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A Vapor-Grown Variable Capacitance Diode

Abstract: Germanium *p-n* junctions have been made which have a large fractional variation of capacitance with voltage and which have promise of being operable at high frequencies. These diodes are produced by a vapor-growth process in which the doping is switched from *n*-type to *p*-type during growth. Capacitances which vary as the reciprocal of voltage over a considerable range have been observed.

This capacitance variation corresponds to a net donor concentration which decreases from its value at the junction approximately as the reciprocal of distance from the junction. At a position corresponding roughly to the edge of the transition region at breakdown, the net donor concentration abruptly increases. This rapid variation of capacitance with voltage and the low series resistance resulting from the discontinuity in doping level should result in high-frequency diodes, if the magnitude of the discontinuity can be increased sufficiently.

Introduction

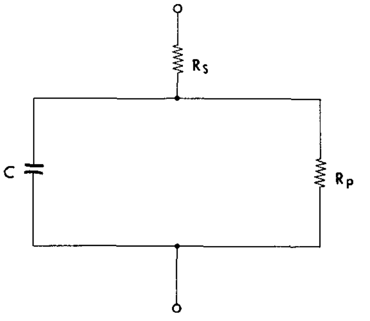
The effect of variation of capacitance of a semiconductor diode with applied voltage has been known for many years. Schottky[1] in 1942 measured this capacitance in metal-semiconductor diodes to help substantiate his theories on the mechanism of rectification. Van der Ziel[2] showed in 1948 how this effect could be used in low-noise frequency conversion and power amplification. Since about 1956, a great deal of effort has been directed toward development of variable-capacitance diodes for power amplification at microwave frequencies.[3-6] The name *parametric amplification* has been coined to denote amplification by use of a variable-reactance device and, as a result, the diodes involved are often referred to as parametric diodes.

A semiconductor diode can be represented by the equivalent circuit of Fig. 1. In this equivalent circuit, R_s represents the bulk resistance of the material on either side of the transition region, C represents the transition capacitance of the junction, and R_p represents a parallel combination of leakage resistance and the non-ohmic barrier resistance. For reverse voltages greater than a few kT/q, the value R_p is effectively the leakage resistance.

The Q of the diode is defined as the ratio of maximum energy stored to rate of energy dissipation. The Q of the diode represented by the equivalent circuit of Fig. 1 can then be shown to be:

$$Q=\frac{\omega C R_p}{1+(R_s/R_p)(1+\omega^2C^2R_p^2)}. \qquad (1)$$

Figure 1 **Equivalent circuit of a variable-capacitance diode.**



Since $R_p \gg R_s$, the maximum value of Q is

$$Q_{\max}=\tfrac{1}{2}\sqrt{(R_p/R_s)} \qquad (2)$$

and occurs at a frequency f', where

$$f'=\frac{1}{2\pi C\sqrt{R_sR_p}}. \qquad (3)$$

In this paper we consider the case where the frequency of operation is much higher than f'. The value of Q, then, at any frequency is

$$Q=\frac{1}{2\pi f R_s C}=\frac{f_c}{f}, \qquad (4)$$

where f_c, the "cut-off frequency," is defined as the frequency at which the Q is unity, or

$$f_c=\frac{1}{2\pi CR_s}\,. \tag{5}$$

The differential capacitance of a variable-capacitance diode can often be represented by a formula of the form

$$C=\frac{C_1}{(V_D-V)^\alpha}\,, \tag{6}$$

where V_D is the "built-in" potential difference and V is the applied voltage. The value of α denotes the nonlinearity of the capacitor and C_1 represents the capacitance for a unity value of (V_D-V).

In a parametric diode designed for use as a subharmonic oscillator, a figure of merit f_m, which represents the maximum frequency at which oscillation is possible, is useful. This can be expressed as

$$f_m=\frac{f_c}{Q_{\min}}\,, \tag{7}$$

where $Q_{\min}$ is the minimum Q required for oscillation. Hilibrand and Beam[3] have shown that

$$Q_{\min}\cong\frac{2}{\alpha}\,, \tag{8}$$

where α is defined by Eq. (6). We can see from this that for maximum f_m the nonlinearity of capacitance with voltage should be as large as possible or the value of α should be large, provided this does not have adverse effects on f_c.

