics concerned with reading will be, and which particular record mode is chosen. For simple electronics, the pulse density on the drum must be lower than the figures quoted above. Twice the information density can be realized with a non-return to zero record scheme.

In general, frequency is no problem up to 3 or 4 megacycles. Core materials are available that have useful permeabilities up to the microwave region. The most important factor effecting response is the bit density, or recorded wave length. All published theoretical work for the sine wave recording treats the losses due to space, gap, coating thickness, etc., as a function of recorded wave length. Core losses in the head are, however, a function of frequency and further, frequency must be considered in relation to the inductance of the coil to be driven.

1. THE RECORDING PROCESS

In order to record information on a magnetic medium, an H field must be produced which will cause a portion of that medium to become magnetized.

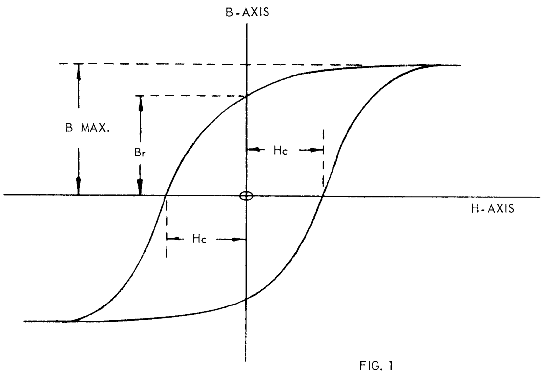

FIG. 1

B-H LOOP

Shown in Figure 1 is a B-H Loop (Hysteresis Loop) with the H axis now called an ampere or current axis, since H is related to current by a constant:

$$H = \frac{.4\pi NI}{L} = KI$$

N = Number of turns

I = Current

L = Length of magnetic path

K = A constant = $\frac{.4\pi N}{L}$

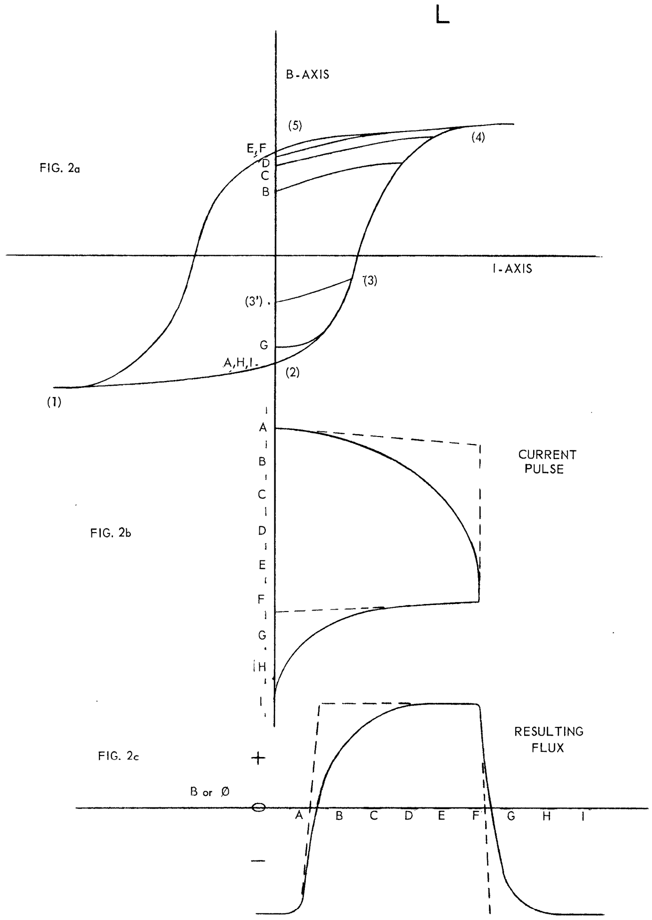

FIG. 2a

FIG. 2b

FIG. 2c

If the B-H Loop shown in Figure 2a represents that of the coating, then the current pulse shown in Figure 2b represents the H field presented to the coating by the gap in the head.

The axis A B C D E F G H I represents time. Figure 2b shows how the H pulse varies with time.

It is assumed that (see Figure 2a) the coating has been previously magnetized by a field represented by (1) so that the state of the coating prior to the current pulse is that represented by (2). If the coating is moving, at every instant the state of the coating will be that represented by (2) when it is presented to the effect of the current pulse.

