THE HUMAN PUPIL SERVOMECHANISM

INTRODUCTION

This Study, performed on a desk-top-sized PACE® TR-10 general purpose analog computer, describes the simulation of pupillary response in the human eye to an increase in light intensity. When the eye is exposed to such an increase in light intensity, the iris contracts, thereby reducing the pupil area and regulating the amount of light falling on the retina.

SYSTEM ANALYSIS

This phenomenon has been investigated by Stark (1) who found that the pupil responds in a frequency range from zero to about 4 cps in a manner which can be described by a transfer function such as

$$L(S) = \frac{0.16e^{-0.18S}}{(1 + 0.1S)^3} \qquad (1)$$

which relates the "pupil-area signal" to retinal flux. The quantity 0.16 is the zero-frequency magnitude; the exponential term represents a pure time delay of 0.18 sec.

This system as described is stable, but if the zero-frequency magnitude is replaced by an adjustable constant, K, there will be a value of this constant which will place the system on the verge of instability. The system then will be self-excited and will produce oscillations termed "induced pupillary hippus".

The prediction by analytical means of K_C--that value of K which causes an oscillatory condition--is complicated greatly by the presence of the exponential factor, $e^{-0.18S}$. While modified versions of common methods (such as the Nyquist diagram) can be used, an analog simulation of the system permits the investigator to determine K_C quickly and directly.

PHYSICAL SYSTEM

The physical system can be described by the simplified block diagram of Figure 1.

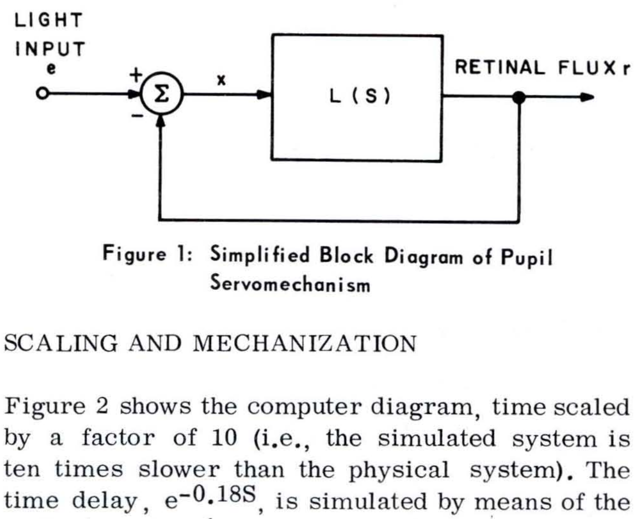


Figure 1: Simplified Block Diagram of Pupil Servomechanism

SCALING AND MECHANIZATION

Figure 2 shows the computer diagram, time scaled by a factor of 10 (i.e., the simulated system is ten times slower than the physical system). The time delay, $e^{-0.18S}$, is simulated by means of the well-known Padé approximation (2)

$$e^{-\tau S} \simeq \frac{840 - 360\,\tau S + 60\,\tau^2 S^2 - 4\,\tau^3 S^3}{840 + 480\,\tau S + 120\,\tau^2 S^2 + 16\,\tau^3 S^3 + \tau^4 S^4} \qquad (2)$$

and also has been time scaled.

