M. F. Choquet
H. J. Nussbaumer

Generation of Synchronous Data Transmission Signals by Digital Echo Modulation

Abstract: This paper shows the basic similarities among various data transmission techniques. It generalizes the concept of how the signal element serves as a unifying feature of various modulation schemes. The present approach permits an extension of the digital echo modulation technique, originally introduced by A. Croisier and J-M. Pierret, to cover most of the cases of digital data transmission. The practical application of digital echo modulation to the design of modems is discussed and examples of digitally implemented modem transmitters are given.

Introduction

The present trend in data transmission is to use large scale integration to decrease both the size and cost of modems. This objective clearly calls for replacing analog signal processing by digital techniques. A significant problem in this respect is to define new signal processing techniques that are well adapted for digital implementation.

In a recent paper[1], A. Croisier and J-M. Pierret described a new approach called *digital echo modulation*, which permits the realization, by digital means, of various filtering and modulation functions in a modem transmitter. This approach is based upon the use of a signal element that is the impulse response of a generalized Nyquist filter[2, 3].

Croisier and Pierret defined this signal element as being

$$s_1(t) = \frac{\sin(\pi f_N t)}{\pi f_N t} \cos 2\pi f_c t.$$

They have shown that, by assigning this signal element to the input data by means of the proper algorithm, digital echo modulation could produce signals similar to those of single sideband and phase modulation and could lead to an efficient implementation with digital circuits. This approach, however, is restricted to a certain class of phase-modulation and single-sideband signals and applies only when the carrier frequency and the signalling rate have some definite relationship.

The purpose of the present paper is to describe an extension of digital echo modulation and to give examples of digitally implemented modem transmitters that utilize this technique.

We shall show how digital echo modulation can be extended to cover most of the practical cases of synchronous data transmission through phase modulation, single-sideband modulation, frequency modulation or a combination of any of these with amplitude modulation. It will also be shown that, with the new extension of digital echo modulation, the restrictions on the relationship between the carrier frequency and the signalling rate are much less severe than in the initial scheme, so that the newer technique can be used in most of the practical cases of modem transmitters.

In this paper, we first briefly review the various linear-modulation schemes and their decomposition into signal elements. The practical use of signal elements for designing modem transmitters is then discussed. Finally it is shown how the digital echo modulation technique can be extended to cover most of the cases of digital data transmission, and practical examples of phase-shift keying, vestigial-sideband and frequency-shift-keying digital echo modem transmitters are given.

Linear modulation

High-performance data transmission systems are usually synchronous and are designed in such a way that data can be recovered in the receiver at regularly spaced intervals. At those sampling times, the received signal is sensed for its amplitude, phase and frequency. Discrimination among a discrete set of possible amplitudes, phases or frequencies permits recovery of the data. This scheme assumes that there is no intersymbol interference, or at least that there is controlled intersymbol interference, if the transmission is to be error-free in the absence of noise. As pointed out previously[1, 4], the linear filtering and modulation functions performed in a modem transmitter can be viewed as produced by a set of elementary signal generators. In the following we shall briefly review the usual modulation schemes and show more precisely how they can be decomposed into a set of elementary signals called *signal elements*.

Let us consider the elementary modem transmitter shown in Fig. 1. In such a modem, where the signalling interval is T, the input data is a sequence of impulses of amplitude a_i at sampling time iT. After low-pass filtering with a filter $G(\omega)$, this signal is product modulated with a carrier of angular frequency ω_c and phase φ. The resulting amplitude-modulated suppressed carrier (AMSC) signal is then band-pass filtered with filter $H(\omega)$. This line signal $e(t)$ represents either a double-sideband or a vestigial-sideband modulated signal, depending upon the particular characteristics of the post-modulation filter.

Assuming the impulse responses of $G(\omega)$ and $H(\omega)$ are respectively $g(t)$ and $h(t)$, $e(t)$ can be expressed as

$$e(t) = \left[\left\{\left[\sum_{i=-\infty}^{+\infty} a_i\delta(t-iT)\right] * g(t)\right\} \cos(\omega_c t + \varphi)\right] * h(t), \qquad (1)$$

where $*$ is the symbol of the convolution product, and

$$e(t) = \left\{\left[\sum_{i=-\infty}^{+\infty} a_i g(t-iT)\right] \cos(\omega_c t + \varphi)\right\} * h(t). \qquad (2)$$

The line signal $e(t)$ can be decomposed into a sum of elementary signals with

$$e(t) = \sum_{i=-\infty}^{+\infty} \left\{a_i g(t-iT) \cos\left[\omega_c(t-iT) + \varphi + \omega_c iT\right]\right\} * h(t) \qquad (3)$$

or

$$e(t) = \sum_{i=-\infty}^{+\infty} a_i e_i(t-iT), \qquad (4)$$

where

$$e_i(t) = [g(t)\cos(\omega_c t + \varphi + \omega_c iT)] * h(t). \qquad (5)$$

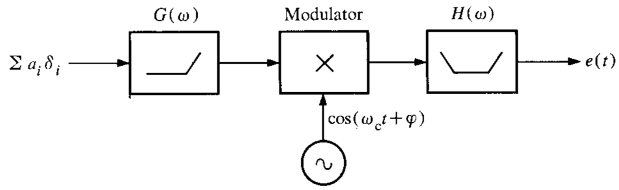
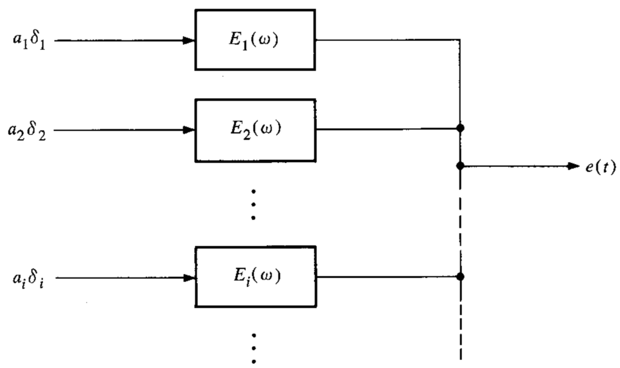


Figure 1 Elementary transmitter.

Figure 2 Signal element generator.



It is clear from (4) that the data transmitter of Fig. 1 is equivalent to that shown in Fig. 2 in which at each sample a_i of the data sequence is associated a signal element $e_i(t)$ delayed by iT. Let us call $E_i(\omega)$ the Fourier transform of $e_i(t)$. The modem transmitter can therefore be viewed as a signal element generator in which a particular signal element $e_i(t)$ is associated with each sampling time iT.

The duration, shape and number of signal elements depend upon the parameters that define the transmitter, i.e., line signalling rate $1/T$, filter characteristics $G(\omega)$ and $H(\omega)$, carrier phase and angular frequency φ and ω_c. From Eq. (5), it can be seen that the signal elements are dependent upon the sampling time iT. This relation can be viewed as the condition of carrier-phase continuity that must be insured when the transmitter in Fig. 1 is replaced by a signal element generator as in Fig. 2.

Let us now extend the previous results to phase-shift keying. We know that an N-phase transmitter can be built by summing the outputs of two elementary transmitters of Fig. 1 with identical filters, where the phases of the carrier signals are related as follows:

$$\left.\begin{array}{ll} \varphi_1 = 0 \ (\text{or } \varphi_0) & \text{for the "in phase" channel} \\ \varphi_2 = \pi/2 \ (\text{or } \varphi_0 + \pi/2) & \text{for the channel in quadrature,} \end{array}\right\} \qquad (6)$$

and the amplitudes of input impulses are assumed to be

$$\left.\begin{array}{l} a_{i1} = K\cos\Phi_{k(i)} \\ a_{i2} = K\sin\Phi_{k(i)}, \end{array}\right\} \qquad (7)$$

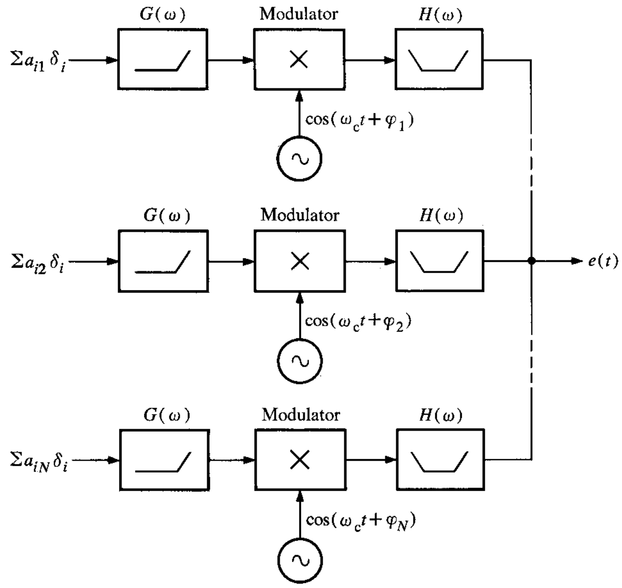


Figure 3 Phase-shift-keying transmitter.

where K is a constant factor, k is an integer and $1 \leq k \leq N$, and $\Phi_{k(i)}$ the carrier phase shift at sampling time, iT being the information-bearing parameter.