The series resistance of the diode can be written, to a good approximation, as

$$R_s=\int_W^t\frac{dx}{Aq\mu(N_D-N_A)}\,, \tag{9}$$

where W represents the width of transition region and t the thickness of the diode, A is diode area, q is electronic charge, μ is carrier mobility and (N_D-N_A) is the net ionized impurity density.

A relationship such as (6) with large α over the range of operation is desired for maximum frequency of operation. In the region extending beyond the edge of the transition region (W) at maximum reverse bias a very large value of N_D-N_A is desired to minimize series resistance.

The purpose of this paper is to describe the characteristics of some vapor-grown germanium diodes which have large nonlinearities in the C vs V relationships and which hold promise of having high operating frequencies. The capacitance characteristics and the variation of net ionized impurity concentrations as functions of position for these diodes are discussed.

In the following section, brief descriptions of the fabrication and measuring techniques are presented. In the subsequent section, the C-V characteristics of these diodes are described and from these results the net impurity distributions are obtained. In Appendix I, the theory is developed for the analysis of the C-V curves in terms of impurity distributions.

Diode fabrication and measurement

Deposited p-n junctions having a large nonlinearity of capacitance with voltage have been made by the closed-tube iodide deposition process described in detail elsewhere in this issue.[7, 8] The seeds are highly doped n-type germanium. One source is n-type germanium and the other, p-type germanium. The temperatures of the zones in the tube are adjusted such that the n-type source, which it is desired to deposit first, is maintained at about 560°C, the seed is about 400°C, and the p-type source is kept at about 30°C above the seed temperature. During the iodide reaction the germanium and impurities are removed from the hot source and deposited in the coolest region—the seed zone. The deposited germanium makes an ohmic epitaxial contact to the n-type seed. To subsequently deposit p-type germanium, the temperature profile is reversed and the germanium and impurities are carried from the p-type source to the seed. The result is a vapor-grown p-n junction.

The wafers are cut into cylinders of about 0.15 cm diameter. Ohmic contacts are made to each side and the resultant diodes are mounted on headers. No attempt has been made to obtain optimum geometry for reduction of series resistance. At present, the control of growth rate (and, therefore, thickness) and doping is not adequate to "tailor make" diodes. Capacitance and Q measurements at 2 Mc/sec are made with the aid of a Boonton Q meter, Type 260-A. To measure capacitance the test signal is decreased to an amplitude of 10 mv so that accurate readings can be made. Because of these small signals, an external detector is required.

Experimental results

Retrograde impurity distributions, which cause a large nonlinearity of capacitance with voltage, have been observed to occur in n-type material in the immediate vicinity of the junction formed when deposition is switched from n-type to p-type impurities. This has been observed in deposits in which either P or As was used for the n-type impurities and either B or Ga was used for the p-type impurity.

Figure 2 shows a plot of log C vs log (V_D-V) for an experimental diode in which the doping on the p-side is much heavier than on the n-side. The curve can be approximated by three straight-line portions and two curved regions. The straight-line portions can each be represented by a formula of the form (6)

$$C=\frac{C_1}{(V_D-V)^\alpha}\,.$$

In Region 1, $C_1=1350$ pf, $\alpha=0.92$.

In Region 2, $C_1=1750$ pf, $\alpha=1.09$.

In Region 3, $C_1=500$ pf, $\alpha=0.5$.

