"SIMULATION OF OXYGEN DYNAMICS RELATIVE TO PURIFICATION OF FRESH WATER"

by

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INTRODUCTION

This study describes the simulation, on both the PACE® TR-48 and TR-20 desk-top general purpose analog computers, of the basic forces affecting the condition of fresh water sewage receivers... biochemical oxygen demands from sewage decomposition and free oxygen content in the receiving waters...and an investigation as to how the resulting dynamic sewage load/oxygen relationship determines, to a great extent, the increasingly important, and often confused, "water pollution status" of the country's rivers, lakes and streams.

In the decomposition of the bulk of sewage components, whether from domestic or industrial sources, oxidation plays an important role. It is recognized, of course, that without exhaustive data regarding particular receiving waters nothing approaching a complete discussion of the oxygen dynamics of fresh and saline water would be possible. The purpose of this simulation is to examine these basic forces in general and to show how they can be simulated with some degree of sophistication on small, general purpose computers. The techniques used then can be extended for use in model building programs with data obtained for specific bodies of water. In this more sophisticated area of simulation, significant strides of real economic and educational value can result.

BENEFITS

Several benefits are provided by this simulation not only in terms of offering greater insight into the dynamics of water pollution, but also in providing basic data for more advanced and sophisticated analyses (such areas are delineated later).

The entire area of water pollution and waste water treatment is receiving increasingly greater attention by the general public as well as by federal and international government agencies. Detergent degradation of water supplies generally, and the Great Lakes/City of Chicago water diversion controversy in particular, are only two of the more publicized aspects of the overall problem.

Through the analog computer simulation of the relatively simple oxygen dynamics of fresh and saline waters, significant information is obtained for laboratory use in determining the important parameters of receiving waters...percentage and type of organic waste, diffusion and discharge patterns, etc. Through curve matching techniques involving dissolved oxygen depletion, effluent rate pattern and strength, reoxygenation rate, etc., it is possible to obtain the "book" on specific bodies of receiving water, which is useful in formulating basic design criteria for the construction and operation of water treatment plants.

PROBLEM DISCUSSION

The basic goals of modern sewage treatment plants can be summarized as follows:

1. to remove the larger and more objectionable settleable solids through screening and sedimentation,
2. to reduce the effluent "load" on the receiving waters to reduce stream degradation to a reasonable level and then maintain it, and
3. to operate efficiently in terms of initial investment and operating costs.

Current design and operation to achieve these goals range from ultra-sophisticated activated-bacteriological treatments to simple, short-duration hold or settling tank procedures.

Enormous economies can be achieved in the design and operation of municipal and industrial sewage treatment plants through the proper interpretation of data concerning the oxygen dynamics of water supplies. The effectiveness of these plants, of course, will determine then the design, operation and associated costs of related water intake treatment plants and, ultimately, overall water quality for all uses.

For a clear understanding of the entire pollution/treatment problem in terms of oxygen dynamics, and for the necessary critical evaluation of existing facilities, it is paramount to recognize that the receiving water itself is the final stage of sewage treatment, the initial steps being the formal treatment plant. This final stage, utilizing the self-purification properties of the water, must be as carefully analyzed and managed as the plant itself.

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Indeed, in the absence of any real, formal treatment plant, the receiving waters are often the only stage of water treatment and their analysis and management become even more critical.

If oxygen conditions in streams receiving treatment effluent are kept within certain limits, the capacity of these streams to digest enormous sewage loads without adverse pollution effects truly is amazing. On the other hand, if oxidation cannot occur because of overloading or other factors, a septic stream is the result. If oxygen depletion is too great, the degree of pollution can range from unfit drinking water through the destruction of wildlife resources and water unfit for bathing to even water unfit for raw industrial use.

Figure 1 shows graphically the relationship between oxygen deficit and septic stream conditions.

