Linear Programming–
Electric-Arc Furnace Steelmaking

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Data Processing Application

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INTRODUCTION

The introduction of linear programming (LP) has produced remarkable and diverse benefits in a number of industries. Recent applications of LP techniques by metal producers -- notably to control costs and quality in alloy blending -- suggest a variety of new applications. The purpose of this manual is to demonstrate the application of LP in the production of steel in electric-arc furnaces -- a process which, because it involves complex blending and quality control, is particularly responsive to LP techniques. The immediate and more obvious LP results enable the steel producer to:

1. Minimize the cost of both initial and supplemental furnace charges
2. Minimize and possibly eliminate off-compositions
3. Maintain accurate scrap inventory records
4. Purchase and sell most economically
5. Evaluate plant operating changes
6. Interpret historical charge data in terms of operating relationships to develop more efficient operation

Contrary to popular belief, little mathematical knowledge or skill is required to formulate an LP model. Nor does the operation of the computer and the analysis of computer results require any advanced technical skill. Linear programming requires nothing more than the expression of all the elements in the process -- plant operating practices, charge materials, specifications, etc. -- in the form of simple linear equations. A general explanation of basic linear programming appears in the IBM data processing application manual An Introduction to Linear Programming (E20-8171), which should be read in conjunction with this manual.

To demonstrate the methods and advantages of LP in steel production, we shall present a typical production problem as a basis for the development of an LP model which can be solved by the IBM 1620/1311 Linear Programming System. With minor modifications the model can be run on any of IBM's LP systems.

The basic process consists of the following phases:

1. An initial charge of scrap and alloying material is melted in a furnace by electrical energy supplied through carbon or graphite electrodes.

2. Oxygen, supplied through lances, is blown through the molten bath to burn off impurities. As a consequence, a slag forms which contains, in addition to the oxidized impurities, a significant quantity of iron oxide and oxides of expensive alloying metals (such as chromium).

3. In alloy steelmaking much of the metallic oxide in the slag is reduced by the addition of silicon -- for example, in the form of high-silicon, low-impurity chrome silicides.

4. In medium-low-, and low-carbon steelmaking, the initial slag is raked and poured off, and a second slag is either formed or placed on the bath. This slag serves to eliminate remaining contaminants and protect the metal bath from contamination by reaction with the furnace atmosphere.

5. When the metal bath is brought to end specifications and temperature, the steel is poured out into ingots, molds, etc.

Because the electric furnace allows close control of both composition and temperature, it is in widespread use in medium-low-carbon steel production and has become the primary producer in stainless and alloy steel production.

The fundamental problem is to produce a specified steel at the lowest possible cost. In order to achieve least-cost production, the producer must consider a complex variety of factors which, immediately or ultimately, contribute to the costs of production. The more obvious variable factors include price, grade, and availability of initial charge scraps, price and quantity of required additives, and heat time (that is, price and quantity of required energy). Less obvious factors that markedly affect costs include refractory erosion, oxygen rate and lance position, and quality control. The least tangible, and possibly the most important, factor that contributes to the formulation of consistently accurate bids (especially for steel orders) is an accurate log of heat histories -- to serve as the basis for predicting operating efficiency and revising operating practices.

PROBLEM ECONOMICS

In most cases a wide variety of scraps, differing in composition, physical condition, and price, are available for the initial charge in the electric-arc furnace. Further, the available quantity of each scrap, as well as its price, fluctuates. The primary economic problem, then, is to determine the composition of an initial charge that will produce the specified steel at least cost. The nature of the initial charge will affect the cost of furnace operation (since different scraps will require different optimum furnace temperature and blow time). Further, the nature of the initial charge, in conjunction with the furnace operation during the melt and decarburization of the charge, affects the cost in terms of relatively expensive reducing and finishing additives.

The crucial interrelation among the several phases of steel production makes it exceedingly difficult to determine the least-cost initial charge, optimum furnace operation, and least-cost supplemental charge. This difficulty is vastly compounded by common fluctuations in the availability of specific scraps, since the alteration of any one component in the initial charge will alter all the relationships required for least-cost production. Heretofore, steel producers employing manual calculation to determine initial furnace charge often used expensive scrap that came close to matching the alloy specification requirements together with expensive pure metals and additives. An increasing number of steelmakers, however, are profiting from the application of linear programming, which enables the producer to examine all possible combinations and quickly determine the most economical furnace charge. Further, by serving to "force" overstocked scrap types in least-cost charges, LP can contribute to the achievement and maintenance of ideal inventory procedures.

SINGLE-FURNACE MODEL FORMULATION

A linear programming model for steel production is a mathematical representation, in the form of linear equations, of all known and estimated factors relevant to the production of the specified steel. To demonstrate the method for formulating such a model, we postulate a specific problem and relatively ideal conditions -- the production of 20,000 lbs. of low-carbon stainless steel from four initially available charge materials. In actual practice a larger number of materials are available to the furnace operator; regardless of their variety and composition (the factors that complicate manual calculation), they can easily be included in the LP model, increasing the model's size but not its complexity.

Input Data Requirements

The following basic data is required to formulate the LP model:

1. Specifications of alloy to be produced

2. Pounds of alloy required

3. Composition analysis of all raw materials

4. Per-pound cost of all raw materials

5. Inventory levels of all raw materials (scrap and reducing and finishing additives)

6. Special raw-material restrictions (for example, ingot weights)

7. Current operating practices (for example, basicity levels)

8. Furnace characteristics (for example, maximum permissible temperature)

Most of this information is available from purchasing, cost accounting, inventory accounting, or other sources and is probably used in existing systems for computing furnace charges. Where exact information cannot be readily obtained, estimates should be made, since it is an easy matter to change the input data and re-solve the problem once an optimal solution has been obtained. Indeed, the rapid calculation of the effect of changes in the input is a prime advantage of the LP approach. Moreover, the accumulation of a log of heat histories will result in increasingly precise estimates.

Example Problem

We wish to produce 20,000 lbs. of steel with the specifications shown in Figure 1. The four initial charge materials available are steel scrap, 430 grade steel scrap, high-carbon ferrochrome, and low-carbon ferrochrome. They may be priced and analyzed as shown in Figure 2.

Since market variations frequently influence the choice of initial charge materials, our model must be responsive to the fifth element in the list of input data requirements: inventory levels. Hence we will assume that the availability of 430 grade scrap and high- and low-carbon ferrochrome is limited to 2000 lbs. each. We can invoke similar limitations, depending on market conditions, to vary the quantities of any of the charge elements at any phase of the process.

We will not postulate here any special raw-material restrictions, though forcing the use of ingot weights may be an important production problem. (This aspect of the problem will be discussed in the section on output basis variables.)