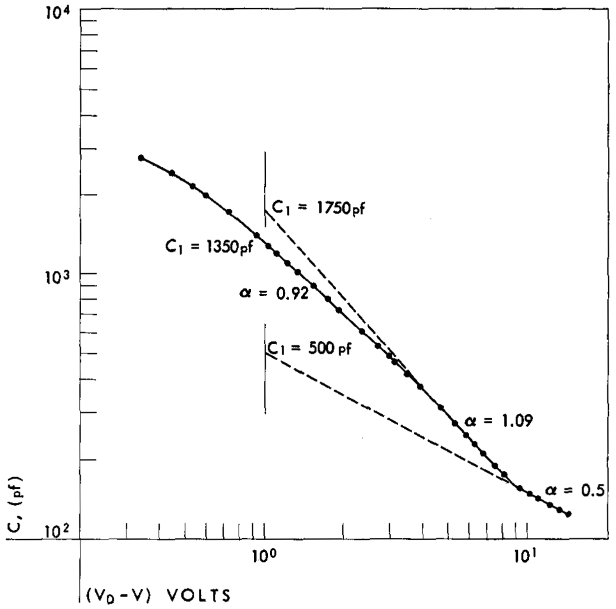


Figure 2 **Log-log plot of C vs (V_D-V) for experimental diode having heavier doping on the *p*-side than on the *n*-side.**

It is shown in Appendix I that for a planar junction, in which the capacitance in a region can be described by a formula of the type (6), the net impurity concentration can be expressed in this same region by

$$N_D-N_A=\frac{1}{\alpha}\frac{\varepsilon}{q}\left(\frac{1}{W_1}\right)^2\left(\frac{W}{W_1}\right)^{-[2-(1/\alpha)]}, \qquad (10)$$

where ε is the permittivity of the semiconductor, W is the width of the transition region at arbitrary voltage and W_1 is the width for (V_D-V) equal to unity.

Jørsboe[10] has shown that for a planar junction the relationship between differential capacitance and transition region width (neglecting mobile charge in the transition region[11]) is

$$C=\frac{\varepsilon A}{W}. \qquad (11)$$

Equation (11) can be used to obtain the value for W_1 and the limits on W from capacitance measurements. If this is done the net donor concentration (atoms/cm²) can be expressed as

$$N_D-N_A=5.7\times 10^{16}\left(\frac{W}{W_1}\right)^{-0.91} \qquad \text{in Region 1,}$$

$$N_D-N_A=7.8\times 10^{16}\left(\frac{W}{W_1}\right)^{-1.09} \qquad \text{in Region 2,}$$

and

$(N_D-N_A)=1.35\times 10^{16}$ in Region 3.

(It is noticed that $\alpha\cong 2-1/\alpha$, since α is so close to unity.)

In the curved regions, the net donor concentration is found point-by-point. The value of α at a point is obtained from the tangent at that point and the value for C_1 is determined from the intercept of the tangent with the line $(V_D-V)=$ unity. This variation in net donor concentration with position is plotted in Fig. 3. It can be seen that there is a discontinuity in N_D-N_A at $W=1.1$ microns. The deposition progressed from right to left. It is thought that the discontinuity corresponds to the reversal of the temperature profile or the attempt to switch from *n*- to *p*-type deposition. After this switch, about 1.1 microns of deposition occurred before the *p*-type deposition actually started. This last portion of the deposit became increasingly *n*-type (at least to within 0.067 micron of the junction) as the deposition progressed.

This discontinuity and subsequent increase in net donor concentration has been observed in many depositions. In some units the retrograde distribution (decrease in net donor concentration with increasing distance from the junction) extends further than the transition region can be expanded before breakdown. In these cases, it is not known whether or not a discontinuity exists. Scatter in the data does not permit an accurate measurement of the "width" of the discontinuity. The discontinuity, however, is judged to be less than 0.1 micron. When the *n*-region is more heavily doped than the *p*-region, the transition region extends into the *p*-side and capacitance measurements indicate that the junction is linearly graded.

The reason for the discontinuity and the retrograde impurity distribution is not well understood. It is suggested that the discontinuity may be a result of the variation in temperature during the reversal of doping. It is thought that the retrograde variation arises from the fact that during the deposition of the first layer, donor atoms deposit out in the second source region more readily than do Ge atoms. As a result, there is a large number of exposed donor atoms in this region on the source, walls, et cetera. When the temperature profile is reversed, these donors react with the iodine as it becomes available and as a result, the vapor (and consequently, the seed) has a progressively greater density of donors. After these donors are exhausted, the *p*-type doping becomes effective.