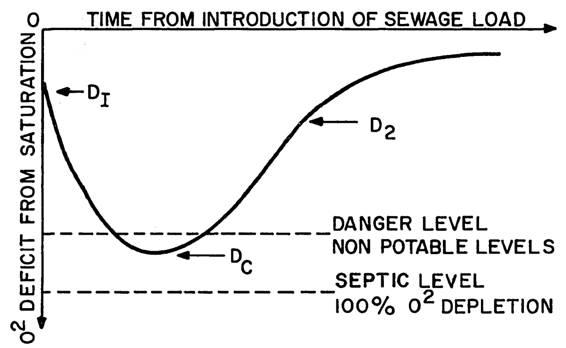


Figure 1: Curve Showing Relationship Between Oxygen Deficit & Stream Degradation
D_I = Initial Deficit , D_c = Critical Deficit
D_2 = Recovery Deficit (D changes sign)

The problem, then, is to determine not only the oxygen content of particular receiving waters but also to analyze the factors which cause oxygen depletion, and to discover sources which either by themselves or through dynamic interaction with others can help to replenish the dissolved oxygen supply.

PROBLEM ANALYSIS

Two basic influences determine the degree of oxygen depletion and pollutional degradation of a receiving water: the capacity it has for oxidizing organic sewage components, as measured by the amount of dissolved oxygen contained per unit volume of the water, and the demands made on it for oxygen by such components, as measured by the Biochemical Oxygen Demand (BOD) per unit volume.†

Water normally contains oxygen in solution in varying percentages depending on 'freshness' and temperature. Saturation values are obtained from true fresh water samples -- rain water, mountain streams, waterfalls, etc. Table 1[1] shows typical values of oxygen saturation in fresh water samples at different temperatures. The values for saline (seawater) samples are approximately 80 per cent of the fresh water values and are similarly temperature-variant.

As this dissolved oxygen in the receiving water is used to supply the oxidation demands of the sewage BOD, additional oxygen must be obtained either from

† Methods for measuring BOD for water samples and their associated rate reaction constants are available in standard Sanitary and Civil Engineering textbooks, and are relatively easy to perform in the laboratory, and well equipped field stations.

TABLE I

Temperature, degrees		Dissolved oxygen, parts per million	Temperature, degrees		Dissolved oxygen, parts per million
Centigrade	Fahrenheit		Centigrade	Fahrenheit	
0	32.0	14.62			
1	33.8	14.33	16	60.8	9.95
2	35.6	13.84	17	62.6	9.74
3	37.4	13.48	18	64.4	9.54
4	39.2	13.13	19	66.2	9.35
5	41.0	12.80	20	68.0	9.17
6	42.8	12.48	21	69.8	8.99
7	44.8	12.17	22	71.6	8.83
8	46.4	11.87	23	73.4	8.68
9	48.2	11.59	24	75.2	8.53
10	50.0	11.33	25	77.0	8.38
11	51.8	11.08	26	78.8	8.22
12	53.6	10.83	27	80.6	8.22
13	55.4	10.60	28	82.4	7.92
14	57.2	10.37	29	84.2	7.77
15	59.0	10.15	30	86.0	7.63

the atmosphere (via aeration) or from other bodies of water rich in oxygen (via stream diversion) to make up the deficiency.

A. Aeration: When using atmospheric reaeration, the rate of oxygen renewal in the water varies with 1.) The amount of oxygen deficit measured from saturation value (which is temperature-variant and, hence, subject to seasonal changes as well as day to day ambient temperature conditions), and 2.) The degree of physical intermixing of atmosphere and water possible for a particular stream flow and configuration (commonly dependent upon the geometric shape of the body of water, its velocity, surface wind action, wave action, and turbulence such as waterfalls, rapids, etc.).

The classic example of artificially inducing a greater mixing of water and atmosphere is the aeration of the Flambeau River at Park Falls, Wisconsin.

B. Stream Diversion: The classic example of using water richer in oxygen to provide the necessary dissolved oxygen to a stream to prevent septic stream condition is the diversion of Lake Michigan by the City of Chicago to flow down the Illinois and Des Plaines rivers.

It should be noted that both of these programs were relatively successful despite their controversial aspects. In each case, their use postponed necessary improvements in existing treatment plants and their relative success in providing reoxygenated water permitted considerable savings in plant investment.

In terms of the overall problem of waste water treatment, however, the successful application of such artificial means -- means not involving formal treatment plants, as such -- illustrates clearly that the task of water purification can be handled quite well by efficient management of oxygen dynamics alone.