Chromium minimum	16.0%
Silicon maximum	1.0%
Manganese maximum	1.0%
Carbon maximum	0.05%

Figure 1. Problem specifications

	Steel Scrap	430 Grade Scrap	High-Carbon Ferrochrome	Low-Carbon Ferrochrome
Cost per lb.	$0.02	$0.075	$0.27	$0.40
Chromium	0	16.0%	55.6%	65.0%
Manganese	1.0%	1.0%	0	0
Silicon	0.2%	0.95%	2.0%	1.0%
Carbon	0.6%	0.12%	8.0%	0.09%
Iron	98.2%	81.43%	34.4%	33.91%

Figure 2. Analysis of materials

The complex thermochemistry and tight controls required in the production of the specified steel introduce problems best handled by an adaptive rather than a static model, especially when the scrap analysis is uncertain.

1. The composition of the initial charge and the amount and variety of reducing and finishing additives are established by a linear program, based on final metal specifications, cost and composition of available charge materials, and plant capacity.

2. Based on carbometer analysis and spectograph analysis of the melt, a new linear program is formulated to determine accurately the quantities of reducing and finishing additives required to achieve the specified steel at least-cost.

For our purposes we need develop only the first of these programs. In practice, the second model can be developed quite easily from the first.

The schematic of the LP model matrix (Figure 3) graphically illustrates the steelmaking process. The detailed model matrix is shown in Figure 4.

Every source of the various elements which make up the final alloy appears at the head of a matrix column, which is called a problem activity. Cost, maximum and minimum specifications, and symbolic designations for the processes which alter the element quantities provided by the sources appear at the ends of the matrix rows, called problem constraints. Consider the first four columns of the blending section of the matrix in

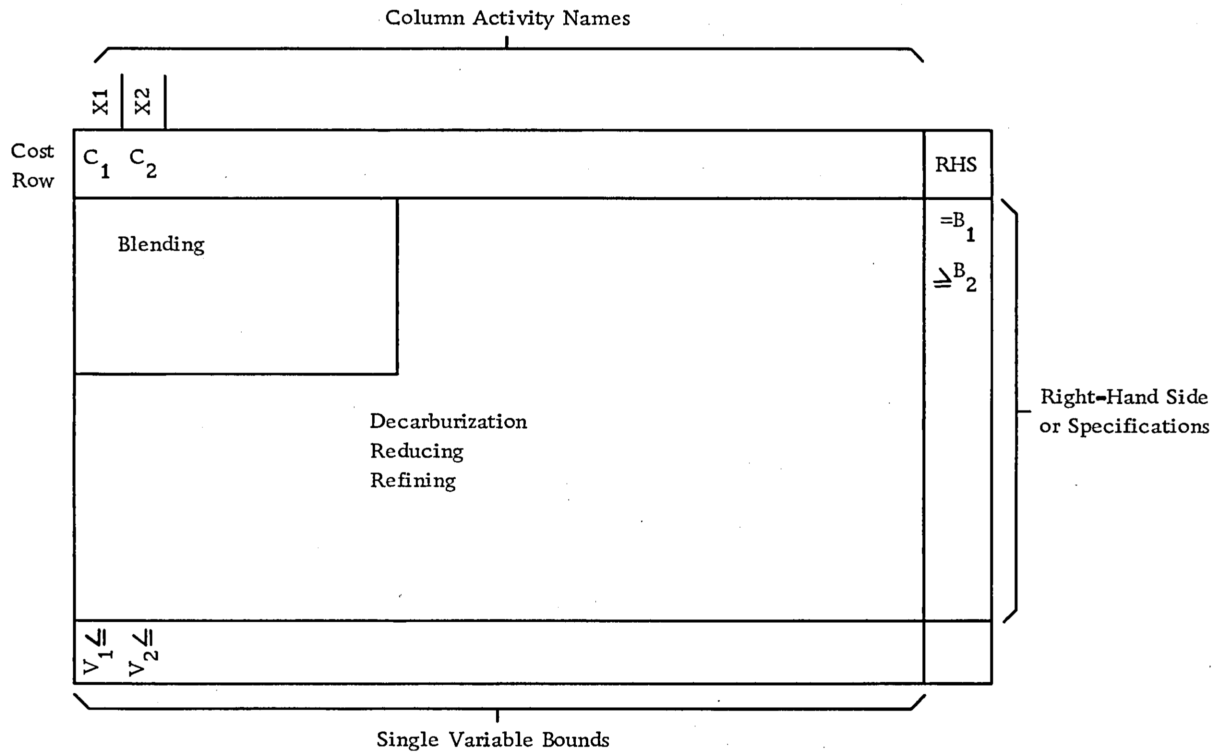

Figure 3. Schematic of LP matrix model -- single-furnace

Figure 4. We have, in effect, transferred the data given in Figure 2 to our matrix. Each of the four sources for the initial charge heads a column and is assigned a symbolic name (mnemonic). Similarly, each of the rows is symbolically named. Figure 5 defines these column and row mnemonics for the blending section of the matrix.

Row Name \ Column Names	No.	STSCP	SP430	HCFCR	LCFCR	CRIT	MNIT	SIIT	CEIT	FEIT	TICW	ISCR	CRSI	RS430	RCFCR	ISFE	LIME	FCFCR	SIS	
Value	1	.02	.075	.27	.40	0	0	0	0	0	0	0	.27	.075	.40	0	.01	.40	0	Cost
CR	2	0	.16	.556	.65	-1														=0
MN	3	.01	.01	0	0		-1													=0
SI	4	.002	.0095	.02	.01			-1												=0
C	5	.006	.0012	.08	.0009				-1											=0
FE	6	.982	.8143	.334	.3391					-1										=0
TOTCHG	7					1	1	1	1	1	-1									=0
CRSLAG	8					1	1				-.074	-1								=0
TOTCRS	9										.074	.95	.39	.17	.65			.65	1	=3400
MN/CR	10						.98							.01					1	=200
FESLAG	11										.075	-1				-1				=0
ENDFE	12									1			.18	.8143	.3391	-.05		.3391		≥16,200
CSPEC	13							-5			5	-.25		12	9	-.25		9		≤100,000
BASE	14							2.14					2.7606				-2			=0
SISPEC	15											-.395	.43	.0095	.01	-.238		.01		≤200
TOTAL	16							-1	-1		1	-.05	.57	1	1	-.05		1		≥20,000
TOTRS4	17		1											1						≤2000
TOTRCF	18				1										1			1		≤2000
				≥ 2000.																

