* Studies included within the EDUCATIONAL classification are, in most cases, derived from the broad spectrum of scientific and engineering computer applications. Their presentation is oriented toward classroom or laboratory use, and they deal with problems appropriate for chemical, electronics, bio-medical, etc., courses. For the convenience of users in specific industrial fields, however, all EDUCATIONAL studies are cross-referenced in the Applications Library Index under the industrial activity(ies) with which they are most closely associated.

# THREE MODE CONTROLLER

## INTRODUCTION

This Study, performed on a PACE® TR-10 desk-top-size general purpose analog computer, describes an investigation of the operation of a three-mode temperature control system. In the diagram of Figure 1, a three mode controller is used to control the output temperature, $T_o$, of a heat exchanger. The measured temperature, $T_m$, is compared to a desired set point temperature, $T_s$, and the difference between these two temperatures results in an error voltage, $\epsilon$. This error voltage

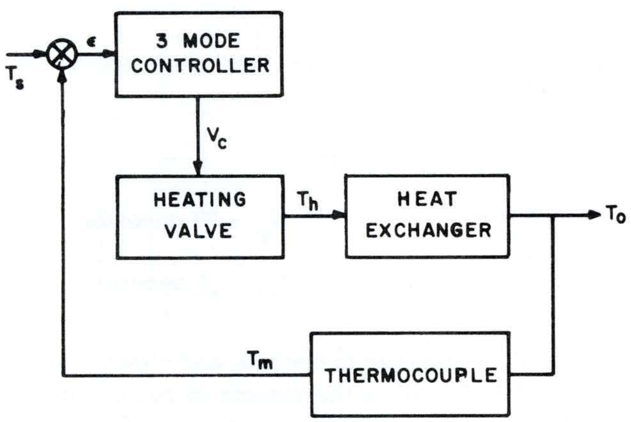


**Figure 1. Simplified Diagram of Three Mode Controller Used to Control Temperature**

is used by the controller to supply a control voltage, $V_c$, to the heating valve. The heating valve, in turn, controls the input heat, $T_h$, to the exchanger. Any changes in $T_o$ are sensed by the thermocouple and fed back to the input of the controller, which is adjusted to provide close control of $T_o$ during both steady state and transient temperature conditions. A simplified block diagram of the three mode controller is shown in Figure 2.

The proportional section of the controller provides straight gain to the error signal. The reset section

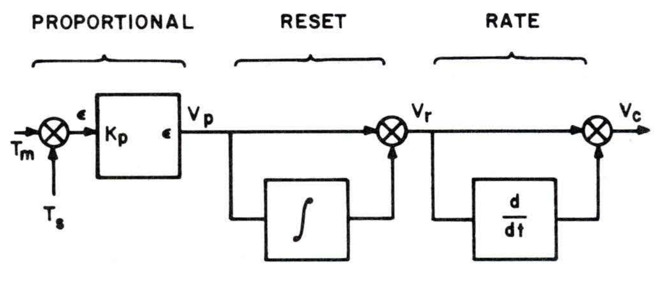


**Figure 2. Block Diagram of Three Modes**

provides the control voltage, $V_c$, with an integral of the error voltage. Its function is to reduce the steady state error signal to zero. The rate section of the controller provides the control voltage with a derivative of the output of the reset section of the controller. Its function is to "anticipate" temperature changes and to smooth out transient response of the temperature control system.

### System Equations

| Differential Form | Transfer Function |
|---|---|
| 1. Heat Exchanger | |
| $\dot{T}_o + \frac{1}{\tau_e} T_o = \frac{1}{\tau_e} T_h$ | $\frac{T_o}{T_h}(S) = \frac{1}{\tau_e S + 1}$ |
| 2. Heating Valve | |
| $\ddot{T}_h + W_n 2 \xi \dot{T}_h + W_n^2 T_h = \frac{W_n^2 V_c}{C}$ | $\frac{T_h}{V_c}(S) = \frac{1}{\frac{S^2}{W_n^2} + \frac{2\xi}{W_n} S + 1}$ |
| 3. Thermocouple | |
| $\dot{T}_m + \frac{1}{\tau_t} T_m = \frac{1}{\tau_t} T_o$ | $\frac{T_m}{T_o}(S) = \frac{1}{\tau_t S + 1}$ |