SYSTEM EQUATIONS

The relationship existing among the various factors involved in oxygen dynamics can be expressed in the following greatly simplified differential equation commonly called the Oxygen Sag Equation:

$$\frac{dD}{dt} = k\ (L_a - y) - r\ D \qquad (1)$$

where D = oxygen deficit, mg/l

D_i = oxygen deficit at time t = o

k = BOD rate coefficient

L_a = BOD load, mg/l

y = total BOD load exerted in time t, mg/l

r = reoxygenation rate coefficient

t = time measured from introduction of L_a

The variable y can be defined further by the following expression:

$$y = L_a\ (1 - e^{kt}) \qquad (2)$$

where $k = k_o e^{C_k (T - To)}$

also $r = r_o e^{C_r (T - To)}$

$L_a = L_o e^{C_L (T - To)}$

and T = water temperature, deg. C

T_o = base temperature for determination of k_o, r_o and L_o, deg. C.

e = natural log base

The values C_k, C_r and C_L are "empiricized" coefficients unique to

1.) flow rate pattern

2.) percentage salinity

3.) percentage inorganic waste and type

4.) type of organic waste

5.) extent of plant life and normal daily and annual cycle

6.) extent of "sludge" deposits (past history of stream)

7.) profile of receiving water (surface area vs. profile areas)

8.) wave action in water

9.) turbulence of flow

10.) surface wind phenomena

11.) tidal action

12.) pattern of sewage diffusion

13.) pattern of sewage discharge.

These thirteen characteristics are those with which the more serious examinations of water/oxygen dynamics are concerned. Outstanding work has been done in the investigation of many of these parameters (to reduce the empirical representation of them) and many advanced studies currently are being carried out with the aid of large and quite sophisticated analog computer programs. †

It should be noted, however, that this general expression Equation (1) of the Oxygen Sag Equation is derived per unit of receiving water volume. It should not be confused with the ultimate total digestion capacity of a stream since flow variation and diffusion patterns play an important role in determining this ultimate value. This is particularly true where the receiving waters are lakes, bays, estuaries or streams with unpredictable or highly variant flows. † For many applications, however, involving receiving waters not subject to such physical disturbances, the general Oxygen Sag Equation is descriptive of the typical oxygen/water balance existing.

One additional coefficient that is encountered occasionally in the analysis of oxygen dynamics is f, referred to as the self-purification constant, which can be expressed as (2)

$$f = r/k \qquad (3)$$

This value provides a rough estimate as to a receiving water's ability to "cleanse" itself. Table II shows typical order-of-magnitude values of f for various types of receiving waters. (2) Figure II shows the variations of k and r with temperature for a particular stream. (1)

TABLE II

Typical Values of f at 20°C

Small ponds & backwaters	0.5-1.0
Sluggish streams, Large lakes	1.0-1.5
Large streams of low velocity	1.5-2.0
Large streams of moderate velocity	2.0-3.0
Swift streams	3.0-5.0
Rapids and waterfalls	above 5

It can be seen that even though the value of f, itself, decreases with increase in temperature (because of increasing BOD rate), the accompanying rise in reoxygenation rate shows that warm water can recover oxygen in solution more quickly than cool water. This basic characteristic of oxygen dynamics can be used effectively in enhancing the efficiency of treatment plants either by increased utilization of artificial heating means or more sophisticated employment of ambient temperature ranges in the receiving water.

† *Typical of the advanced work being done in this area is the study entitled "Analog Computation of the Dispersion of Pollutants in the Delaware Estuary" by Dr. J. O'Connor, L. L. Falk, and Rl G. E. Franks*