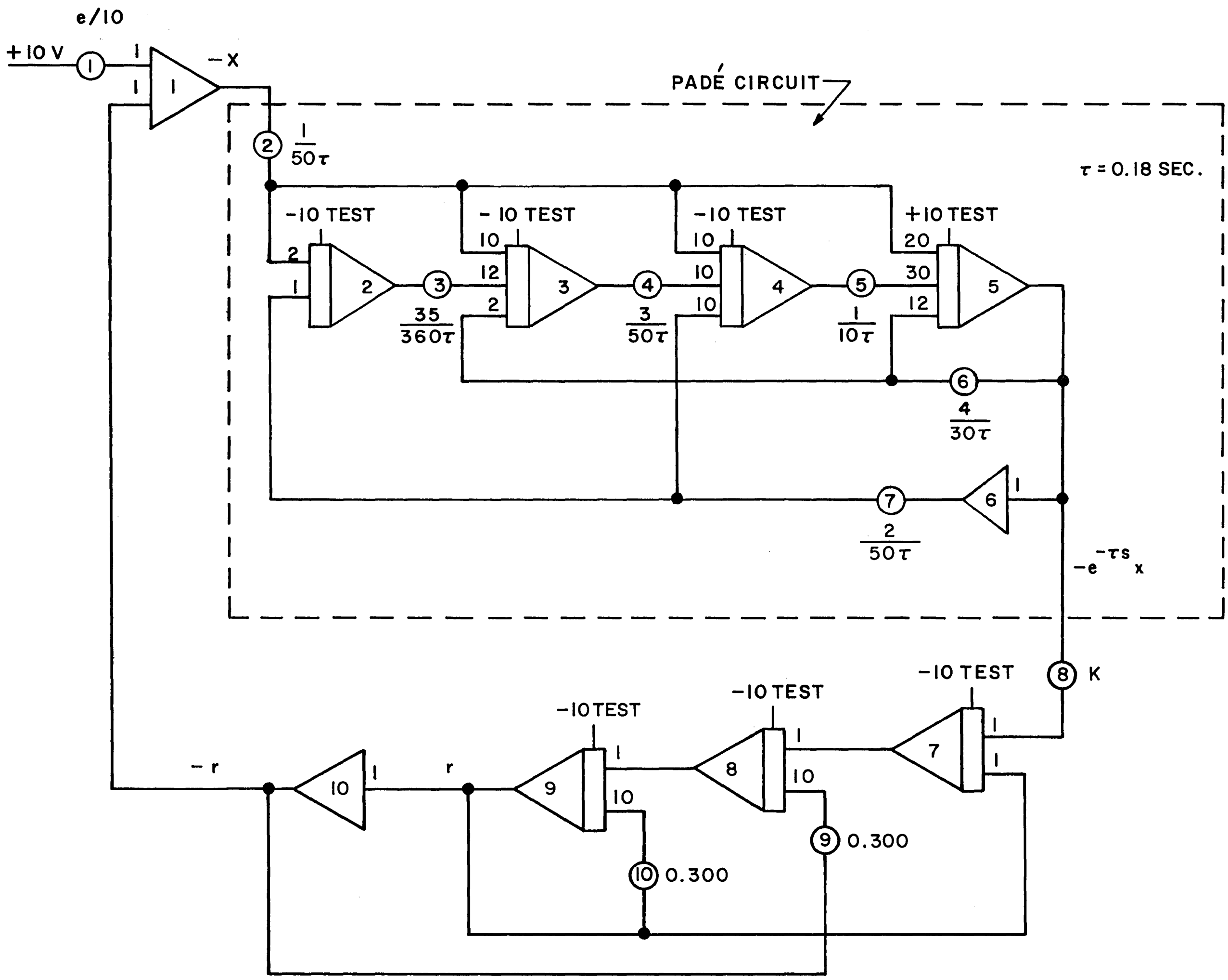


Figure 2: Scaled Computer Diagram

The potentiometer and amplifier assignment sheets are shown in Figures 3 and 4.

PROBLEM Pupil Servo
DATE

POT NO.	PARAMETER DESCRIPTION	SETTING STATIC CHECK	STATIC CHECK OUTPUT VOLTAGE	SETTING RUN NUMBER I	NOTES	POT NO.
1	e/10	0.200	———	——→		1
2	1/50τ	0.111	———	——→		2
3	35/360τ	0.639	———	——→		3
4	3/50τ	0.333	———	——→		4
5	1/10τ	0.556	———	——→		5
6	4/30τ	0.741	———	——→		6
7	2/50τ	0.222	———	——→		7
8	K	0.160			Parametric Variable	8
9	Constant	0.300	———	——→		9
10	Constant	0.300	———	——→		10

Figure 3: TR-10 Potentiometer Assignment Sheet

PROBLEM Pupil Servo
DATE

AMP NO.	FB	OUTPUT VARIABLE	STATIC CHECK				NOTES
			CALCULATED		MEASURED		
			INTEGRATOR INPUT SUM	OUTPUT	INTEGRATOR INPUT SUM	OUTPUT	
1	Σ	x		-8.00			
2	∫	Padé Circuit	0.444	+10			
3	∫	Padé Circuit	53.0*	+10			
4	∫	Padé Circuit	46.6*	+10			
5	∫	$-e^{-S}x$	60.1*	-10			
6	Σ	$+e^{-S}x$		+10			
7	∫	Pupil circuit	+8.40	+10			*Check amplifier gain must be -1/10
8	∫	Pupil circuit	-20.0*	+10			
9	∫	r	+40.0*	+10			
10	Σ	-r		-10			

Figure 4: TR-10 Anplifier Assignment Sheet

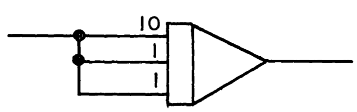
RESULTS

By applying a step function corresponding to a rapid change in light intensity input and assigning a different value of K for each run, one can quickly find the value of K_c. In this problem, the value found was $K_c = 1.97$. A pencil and paper analysis based on approximating $e^{\tau s}$ by the first three terms of its series expansion yields the result $K_c = 1.82$, with an oscillation frequency of about 66 cycles per minute.

EQUIPMENT COMPLEMENT

The major pieces of equipment necessary for this simulation include: 10 Operational Amplifiers (7 of which are used as integrators) and 10 potentiometers.

Notes: 1. Gains of 2, 12, 20, etc. can be obtained with parallel combinations of standard gains of 1 and 10. For example, a gain of 12 is achieved with two gains of 1 and one gain of 10 in parallel. Thus



2. To obtain $K > 1$, change the gain from 1 to 10 of the channel of amplifier #7 which is fed by potentiometer #8.

3. This study was derived from a problem in ''Principles of Electronic Instrumentation by Lynch and Truxal, McGraw-Hill Book Company, Inc., New York, 1962, pp. 682-685.

REFERENCES

(1) Stark, L., ''Stability, Oscillation and Noise in the Human Pupil Servomechanism'', Bio-Medical Electronics Issue, Proc. IRE, Vol. 47, No. 11, pp. 1925-1939, Nov. 1959.

(2) Rogers, A.E. and Connolly, T.W., ''Analog Computation in Engineering Design'', McGraw-Hill Book Company, Inc., New York, N.Y., 1960, Appendix 4.