Applying (4) and (5), the line signal $e(t)$ of Fig. 3 becomes

$$e(t) = \sum_{i=-\infty}^{+\infty} a_{i1}e_{i1}(t-iT) + \sum_{i=-\infty}^{+\infty} a_{i2}e_{i2}(t-iT), \quad (8)$$

where $e_{i1}(t)$ and $e_{i2}(t)$ are the signal elements for each modulator as defined by (5). Using (6) we can write

$$e_{i1}(t) = [g(t) \cos (\omega_c t + \omega_c iT)] * h(t) \quad (9)$$

$$e_{i2}(t) = -[g(t) \sin (\omega_c t + \omega_c iT)] * h(t). \quad (10)$$

Introducing (7) and assuming $K = 1$, $e(t)$ becomes

$$e(t) = \sum_{i=-\infty}^{+\infty} \{g(t-iT) \cos \Phi_{k(i)} \cos [\omega_c(t-iT) + \omega_c iT]\} * h(t)$$
$$= \sum_{i=-\infty}^{+\infty} \{g(t-iT) \sin \Phi_{k(i)} \sin [\omega_c(t-iT) + \omega_c iT]\} * h(t) \quad (11)$$

or

$$e(t) = \sum_{i=-\infty}^{+\infty} \{g(t-iT) \cos [\omega_c(t-iT) + \Phi_{k(i)} + \omega_c iT]\} * h(t), \quad (12)$$

which may be written as

$$e(t) = \sum_{i=-\infty}^{+\infty} \sum_{j=1}^{+N} a_{i,j}e_{i,j}(t-iT), \quad (13)$$

where

$$e_{i,j}(t) = [g(t) \cos (\omega_c t + \Phi_j + \omega_c iT)] * h(t) \quad (14)$$

with

$$\left.\begin{array}{ll} a_{i,j} = 0 & \text{for } j \neq k \\ a_{i,k} = 1 & \end{array}\right\}. \quad (15)$$

Equations (13) to (15) mean that a conventional phase-shift-keying transmitter can still be viewed as a signal element generator, as shown in Fig. 3. This generator is designed to send N sets of signal elements according to the N possible phase shifts. At each sampling time, one among the N possible signal elements is selected as a function of the input data. It should be noted that this phase-shift-keying modulator uses the same basic signal elements as the AMSC and VSB transmitters.

The particular case of a frequency-shift-keying transmitter, called a *linear FSK*, can be treated by using the same signal elements as for amplitude and phase modulation.

We shall again restrict ourselves to linear modulation and shall consider a binary frequency-shift keying scheme in which the carrier can take one of two possible angular frequencies ω_M or ω_s at sampling times.

This case has been treated by Sunde[5] who has shown that when the frequency shift is equal to the signalling rate, an FSK transmitter that satisfies Nyquist requirements could be built.

As pointed out by Sunde[5], the frequency-shift-keyed signal can be written as

$$e(t) = \sin \omega_D t \sin (\omega_c t + \theta) + s_1(t) \cos (\omega_c t + \theta) \quad (16)$$

with $\omega_D = \dfrac{\omega_M - \omega_s}{2}$

and $\omega_c = \dfrac{\omega_M + \omega_s}{2}$.

At sampling points $t = iT$, $\omega_D iT = i\pi$. In this case Bennett and Davey[6] have shown that the instantaneous angular frequency deviation $\omega(t)$ from midband becomes

$$\omega(iT) = (-1)^{i+1} \frac{\omega_D}{s_1(iT)}.$$

Let us write $s_1(t)$ as

$$s_1(t) = \sum_{i=-\infty}^{+\infty} (-1)^{i+1} a_i g(t-iT),$$

with $a_i = \pm 1$ in the case of binary transmission.

Assuming $g(t)$ satisfies the requirements for zero intersymbol interference,

$g(iT) = 0$ for $\quad i \neq 0$

and $g(0) = 1$.

Under these conditions, the frequency deviation at sampling times becomes $\omega(iT) = a_i\omega_d$, which satisfies the conditions for FSK transmission without intersymbol interference. The frequency-shift-keyed signal can therefore be written as

$$e(t) = \sin \omega_d t \sin (\omega_c t + \theta) + \left[\sum_{i=-\infty}^{i=+\infty} (-1)^{i+1} a_i g(t - iT)\right] \cos (\omega_c t + \theta). \qquad (17)$$

The first term of (17) corresponds to two tones transmitted respectively at mark and space frequencies. The second term of (17) represents a double-sideband suppressed-carrier modulator. This means that an FSK transmitter as defined by Sunde can be reduced to a generator that produces two tones and signal elements $e_i(t)$ such that

$$e_i(t) = g(t) \cos (\omega_c t + \theta + \omega_c iT).$$

The two corresponding implementations are shown respectively in Figs. 4(a) and 4(b).

Signal elements

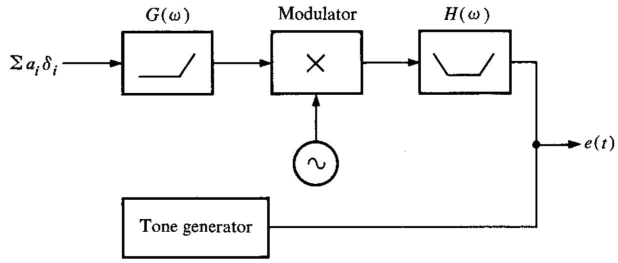
We have pointed out in the preceding section that linear amplitude, phase and frequency modulation schemes can be decomposed into a set of elementary signals $e_i(t)$ called signal elements. More precisely, a modem transmitter can be viewed as a signal element generator. This generator assigns to the data the signal elements required to characterize the various modulation and filtering functions to be performed. In this approach, the filters and modulators used in a conventional modem transmitter are replaced by a memory that stores the set of signal elements. Logic circuitry allows one to fetch and combine the signal elements as a function of input data according to the algorithm that characterizes the modulation (Fig. 5).

This approach is useful in pointing out the basic similarities among various modulation schemes usually viewed as fairly different[12]. The interest of this approach however is not purely theoretical because it suggests a new way of implementing modem transmitters.

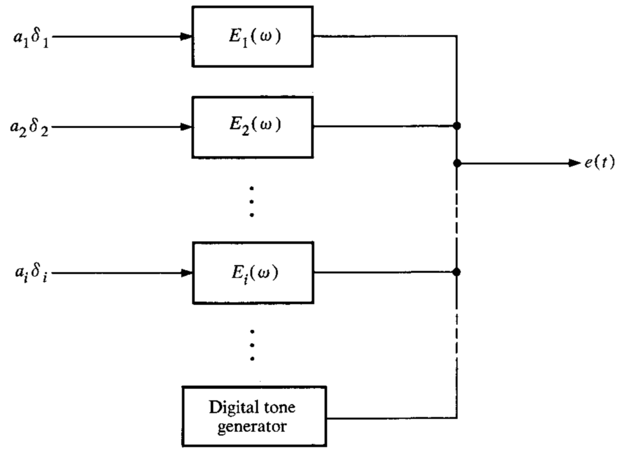
Such a transmitter is realizable and could be entirely implemented in digital circuits, thereby leading to an attractive LSI design provided the number of signal elements stored can be kept reasonably small. This requirement means that the transmitter must be designed so as to need as few signal elements as possible, and the signal elements must be time bounded.

Let us first show now that the total number of required signal elements can be kept very small if certain relations between the signalling rate and the line-signal center frequency are respected.