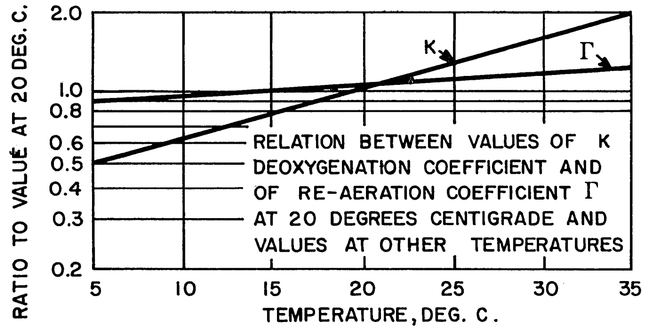
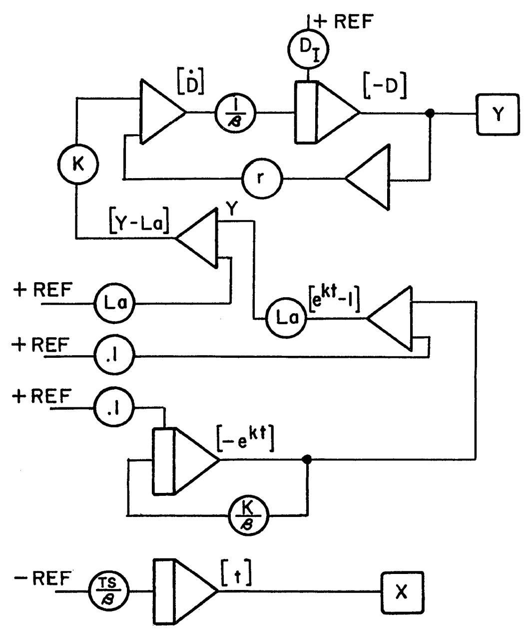


Figure 2: Relation Between k and r at Various Temperatures

ANALOG SIMULATION PROGRAMS

The analog computer programs used to simulate the Oxygen Sag Equation are straight-forward. Three of these programs, illustrating the various degrees of sophistication possible and the various complement of hardware required, are presented unscaled in Figures 3, 4 and 5. A completely scaled and working diagram for the circuit shown in Figure 4 is shown in Appendix A.



TS = TIME SCALE FACTOR RAMP FACTOR

Figure 3: Straightforward Mechanization of Basic Oxygen Sag Equation

RESULTS

Typical of the results obtained are the curves shown in Figures 6 and 7. Figure 6, obtained from runs of the computer circuit of Figure 4