Figure 2c represents the flux, ø, or the flux density, B (these are related by a constant, the cross-sectional area of the coating) that is recorded on the coating under the above-mentioned conditions.

The flux pulse in Figure 2c is obtained as follows: If a field is presented to the coating that drives it from (2) to (4) in Figure 2a, then when this field is removed, the coating will be left in the state represented by (5). However, if a field only large enough to drive the coating from (2) to (3) is presented and then removed, the coating will be left in the state represented by (3').

From the foregoing discussion, it can be seen that the letters A thru I on Figure 2a represent the conditions in which the different portions of the coating will be left when presented with the current or H pulse shown. It is these conditions, the flux or B from Figure 2a, that are plotted against the A thru I points on 2b to obtain 2c.

It can be seen from Figure 2b and 2c that the flux that is recorded resembles in shape the field that causes the recording. A sharp record pulse will record a sharp flux pulse on the coating (the dotted lines in 2b and 2c).

This article so far has dealt with rather idealized conditions in that a point by point analysis of a coating with an assumed magnetic history is made. In actual practice, although the coating can be pre-biased as assumed, the field presented to the coating by the head is not as clear cut as that assumed in the foregoing discussion. This article and a portion of that to follow will be concerned only with recording a single pulse on a pre-biased coating. The cases dealing with recording a series of pulses and also information follow this.

Figure 3 represents approximately the field present in air in front of the gap of a recording head when current is passing thru the head coil. The view shown is a cross section of the head perpendicular to the gap.

The field deep in the gap is of strength 1 (or 100%). The field decreases with distance from the gap. The field for a given strength also becomes wider in front of the gap.

If a coated surface is placed in front of the gap at a distance from the head equal to one gap length (i.e. at 1.0 mil space from a 1.0 mil gap or at 0.5 mil for a 0.5 mil gap), then the maximum field that the edge of the coating nearest the head can encounter is about 28% of that present deep in the head gap.

Consider that the field deep in the gap is steady at a value corresponding to the maximum portion of the current pulse shown in Figure 2b. Obviously, if only 28% of this field strength is presented to the coating, then the coating is only being driven to point (3) and consequently will settle down to point (3') when this field is removed. This portion of the coating would be at point (3'), while all other surrounding coating is at point (2). This difference in flux density is small compared to what is possible if the portion of coating in question were left at point (5).

Playback is proportional to the rate of change of flux through the head coil. If the change in flux is greater in the same interval of time, then a greater playback will result.

The above discussion indicates, therefore, that when recording for maximum playback, it is best to saturate the coating to obtain the maximum rate of change of flux on playback. To saturate the coating, a field pulse of the amplitude shown in Figure 2b must be presented *to the coating*. From Figure 3, in order to present such an amplitude to the coating, a field much greater than this must be created in the head gap. In this case, a field about 3.6 times as great must be produced in the gap in order that 28% of it will saturate the coating.

It can be seen in Figure 2 that with a B-H Loop as shown and a current (or H) pulse as shown, small values of current will produce small changes in the magnetization of the coating. Since this current pulse is 28% in amplitude when referred to the deep gap field, the point represented by (3) is about 0.1 in amplitude. This amplitude causes a significant change in the magnetization of the coating, since it leaves the coating at point (3').

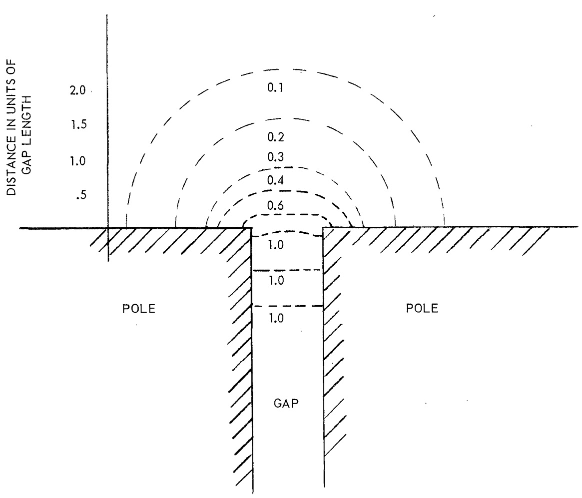

FIG. 3

If the coating in Figure 3 is one gap space from the gap (line 1.0 on the vertical scale), then the distance over which it experiences a field strength equal to point (3) in Figure 2a, which is represented by the 0.1 line of Figure 3, is *four times* as wide as the gap.