We have seen in the preceding section that at each sampling time iT, the transmitter must generate one or



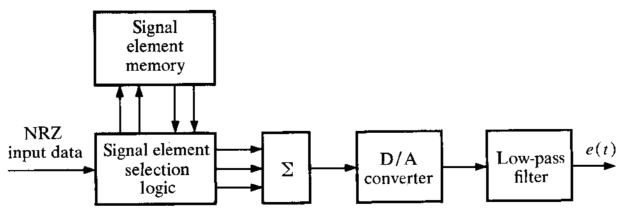
(a)



(b)

Figure 4 FSK transmitter, (a) Sunde's model of FSK transmitter and (b) Equivalent Sunde's model.

Figure 5 General scheme of digital transmitter.



several signal elements $e_i(t)$. In the case of N-level discrete modulation, a maximum of N signal elements must be stored for each sampling time.

In the general case, the signal elements corresponding to a given sampling time are different from those required at other sampling times. However, the number of different signal elements is finite if the following relation exists between the sampling rate $1/T$ and the angular frequency ω_c:

$$\frac{\omega_c T}{2\pi} = \frac{P}{Q} \quad \text{with } P \text{ and } Q \text{ integers.} \qquad (18)$$

In this case, the signal elements corresponding to a given data value are a periodic function of the sampling time iT as shown by the basic definition formula

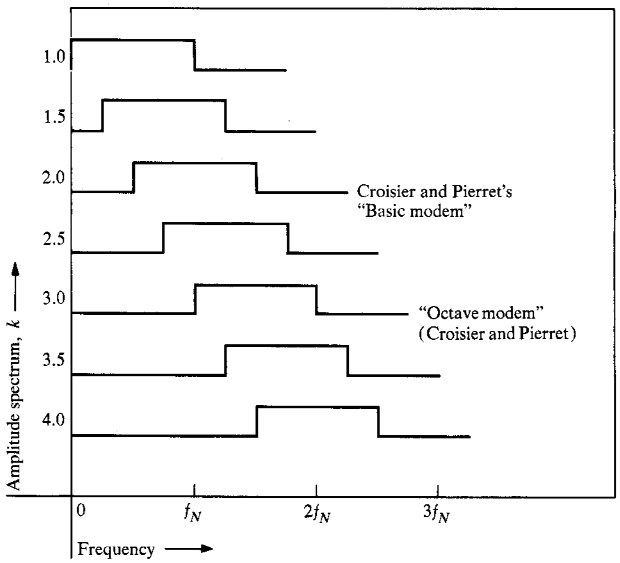
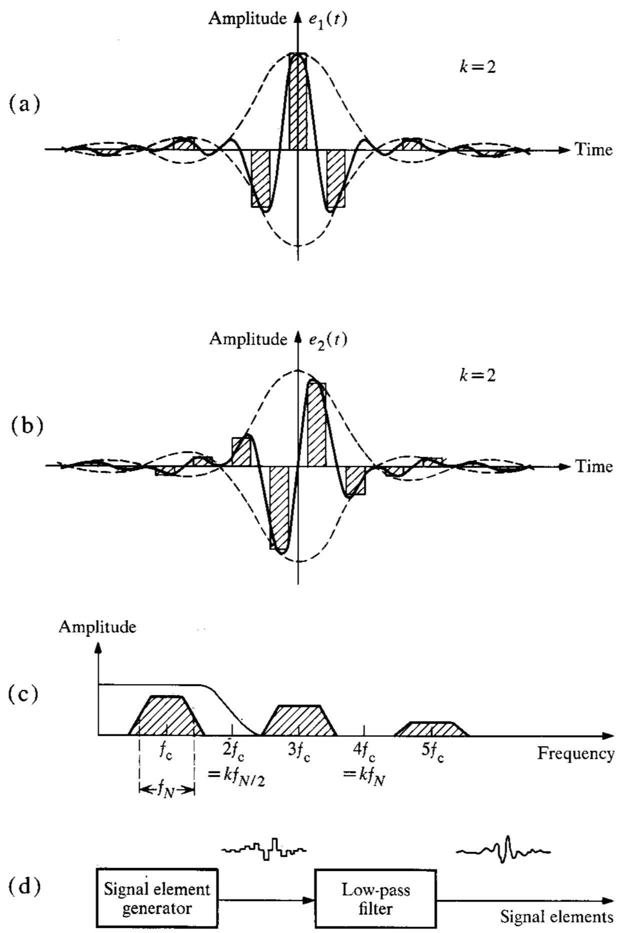


Figure 6 Frequency spectra of generalized Nyquist-type signal elements.

Figure 7 Digital generation of the cosine and sine signal elements ($k = 2$).



$$e_i(t) = [g(t) \cos (\omega_c t + \varphi + \omega_c iT)] * h(t). \quad (5)$$

The second condition for a finite storage capacity is that $e_i(t)$ be time limited. This requirement leads one to approximate the ideal spectrum by using finite signal elements that are derived from the "theoretical" signal elements of infinite duration by using a truncating window[7]. The storage requirements for such signal elements may be in practice extremely limited.

In many practical cases, the filters in the modulation channels can be reduced to a simple low-pass filter and the center frequency f_c of the modulated signal can be related to the line signalling rate f_N by the following equation:

$$2f_c = kf_N,$$

where

$$k = 1, 1.5, 2, \cdots, (1 + M/2)$$

and

$$f_c = \omega_c/2\pi.$$

In those cases we shall show in the next section that the modem transmitter can be built by using only two signal elements $e_1(t)$ and $e_2(t)$, which are orthogonal in the time domain. These signal elements are

$$e_1(t) = \frac{\sin (\pi f_N t)}{\pi f_N t} x(t) \cos 2\pi f_c t \quad (19)$$

and

$$e_2(t) = \frac{\sin (\pi f_N t)}{\pi f_N t} x(t) \sin 2\pi f_c t. \quad (20)$$

These functions will be referred to in the following discussion as "cosine" and "sine" signal elements. With these two signal elements the results of Croisier and Pierret[1] can be extended to half-integer values of k. Cases corresponding to the other values of k that comply with Eq. (18) are feasible but would require more than two signal elements. These cases will not be treated in this paper.

Comparing these two signal elements with the general expression given by

$$e_i(t) = [g(t) \cos (\omega_c t + \varphi + \omega_c iT)] * h(t), \quad (5)$$

we see that $H(\omega) \equiv 1$ and

$$g(t) = \left(\frac{\sin (\pi f_N t)}{\pi f_N t}\right) x(t) = g_0(t)\, x(t).$$

The Nyquist signal element $g_0(t)$[1,11] satisfies the condition of no intersymbol interference $g(iT) = 0$ for $1 \neq 0$ (where $T = 1/f_N$). Its Fourier transform $G_0(\omega)$ is a rectangular spectrum extending from $f = 0$ to $f_N/2$. Multiplication of $g_0(t)$ by a truncating window $x(t)$ of finite duration allows to obtain a realizable signal element $g(t)$ with no intersymbol interference at sampling times.

In the frequency domain, the process described above corresponds to the convolution $G(\omega) = G_0(\omega) * X(\omega)$, which widens the frequency spectrum of $G_0(\omega)$. In practice, we have a smooth roll-off rather than a sharp transition at the edge of this spectrum and some residual components at high frequencies.

The choice of $x(t)$ is essential in obtaining the best compromise between the signal element extensions in time and frequency domains.

The signal elements $e_1(t)$ and $e_2(t)$ can be considered as low-pass filter responses modulated by two orthogonal carriers. This corresponds to a "band-pass" spectrum with symmetrical roll-off. For each permissible k with a given f_N, there is a corresponding frequency spectrum shown schematically in Fig. 6. Elements $e_1(t)$ and $e_2(t)$ are shown respectively in Figs. 7(a) and 7(b) for $k = 2$.

The cases discussed in this paper can be seen as a generalization of those presented by Croisier and Pierret, where only signal elements $e_1(t)$ and integer values of k are used.

The design of digital echo modems will call for generating the signal elements $e_1(t)$ and $e_2(t)$ in the modem transmitter. These signal elements can be stored and generated in a number of ways, either by purely digital means or by pulse amplitude modulation (PAM). In the case of a purely digital implementation, the signal elements can be encoded, for instance, by pulse code modulation (PCM) or delta modulation (ΔM).