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4. Controller

A. Proportional

$$K_p (T_s - T_m)(C) = V_p \qquad \frac{V_p}{(T_s - T_m)} = K_p$$

B. Reset

$$V_p + \frac{1}{K_r}\int_0^t V_p\, dt = V_r \qquad \frac{V_r}{V_p}(S) = \frac{K_r S + 1}{K_r S}$$

C. Rate

$$\frac{1}{\alpha} V_r + \frac{1}{\alpha K_d}\int (V_r + V_c)\, dt = V_c \qquad \frac{V_c}{V_r}(S) = \frac{K_d S + 1}{\alpha K_d S + 1}$$

## Range of Parameters and Variables

1. Controller

| | | | |
|---|---|---|---|
| $K_p$ | = | Proportional Gain | $0 < K_p \leq 10$ |
| $K_r$ | = | Reset Time | $\infty \geq K_r \geq 1$ sec. |
| $K_d$ | = | Rate Time | $\infty \geq K_d \geq .1$ sec. |
| $\alpha$ | = | Compensation Ratio | $= \frac{1}{10}$ |
| $T_s$ | = | Set Point Temperature | $0 \leq T_s \leq 100°$ |
| $V_p$ | = | Proportional Voltage | $0 \leq V_p \leq 10$ v |
| $V_r$ | = | Reset Voltage | $0 \leq V_r \leq 10$ v |

2. Exchanger

| | | | |
|---|---|---|---|
| $T_o$ | = | Output Temperature | 0 to 100° F |
| $T_h$ | = | Input Temperature | 0 to 100° F |
| $\tau_e$ | = | Time Constant | = 2 sec. |

3. Thermocouple

| | | | |
|---|---|---|---|
| $T_m$ | = | Thermocouple Output | 0 to 100° F |
| $\tau_t$ | = | Time Constant | = 1 sec. |

4. Valve

| | | | |
|---|---|---|---|
| $W_n$ | = | Natural Frequency | $= \sqrt{18}$ radians/sec. |
| $V_c$ | = | Valve Input Voltage | 0 to 10 v |
| $\xi$ | = | Damping Factor | = .7 |
| C | = | Control Constant | = 1 V/10°F |

## Problem Statement

Develop a computer program to investigate the operation of the control system operating at different set points and different values of proportional gain, reset, and rate. Exchanger output temperature and temperature error should be plotted with respect to time.

Develop the following:

a) Scaled Equations

b) Computer Circuit Diagram

c) Amplifier and Attenuator Sheets for the Runs Given Below

d) Static Check

## Data

### First Run

$T_s = 20°$ F $\qquad K_r = \infty$ sec.

$K_p = 5 \qquad K_d = .1$ sec.

Place the computer in the operate mode and slowly raise the temperature set point to 20° F. Note the temperature error on the computer voltmeter. Slowly raise the temperature set point to 50° F and again note the temperature error.

### Second Run

$T_s = 20°$ F $\qquad K_r = 20$ sec.

$K_p = 5 \qquad K_d = .1$ sec.

Repeat run #1 with the rate time set at 20 seconds.

### Third Run

$T_s = 20°$ F $\qquad K_r = 20$ seconds

$K_p = 5 \qquad K_d = .1$ second

Place the computer in operate and slowly raise $T_s$ to 20° F. Allow the system to reach steady state operation and then apply a 1° F step to the temperature set point. Record $T_o$ with respect to time for each of the following values of $K_d$:

0.1 sec, 0.5 sec., 1 sec. and 2 sec.

Observe the speed of response of $T_o$ and the amount of time it takes $T_o$ to reach the steady state value.