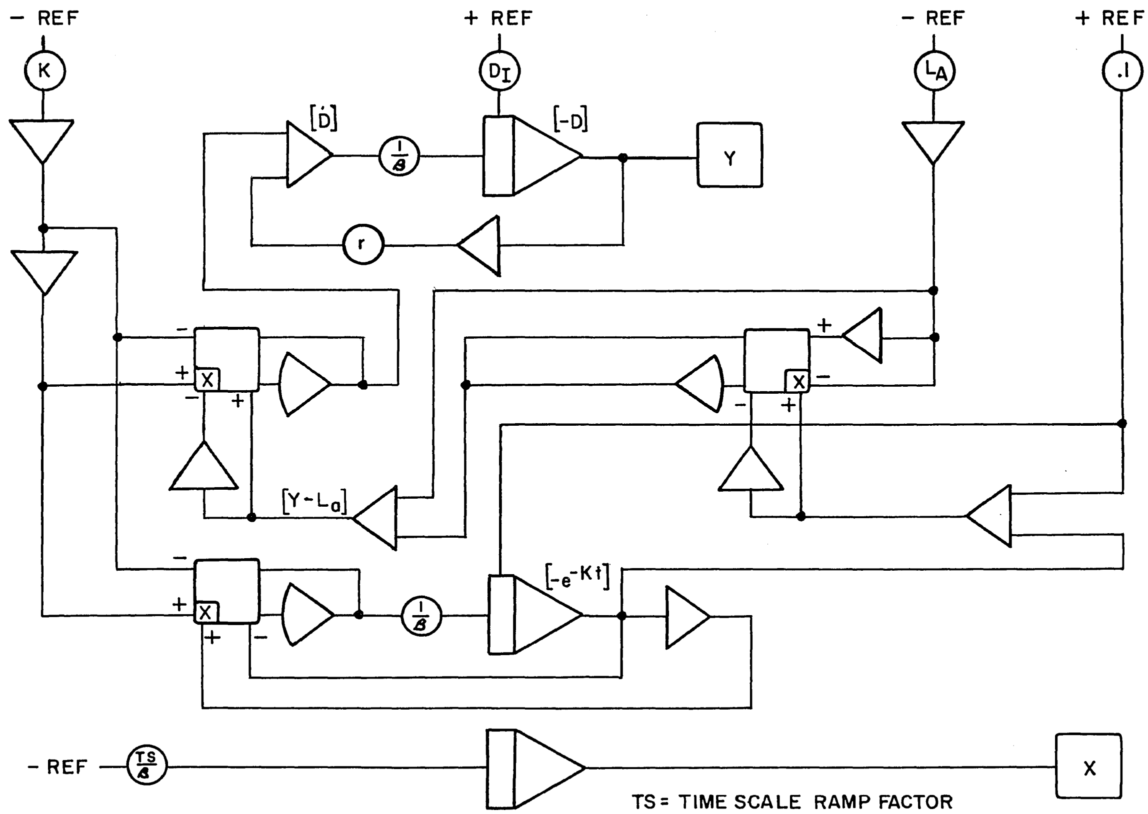


Figure 4: Use of Multipliers to Enable Coefficients, k and L_a, to be Set on Single Potentiometers

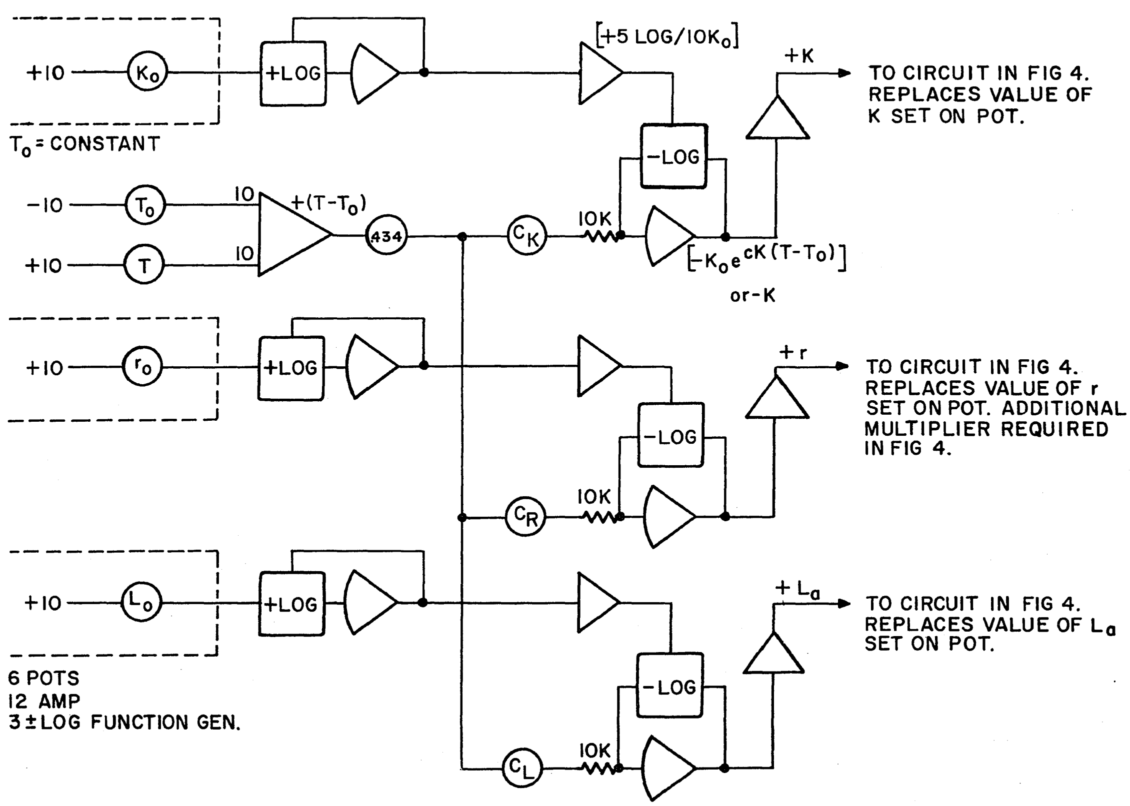


Figure 5: Analog Computer Circuit Permitting Adjustments in Temperature Variation as well as Inclusion of Parameters C_k, C_r, C_L Unique to Particular Bodies of Water

where L_a was the variable parameter, illustrates the time required for a constant-temperature receiving water to replenish its dissolved oxygen content after various BOD requirements.

Figure 7, obtained from runs of the computer circuit of Figure 4, shows similar dissolved-oxygen vs. time curves for receiving water at several different temperatures.