We shall present here the PAM approach for generating $e_1(t)$ and $e_2(t)$. By definition, these signal elements have an upper frequency limit well below $2f_c$. They can therefore be entirely defined by amplitude samples taken at the rate $4f_c$. Entering the samples into a low-pass filter with cutoff frequency $2f_c$ will permit exact reproduction of signal elements.

Figure 7 illustrates the process. It shows that the spectrum of the pulse train contains that of the desired signal plus an infinity of side lobes around $3f_c$, $5f_c \cdots$. The low-pass filter removes these side lobes.

This process has been called "digital echo modulation" because the pulse train may be considered as a central "data" pulse with added echoes. This method of signal element generation is particularly efficient because the pulse trains may be obtained with very accurate timings and amplitudes.

The truncating problem has been treated by Croisier and Pierret[1] for the case of the cosine signal element $e_1(t)$ and $k = 2$. Figure 8 shows the out-of-band attenuation obtained in this case as a function of the number of echoes for various conditions of "excess bandwidth" over the ideal Nyquist case. It can be seen that the Nyquist limit can be approached very closely with a limited number of echoes.

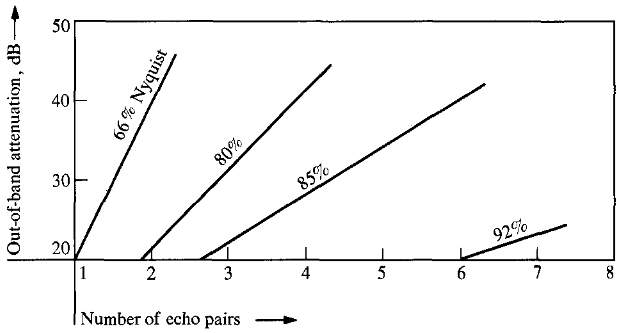


Figure 8 Number of echoes necessary for a given filtering performance of "basic modems".

Digital echo phase modulation

• *Two-phase modulation*

Let us consider again the transmitter shown in Fig. 1. Such a transmitter is a two-phase modulation transmitter provided $H(\omega) = 1$ within the transmission band. In the case of binary transmission, the incoming data signal is sampled to generate a sequence of impulses $a_i\delta(t - iT)$ at signalling rate $1/T$. Each impulse has an amplitude $+1$ or -1 and represents one bit of the data sequence.

Let us assume now that $G(\omega)$ is a low-pass filter with symmetrical roll-off around the Nyquist frequency $f_N/2 = 1/2T$. Its impulse response $g(t)$ can be written as

$$g(t) = x(t)\,\frac{\sin \pi f_N t}{\pi f_N t}.$$

Referring to (5), the signal elements become

$$e_i(t) = x(t)\,\frac{\sin \pi f_N t}{\pi f_N t} \cos\,(\omega_c t + \varphi + \omega_c iT).$$

We have defined in the previous section two kinds of signal elements that correspond to the cases where $2f_c = kf_N$. Assuming that the phase of the carrier is zero, we have

$$e_i(t) = x(t)\,\frac{\sin \pi f_N t}{\pi f_N t} \cos\,(k\pi f_N t + ki\pi).$$

For integer values of k, with $k = p$, we have:

$$e_i(t) = (-1)^{pi} x(t)\,\frac{\sin \pi f_N t}{\pi f_N t} \cos k\pi f_N t \qquad (21)$$

and for fractional values of k, with $k = p + 1/2$,

$$e_i(t) = (-1)^{pi} x(t)\,\frac{\sin \pi f_N t}{\pi f_N t} \cos\,(k\pi f_N t + i\pi/2). \qquad (22)$$

It can now be seen that, not considering the sign, the set

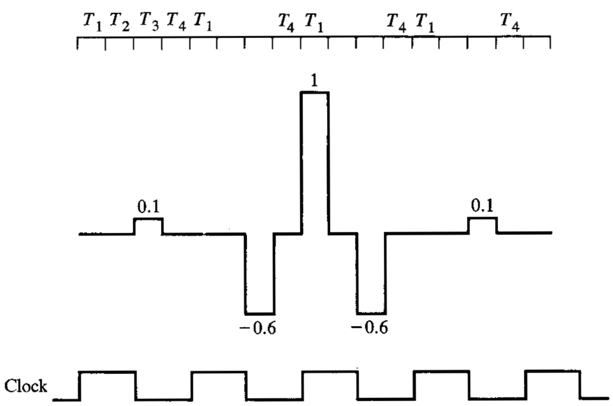
Table 1 Two-phase modulation algorithm with k integer. $k = p$.

Input bit			*Bit number i*			
	−2	−1	0	+1	+2	+3
0	$-e_1$	$-(-1)^p e_1$	$-e_1$	$-(-1)^p e_1$	$-e_1$	$-(-1)^p e_1$
1	e_1	$(-1)^p e_1$	e_1	$(-1)^p e_1$	e_1	$(-1)^p e_1$

Table 2 Two-phase modulation algorithm with k fractional. $k = p + 1/2$.

Input bit			*Bit number i*			
	−2	−1	0	+1	+2	+3
0	e_1	$-(-1)^p e_2$	$-e_1$	$+(-1)^p e_2$	e_1	$-(-1)^p e_2$
1	$-e_1$	$+(-1)^p e_2$	$+e_1$	$-(-1)^p e_2$	$-e_1$	$+(-1)^p e_2$

Figure 9 Digital "basic" signal element ($k = 2$).



of signal elements reduces to only two signal elements $e_1(t)$ and $e_2(t)$ with

$$e_1(t) = x(t)\,\frac{\sin \pi f_N t}{\pi f_N t}\cos k\pi f_N t$$

and (23)

$$e_2(t) = x(t)\,\frac{\sin \pi f_N t}{\pi f_N t}\sin k\pi f_N t.$$

The two-phase modulation scheme of Fig. 1 is therefore equivalent to sending at each clock period T either one of the two signal elements $e_1(t)$ or $e_2(t)$ with a polarity that depends upon the input data and the sampling time. More precisely, the encoding algorithms for integer and fractional values of k are given respectively by Tables 1 and 2.

A practical design of the digital echo two-phase modulator is now straightforward: the filters and modulators in the conventional approach are replaced by a simple pulse-train generator that generates the sampled representation of the two signal elements $e_1(t)$ and $e_2(t)$. In the case of $k = 2$, each signalling period is divided into one bit interval, one echo interval and two idle intervals.

During the bit time T_1 and the echo time T_3, the signal amplitude is given by a particular weighting of the bits contained in a shift register. During idle times T_2 and T_4, the signal amplitude is forced to zero because of timing pulses. At the end of the signalling interval, the data is moved one bit in the shift register and the sequence is resumed. The encoder can easily be designed in such a way that the weighting resistors are never idle. This design allows one to avoid using analog switches and to drive the weighting resistors directly by the logic levels.

Practical implementation of a two-phase modem transmitter for $k = 2$ is shown in Fig. 10. Because k is an integer, the modem transmitter can be built by using a single signal element, as represented in its sampled form in Fig. 9. In the case of the PAM implementation, the precision of the weighting resistors in Fig. 10 depends on the desired out-of-band rejection and the accuracy on the carrier phase. In practice, for 30 to 40 dB out-of-band rejection and up to 8-phase modulation, an accuracy of about 1 percent on the resistors is required.

It should be noted that the low-pass filter connected to the output of the encoder has the sole purpose of eliminating the upper side lobes of the data spectrum. Because the first and second lobes are spaced wide apart, this filter is not critical for the modem operation and can be made very simple and inexpensive. The transmitter of Fig. 10 could, for instance, encode a 1800-bps data signal into a two-phase modulated signal centered at 1800 Hz with a Nyquist bandwidth (bandwidth between 6 dB points) of 1800 Hz. Effective bandwidth, defined as the bandwidth between 40 dB points, is about 2700 Hz (Fig. 8).

If the transmitter had been implemented with three pairs of echoes, the bandwidth between 30 dB points would have been 2250 Hz.

It should be noted that the encoder generates pulses of finite duration iT instead of impulses. This results in a distortion of the data spectrum by a factor of $A(f) = \sin \pi f T_i / \pi f T_i$. This amplitude distortion is moderate and correction is easily made. (For $k = 2$, the "6 dB" points are changed to 5.28 and 7.2 dB relative to midband.) This distortion can be compensated for by an appropriate filter in the receiver. It can also be compensated by an appropriate predistortion of the signal elements $e_1(t)$, which can be practically achieved through a small modification of the encoders of Fig. 10.