### Fourth Run

Try to adjust the controller for optimum operation with a set point of 20° F and a temperature step of 1° F. Optimum operation is considered to be quick response to a step input with little temperature overshoot and a very small static error. The system also should be stable and not tend to oscillate. Record the values of $K_p$, $K_r$ and $K_d$ for what is considered optimum response.

## Introduction to Solution

The system equations given above are re-written so that the highest order derivative is on the left side of the equal sign. A basic computer diagram is now sketched out to solve the system equations. This diagram is drawn without regard for equation scaling. It is used to find out roughly how much equipment is needed and to gain an understanding of any equation manipulation which might result in a more simplified computer model.

A rough computer program is given in Figure 3. Using this circuit, the following equipment complement will be required:

- 6 Integrators
- 4 Summing Amplifiers
- 3 Inverters
- 19 Attenuators (6 of which are used for static test IC's)

## Scaled Equations

The first step in obtaining scaled equations is to rewrite the system equations so that the highest derivative appears on the left side of the equation

1. $\dot{T}_o = \frac{1}{\tau_e} T_h - \frac{1}{\tau_e} T_o$ (Exchanger)

2. $\ddot{T}_h = \frac{W_n^2 V_c}{C} - W_n 2 \xi \dot{T}_h - W_n^2 T_h$ (Valve)

3. $\dot{T}_m = \frac{1}{\tau_t} T_o - \frac{1}{\tau_t} T_m$ (Thermocouple)

4. A. $K_p (T_s - T_m)(C) = V_p$ (Proportional)

   B. $V_r = V_p + \frac{1}{K_r} \int_o^t V_p \, dt$ (Reset)

   C. $V_c = \frac{1}{\alpha} V_r + \frac{1}{\alpha K_d} \int_o^t (V_r - V_c) \, dt$ (Rate)

Next, scale factors are obtained by dividing the reference voltage by the maximum value of the variable.