Figure 4. LP matrix model -- single-furnace problem

Item Name	Column Mnemonic
Steel scrap	STSCP
430 grade scrap	SP430
High-carbon ferrochrome	HCFCR
Low-carbon ferrochrome	LCFCR
Chromium initially charged	CRIT
Manganese initially charged	MNIT
Silicon initially charged	SIIT
Carbon initially charged	CEIT
Iron initially charged	FEIT
Total initial charge weight	TICW

Element or Control Name	Row Mnemonic
Price per pound (of initial charge materials)	VALUE
Chromium	CR
Manganese	MN
Silicon	SI
Carbon	C
Iron	FE
Total elements charged	TOTCHG

Figure 5. Mnemonic table -- blending section of matrix

COST CONSTRAINT (Objective Function)

The first problem constraint row (1) incorporates the price per pound of each scrap and additive. Hence the cost of the specified steel may be expressed by the linear equation:

0.02 STSCP + 0.075 SP430 + 0.27 HCFCR ... = COST,

where in each term the coefficient is the price per pound, and the mnemonic is the quantity in pounds, of the material to be charged. The solution will give the quantity of each material required to produce the specified steel at minimum cost.

CHARGE MATERIAL SUPPLY EQUATIONS

Rows 2 through 6 establish the quantities of each element in the initial charge material. The second row, for instance, establishes the quantity of chromium in the initial charge. It is a linear summation of the pounds of chromium per pound of material in each of the scraps to be blended. Since steel scrap contains no chromium, a zero appears as the coefficient for STSCP in the CR equation. As Figure 2 indicates, 430 grade scrap, high-carbon ferrochrome, and low-carbon ferrochrome contain 16%, 55.6%, and 65% chromium, respectively. We can, consequently, express the chromium in the initial charge as follows:

0. STSCP + 0.16 SP430 + 0.556 HCFCR + 0.65 LCFCR = CR,

where the mnemonics are the variable quantities of raw material to be computed. We then provide a problem activity column for the total chromium in the initial charge (CRIT). Thus,

0. STSCP + 0.16 SP430 + 0.556 HCFCR + 0.65 LCFCR - 1. CRIT = 0.

Similarly, the quantities of manganese, silicon, carbon, and iron in the initially available materials are indicated in matrix rows 3, 4, 5, and 6, and a total-element-in-initial-charge column is formed for each. The final factor in the first section of the matrix is a problem activity column for the total initial charge weight (TICW) and a constraint row (7) indicating that the total weight of all the separate elements charged minus the total weight of the initial charge equals zero:

CRIT + MNIT + SIIT + CEIT + FEIT - TICW = 0.

The limitation of 430 grade steel and low-carbon ferrochrome to 2000 lbs. each appears in the last two constraint rows of the matrix (17 and 18); but the limitation of high-carbon ferrochrome, because it is employed at only one phase in the process, is introduced in the column devoted to that material. The number of rows directly affects calculation time and, further, is the determinant of machine capacity. Therefore, this feature -- the ability to bound any single activity without employing a row -- makes an important contribution to the computer's speed and problem capacity, and becomes particularly useful when solving multifurnace models.

In order to blend an initial charge properly we must consider not only the final specifications of the desired steel but also changes in the total weight of each of the initially charged elements resulting from the decarburization, reducing, and refining processes.

SPECIFICATION AND CONTROL CONSTRAINTS

Recent research in the thermochemistry of steel production makes it possible to predict and allow for reactions and losses that occur during decarburization and reducing. Such research emphatically demonstrates that special factors -- hearth material, heat size, variations in initial charge metal percentages, etc. -- contribute to the empirical result. Yet a sufficiently reasonable correlation between calculated estimates and end metal composition has been observed to justify the use of such estimating techniques in production. Consequently, we have employed a number of relationships which hold in the production of low-carbon steel in order to properly model the total process.

Recently researchers have formulated linear models for oxygen and electrical energy consumption in steel production (see reference 1 in the Bibliography). Additional work has made it possible to introduce such equations into a linear program for steel production, but we shall assume, in the sample problem analyzed here, that the oxygen and energy consumed will be determined separately. Appropriate equations should be introduced once they have been determined for a particular plant.

Clearly, specific plant practices determine the relationships which govern the reduction and finishing of steel; observed variations from the formulas employed here can easily be introduced into the series of problem activities and problem constraints that make up the second section of the LP matrix. Figure 6 provides a table of mnemonics for this section.

Chromium Specification Constraints

Since the end metal must contain at least 16% chromium, and we propose to produce 20,000 lbs. of steel, at least 3200 lbs. of chromium must be present in order to meet the specification. Manganese, which is limited to 1% (or 200 lbs.) of the end metal, tends, when present in small quantities, to behave as chromium does; so it is convenient to treat it in the same constraint row. We may formulate an expression for all the chromium plus manganese (which we shall call "modified chromium" as follows:

> Total modified chromium =
> (Modified chromium in the bath after the blow) +
> (Modified chromium reduced from the slag) +
> (Chromium from chrome silicide additive) +
> (Modified chromium from refining additives) = at least 3400 lbs.

Slag Chromium

When the furnace temperature and the carbon content at the end of the blow are known, the chromium content of the bath may be determined (see reference 2). Let us assume a furnace temperature of 3300^{o}F. with a carbon content limited by a specification of 0.05%. Under these conditions the bath, after the blow, will contain approximately 8% chromium

Item Name	Column Mnemonic
Modified chromium in the slag	ISCR
Chrome silicide additive at refining stage	CRSI
430 grade scrap at refining stage	RS430
Low-carbon ferrochrome at refining stage	RCFCR
Iron in the slag	ISFE
Lime at refining stage	LIME
Low-carbon ferrochrome at finishing stage	FCFCR
Slack in the chromium and manganese specifications	SIS

Item Name	Row Mnemonic
Chromium-oxidized-to-slag relationship	CRSLAG
Total modified chromium specification constraint	TOTCRS
Total manganese specification constraint	MN/CR
Iron-oxidized-to-slag relationship	FESLAG
Total iron specification constraint	ENDFE
Total carbon specification constraint	CSPEC
Basicity relationship	BASE
Total silicon specification constraint	SISPEC
Total end metal	TOTAL
Inventory limitation on 430 grade scrap	TOTRS4
Inventory limitation on low-carbon ferrochrome	TOTRCF

Figure 6. Mnemonic table -- specification and control section of matrix

and manganese, and the balance of the charged chromium (and manganese) will have been oxidized to the slag. Further, we may assume, for practical operating purposes, that the ratio between the chromium and the manganese (approximately 16:1) remains constant for the bath and the slag. Finally, at this temperature 7.5% of the initial charge weight (TICW) will have been oxidized so that bath weight after oxidation = 0.925 TICW. Consequently, we can establish the amount of modified chromium (that is, chromium+manganese) in the slag (see reference 3). We can now add to the matrix an equation (row 8) that represents the modified chromium in the slag at the end of the blow:

Modified chromium in slag (CRSLAG) =
(Total Chromium initially charged (CRIT)) +
(Total Manganese initially charged (MNIT)) -
(8% bath weight after metallic oxidation),

where bath weight equals 92.5% of total charge weight (TICW). If we establish a new problem activity variable for the chromium and manganese in the slag (ISCR) and solve for zero right-hand side, we have the following linear representation of the quantity of chromium and manganese in the slag:

CRIT + MNIT - (0.08 x 0.925 x TICW) - ISCR = 0,

or

CRIT + MNIT - 0.074 TICW - ISCR = 0.