- *N-phase modulation*

Digital echo modulation can be extended easily to cover the cases of N-phase modulation. As shown previously,

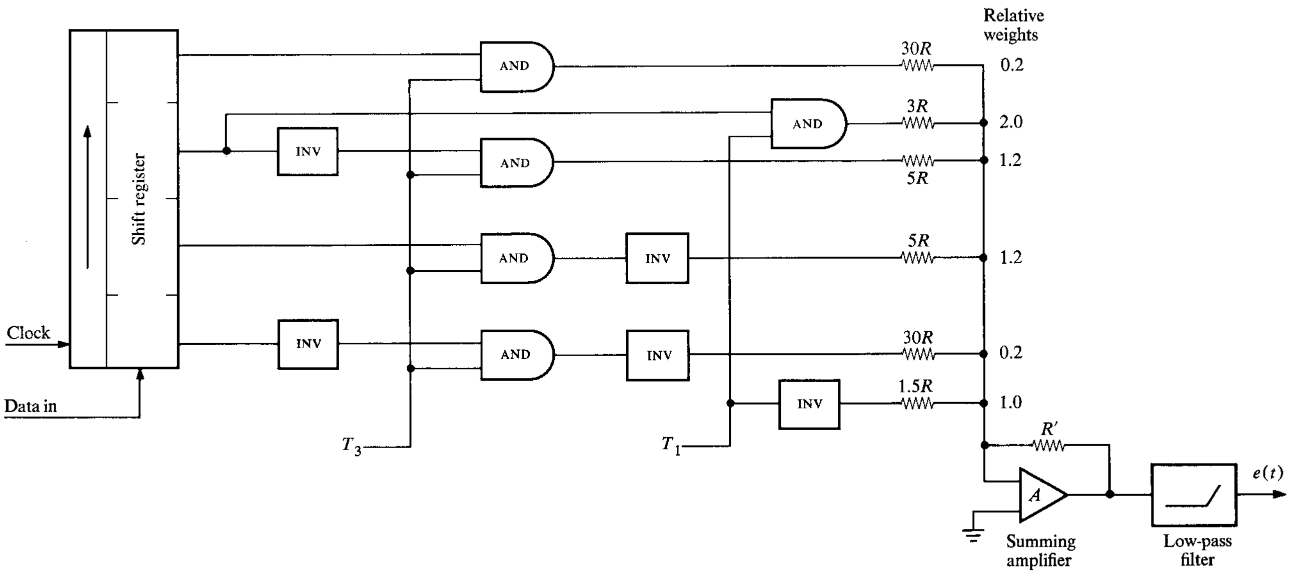


Figure 10 Two-phase transmitter ($k = 2$).

an N-phase modulator can be built by adding the outputs of N elementary two-phase modulators operating with carrier phases φ_1, $\varphi_2 \cdots \varphi_N$, respectively. If Q signal elements are required for one channel, Eq. (18), NQ signal elements will be required for an N-phase modulator. In practice, the number of signal elements that have to be generated may be significantly reduced by properly selecting the ratio between the carrier frequency and the line signalling rate. In the case of equally spaced phases, the number of signal elements reduces to the least common multiple of N and Q. Instead of storing NQ signal elements corresponding to the N different carrier phases, it is often less expensive to store only $2Q$ signal elements corresponding to two orthogonal channels from which the NQ signal elements are derived. In the following, we shall restrict ourselves to presenting this approach in the case of the PAM implementation.

The signal elements can be written as

$$e_i(t) = x(t)\,\frac{\sin \pi f_N t}{\pi f_N t}\cos(\omega_c t + \varphi + \omega_c iT).$$

As pointed out previously, the modem transmitter will store only two sets of orthogonal signal elements $e_{i1}(t)$ and $e_{i2}(t)$, one for each of the two orthogonal channels:

$$e_{i1}(t) = x(t)\,\frac{\sin \pi f_N t}{\pi f_N t}\cos(\omega_c t + \omega_c iT)$$

and (24)

$$e_{i2}(t) = x(t)\,\frac{\sin \pi f_N t}{\pi f_N t}\sin(\omega_c t + \omega_c iT).$$

Under these conditions, a signal element $e_i(t)$ with carrier phase φ_j will be derived from $e_{i1}(t)$ and $e_{i2}(t)$ by performing the operation

$$e_i(t) = e_{i1}(t)\cos\varphi_j - e_{i2}(t)\sin\varphi_j. \tag{25}$$

Let us now apply this approach to four-phase modulation. In this case, the input data sequence can be divided into two interleaved data sequences a_i and b_i with each data sequence corresponding to one of two orthogonal channels. In the case of a modem with no delay between the two orthogonal channels, we will assign one out of four signal elements to each dibit $a_i b_i$ and the line signalling rate $f_N = 1/T$ will be one-half of the data signalling rate. Since the phases of the four signal elements must be $\pi/2$ apart, $e_i(t)$ can be derived from $e_{i1}(t)$ and $e_{i2}(t)$ by encoding the binary data as

$$e_i(t) = \frac{a_i}{\sqrt{2}}\,e_{i1}(t) - \frac{b_i}{\sqrt{2}}\,e_{i2}(t).$$

Introducing the relation $2f_c = kf_N$ and using the definition of e_1 and e_2 in (23) with integer valued p, we get

$$e_{i1}(t) = (-1)^{pi}e_1$$

$$e_{i2}(t) = (-1)^{pi}e_2$$

with $k = p$. and

$$e_{i1}(t) = -\,(-1)^{pi}e_2$$

$$e_{i2}(t) = (-1)^{pi}e_1.$$

with $k = p + 1/2$.

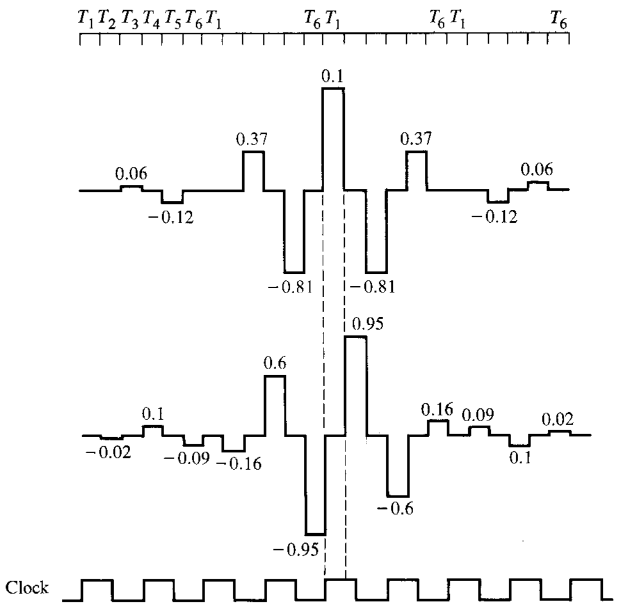


Figure 11 Digital signal elements ($k = 3$).

A practical implementation of a four-phase modulation transmitter can now be derived. Let us assume $2f_c = kf_N$ with $k = 3$. Such a case corresponds for instance to that of a transmitter which operates at 2400 bps with a carrier centered at 1800 Hz and a Nyquist bandwidth (6-dB points) of 1200 Hz (one of the two standards recommended by the CCITT for 2400 bps transmission). Under these conditions the orthogonal signal elements become

$$e_1(t) = (-1)^i x(t) \frac{\sin \pi f_N t}{\pi f_N t} \cos 3\pi f_N t$$
$$e_2(t) = (-1)^i x(t) \frac{\sin \pi f_N t}{\pi f_N t} \sin 3\pi f_N t. \qquad (26)$$

It is apparent from (26) that only two signal elements are required in the modem transmitter if the polarity of every other dibit is reversed in the encoder. The cosine and sine signal elements $e_1(t)$ and $e_2(t)$ are shown in their sampled form on Fig. 11. It should be noted that the echoes of the two signal elements are interleaved, so that the design of the encoder is relatively simple. The practical implementation is shown in Fig. 12. It is very similar to that of a two-phase modem and therefore needs not be discussed further.