Figure 4 shows, to scale, the pulse that would be recorded on a stationary drum, pre-biased to negative saturation, if

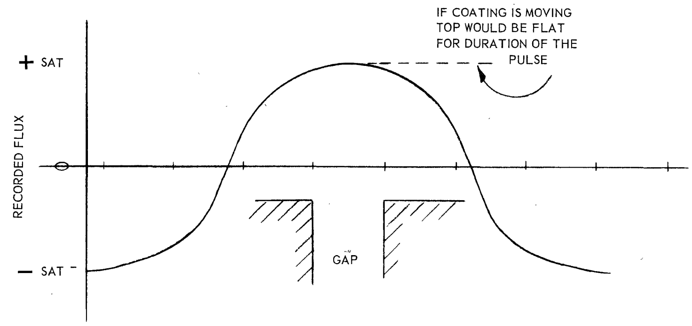

FIG. 4

a current pulse large enough to barely saturate the coating were passed through a head one gap length from the drum. The points between which the flux starts to change are about seven gap lengths apart. The pulse is thus spread on the coating.

If the coating were considered to be moving while this pulse were being written, the top would be flat as shown in Figure 4. Any pulses written, therefore, are longer than the gap spacewise and longer than the write pulse timewise.

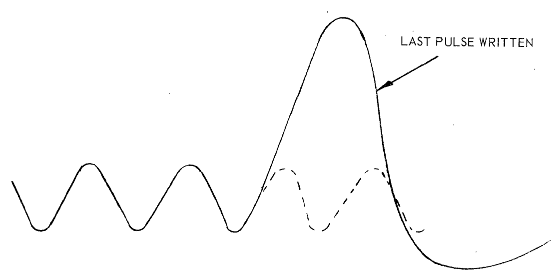

FIG. 5

The amount of spreading is increased to a certain degree when the coating is moving, since it takes a finite time for the current, hence the H field, to rise to maximum value. While this field is increasing, the coating is subjected to various field strengths a large distance along the head from the gap. If a previous pulse had been written, a portion of that written pulse would be subjected to the field effects of the next pulse being written. It would be less sharply defined and in part reduced in magnitude. This crowding effect is indicated in Figure 5. This figure represents the playback of a series of pulses written so that crowding can be observed. The last pulse is not crowded and is wider and higher in amplitude. The dotted line indicates what would have happened if the last pulse were followed by others of the same frequency.

II. DE-MAGNETIZATION

An effect exists that tends to de-magnetize the recorded pulse. This effect becomes more pronounced as the pulse length decreases. It is most harmful to the higher pulse densities.

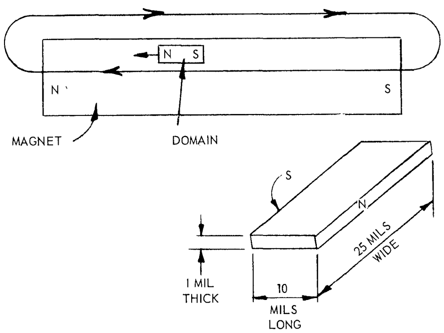

FIG. 6

Figure 6 shows a cross section thru a bar magnet. The field of the magnet inside of the magnet has a direction from the south pole to the north pole. The magnet may be considered to be a lump of smaller magnets or domains, each of which has a north and south pole. The field outside of the magnet has a direction from north to south. Therefore, this external field will tend to de-magnetize some of the domains that it passes near. This has the effect of de-magnetizing the magnet.

For a given strength of magnet, the shorter the distance from north to south pole, the greater is the H. (H is magnetomotive force per length). Therefore, the de-magnetizing field is greater.

The proportions of a recorded bit at 100 pulses per inch are shown in Figure 6. It is obvious that the higher pulse densities are subject to a large de-magnetization field.

This de-magnetization effect is coupled with the pulse spreading mentioned previously in tending to reduce the strength of recorded magnets at higher pulse densities.

III. HIGH DENSITY RECORDING

The article to now has concerned the recording of an isolated bit on a coating. It has been shown that such a bit will spread a considerable amount. In order to realize high density recording, the width of a static pulse must be made narrower. (A static pulse is defined as a pulse put on a pre-biased coating when the coating is not moving relative to the head. See Figure 4).