This, then, is the first equation in the second section of the matrix (row 8).

Total Chromium Specification

We can now formulate a linear inequality expressing the specifications constraint, that final chromium and manganese from all sources must be equal to or greater than 3400 lbs.

Since, as we have seen, 8% of the bath after the blow -- or 0.074 of the total initial charge weight (TICW) -- is modified chromium, 0.074 is the coefficient of TICW in the total modified chromium row (TOTCRS).

We may further assume that achievement of the proper basicity (to be discussed below) will result in the reduction of 95% of the modified chromium in the slag (ISCR) (see reference 3). Hence 0.95 is the coefficient of ISCR in the TOTCRS row.

Of the various additives which may be used to reduce chromium from the slag we will use a chrome silicide composed of 39% chromium, 43% silicon, and 18% iron, which costs 27 cents a pound. This new problem activity variable is mnemonically labeled CRSI. The amount of silicon required for reduction is the stoichiometric equivalent of the chromium reducible from the slag (under the specified control factors), and the chromium content of the chrome silicide passes, for all practical purposes, entirely into the bath. Hence we add the coefficient 0.39 (lbs. of chromium per lb. of additive) for CRSI as a further source of chromium in the TOTCRS row.

Finally, additional quantities of 430 grade scrap and low-carbon ferrochrome may be added in the reducing stage, and low-carbon ferrochrome added again in the finishing stage, to bring the bath to specification. Though these materials already appear in section one of the matrix, we must assign new mnemonics in order to distinguish between the quantities used in the initial charge and the quantities that must be added as refining and finishing agents. Hence we add three new problem activities: RS430, RCFCR, and FCFCR. (See Figure 6.)

The scraps added at this stage provide additional chromium and manganese. The 430 grade scrap contributes 0.16 lb. of chromium, and 0.01 lb. of manganese, per pound, and the low-carbon ferrochrome (at both the refining and the finishing stages) provides 0.65 lb. of chromium per pound. Consequently, we can formulate the following inequality as the specification constraint for the total quantity of chromium and manganese in the end metal:

$$0.074\ TICW + 0.95\ ISCR + 0.39\ CRSI + 0.17\ RS430 + 0.65\ RCFCR + 0.65\ FCFCR \geq 3400 \text{ lbs.}$$

We then constrain the final total manganese (which, from experience, will equal 98% of initial manganese charged plus all subsequent manganese additions) to a maximum of 200 lbs. in matrix row 10, thus establishing the minimum of 3200 lbs. to meet the 16% chromium specification for the final metal.

One more factor will contribute to accurate control of the expensive chromium in the end metal. As the equations now stand, extra chromium will be forced into the solution if there is less than 200 lbs. of manganese in the end metal. In order to avoid this we establish a slack column activity (labeled SIS) which represents the difference between the total manganese from all sources actually present and the 200 lbs. maximum manganese constraint. As a result, the solution will demand only the 3200 lbs. of chromium required by the specification, because the slack will complete the equation, so that:

(Chromium from all sources) +
(Manganese from all sources) +
(200-Manganese from all sources (SIS)) = 3400 lbs. modified chromium.

Hence the final chromium and manganese constraint rows (rows 9 and 10) include the slack SIS.

Iron Specification Constraint

The total iron in the end metal may be expressed as follows:

(Initially charged iron) -
(Iron not reduced from slag) +
(Iron from reducing additions) +
(Iron from finishing additions) = specified iron content (ENDFE)

Slag Iron

A number of relationships have been suggested which provide, either directly or indirectly, an estimate of the amount of iron oxidized during the blow. For this model we have used a relation involving the ratio of the weight of metal in metallic oxides in the slag at the end of the oxygen blow to the total initial charge weight and the final bath carbon content (see reference 3). This relationship, which was employed in the formulation of the chromium-to-slag equations, indicates that at 3300^{o}F. and 0.05% carbon bath content, 7.5% of the initial charge weight will be oxidized to slag. Since we have already established a problem activity for the quantity of chromium and manganese oxidized to slag during the blow, it is a simple matter to arrive at the quantity of iron oxidized to slag. Iron in the slag, a new problem activity (ISFE), equals 7.5% of the total initial charge (TICW), less the chromium and manganese in the slag (ISCR). This is expressed in constraint row 11 as:

$$0.075\ TICW - ISCR - ISFE = 0.$$

Though the iron-from-slag recovery rate varies from plant to plant depending upon operating practice, final specifications, and basicity, we will assume a recovery rate of 95% for the conditions that determine our model. It is now possible to formulate an inequality for end iron specification, which, though not essential to the linear program, may provide the producer with useful insights into the complex relationships among the various control factors in the process.

Since our final bath weight of 20,000 lbs. will contain approximately 81%, or 16,200 lbs., of iron, we can collect all the iron sources into an expression of the end iron specification:

$$FEIT + 0.18\ CRSI + 0.8143\ RS430 + 0.3391\ RCFCR - 0.05\ ISFE + 0.3391\ FCFCR \geq 16,200 \text{ lbs.}$$

where the coefficients of CRSI, RS430, RCFCR, and FCFCR represent the percentage of iron in these additions.

Carbon Specification Constraint

Since the carbon specification of 0.05% for the final bath limits the total weight of carbon to 0.0005 x 20,000 = 10 lbs., we can introduce a carbon constraint to insure that the reducing and refining additives do not raise the carbon content of the bath above the 0.05% achieved by the blow. Thus:

(Carbon from the initial charge not burnt off) +
(Carbon from refining additives) +
(Carbon from finishing additives) ≤ 10 lbs. carbon

For our purposes, we may calculate the weight of the bath after reduction as equal to the weight of the initial charge (TICW) less the weight of the initially charged silicon (which is oxidized) and the chromium, manganese, and iron that remains in the slag. Bath weight after reduction, then, equals:

$$TICW - SIIT - 0.05\ ISCR - 0.05\ ISFE.$$

To constrain the carbon to a maximum of 10 lbs. we expand the above inequality limiting the total carbon left in the bath and contained in reducing and finishing additives to the required specification:

$$0.0005\ (TICW - SIIT - 0.05\ ISCR - 0.05\ ISFE) + 0.0012\ RS430 + 0.0009\ RCFCR + 0.0009\ FCFCR \leq 10.$$

Since the product of small coefficients (e.g., .0005 x .05) might cause numerical problems, we multiply the equation by 10,000, which eliminates the problem without altering the solution:

$$5\ (TICW - SIIT - 0.05\ ISCR - 0.05\ ISFE) + 12\ RS430 + 9\ RCFCR + 9\ FCFCR \leq 100{,}000,$$

or,

$$5\ TICW - 5\ SIIT - 0.25\ ISCR - 0.25\ ISFE + 12\ RS430 + 9\ RCFCR + 9\ FCFCR \leq 100{,}000.$$

Silicon Specification Constraint

Though silicon is not required in the end metal for any alloying purpose, all the silicon used to reduce the metallic oxides in the slag may not be consumed. However, the amount of silicon permissible in the end metal is limited by specification to a maximum of 1%, or 200 lbs. Therefore, we must establish a constraint row that limits the total silicon present in the end metal from all sources to a maximum of 200 lbs. We may, at the outset, assume that all silicon present in the initial charge is oxidized during the melt. The silicon specification constraint may then be expressed as follows:

(Silicon from silicide additive) +
(Silicon from refining and finishing additions) -
(Silicon required to reduce 95% of modified chromium from slag) -
(Silicon required to reduce 95% of iron from slag) ≤ 200 lbs.