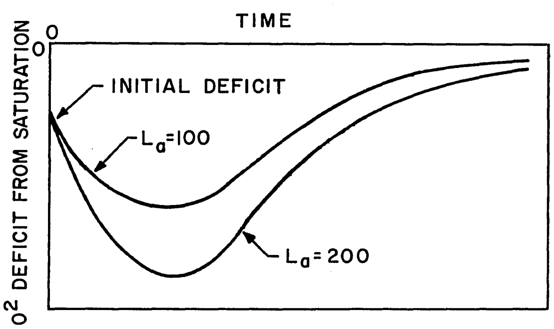


Figure 6: Effect of Various BOD Loads on Dissolved Oxygen Recovery Time. Temperature Constant

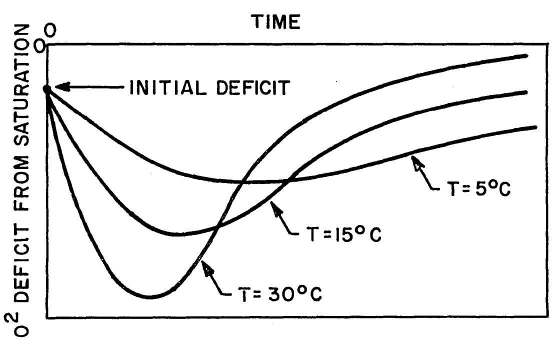


Figure 7: Effect of Various Temperatures on Dissolved Oxygen Recovery Time, BOD Load Constant

Briefly summarized, the following observations can be made regarding the results obtained from this study:

1. With a constant BOD load -- e.g. regular as opposed to intermittent bulk discharges of sewage or waste, etc. -- the higher the temperature of the receiving water, the more rapid <u>and</u> the more complete the depletion of dissolved oxygen from that water, but the quicker the recovery rate of oxygen back into solution.

2. With a constant temperature, the higher the load the more rapid and complete the depletion of dissolved oxygen, as above, and again the quicker the recovery rage.

3. A significant area of optimization exists as to loading vs. temperature, particularly in regard to effluent rate pattern and strength. In this regard, Figure 1 illustrates several important criteria of the oxygen deficit curve.

4. Where measurements have been made from field samples, it is possible by curve matching to determine several important characteristics of the receiving water, such as reoxygenation rate coefficient at base temperature, r_o, BOD rate coefficient at base temperature, k_o, and value of self-purification constant at base temperature, f_o, as well as those aspects of water/oxygen dynamics relative to salinity, plant life, organic and inorganic waste, etc., (C_k, C_r, C_L). These characteristics then can be related to ambient temperature ranges in the receiving water.

This method of curve matching makes it possible to obtain the "book" on a particular body of receiving water and to use this complete oxygen dynamics analysis as basic design criterion for the construction and operation of a treatment plant.

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CONCLUSION

This analog computer simulation of relatively simple oxygen dynamics offers several important advantages. Educationally, it provides an insight into stream pollution itself that has not been completely available in the past by giving a new look at the factors both effecting and affecting such pollution.

In the laboratory, the simulation can be put to use in rapidly determining the important parameters of receiving waters either to form the basis for selection or rejection of a particular body of water or to provide information as to the amount of preconditioning necessary in the water to meet the requirements of projected sewage loads.

Of the utmost importance, however, is the fact that this simulation, which can be performed with a fair degree of sophistication on a small, general purpose analog computer, provides a convenient and inexpensive means to obtain complete oxygen dynamics analyses. This data, properly interpreted and applied to water and sewage problem areas, can lead to vast improvements in the overall efficiency of treatment plants and in the quality of water for all uses, industrial and private.

APPENDIX A

Computer Mechanization of Oxygen Sag Equation

$$\frac{dD}{dt} = k \ (L_a - y) - r \ D \quad (1)$$

where $y = L_a (1 - e^{kt})$

Conversion Factors

1 ppm = 1 mg/liter - 8.34 lb/million gallons

D_i = 3 mg/liter (max. assumed)

D_{mx} = 15 mg/liter (max. assumed)

L_a, max. = 500 mg/liter (max. assumed, theoretically unlimited)

L_a, typical = 20-400 mg/liter

y, max. = 500 mg/liter (max. assumed, theoretically unlimited)

y, typical = 200-400 mg/liter

k, typical = 0.25-0.75/day

r, typical = 0.25-1.00/day

Scale Factors

Variable	Maximum Value	Scale Factor	Computer Variable
$\dot{D}$	200 lb/mg	10/200	$[0.05 \dot{D}]$
D	200 lb/mg	10/200	$[0.05 D]$
y	500 mg/l	10/500	$[0.02 y]$

The potentiometer sheet is shown in Table III.