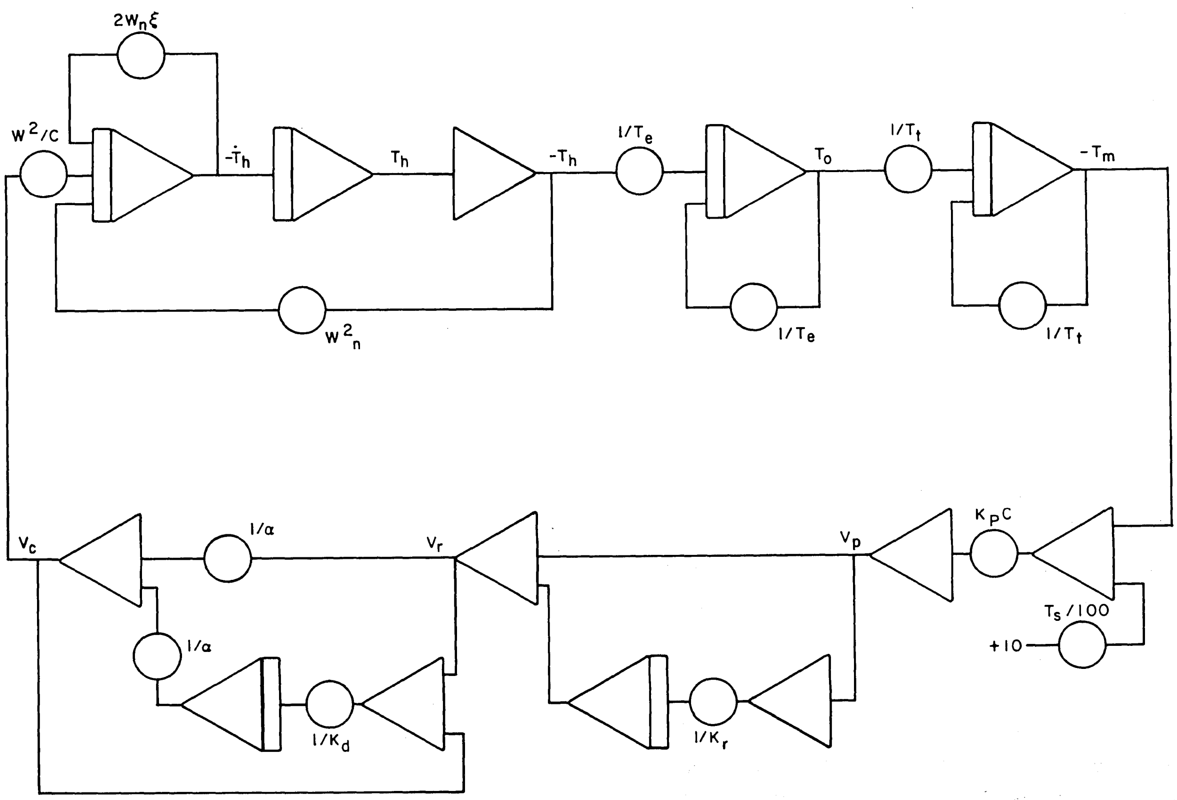


Figure 3. Unscaled Computer Diagram

| Variable | Max. Value | Scale Factor | Computer Variable |
|---|---|---|---|
| $T_o$ | 100° F | $\frac{10}{100}$ | $\left[\frac{1}{10} T_o\right]$ |
| $T_h$ | 100° F | $\frac{10}{100}$ | $\left[\frac{1}{10} T_h\right]$ |
| $V_c$ | 10 V | $\frac{10}{10}$ | $\left[V_c\right]$ |
| $T_m$ | 100° F | $\frac{10}{100}$ | $\left[\frac{1}{10} T_m\right]$ |
| $T_s$ | 100° F | $\frac{10}{100}$ | $\left[\frac{1}{10} T_s\right]$ |
| $V_p$ | 10 V | $\frac{10}{10}$ | $\left[V_p\right]$ |
| $V_r$ | 10 V | $\frac{10}{10}$ | $\left[V_r\right]$ |
| $\dot{T}_o$ | 10°/sec | $\frac{10}{10}$ | $\left[\dot{T}_o\right]$ |
| $\dot{T}_m$ | 10°/sec | $\frac{10}{10}$ | $\left[\dot{T}_m\right]$ |
| $\dot{T}_h$ | 10V/sec | $\frac{10}{10}$ | $\left[\dot{T}_h\right]$ |

The following scaled voltage equations were obtained:

1. $\left[\frac{\dot{T}_o}{10}\right] = \left(\frac{1}{\tau_e}\right)\left[\frac{1}{10} T_h\right] - \left(\frac{1}{\tau_e}\right)\left[\frac{1}{10} T_o\right]$ (Exchanger)

2. $\left[\frac{\ddot{T}_h}{10}\right] = 20\left(\frac{W_n^2}{200\ C}\right)\left[V_c\right] - 10\left(\frac{2\ W_n \xi}{10}\right)\left[\frac{\dot{T}_h}{10}\right] - 20\left(\frac{W_n^2}{20}\right)\left[\frac{T_h}{10}\right]$ (Valve)

3. $\left[\frac{\dot{T}_m}{10}\right] = \left(\frac{1}{\tau_t}\right)\left[\frac{1}{10} T_o\right] - \left(\frac{1}{\tau_t}\right)\left[\frac{1}{10} T_m\right]$ (Thermocouple)

4. $-10\left(C\ K_p\right)\left[\frac{1}{10} T_m - \frac{1}{10} T_s\right] = \left[V_p\right]$ (Proportional)

$\left[V_r\right] = +\left[V_p\right] + \left(\frac{1}{K_r}\right)\left[\int_o^t V_p\, dt\right]$ (Reset)

$\left[V_c\right] = +\frac{1}{\alpha}\left[V_r\right] + \frac{1}{\alpha}\left[K_d\right]\left[\int_o^t (V_r - V_c)\, dt\right]$ (Rate)

In the above equations --

The numbers preceding the brackets are amplifier gains;

The quantities in the square brackets are computer voltages;

The coefficients in the curved brackets are attenuator settings.