Some narrowing of the recording pulse is brought about by decreasing the operating space between the gap and the head. It can be seen from Figure 3 that if the space is half of the gap, about 42% of the deep gap field is presented to the coating surface. This means that a smaller deep gap field is required to saturate the coating surface. Consequently, the field strength represented by the 0.1 line on Figure 3 will be smaller and the amount of spreading will be slightly less.

Some of the parameters affecting the spreading of a recorded pulse have been considered. It can be seen from Figure 3 that operation of a head at a space from the coating greater than the gap between poles is deleterious because of the rapidity with which the field falls off in front of the head.

In order to record narrow pulses, four conditions are required:

1. A gap that is much narrower than the smallest wave length to be recorded.
2. A space that is about the same or smaller than the gap.

Some further improvement in the width of a recorded pulse can be made by a basic improvement in the coating. So far only the surface of the coating nearest the gap has been considered. If Figure 3 represents a 1.0 mil gap head spaced 1.0 mil from a coating, 1.0 mil thick, then the portion of the coating that is furthest from the head experiences a field strength that is only about one half that of the front edge of the coating. Thick coatings are not effective in increasing the high pulse density playback, since it is only the portion of coating near the surface that contributes to the signal. This thickness effect tends to cause a rather sloppy pulse to be recorded rather than a sharp pulse such as indicated in Figure 4. By increasing the write current, more of the coating further away from the head becomes more strongly magnetized and leads to slightly greater playback. However, this increase in write current causes the pulse to spread further according to the mechanisms discussed under pulse spreading.

3. Coatings that are thin compared to the gap length.

Assume a thin magnetic coating spaced one gap length from a recording head. A static pulse then would be as represented by Figure 4. Following the same reasoning and referring to Figure 3 will show that Figure 7b will result from consideration of Figure 7a. Figure 7a represents a normal hysteresis loop and a square hysteresis loop, both with the same Hc and Br. The obvious benefit in sharpness of a recorded pulse is shown in 7b. The sharp sides of the

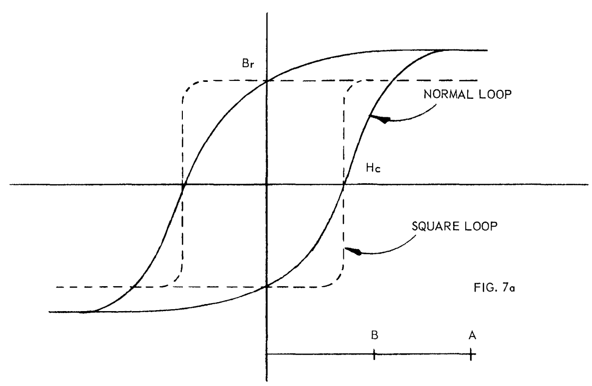

FIG. 7a

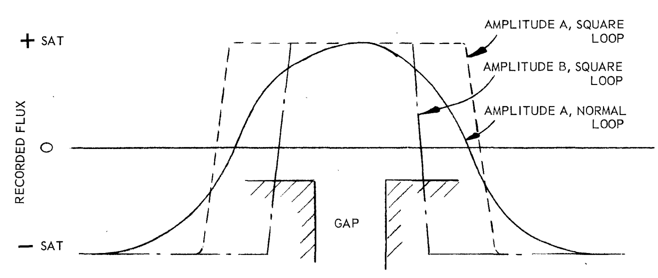

FIG. 7b

pulses representative of the square loop come about because the magnetization of the square material does not change until a field at least equal to Hc is presented to the material. The change is then very abrupt. Therefore, the small fields further along the head will not affect the coating. This reduces the effect of spreading. A smaller write current is required, since a write current or field amplitude as large as B in Figure 7a will saturate the square coating, whereas it will not saturate the normal coating. The advantage in a smaller write current is seen in Figure 7b in that the written pulse is not only considerably sharper, but also considerably narrower than the others.

4. Coating material with essentially a square hysteresis loop.

It should be noted that all the effects so far discussed are wave length effects. They are present at both high and low frequencies, since the wave length that is recorded is related to frequency by the surface speed of the drum.

The frequency response of the head is controlled by several factors. One of these is the permeability of the core material. This permeability falls off with frequency. However, with present day head core material, this fall off of permeability is not significant below a megacycle.