The scaled computer diagram is shown in Figure 8.

TABLE III

Pot	S1	S2	S3	S4	Parameter
1	.8	.8	.6	.6	K
2	.1	.1	.1	.1	Di
3	.100	.125	.125	.125	La
4	.7	.7	.5	.7	r
5	CONS	TANT			.25
6	CONS	TANT			.20
7	CONS	TANT			.1
8	CONS	TANT			1.0
9	CONS	TANT			.5
20	TIME	BASE	GENE	RATOR	.065

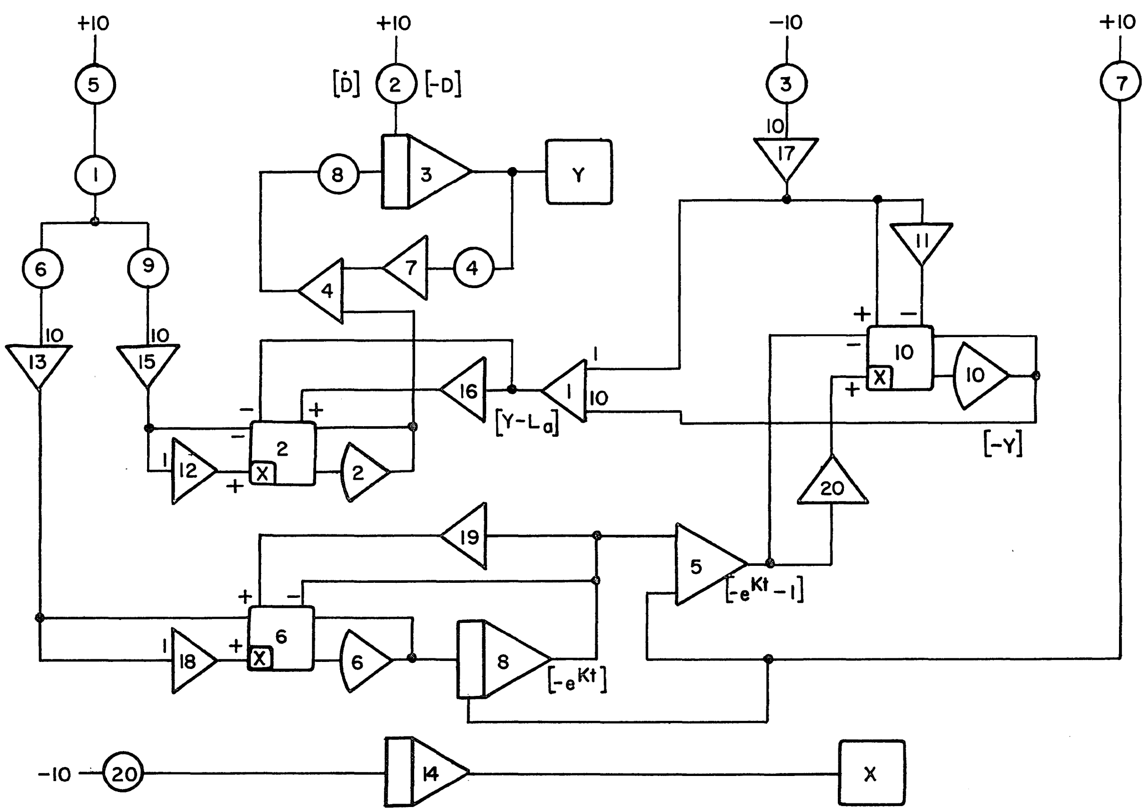

Figure 8: Computer Diagram

BIBLIOGRAPHY

1. Urquhart, L. C.: Civil Engineering Handbook, McGraw-Hill, New York, 1959.

2. Fair, G. M., and J. C. Geyer: Water Supply and Waste Water Disposal, John Wiley & Sons, Inc., New York, 1954.