We use the chrome silicide (CRSI) as a reducing agent in this model. The quantity of added silicon that is not oxidized during the reduction of metal from the slag must be calculated and constrained to the final silicon specification. In order to determine the amount of silicon needed to reduce 1 lb. of chromium to the bath, we calculate the amount of silicon stoichiometrically equivalent to 1 lb. of chromium, as follows:

$$2Cr_2O_3 + \underset{84}{3Si} \longrightarrow \underset{208}{4Cr} + 3SiO_2,$$

where the atomic weight of Si is 28 and Cr, 52 (see reference 4). Thus, the reduction of 1 lb. of chromium requires 84/208, or 0.404 lb., of silicon. This is supplied by the chrome silicide additive (CRSI) containing 43% silicon. Similarly, the reduction of 1 lb. of iron requires 0.251 lb. of silicon, and the reduction of 1 lb. of manganese requires 0.255 lb. of silicon. In this model we have assumed a ratio, limited by the final metal specifications, of 16 parts chromium to 1 part manganese in the slag (ISCR). We may therefore compute the composite silicon requirement as follows:

$$\frac{16(0.404) + 0.255}{17} = 0.395 \text{ lb. of silicon to reduce 1 lb. of modified chromium (chromium + manganese) from the slag.}$$

Further, our model has assumed conditions that result in the reduction of 95% of the metals in the slag. We may now formulate the silicon specification constraint (row 15):

$$0.43\ CRSI + 0.0095\ RS430 + 0.01\ RCFCR + 0.01\ FCFCR - (0.95 \times 0.395)\ ISCR - (0.95 \times 0.251)\ ISFE \leq 200 \text{ lbs.},$$

or

$$0.43\ CRSI + 0.0095\ RS430 + 0.01\ RCFCR + 0.01\ FCFCR - 0.375\ ISCR - 0.238\ ISFE \leq 200 \text{ lbs.}$$

However a safety factor will be incorporated into this equation because the reducing efficiency of the silicon varies with different heat (and hence slag) volumes and so more silicon may be required for larger heats.

Therefore the (-0.375) coefficient of ISCR (chromium + manganese in slag) has been changed to (-0.395). The additional factor of -0.02 ISCR = (-0.395 ISCR) - (-0.375 ISCR) is the safety factor.

This safety factor is not strictly needed for this size heat (20,000 lbs.) but has been included for illustrative purposes.

The silicon specification constraint becomes:

$$0.43\ CRSI + 0.0095\ RS430 + 0.01\ RCFCR + 0.01\ FCFCR - 0.395\ ISCR - 0.238\ ISFE \leq 200 \text{ lbs.}$$

Basicity Constraint

One of the principal operating factors we have assumed in the formulation of this matrix model is the relation between basicity and metallic oxides in the slag which has been observed for a variety of furnace volumes.

For the relatively small (10 ton) heat considered here, the reduction of 95% of the metallic oxides from the slag occurs when the slag basicity -- which we shall assume to be the ratio of lime (CaO) plus magnesia (MgO, from refractory erosion) to silica (SiO_2) -- equals 1.5; that is,

$$\frac{CaO + MgO}{SiO_2} = 1.5.$$

The quantity of lime, added in the reducing stage to achieve proper basicity, is a new variable for which a new problem activity (LIME) is formulated. Its cost is taken as $20 per ton, or 1¢ per lb.

The silica (SiO_2) in the furnace is formed from the oxidation of initially charged silicon (SIIT) and the oxidation of the silicon in the chrome silicide reducing agent (CRSI). The amount of magnesia (MgO) from refractory erosion is equal to the amount of silica formed by the oxidation of initially charged silicon (SIIT). The equation

$$\underset{28}{Si} + \underset{32}{O_2} \longrightarrow \underset{60}{SiO_2}$$

indicates that 1 lb. of silicon produces 60/28 or 2.14 lbs. of silica. Therefore, the silica formed from initially charged silicon equals 2.14 SIIT, and consequently the amount of magnesia (MgO) formed is also 2.14 SIIT.

Now, each pound of chrome silicide (CRSI) contains 0.43 lb. of silicon. Hence each pound of the additive results in 0.43 x 2.14 or 0.9202 lb. of silica. We may therefore formulate the following set of equations to arrive at a linear representation of the required basicity constraint of 1.5:

$$\frac{LIME + MgO}{SiO_2} = 1.5 = 3/2.$$

Since

$$SiO_2 = 2.14\ SIIT + 0.9202\ CRSI,$$

we obtain

$$\frac{LIME + MgO}{(2.14\ SIIT + 0.9202\ CRSI)} = 3/2.$$

Further,

$$MgO = 2.14\ SIIT;$$

hence,

$$\frac{LIME + (2.14\ SIIT)}{(2.14\ SIIT + 0.9202\ CRSI)} = 3/2,$$

or,

$$2\,(LIME + 2.14\ SIIT) = 3\,(2.14\ SIIT + 0.9202\ CRSI),$$

or,

$$0 = 2.14\ SIIT + 2.7606\ CRSI - 2\ LIME.$$

Total End Constraint

Our model requires only the addition of an end metal constraint which sets the sum of all the elements in the final bath equal to the quantity desired, in this case at least 20,000 lbs.

We simply sum the initial charge weight (TICW) and all additions remaining in the final bath (RS430, RCFCR, FCFCR), plus the portion of the chrome silicide (CRSI) that remains (that is, 0.18 Fe + .39 Cr), and subtract the quantities of the initial charge which were oxidized (SIIT and CEIT) or unrecovered from slag (0.05 ISCR + 0.05 ISFE). The end total equation, then, is:

$$TICW - SIIT - CEIT - 0.05\ ISCR - 0.05\ ISFE + 0.57\ CRSI + RS430 + RCFCR + FCFCR \geq 20{,}000\ \text{lbs.}\ (TOTAL).$$

MULTIFURNACE MODEL FORMULATION

Having once designed a basic single-furnace matrix for the production of any specified steel, it is an easy matter to design a multifurnace model. Such a model allows the producer to compute least-cost charges for a number of furnaces (or consecutive charges for the same furnace) simultaneously, even if different alloys are specified for each furnace. As Figure 7 indicates, a multifurnace model consists of a set of submatrices, each of which has the appropriate constraint rows to meet alloy specification, and each of which has a unique designation for its column activities. That is, the raw material mnemonics for furnace 1 may be prefixed with a numeral 1, those for furnace two are prefixed with a 2, and so on. Thus, in a single computer run the producer may determine how much steel scrap to use in each furnace, how much 430 grade scrap, how much silicide additive, etc.