IV. PLAYBACK CONSIDERATIONS

There are two prime factors involved in high pulse density recording. The first is getting a sharp, well-defined pulse on a drum. The second is detecting this pulse.

The main consideration on playback is capturing as much of the flux of the recorded magnet as possible. Playback voltage is proportional to the number of turns and the rate of change of flux through these turns. Thus for large playback voltages, the greater the number of turns the better.

Since the same head is used for both recording and playback, it would seem that one with a large number of turns would be beneficial. Although a large number of turns would reduce the write current required (since ampere turns is proportional to H), a serious frequency response problem would be introduced.

The voltage across an inductance is defined as:

$$e = -L \frac{di}{dt}$$

Where:

e = volts

di = change in current, amperes

dt = change in time, seconds

L = inductance, henries

Assume that it requires 15 amp turns to write. Assume further that it is desired to write with current pulses that are 0.5 microseconds in duration. It is desirable that the current rise to its full value in considerably less time than the duration of the current pulse.

If a head of 50 turns has an inductance of 70 microhenries, a head of 200 turns will have an inductance of about 1120 microhenries. This is because the inductance varies as the number of turns squared. For this illustration, the physical and magnetic properties of the two heads are identical, the number of turns only being varied.

A rise from zero amperes to 0.3 ampere may be caused in the 50 turn head in one tenth of a micro-second (the entire duration of the write pulse is five tenths of a micro-second). To maintain the constant ampere turns, it will be desired that the current change in the 200 turn head be from zero to .075 ampere in one tenth of a microsecond.

Substituting the above values in the equation for the voltage across an inductance, it will be seen that to produce such a current rise will require a step voltage applied to the 50 turn head of 210 volts and one of 840 volts to the 200 turn head.

The larger inductance is much harder to drive at high frequency. High inductance heads are inherently low frequency devices, since it takes longer to cause a current change in them with reasonable voltages.

The design of a read-record head is thus a compromise between reasonable record characteristics and usable playback characteristics. Of the two considerations, the record characteristics are more important when a head is used for both read and write, since a sharp, well-defined pulse must be placed on the coating before anything may be played back.

Since playback depends on capturing the flux of the recorded magnet and passing it through a large number of turns, the prime considerations for efficiency in playback would be:

1. A head gap narrower than the smallest wave length to be played back.
2. A head to coating space as small as possible.
3. As many turns as possible on the playback head.

Since the head is constructed of magnetic material, a very narrow gap head would appear almost like a solid block of magnetic material across the front gap. The object of a read head is to cause as much of the recorded flux as possible to pass through the head coil. This requires that the reluctance of the front gap be larger than the reluctance

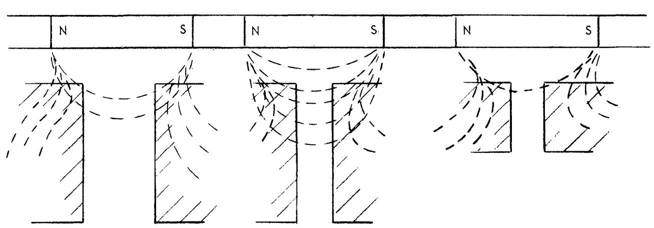

FIG. 8a FIG. 8b FIG. 8c

around through the head. This is illustrated in Figure 8 where three different gaps are shown in the vicinity of a certain wave length magnet. Figure 8a shows a deep wide gap with some of the magnet flux closing in front of the gap but most of it going around through the pole pieces.

Figure 8b represents the same pole piece except that they are spaced only half as far apart. It can be seen that more of the flux finds it easier to close across the front gap and less flux will go through the pole pieces. In Figure 8c, the depth of the gap has been reduced; otherwise, it is the same as 8b. In this case the reluctance of the front gap has been increased and, as in 8a, more of the flux finds it easier to go through the poles, and consequently through the coil, than closes in the front gap.

4. Careful balance between the reluctance of the front gap and the reluctance through the head pole pieces and coil.

V. METHODS OF REPRESENTING INFORMATION ON MAGNETIC COATINGS

Binary representation of alpha-numeric information (numbers and letters) is the most commonly used scheme in computer work. Binary arithmetic is a representation using powers of two. This choice can be shown to be most applicable to the use of relays, tubes, cores, magnetic recording, etc.