## PROBLEM MECHANIZATION

The potentiometer and amplifier assignment sheets are shown in Figures 4 and 5. The scaled computer diagram is shown in Figure 6.

## Static Check

To check the scaled equations, computer circuit, patching and equipment, a static check is prepared using the original equations.

Let

| | |
|---|---|
| $T_h = 20°$ F | $\dot{T}_h = 10°$ F/sec. |
| $T_o = 15°$ F | $K_p = 5$ |
| $T_m = 10°$ F | $\tau_e = 2$ sec. |
| $T_s = 20°$ F | $\tau_t = 1$ sec. |
| $V_c = 2$ v | C = 1 V/10° F |

$$\frac{1}{K_r}\int_o^t V_p\, dt = -1\ \text{V}$$

$$\frac{1}{K_d}\int_o^t (V_r + V_c)\, dt = +3.80\ \text{V}$$

Substituting these values into the original equations and solving for the highest derivatives:

1. $\dot{T}_o + \frac{1}{\tau_e} T_o = \frac{1}{\tau_e} T_h$ (Exchanger)

$\dot{T}_o + \frac{1}{2} \times 20° = \frac{1}{2} \times 15°$

$\dot{T}_o = 2.5°/\text{sec.}$

2. $\ddot{T}_h + W_n \cdot 2\ \xi\ \dot{T}_h + W_n^2\ T_h = \frac{W_n^2\ V_c}{C}$ (Valve)

$\ddot{T}_h + 18 \cdot 2 \cdot 0.7 \cdot 10 + 18 \cdot 20 = \frac{18.2}{1/10}$

$\ddot{T}_h = 360 - 59.5 - 360 = -59.5°$ F/sec/sec.

3. $\dot{T}_m + \frac{1}{\tau_t} T_m = \frac{1}{\tau_t} T_o$ (Thermocouple)

$\dot{T}_m + 1 \cdot 10 = 1 \cdot 15$

$\dot{T}_m = 5°$ F/sec.

4. Controller

A. $K_p\ (T_s - T_m)\ (C) = V_p$ (Proportional)

$5\ (20° - 10°)\ \frac{1}{10} = V_p$

$V_p = 5\text{V}$

B. $V_p + \frac{1}{K_r}\int_o^t V_p\, dt = V_r$ (Reset)

$5 + -1\text{V} = V_r$

$V_r = 4\text{V}$

C. $\frac{1}{\alpha} V_r + \frac{1}{\alpha K_d}\int (V_r + V_c)\, dt = V_c$ (Rate)

$10.4 - 10 \cdot 3.8 = V_c$

$V_c = +2\text{V}$

PROBLEM 3 Mode Controller

| POT NO. | PARAMETER DESCRIPTION | SETTING STATIC CHECK | SETTING RUN NO. 1 | SETTING RUN NO. 2 | SETTING RUN NO. 3 | NOTES | POT NO. |
|---|---|---|---|---|---|---|---|
| | $C\ K_p$ | .500 | .500 | .500 | .500 | | |
| | $1/K_r$ | .100 | 0 | .100 | .100 | | |
| | $1/K_d$ | 1.00 | 1.00 | 1.00 | .500 | | |
| | $W^2/200\ C$ | .900 | → | → | → | | |
| | $W^2/20$ | .900 | → | → | → | | |
| | $2\ \xi\ W/10$ | .594 | → | → | → | | |
| | $1/\tau_e$ | .500 | → | → | → | | |
| | $1/\tau_e$ | .500 | → | → | → | | |
| | $1/\tau_t$ | 1.00 | → | → | → | | |
| | $1/\tau_t$ | 1.00 | → | → | → | | |
| | $T_s/100$ | .200 | → | → | → | | |
| | I. C. Amp 04 | .100 | 0 | → | → | | |
| | I. C. Amp 07 | .380 | 0 | → | → | | |
| | $T_{h_o}/100$ | .100 | 0 | → | → | | |
| | $T_{h_o}/100$ | .200 | 0 | → | → | | |
| | $T_{o_o}/100$ | .150 | 0 | → | → | | |
| | $T_{m_o}/100$ | .100 | 0 | → | → | | |