Ideally, the model should be solved with no inventory constraints in order to determine optimal solutions. However, such constraints can be introduced, either into the submatrix to prevent violations of feasible operating practice (such as too much chromium in initial charge) or into the multifurnace model (reflecting irremedial inventory limitations) -- this procedure will then permit computation of the optimum distribution of available stock for least-cost overall production. The most obvious advantage of the multifurnace LP model is that, in a minimum of computer time, it allocates from all available inventory supplies to all the furnaces at optimal levels, resulting in the least-cost use of both stocks and furnaces. Further, re-solutions based on output report suggestions will respond to overall considerations of inventory and costs rather than to a single heat problem. Quite conceivably, a multifurnace model solution may indicate that a nonoptimal solution for one furnace will allow the best overall use of inventory and furnace capacity.

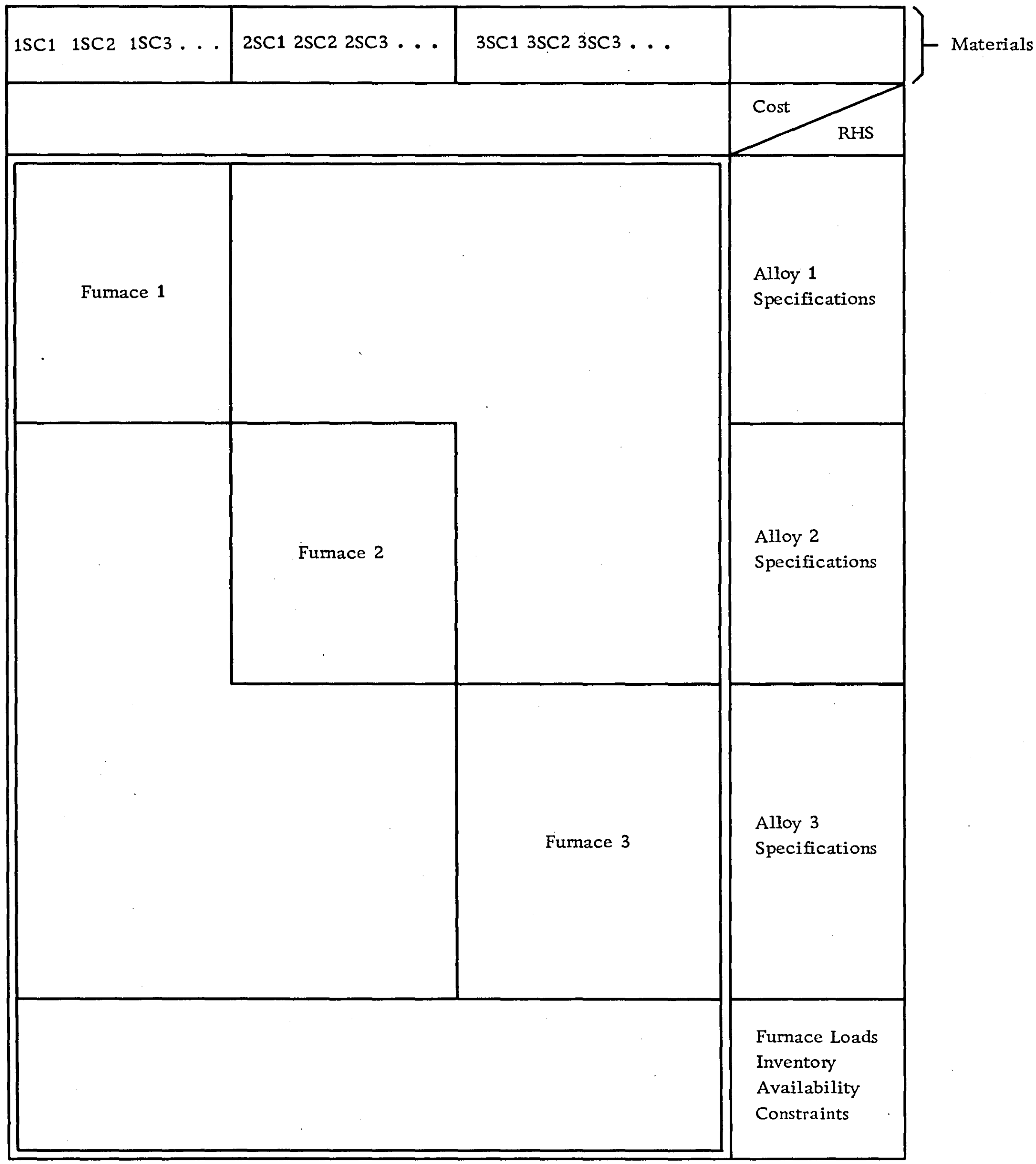

Figure 7. Schematic of multifurnace matrix model

SUMMARY

The foregoing discussion has demonstrated the applicability of linear programming techniques in electric furnace steel production. To that end we constructed an LP model designed to solve a typical production problem. Though we simplified the problem by assuming that only four scraps and two additives were available, the LP model formulated here can be expanded readily to include the large variety of scraps usually available to the furnace operator, as well as alternative combinations of reducing and finishing additives. Indeed, the usefulness of LP increases as the variety of available raw materials increases.

A number of furnace operators have found, in fact, that the blending section of the matrix, used independently of the process section, can introduce cost savings of sufficient magnitude to justify the standard application of LP techniques in the industry. Blend matrices, which are immediately applicable, can later be expanded to reflect process variations. The IBM data processing application manual Linear Programming--Aluminum Alloy Blending (E20-0127-0) provides further information on the possibilities of LP applications in alloy blending problems.

Construction of the basic LP model entails little more than organizing, in a special format, the data historically used in calculating furnace charges. Once initially constructed and converted to an input medium for computer processing, the model becomes a master record. It can be updated regularly to account for new conditions such as the addition or deletion of activities, changes in inventory constraints, changes in costs, changes in specifications, etc.

OUTPUT REPORTS

The linear program will employ the input data to compute a variety of output reports. We are here principally concerned with four basic reports which the computer delivers.

The first of these is called the basis variables (BASIS. VARBLS) report. It provides a list of all the activities, indicating (in this problem) not only the quantities of each raw material required but also the levels of each separate element charged and the distribution of elements caused by furnace thermochemistry.