The basic idea of binary notation as compared to decimal notation is outlined in Figure 9. Some very comprehensive discussions are available in the references listed in the bibliography.

C B A — DECIMAL POINT

POWER OF BASE 4 3 2 1 0 .

DECIMAL NUMBER 0 0 0 1 6 . $= 1\times10^1 + 6\times10^0 = 10 + 6 = 16$

BINARY NUMBER R (16) 1 0 0 0 0 . $= 1\times2^4 + 0\times2^3 + 0\times2^2 + 0\times2^1 + 0\times2^0 = 16$

BINARY NUMBER (1) 0 0 0 0 1 . $= 1\times2^0 = 1$

BINARY NUMBER (2) 0 0 0 1 0 . $= 1\times2^1 + 0\times2^0 = 2$

BINARY NUMBER (3) 0 0 0 1 1 . $= 1\times2^1 + 1\times2^0 = 2 + 1 = 3$

BINARY	DECIMAL
0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6
0111	7
1000	8
1001	9
1010	10
1011	11
1100	12
1101	13
1110	14
1111	15

FIG. 9

Figure 9 shows the positional notation used in both decimal and binary systems. The decimal system operates on powers of ten. Position A on Figure 9 represents ten to the zero power. Position B represents ten to the first power, etc. The decimal system uses ten symbols along with the positional determination.

The binary system uses two symbols with the positional notation. Its efficiency in computer work comes from the fact that the two different symbols used in binary arithmetic can be represented by:

1. A relay, closed or open
2. A tube, conducting or cut off
3. A switch, closed or open
4. A core, magnetized one way or the other
5. A light, on or off
6. A magnet, recorded in one polarity or the other
7. A hole or no hole in paper tape

Figure 9 indicates the notation involved in representing 16 in the decimal and binary systems as well as the decimal equivalent of several other binary numbers. From this, the extension of binary notation will be obvious.

Note that due to the small base (2) of the binary system, many more positions are needed to represent a given quantity. For instance, five positions are needed to represent 16.

In a computer, in order to represent alphabetic information, exactly the same scheme can be used if other information is included in a "word" to indicate that it is alphabetic or numerical information. This is done by proper timing in the computer.

Binary information can be recorded on a drum as spots that are magnetized with different polarities. The order and form of these magnetized spots can be arranged several ways in order to represent any given information.

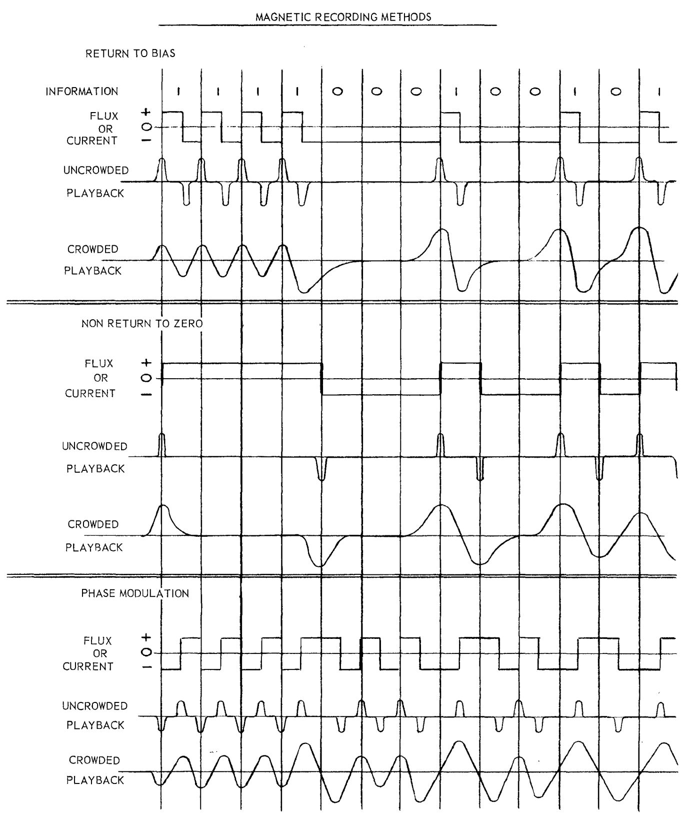

FIG. 10

Figure 10 indicates three common recording schemes. All three schemes represent the information written across the top. Many variations of the basic schemes are possible.