Figure 4. TR-10 Potentiometer Assignment Sheet

PROBLEM 3 Mode Controller

DATE

| AMP NO. | FB | OUTPUT VARIABLE | STATIC CHECK CALCULATED INTEGRATOR INPUT SUM | STATIC CHECK CALCULATED OUTPUT | STATIC CHECK MEASURED INTEGRATOR INPUT SUM | STATIC CHECK MEASURED OUTPUT | NOTES |
|---|---|---|---|---|---|---|---|
| 1 | | $-\epsilon$ | | -1V | | | |
| 2 | | $+10\epsilon$ | | +5V | | | |
| 3 | | $-10\epsilon$ | | -5V | | | |
| 4 | INT | $-\frac{10}{K_r}\int \epsilon\, dt$ | +0.05* | -1.00 | | | |
| 5 | | $-V_r$ | | -4.00 | | | |
| 6 | | $V_c - V_r$ | | +2.00 | | | |
| 7 | INT | $-\frac{1}{T_r}\int (V_c - V_r)$ | -0.20* | +3.80 | | | |
| 8 | | $V_c$ | | +2.00 | | | |
| 9 | INT | $-\dot{T}_h/10$ | + .594* | -1.00 | | | |
| 10 | INT | $T_h/10$ | +0.10* | +2.00 | | | |
| 11 | | $-T_h$ | | -2.00 | | | |
| 12 | INT | $T_o$ | -0.25** | +1.50 | | | |
| 13 | INT | $-T_m$ | -0.05** | -1.00 | | | |
| 14 | | | | | | | |
| 15 | | | | | | | |
| 16 | | | | | | | |
| 17 | | | | | | | |
| 18 | | | | | | | |
| 19 | | | | | | | |
| 20 | | | | | | | |

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*Check Amp Gain = 1/10 **Check Amp Gain = -1