The flexibility of the digital echo modulation approach can be further illustrated by examining the case of eight-phase modulation. In this case, the input data sequence must be divided into three interleaved data sequences a_i, b_i, c_i. At each sampling time iT, one out of eight signal elements $\pi/4$ apart must be generated in accordance with the particular combination of a_i, b_i, c_i and the line signalling rate f_N will be one-third of the data signalling rate.

As in the case of four-phase modulation, we shall present here only the approach based upon the use of two orthogonal sets of signal elements $e_{i1}(t)$ and $e_{i2}(t)$.

The eight signal elements can be generated with the two orthogonal signal elements $e_{i1}(t)$ and $e_{i2}(t)$ provided each of these signal elements can take two different amplitudes. In the case of binary transmission, with a_i, b_i and c_i being ± 1, the reader may easily verify that the signal elements can be derived from $e_{i1}(t)$ and $e_{i2}(t)$ by encoding the binary data as:

$$e_i(t) = [b_i e_{i1}(t) + c_i e_{i2}(t)]\left(\cos\frac{\pi}{8} + \sin\frac{\pi}{8}\right) + [a_i b_i e_{i1}(t) - a_i c_i e_{i2}(t)]\left(\cos\frac{\pi}{8} + \sin\frac{\pi}{8}\right). \qquad (27)$$

Let us now consider an eight-phase modem transmitter with $2f_c = kf_N$ and $k = 2$. This would, for instance, correspond to a modem operating at 4800 bps with a carrier centered at 1600 Hz and a Nyquist bandwidth (6 dB points) of 1600 Hz. Under these conditions, the signal elements become

$$e_{i1}(t) = x(t) \frac{\sin \pi f_N t}{\pi f_N t} \cos 2\pi f_N t$$
$$e_{i2}(t) = x(t) \frac{\sin \pi f_N t}{\pi f_N t} \sin 2\pi f_N t, \qquad (28)$$

which shows that only two signal elements are required in the transmitter. These signal elements are represented in Figs. 7(a) and 7(b).

The practical implementation of the transmitter is shown in Fig. 13. The input data stream is divided by logic circuits into two sequences according to (27). These two sequences are encoded separately by sine and cosine encoders similar to those described for two-phase and four-phase modulation. The eight-phase signal is obtained by summing encoded sequences b_i, c_i and $a_i b_i$, $-a_i c_i$ weighted respectively by $\cos(\pi/8) + \sin(\pi/8) = 1.307$ and $\cos(\pi/8) - \sin(\pi/8) = 0.541$.

Digital echo vestigial-sideband modulation

As already mentioned in the section on linear modulation, vestigial-sideband modulation is a case of amplitude modulation with a particular shaping of the post modulation filter $H(\omega)$ (Fig. 1). In the case of a vestigial-sideband system with symmetrical spectrum, the filtering process caused by $H(\omega)$ is shown on Fig. 14: the baseband signal with cut-off frequency f_N is double-sideband modulated around a carrier frequency f_0. The upper or lower side lobes are removed by the post-modulation filter $H(\omega)$. The filtering results into a symmetrical vestigial-sideband spectrum with center frequency $f_c = f_0 \pm f_N/2$.

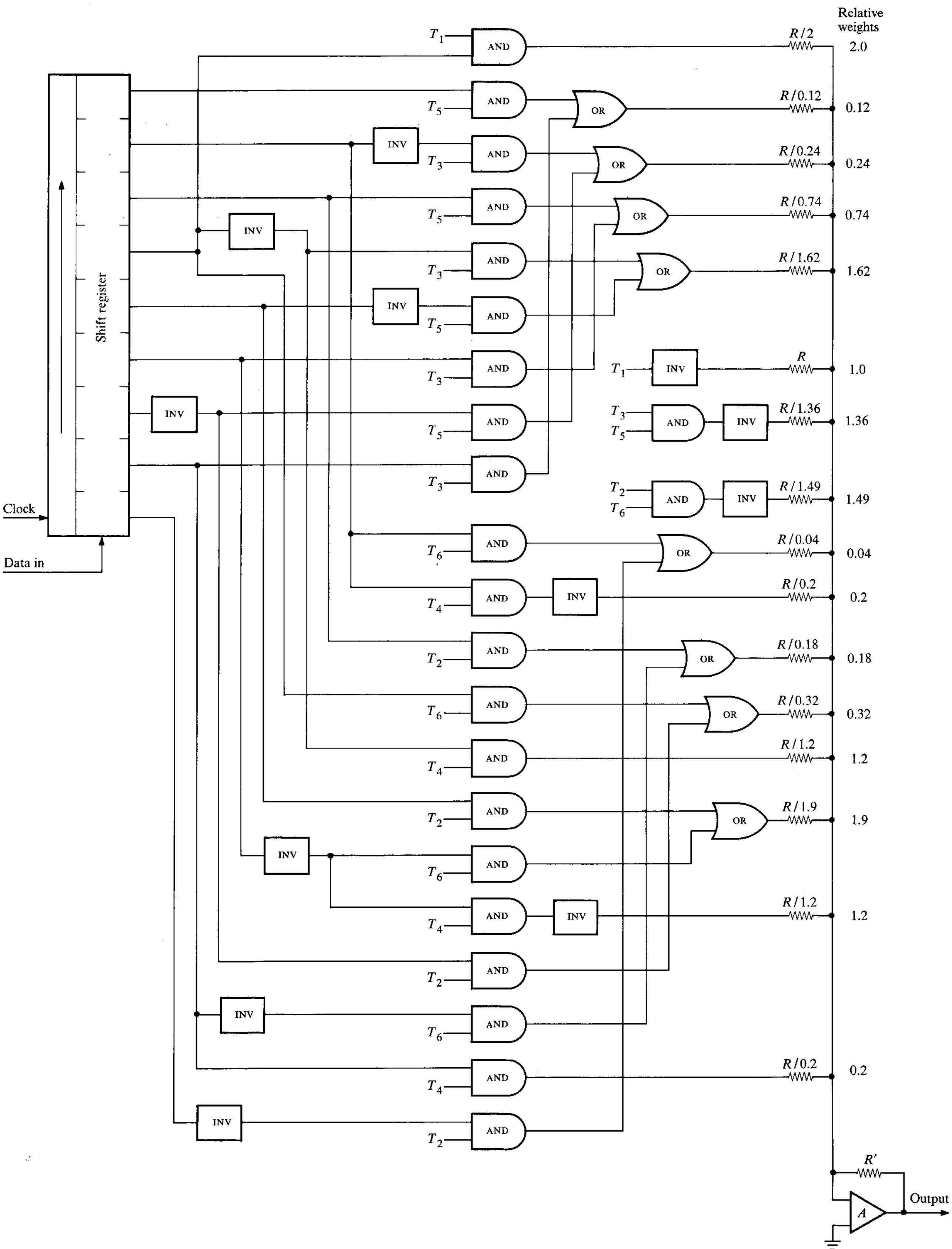


Figure 12 Four-phase transmitter ($k = 3$).