RETURN TO BIAS (RB)

This method uses a fixed length positive current pulse to represent a "one" and a constant negative current to represent "zero".

Frequency Requirements

The RB method is a wide band recording method, since a single "one" in the middle of a long string of "zeros" represents an isolated narrow pulse. The upper basic frequency limit is represented by all "ones" being recorded. The playback amplitude will decrease as the pulses are crowded together (all "ones"). Thus with this method, a wide frequency range is needed and a variety of playback amplitudes must be handled if the pulse density is high.

The read amplifier must have sufficient gain to pick up the all "ones" condition, yet at the same time must not overload when it receives a single "one" which may be greater in amplitude, depending on pulse packing.

Another problem with this method is that if the pulse density is high, a single "one" will tend to spread out and be longer in time than a "one" that is crowded. This leads to some ambiguity in reading the bits on either side of the isolated "one".

If the pulse density is low, it is possible that noise could be interpreted as a "one" when the information should be "zero".

Information can read back by detecting positive or negative peaks, since each one is present when a "one" is written. The absence of a peak represents a "zero".

NON RETURN TO ZERO (NRZ)

This method uses a positive current pulse for as long as "ones" are being written. A negative current is maintained whenever the information is "zero".

Frequency Requirements

The NRZ system is also a wide band recording method and is subject to the same noise, errors and ambiguity that the RZ system is.

However, the highest frequency (highest pulse density) is half that of the RZ system under the same conditions of speed and information.

This means that under crowded conditions, a given head drum system can store twice the amount of information as the RZ system without a further decrease in playback amplitude because of pulse crowding.

Information can read back by triggering a Flip Flop with the playback. As long as the read Flip Flop is in one state, the information is "ones"; if it is in the opposite state, "zeros" are being read.

One disadvantage of this scheme is that the long wave lengths that it is possible to record can spread out and be detected by heads on an adjacent track. This can be seen from consideration of the field around a long bar magnet.

PHASE MODULATION (PM)

This system is based on the idea that something must be written in each bit cell in order to do away with the ambiguity caused by an isolated pulse and to eliminate noise errors. See Figure 13 for record scheme.

Frequency Requirements

The PM system is a narrow band system. The only fundamental frequencies involved are contained in one octave. (Freq. "A" and twice Freq. "A"). The playback amplifiers may, therefore, be narrow band devices in order to help eliminate noise.

A positive flux change in the middle of a bit represents a "one". A negative flux change in the middle of a bit represents a "zero". The flux changes at the bit boundary are extraneous. This, as do most other systems, requires a clock or sprocket series from the drum in order to gate with the playback signal to determine the information.

OTHER METHODS

Many other methods exist for representing information. The three methods outlined above, however, represent the extremes in frequency requirements.

As was shown in the section on binary representation, a large number of bits are needed to represent a large number. This shortcoming is removed by a digit parallel-word serial use of computers. Parallel-serial use is as follows:

If it is desired to represent in binary fashion all ten number symbols used in the decimal notation, only four binary bits are needed. This is shown below:

Decimal	Binary	Decimal	Binary
0	0000	5	0101
1	0001	6	0110
2	0010	7	0111
3	0011	8	1000
4	0100	9	1001

If four bits are taken at a time therefore in parallel, each group of four could represent one decimal digit. Instead of representing 16 in a serial fashion (one bit at a time) thus

16 10000

It could be done as follows:

Decimal	1	6
Binary	1	0
	0	1
	0	1
	0	0

where a column of binary bits, with the decimal point at the top, represents one decimal bit. Thus by reading four bits in parallel with the groups of four bits being taken in a serial fashion, an economy is realized in the time taken to "read" large binary numbers.

VI. EMPIRICAL GENERALIZATIONS

This discussion has dealt primarily with the individual effects of several parameters on the transducing efficiency of recording heads. In practice all parameters have effect simultaneously. Some empirical generalizations will be given.

Test results from the use of a given shape current pulse of fixed duration will not necessarily hold for changes in these two parameters when everything else is held constant.