Figure 5. TR-10 Amplifier Assignment Sheet

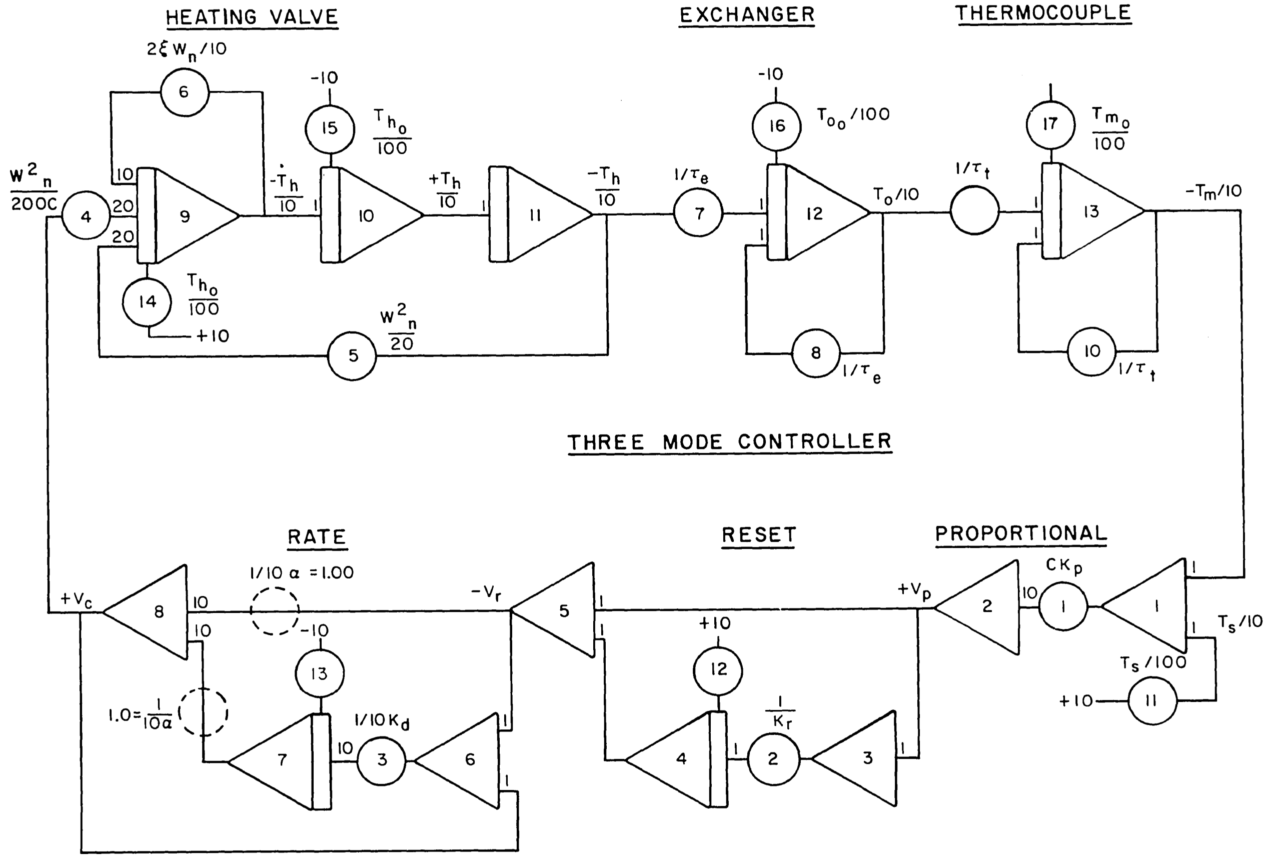


Figure 6. Scaled Computer Diagram

It should be noted that some of the above values are integrator inputs which cannot be checked directly. The output of an integrator in RESET is simply its initial condition and is independent of the inputs. Therefore, the inputs from the integrator are transferred temporarily to a spare summing amplifier, read, and then patched back into the integrator where they belong. By doing this for each integrator in turn, the values of all the integrator inputs can be verified.

When temporarily transferring inputs from an integrator to a summer, use the actual integrator input resistors. This insures that both the leads and input resistors are checked. Since a summer with a 10K feedback has 1/10 the gain that an integrator with the same input resistor would have, the output of the amplifier used for checking is 1/10 (TOTAL INPUT TO INTEGRATOR).

## Conclusion

When the three mode controller is operated without reset, there will be an error between the set point temperature and the exchanger temperature. The exchanger output temperature will always be lower than the set point temperature.

If reset is added to the temperature control, the error between the set point temperature and the exchanger temperature will eventually be zero. The amount of time that it takes the temperature error to go to zero depends on the reset time. A high value of reset time will result in a long time for the temperature error to reach zero and vice-versa. However, if the reset time is made too short, the system will tend to be unstable.

As rate is added to the control signal, the response of the system to a temperature step will be much faster. The output temperature of the exchanger will rise more quickly to its new value and will reach. steady state operation faster. If too much rate is added, the system will tend to oscillate. A graph of exchanger output temperature versus time for different rate times is given in Figure 7.

In this problem, the temperature set points and temperature steps were purposely kept low in order to operate all of the equipment in its linear range. Although the values used are not realistic, the basic operation of the three mode controller is. After running this problem, one should have a thorough understanding of how a three mode controller operates and how it may be simulated on an analog computer.

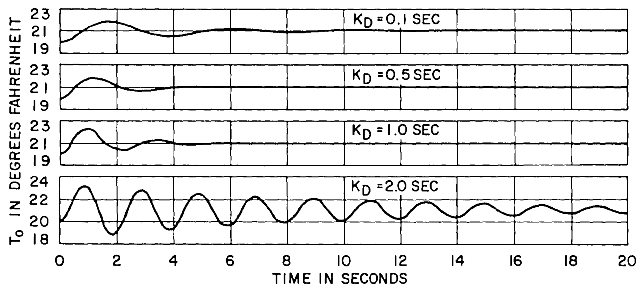


Figure 7. Exchanger Output Temperature vs Time