The second is the slacks report, which provides a list of all the right-hand-side mnemonics and indicates for each linear inequality the difference (if any) between its upper or lower bound and the optimal solution. Whenever the row is an equality, or the inequality is solved at a bound, the slacks report provides a figure called the simplex multiplier, which is significant to the next report.

The third report, called the DO. D/J report, comprises two parts. The first part lists all the column activities -- raw materials in this case -- which are employed at a bound. Most often the bound is zero, and the report indicates for each material its current cost and the amount its cost must drop before it reaches a level at which it may be introduced into the basis, that is, the optimal solution of the problem. Sometimes an upper bound constrains the raw material and the DO. D/J report indicates the highest price at which that material would remain in the optimal solution.

The second part of the DO. D/J report provides a list of all the row (that is, right-hand-side) mnemonics, and for each equation (and each inequality solved at one of its bounds) indicates the cost (which is the simplex multiplier in the slacks report) of changing the right-hand side of the expression by one unit.

Finally, the output includes the cost range (COST. R) report, which indicates for each activity (column) employed in the optimal solution the following data: current cost, highest cost before its quantity in the optimal solution changes, what other activity would enter the solution at that highest cost, lowest cost before its quantity in the optimal solution changes, what other activity would enter the solution at that lowest cost. Each of these reports is discussed and illustrated below.

Basis Variables Report

The IBM 1620/1311 LP system provides the optimal solution shown in Figure 8 for the problem we have formulated.

NAME	ACTIVITY LEVEL
STSCP	13669.628
SP430	2000.000
HCFCR	2000.000
LCFCR	1360.449
CRSI	1328.567
LIME	1940.761
FCFCR	639.551

Figure 8. Basis variables report -- optimal solution

The total raw material cost (excluding oxygen and electrical energy) of 20, 000 lbs. of steel comprising these materials is $2, 141. 51, or 10. 71¢ per pound. If this solution is to be implemented without change, it can be disseminated immediately to both the inventory accounting department and the furnace operator. The inventory accounting department employs the record of raw materials consumed to maintain updated inventory records. The furnace operator employs the solution as a work order to be followed in charging the furnace.

At this point, the model may easily be adjusted to deal with a special raw material restriction, such as ingot weight scrap supplies. If, for example, steel scrap (STSCP) is stocked in 200 lb. ingot weights, the operator simply rounds off the optimal solution for steel scrap to the nearest multiple of 200 and re-solves the problem. For this problem he would re-solve after imposing a 13,800 lb. activity level for steel scrap. He cannot round to 13,600 lbs. because only 6000 lbs. of other basic materials are available to meet the total of at least 20,000 lbs. of end metal required. Reducing the amount of steel scrap, then, would force the use of expensive refining additives to meet the end metal requirement. Imposing ingot weight constraints can upset the balance of materials that meet the specifications of the end metal. Consequently, it may become necessary to obtain several solutions, with constraint changes suggested by the previous solution. However, this approach to controlling ingot allocations has proved highly effective in actual application.

The basis variables report provides -- in addition to the specific quantities of raw materials required, discussed above -- the quantity of each element in the initial charge, the total quantity of all materials initially charged, and the quantities of metal oxidized to the slag (Figure 9).

NAME	ACTIVITY LEVEL	
CRIT	2316.292	Elements initially charged
MNIT	156.696	Elements initially charged
SIIT	99.944	Elements initially charged
CEIT	245.642	Elements initially charged
FEIT	16201.503	Elements initially charged
TICW	19020.077	
ISCR	1065.502	Metals oxidized
ISFE	361.003	Metals oxidized

Figure 9. Basis variables report -- initial charge and slag

This additional information is extremely useful and may be employed in a variety of re-solutions. For example, we shall see further on that the limitation of 2000 lbs. maximum on three of the input materials tends to force the price of the end metal up. The analysis of the DO. D/J report (discussed further on) indicates that additional quantities of 430 grade scrap would result in a lower total price for our metal. Hence, we can resolve the problem with the limitation on 430 scrap removed. Such a solution reveals that 17,193 lbs. of 430 scrap is required in an optimal solution where 20,000 lbs. of end metal would cost only $1,720.83, a remarkable 25% saving. On the other hand, almost 400 additional lbs. of chromium would be oxidized to the slag, which might introduce a serious slag viscosity problem and result in unfeasible plant practice. The new solution indicates, as well, the appropriate price to pay for new inventory. Were slag viscosity not a problem in this case, the purchase of additional 430 grade scrap (if available) at a 2¢ per pound premium would still result in a saving of $116 in total cost:

$2141.51 - $1720.83 = $420.68

15,193 lbs. RS430 steel scrap x 2¢ premium = $303.86

Net saving = $116.82.

Now let us consider the effect on the solution if, rather than 430 grade scrap, high-carbon ferrochrome were available in unlimited quantities. The basis variables report in such a solution reveals that 4371 lbs. of high-carbon ferrochrome is required, with a total end metal cost of $1978.38. However, we again discover a substantial increase in the amount of chromium oxidized to the slag, in this case 347 lbs. Further, and quite significantly, this re-solution indicates that 432 lbs. of carbon will be charged rather than the 245.6 lbs. indicated by our original solution. The cost of the additional blow time required to oxidize that additional carbon may well overbalance the apparent saving realized through the use of additional high-carbon ferrochrome.

Whatever the specific situation, the producer has before him in the basis variables report a variety of information upon which to base his final decision. Consequently, it may be useful initially to solve the problem with no material constraints at all. Such a solution will indicate the lowest possible cost, but additional information provided by the basis variables report will indicate violations of good plant practices -- too much chromium initially charged, too much carbon initially charged, too much metal oxidized to slag -- and, further, reveal which raw materials should be constrained in order to achieve feasible furnace operation. Finally, inventory and availability constraints may be added to produce the best solution that can be achieved in actual practice.

Slacks Report

The slacks report, shown in Figure 10, indicates for each linear inequality the difference, if any, between its upper or lower bound and the optimal solution. For each equality, or inequality solved at a bound, it provides a figure -- the simplex multiplier -- that reveals the cost of changing the right-hand side of the equation or inequality by one unit.

NAME	ACTIVITY LEVEL	SIMPLEX MULT.
TOTCRS	.000	.665-
MN/CR		.665
ENDFE	439.467	
SISPEC	129.113	
TOTRCF		.031
TOTRS4		.046
VALUE	2141.513	

Figure 10. Slacks report

For example, the TOTCRS figures indicate that the total amount of chromium and manganese was expressed as an equality in the original matrix (the slack is 0), and the simplex multiplier indicates that if the equation demanded slightly less chromium, the total price of the end metal would decrease 66.5¢ per pound. (The same indication will occur more graphically in the DO.D/J report.) This is a rather high simplex multiplier for a problem of this sort, and it might be appropriate to re-solve the problem with slightly relaxed chromium specification, say 15.8% instead of 16%, to discover rapidly for the customer's information the cost of the specified quality.