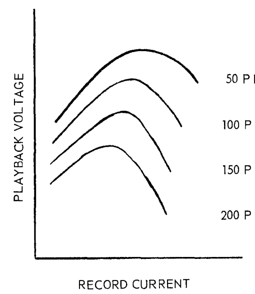

FIG. 11

There is in general no fixed record current that will be optimum for all practical pulse densities, all other variables being held constant. Figure 11 shows the playback variation as a function of current and pulse density. The choice of record current must be dependent on the wave lengths to be recorded. This is reasonable in the light of pulse spreading, since a smaller current will cause less spreading and will thus influence adjacent pulses to a smaller degree. It is reasonable, therefore, that higher pulse densities require less current for maximum playback. When an application requires selective alteration of data with no intervening erase cycle, it is a further requirement that the current used for the higher densities be sufficient to erase or write over the lower density recordings.

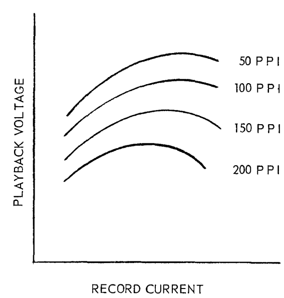

FIG. 12

The particular recording mode (RB, NRZ, Phase Modulation) also has an effect on the shape of the curves. Figure 12 shows the general shape of the playback-current-pulse density curves under phase modulation conditions. The variation caused by current changes can be seen to be somewhat less than that in Figure 11 which, in general, represents a return to bias system.

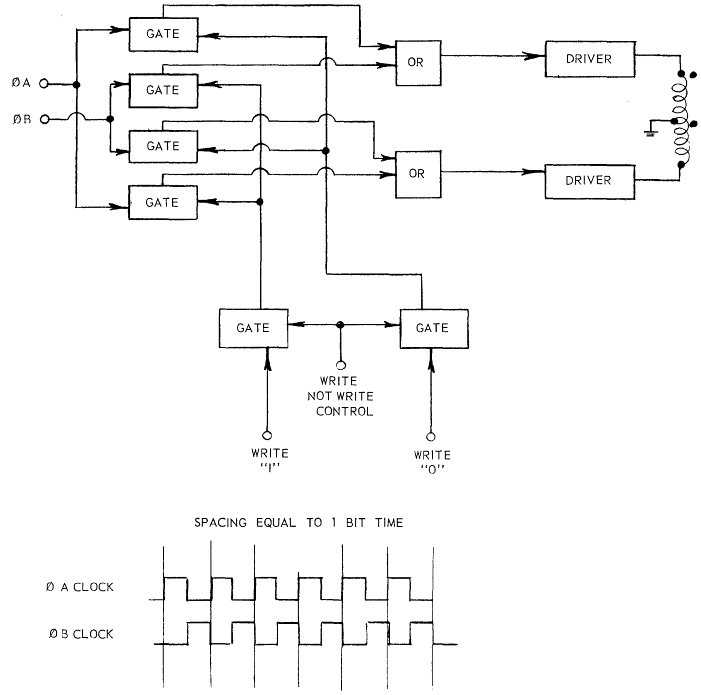

FIG. 13

In general, there is a limit to the extent that playback can be increased by increasing the number of turns. As the turns are increased, the inductance and distributed capacitance are both increased. This lowers the resonant frequency of the head and lowers the frequency at which the head starts to appear capacitive. Thus a large increase in the number of turns can prove very disappointing in playback. Since the transducing efficiency of the head on playback is based on the amount of flux available to thread the coil, a wider head and wider recorded track will increase playback. This is, of course, at the expense of a larger inductance in the head coils both for write and read.

VII. TERMS

mmf — Magnetomotive Force

H — Magnetomotive Force per unit length. This is force that produces the flux.

Ø — Flux

B — Flux density. Flux lines per unit area.

R — Reluctance. A measure of how easy it is to produce flux with a given H.

"Ohms Law" of the magnetic circuit:

mmf = Ø R

Note the resemblance to:

E = I R

u — Permeability. This is the ratio of B/H. A high permeability material produces more flux per unit cross sectional area than a low permeability material for the same H.

B-H Loop — This is the instantaneous plot of B vs. H, and in general has the form shown in Figure 1.

B max. — The maximum flux density that can be produced in the material regardless of how much drive (H) is provided.

Br — The remnance or the flux density that is retained by the material when the drive (H) has been reduced to zero after having been sufficiently high to saturate the material.

Hc — Coercivity, coercive force. This is the H that is needed to reduce the remnance to zero. Note that this H is the opposite polarity from that which caused the remnance.

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