An appropriate decrease in area at the edge of the transition region with applied voltage as might occur for a jagged interface has been ruled out as a cause for the observed *C-V* behavior. This behavior has been observed for depositions which occurred on the (111), (211) and (110) faces, and although the (211) and (110) faces might conceivably have a deposited surface rough (nonplanar) enough to account for the observed *C-V* characteristics over small ranges of voltage, the (111) face is smooth enough to make this effect negligible.

In addition, diodes made by depositing first *p*- and then *n*-type Ge do not show this effect, and the value of α in equation (6) is very nearly equal to 1/3, indicating these junctions are linearly graded.

As mentioned above, no attempt was made to optimize geometry to obtain high-frequency devices. For the case of the diode described here the series resistance was meas-

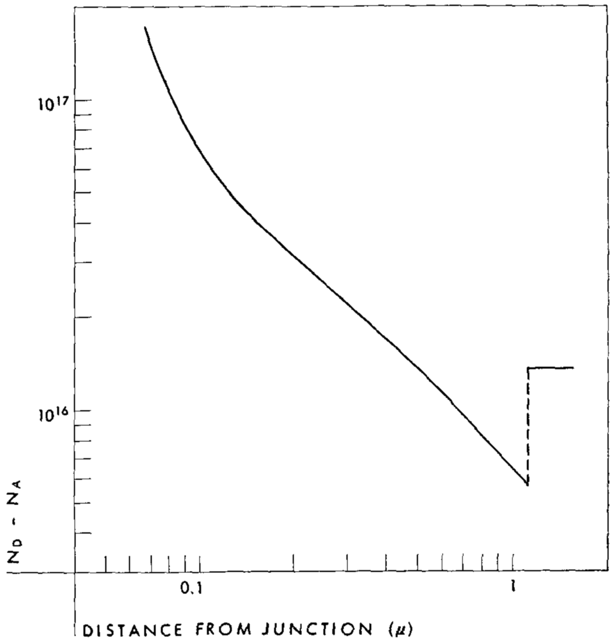


Figure 3 **Variation of net donor concentration with distance from junction in experimental diode.**

ured to be 8 ohms. This high value of series resistance is a result of a high resistivity and excessive thickness. At a reverse voltage of 5 v, the value of f_c is 70 mc, and f_m is slightly less than half this value. These values compare very poorly with those obtainable with diodes made by other processes, but with further work it seems entirely possible that variable-capacitance diodes with large figures of merit can be produced by this method.

Summary

Variable-capacitance diodes which have a large fractional variation of capacitance with applied voltage have been made by the vapor-growth process. This variation is caused by a retrograde impurity distribution.

At a distance from the junction approximately equal to the width of the transition region at the breakdown voltage, the net donor concentration abruptly increases and remains constant beyond this point. This large donor concentration results in a low series resistance.

The retrograde impurity distribution and the magnitude of the discontinuity in net donor concentration are at present not sufficiently controllable to yield variable-capacitance diodes having figures of merit comparable to those produced by other means.

Acknowledgments

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Appendix I: Derivation of the expression relating C-V characteristics to the impurity density distribution.

Consider a p-n junction in which $N_D - N_A$ in the n-side is much less than $N_A - N_D$ on the p-side at the edges of the transition region. In this case, the space-charge widening with voltage will occur predominately on the n-side. The variation of transition region width with voltage and therefore the variation of capacitance with voltage will be functions of the width of the transition region and the net donor density at the edge of the transition region.

Let us consider a planar junction in which formula (11) is valid. The space charge density (ρ) can be approximated as $\rho = q(N_D - N_A)$ in the transition region and $\rho = 0$ beyond this.

At the edge of the transition region the differential charge (dQ) is

$$dQ = q(N_D - N_A) A\,dW. \tag{I-1}$$

With the aid of Eq. (I-1), and the definition of nonlinear differential capacitance $C = \left|\frac{dQ}{dV}\right|$ the capacitance can be expressed

$$C = \left| q(N_D - N_A) A \frac{dW}{dV} \right|. \tag{I-2}$$

Making use of Eq. (11), the formula

$$(N_D - N_A) = \frac{C^3}{q\varepsilon A^2} \left(\frac{dC}{dV} \right)^{-1} \tag{I-3}$$

can be obtained.[12]

It can be seen from the above that C would be expected to be a continuous, monotonic function of voltage. As a result, a discontinuity in $N_D - N_A$ would be manifested by a discontinuity in slope of the capacitance-voltage curve (Eq. I-3).