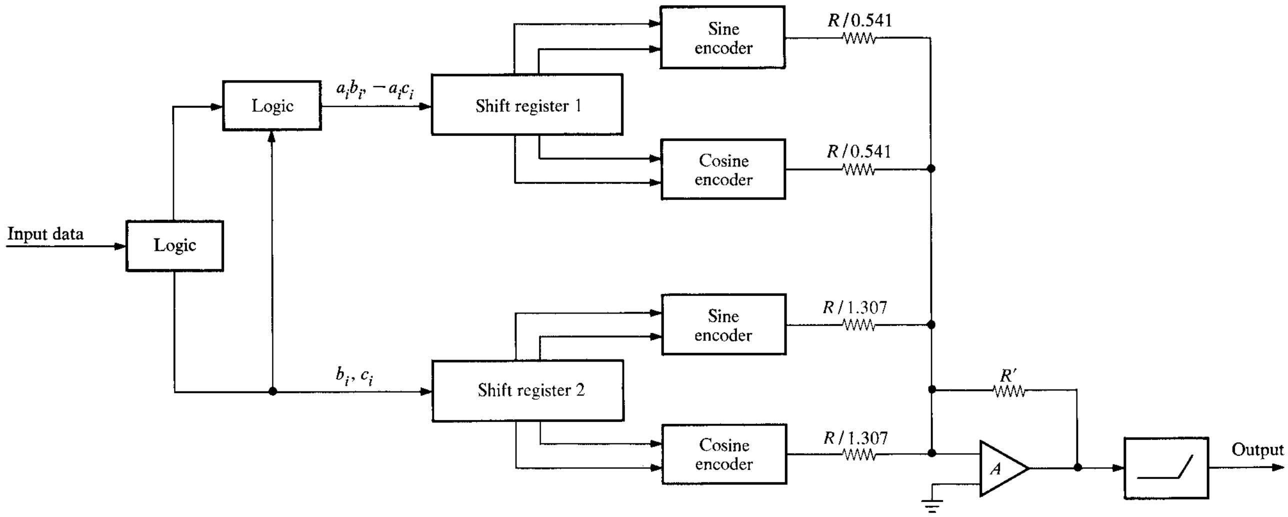
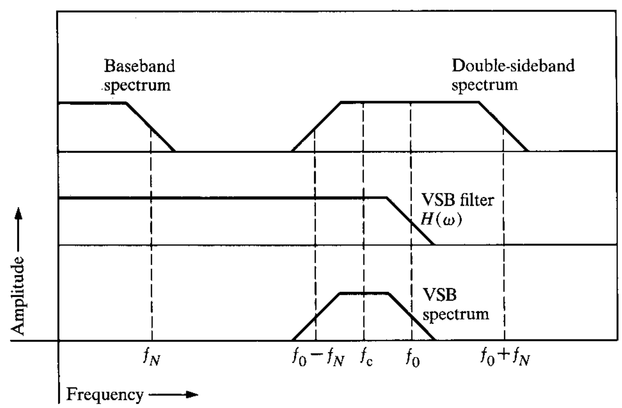


Figure 13 Digital eight-phase transmitter.

Figure 14 Double sideband and VSB spectrums.



Let us select the lower sideband $f_c = f_0 - f_N/2$. Under these conditions, the double-sideband signal element can be written from (5):

$$e_{i0}(t) = x(t)\,\frac{\sin 2\pi f_N t}{2\pi f_N t}\cos\,(2\pi f_0 t + \varphi),$$

where the first two factors represent the low-pass response $g(t)$ and $\varphi = \varphi_0 + 2i\pi f_0 T$, φ_0 being the carrier phase and $1/T = 2f_N$ the signalling rate.

Since $G(\omega)$ and $H(\omega)$ are both real, and with the assumption that both have the same roll-off factor $x(t)$, the VSB filtered signal element is given by

$$e_i(t) = e_{i0}(t) * h(t) = x(t)\,\frac{\sin \pi f_N t}{\pi f_N t}\cos\,(2\pi f_c t + \varphi) \quad (29)$$

with $\varphi = \varphi_0 = 2i\pi T f_0$.

Table 3 VSB signal element assignment with k odd $= 2p + 1$.

Bit values	$t^a = -2$, $i = -2$	-1, -1	0, 0	$+1$, $+1$	$+2$, $+2$	$+3$, $+3$
0	$-e_1$	$-(-1)^{p+1}e_1$	$-e_1$	$-(-1)^{p+1}e_1$	$-e_1$	$-(-1)^{p+1}e_1$
1	e_1	$(-1)^{p+1}e_1$	e_1	$(-1)^{p+1}e_1$	e_1	$(-1)^{p+1}e_1$

[a]The time unit is the reciprocal of the binary data rate.

Table 4 VSB signal element assignment with k even $= 2p$.

Bit values	$t^a = -2$, $i = -2$	-1, -1	0, 0	$+1$, $+1$	$+2$, $+2$	$+3$, $+3$
0	$+e_1$	$(-1)^p e_2$	$-e_1$	$(-1)^p e_2$	$+e_1$	$-(-1)^p e_2$
1	$-e_1$	$(-1)^p e_2$	$+e_1$	$-(-1)^p e_2$	$-e_1$	$(-1)^p e_2$

[a]The time unit is the reciprocal of the binary data rate.

Table 5 Four-phase signal element assignment with any integer value of k.

Bit values	$t^a = -2$, $i = -1$	0, 0	$+2$, $+1$
0 0	$(-1)^k(-e_1 + e_2)$	$-e_1 + e_2$	$(-1)^k(-e_1 + e_2)$
0 1	$(-1)^k(-e_1 - e_2)$	$-e_1 - e_2$	$(-1)^k(-e_1 + e_2)$
1 0	$(-1)^k(+e_1 + e_2)$	$+e_1 + e_2$	$(-1)^k(+e_1 + e_2)$
1 1	$(-1)^k(+e_1 - e_2)$	$+e_1 - e_2$	$(-1)^k(+e_1 - e_2)$

[a]The time unit is the reciprocal of the binary data rate.

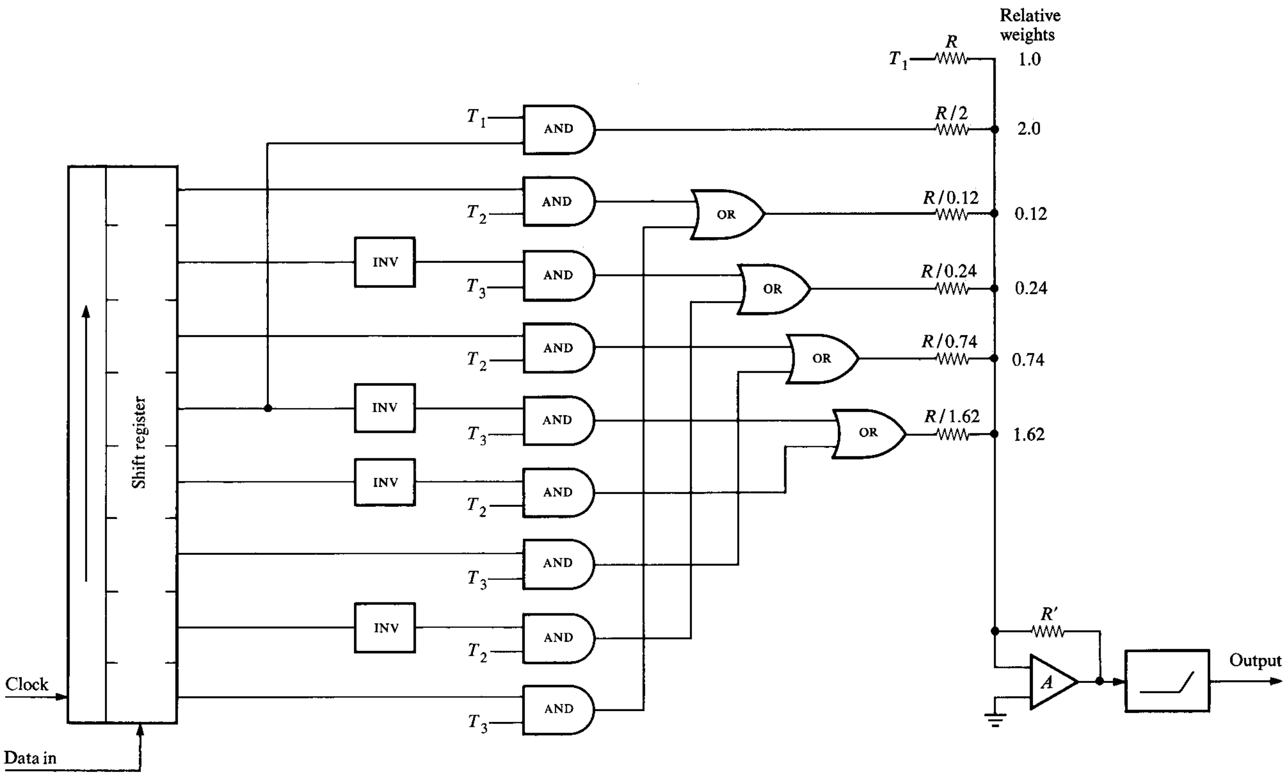


Figure 15 VSB transmitter.