The report reveals, further, that the total iron (ENDFE) in the optimal solution exceeds the lower bound of 16,200 lbs. by 439.467 lbs., and similarly, that the total silicon in the optimal solution falls 129.113 lbs. below its upper bound of 200 lbs. But neither of these indications is as interesting as those that follow in this particular problem.

The slack report indicates that all of the allowable low-carbon ferrochrome (2000 lbs.) and 430 grade scrap (2000 lbs.) have been used in the optimal solution (hence no slack appears for either). The simplex multipliers indicate that in the neighborhood of the optimal solution, slightly more low-carbon ferrochrome will reduce the total price at the rate of 3.1¢ per pound, while slightly more 430 scrap will reduce the total price at the rate of 4.6¢ per lb. These figures suggest that considerable savings might be realized if more 430 scrap and low-carbon ferrochrome were made available.

The final line in the slack report -- VALUE -- is the cost of the specified metal: $2,141.513.

VBLS NAME	CURRENT COST	REDUCED COST	BASIS VALUE
HCFCR	0.270	0.098	0.368
RS430	0.075	0.031	0.044
RCFCR	0.400	0.000	0.400

ROWS NAME	INCR B VALUE	DECR B VALUE
TOTCRS		0.665
TOTRCF	0.031	
TOTRS4	0.046	

Figure 11. DO.D/J report

DO. D/J Report

The DO. D/J report (Figure 11) consists of two parts. The first may be thought of as listing all the problem activities (columns) that enter the solution at a bound. Most often that bound is zero, an indication that the material, at its specific price, is not used in the optimal solution. In our model, however, we also established an upper bound on the quantity of high-carbon ferrochrome (HCFCR) available for use, and the appearance of HCFCR in the DO. D/J report reveals that it is present in the solution at its upper bound -- 2000 lbs. The report indicates that the bound on high-carbon ferrochrome is, in fact, forcing the price up, that even if the current cost of that material were up to 9.8¢ higher, for a total cost of 36.8¢ per pound, additional high-carbon ferrochrome would result in a lower total cost for the end metal.

The next line in the DO. D/J report indicates that no grade 430 scrap is used in the refining stage. In fact, as the basis variables report (Figure 8) revealed, all 2000 lbs. of available 430 grade scrap is initially charged. The DO. D/J report, however, points out that some 430 grade scrap would be used at the refining stage if the price dropped from 7.5¢ to 4.4¢ per pound. Further, the next line indicates that no low-carbon ferrochrome is employed at the refining stage, though the figures reveal that the price of the material would allow it to be so employed. In fact, the report demonstrates that the competition for the least-cost distribution of the available low-carbon ferrochrome ended in a tie. As the basis variables report (Figure 8) indicates, the 2000 lbs. of low-carbon ferrochrome was divided between an initial charge (LCFCR) of 1360.449 lbs. and a finishing charge (FCFCR) of 639.551 lbs.

The second section of the DO. D/J report makes graphic some of the information from the slacks report. The first line indicates that if less chromium (TOTCRS) were specified, the total price (VALUE) would drop at the rate 66.5¢ per lb., in the neighborhood of the optimal solution. This figure provides an immediate indication of the price of quality -- it suggests again that a slight relaxation of specifications may result in significant cost reduction.

The last two lines in the DO. D/J report reveal that the bounds on 430 grade scrap and low-carbon ferrochrome are forcing the price up. The report indicates that in the neighborhood of the optimal solution an increase of 1 lb. of low-carbon ferrochrome would result in a 3.1¢ saving and an increase of 1 lb. of 430 grade scrap would result in a saving of 4.6¢. We have already discussed the possibilities for action suggested by such information. Re-solution without constraints provides the producer with accurate data upon which to base scrap purchasing decisions.

NAME	CURRENT COST	HIGHEST COST	HI-VAR	LO-VAR	LOWEST COST
STSCP	0.020	0.535	RS430	TOTAL	0.023-
SP430	0.075	0.106	RS430		INFINITY-
LCFCR	0.400	0.415	TOTRCF	RS430	0.382
CRSI	0.270	INFINITY		TOTRCF	0.249
LIME	0.010	INFINITY		TOTRCF	0.005-
FCFCR	0.400	0.400	RCFCR	TOTRCF	0.371

Figure 12. Cost range report

Cost Range Report

The quantity of each raw material required (given in the basis variables report) will remain unchanged within the cost range indicated by the COST. R report (Figure 12). For example, 13,669.628 lbs. of steel scrap would be required in an optimal solution even if this material cost up to 53.5¢ per lb. instead of 2¢. In the event that steel scrap did in fact exceed 53.5¢, some of it would be replaced by 430 grade scrap added at the refining stage (RS430). The lowest point in the cost of steel scrap -- that is, the point at which some other variable would enter the solution -- is a negative number and hence is irrelevant to this problem. Note, however, that the extremely large highest-cost figure for steel scrap results from the limitation of the other three raw materials to 2000 lbs. each. In fact, when the same problem is solved with no constraint on the quantity of 430 grade scrap available, the cost range report shows that the highest cost of steel scrap is 6.1¢, at which point low-carbon ferrochrome enters the solution at the refining stage.

Similarly, the cost range for initially charged 430 grade scrap is 10.6¢ to 0¢. If the material were to exceed 10.6¢ per lb., some of it would be employed at the refining stage (where, currently, none of it is used). There is no feasible lowest cost for this material.

The cost range report provides a good measure of sensitivity to price changes, since it indicates at what prices the optimal solution will change, and what raw materials may be used most appropriately to substitute for unavailable or overpriced stock. In our problem, the most interesting information in the report is the lowest cost of the chrome silicide additive (CRSI). The additive will be used no matter how expensive it is (because we have provided no other source of silicon), but if it drops below 24.9¢ (a drop of only 2.1¢) it will cause a change that will reduce the amount of 40¢ per lb. low-carbon ferrochrome required -- additional chrome silicide supplying a portion of the required chromium. Hence, the producer is warned that a good purchase in chrome silicide, at the price indicated, for the production of this particular steel may be more profitable than a purchase of additional low-carbon ferrochrome, for instance. In any case, he can provide himself with detailed and accurate data by re-solving the problem using the lowest-cost price of chrome silicide.

Summary

The various output reports not only inform the producer of the specific optimal solution of the problem at hand but also alert him to a variety of relationships any one of which may profoundly influence the total cost of the end metal. The computer enables the producer to rapidly re-solve the same problem with a number of variations suggested by the output reports. He can, in effect, use the program as a model to aid in the solution of a series of different problems. What if the price of each raw material varies? What if certain inventory purchases are possible at specific prices? What if quality controls vary? The LP program provides data which enables the producer to make the most judicious policy decisions in matters of furnace charging, quality control, inventory control, purchasing, and product research. It makes possible continuous management study -- resulting in decreased costs, increased efficiency, and maximum profits.

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