Equation (I-3) in conjunction with Eq. (11) determines the net donor concentration as a function of position in the region near the metallurgical junction.

If the transition region spread with voltage is predominately on the p-type side of the junction, the same equations are valid if $(N_D - N_A)$ is replaced by $(N_A - N_D)$.

It must be emphasized that these equations are valid for arbitrary functions of doping with distance from the metallurgical junction.

In many cases, observed variation of capacitance with voltage can be expressed for an appreciable range in voltage by a formula of the form (6)

$$C = \frac{C_1}{(V_D - V)^\alpha}.$$

For these cases, the term $(dC/dV)^{-1}$ of Eq. (I-3) can be evaluated and Eq. (I-3) becomes:

$$N_D - N_A = \frac{C_1{}^2}{\alpha q \varepsilon A^2} \left(\frac{C}{C_1} \right)^{2-1/\alpha}. \tag{I-4}$$

Combining Eqs. (I-4) and (11) and letting $W = W_1$ at $V_D - V = 1$, we obtain

$$N_D - N_A = \frac{1}{\alpha}\frac{\varepsilon}{q}\left(\frac{1}{W_1}\right)^2\left(\frac{W}{W_1}\right)^{-[2-(1/\alpha)]}, \tag{I-5}$$

which expresses the net donor concentration as a function of distance from the metallurgical junction in the range where Eq. (6) is valid. It is often possible to plot the log C vs log $(V_D - V)$ relationship by a series of straight lines. Then a number of relationships of Eq. (6) are valid, each in a limited region.

It should be pointed out that although Eqs. (I-4) and (I-5) are valid for regions where a plot of log C vs log $(V_D - V)$ gives straight lines, the converse is not true. A particular power law relationship of $N_D - N_A$ with distance, extends nearer the junction than the corresponding power law relationship of capacitance.

If one side of the junction is not heavily doped compared to the other, an error will result in using these formulae. The maximum error will occur for equal dopings on either edge of the transition region and the calculated concentration will be one-half the actual density. This can be seen from Eq. (I-1) where $(N_D - N_A)$ would be replaced by $(N_D - N_A)/2$.

Appendix II: Degree of nonlinearity in variable-capacity diodes.

Equation (6) can be expanded in a Taylor series about some operating voltage V_0 and the first two terms are

$$C = C_0 + \alpha C_0 \frac{(V - V_0)}{(V_D - V_0)}, \tag{II-1}$$

where C_0 is the capacitance at an applied bias voltage V_0. From this, we can see that α is a measure of the nonlinearity of capacitance with voltage.

The question arises as to what values of α are permissible. Negative values of α indicate that W is decreasing with applied voltage. This is not possible according to the physics of semiconductor diodes. A zero value of α corresponds to no variation in space-charge width with voltage. This is the case for classical linear capacitors. From Eq. (I-5), it is seen that there is no upper limit for α if the variation in $N_D - N_A$ can be adjusted appropriately. Values of α greater than or equal to 1, however, are not possible in the limit as $V_D - V$ approaches zero, since then an infinite amount of charge is contained in an infinitesimal region, i.e.,

$$Q = \int_0^{\delta} qA(N_D - N_A)\,dX = \infty \qquad \text{for } \alpha \geq 1\,. \tag{II-2}$$

References and footnotes

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10. H. Jørsboe, *Proc. IRE,* **47,** 591 (1959).
11. This is a reasonably good approximation for germanium junctions except at high current densities.
12. Equation (I-3) is equivalent to the equation

$$(N_D - N_A) = \frac{2}{q\varepsilon A^2}\frac{dV}{d(1/C^2)},$$

obtained by Schottky (Ref. 1).

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