Introducing $f_0 = f_c + f_N/2$, the VSB signal element can be rewritten as:

$$e_i(t) = x(t)\,\frac{\sin 2\pi f_N t}{2\pi f_N t}\cos(2\pi f_0 t + \varphi) + x(t)\,\frac{\sin 2\pi f_N t}{\pi f_N t}\sin(2\pi f_0 t + \varphi).$$

It can then easily be verified that synchronous demodulation (multiplication and removal of the upper frequency components) of $e_i(t - iT)$ by the carrier $\cos(2\pi f_0 t + \varphi_0)$ yields the expected baseband Nyquist type signal element

$$g(t - iT) = x(t - iT)\,\frac{\sin 2\pi f_N (t - iT)}{2\pi f_N (t - iT)}.$$

We now introduce $2f_c = kf_N$ and the signal elements (29) become

$$e_i(t) = x(t)\,\frac{\sin \pi f_N t}{\pi f_N t}\cos\left[k\pi f_N t + \varphi_0 + i(k+1)\frac{\pi}{2}\right]. \quad (30)$$

If we want to have only one or two different signal elements (one sine and one cosine), we must choose an integer value of k, and assuming the phase of the carrier is zero, we have

$$e_i(t) = (-1)^{i(p+1)} x(t)\,\frac{\sin \pi f_N t}{\pi f_N t}\cos k\pi f_N t$$

for odd values of k, ($k = 2p + 1$), and

$$e_i(t) = (-1)^{pi} x(t)\,\frac{\sin \pi f_N t}{\pi f_N t}\cos(k\pi f_N t + i\pi/2)$$

for even values of k, ($k = 2p$).

Tables 3 and 4 give the encoding algorithms for these two cases, while Table 5 gives, for the sake of comparison, the algorithm for a four-phase modulation of equal data rate and frequency spectrum. The signal elements $e_1(t)$ and $e_2(t)$ are the same in all three cases; they are given by (23).

As already pointed out by Croisier and Pierret[1], only one single signal element is required when k is odd.

Practical implementation of a vestigial-sideband transmitter is very similar to that of N-phase modulation transmitters since in some cases the same signal elements are used. A VSB transmitter for $k = 3$ is shown in Fig. 15. This case would correspond for instance to that of a transmitter operating at 2400 bps with a spectrum centered at 1800 Hz and a Nyquist bandwidth (6 dB points) of 1200 Hz. The corresponding signal element is shown on Fig. 11. It can be seen that this VSB transmitter bears close resemblance to the four-phase transmitter of Fig. 12 which operates at the same speed.

This result was to be expected as the various modulation schemes in digital echo modulation use the same signal elements and differ only in the way these signal elements are assigned to the data.

This unique feature of the digital echo modulation could make it possible to build economically "universal" modem transmitters thanks to the elimination of all sophisticated analog filters in the transmitter.

Digital echo frequency modulation

It has been shown in the section on linear modulation that the linear frequency-shift-keyed modem as defined by Sunde could be reduced to a two-phase modulator and a two-tone generator. In the case of a low-pass filter with symmetrical roll-off around the Nyquist frequency, the frequency-shift-keyed line signal becomes

$$e(t) = \sin \pi f_N t \sin (\omega_0 t + \varphi) + \left[\sum_{i=-\infty}^{+\infty} a_i x(t) \frac{\sin \pi f_N (t - iT)}{\pi f_N (t - iT)} \right] \cos (\omega_0 t + \varphi). \quad (31)$$

The second term, which corresponds to a two-phase modulation, can be generated as described in the section entitled "two-phase modulation." Under these conditions, if $2f_c = kf_N$, and $k = 2$, the transmitter of Fig. 10 could be considered as an FSK transmitter operating at 1800 bps with a center frequency of 1800 Hz and a Nyquist bandwidth(6 dB points) of 1800 Hz provided two tones are generated at frequencies of 900 and 2700 Hz with the proper amplitude. These two tones could be generated conventionally. However it is often more practical to generate digitally PAM pulses which, when fed into the transmitter low-pass filter, permit the reconstitution of the two tones. In this case, and if the carrier phase φ is zero, the transmitter must store a sampled representation of

$$\sin \pi f_N t \sin k \frac{\pi}{2} f_N t.$$

Multilevel transmission

Digital echo modulation can easily be extended to cover the cases of multilevel transmission. This can be done either by storing in the transmitter the signal elements corresponding to the various levels or by properly weighting normalized signal elements during the signal-element generation. In the case of the PAM implementation the data is distributed to n parallel pulse train generators. The outputs of the n generators are weighted by the factors 1, 2, 4, $\cdots$ and summed. The result is a signal element with 2^n levels, carrying n bits of information.

A particularly interesting case is that of correlative encoding[8–10], such as duobinary transmission. Such a scheme can be implemented by feeding a multilevel digital echo modulator with the output of a correlative encoder. It is, however, more efficient to combine the correlative encoder with the digital echo modulator. This corresponds to storing in the transmitter new signal elements. In the case of partial response, the new signal elements are:

$$e_i(t) = [(g(t) - g(t - 2T)) \cos (\omega_c t + \varphi + \omega_c iT)] * h(t).$$

Because of the additional filtering performed by the correlative encoding, the signal elements can be represented by a small number of samples, thereby leading to a very simple hardware implementation.

Practical results

Digital echo modulation has already been used for implementing a number of modems (the IBM 3978 modem family). Some practical results obtained with this technique have been discussed by Croisier and Pierret[1] in the case of "cosine" signal elements. Extensive experimentation and simulation have shown that, with a very limited amount of digital hardware, digital echo modulation permitted the generation of data transmission signals with good accuracy and stability.

Conclusions

Digital echo modulation has been extended to cover most of the practical cases of data transmission. The various linear modulation schemes have been reviewed and the corresponding signal elements have been derived. It has been shown how practical modem transmitters can be built by using time domain representations of those signal elements.

The initial digital echo modulation has been extended to cover all keyed linear-modulation schemes where the ratio of carrier frequency to signalling rate is a simple fraction.

Since this last limitation is not very severe, many practical modem transmitters can now be built by using the digital echo modulation technique.

Digital echo modem transmitters can be implemented by storing the signal elements as PAM, PCM or delta-modulation samples. With PCM- or delta-encoded signal elements, modem transmitters can be built using large scale integration. This makes it possible to design low cost multispeed, multifunction modem transmitters with performance equal to or better than that of conventional modems as far as bandwidth efficiency and residual distortion are concerned.

Acknowledgments

We wish to thank J-M. Pierret, who first proposed the initial scheme for digital echo modulation, and A. Croisier, A. Desblache and the members of the IBM La Gaude Laboratory modem technology group for their contributions.

References

1. A. Croisier and J-M. Pierret, "The Digital Echo Modulation" *IEEE Trans. Communication Technology.* **COM-18** No. 4, 367–376 (1970).
2. M. Humbert, "Four-phase Digital Echo Modulation Technique," *CCITT Communication*, Sp A number. 150-E (October 30, 1967).
3. A. Croisier and J-M. Pierret, "High Efficiency Data Transmission through Digital Echo Modulation," Conference Record, 1969 IEEE International Conference on Communications, pp. 29, 9–29, 14, June 9–11, 1969, Boulder, Colorado.
4. J. W. Smith, "A Unified View of Synchronous Data Transmission System Design," *Bell System Tech. J.* **47** No. 3, 273–300 (1968).
5. E. D. Sunde, "Ideal Binary Pulse Transmission by AM and FM," *Bell System Tech. J.* **38,** 1357–1426 (1959).
6. W. R. Bennett and J. R. Davey, "Data Transmission," McGraw-Hill Book Company, 1965, Ch. 5, pp. 70–77.
7. F. F. Kuo and J. F. Kaiser, "Systems Analysis by Digital Computer," Wiley, 1966, Ch. 7, pp. 228–243.
8. A. Lender, "The Duobinary Technique for High Speed Data Transmission," *IEEE Trans. On Communications and Electronics.* **82,** 214–218 (1963).
9. E. Gorog and C. M. Melas, "Digital Data Transmission System with Optimum Bandwidth Utilization," Proc. IEEE Tenth Nat. Commun. Symposium, pp. 330–341, October 1964, Utica, New-York.
10. E. R. Kretzmer, "Generalization of a Technique for Binary Data Communication," *IEEE Trans. on Communication Technology.* **COM-14,** pp 67–68, Feb. 1966.
11. H. Nyquist, "Certain Topics in Telegraph Transmission Theory," *AIEE Trans.*, **47** 617–644 (1928).
12. J. R. Davey, "Digital data signal space diagrams," *Bell System Tech. J.* **43** 2973–2984 (1964).

Received February 3, 1971

The authors are located at the IBM Centre d'Etudes et Recherches, 06 La Gaude, Alpes Maritimes, France.