Linear Programming—Ice Cream Blending

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INTRODUCTION

This manual explores the application of linear programming (LP) in ice cream blending. Linear programming is becoming more and more important as a decision-making tool in a variety of industries — notably, alloy blending, feed mixing, cotton blending, and food blending. For some time it has been standard in the petroleum industry, where it is used in all phases of refinery scheduling and control as well as in refinery expansion studies.

Whenever a process involves the blending of a variety of materials within specification constraints, the application of linear programming techniques enables the producer to determine rapidly the optimal, or least-cost, blend. The usefulness of LP increases as the number of materials and the complexity of the industrial process increase .

The production of ice cream involves precisely the sort of complex blending and control problem that LP is designed to solve, and the purpose of this manual is to demonstrate the application of this solution method to ice cream blending. This is accomplished through use of a typical production problem that illustrates the principles and advantages of LP in a specific situation.

An LP model is developed that can, with minor modifications, be solved by any IBM linear programming system. Among its more immediate and obvious benefits, as will be shown, linear programming helps the producer to:

- Minimize the cost of an ice cream blend
- Control the use of ingredients in inventory
- Reduce and idealize inventory levels
- Reduce waste
- Eliminate off-standard blends
- Purchase and sell most economically

Contrary to popular belief, little mathematical knowledge or skill is required to formulate an LP model. Nor do the operation of the computer and the analysis of computer results require any advanced technical skill. All that is required is the expression, in the form of simple linear equations or inequalities, of all the elements in the process — such as ingredients, specifications, total quantity desired, and so on.

A general explanation of basic linear programming appears in IBM data processing application manual An Introduction to Linear Programming (E20-8171), which should be read in conjunction with this manual.

PROBLEM PROFILE AND ECONOMICS

The basic ice cream production process begins with the blending of ingredients in the mixing vat and ends in the hardening room where packaged products are frozen before shipment. We shall here consider only the methods for arriving at an optimal blend, though LP can be used to provide data as a basis for packaging and distribution decisions as well.

Essentially, the blend problem requires the calculation of the appropriate quantities of a variety of different ingredients to be used to satisfy a basic recipe. The basic recipe varies, depending on the nature and quality of the product; and for any given product, the recipe may allow ranges in the specifications — for instance, it may require a minimum of 10% and a maximum of 16% butterfat.

The fundamental problem is to produce the specified ice cream at the lowest possible cost. To achieve this, the producer must consider a complex variety of factors contributing to costs. The more obvious of these include price and availability of blend ingredients. Since a variety of ingredients may be used to introduce the elements required by the recipe and since price and availability of these ingredients may fluctuate, the determination of an optimal blend by manual methods is quite difficult. Further, since ingredients often contribute <u>fractional</u> quantities of any recipe element — such as, say, sweetness — the alteration of any one component in the blend because of variations in price may alter <u>all</u> the relationships required for least-cost production, vastly compounding the difficulty of manual calculation.

In view of such difficulties, an increasing number of producers are examining closely the application of linear programming, which allows rapid examination and evaluation of all possible blend combinations and a quick determination of the most economical mix. Further, by "forcing" overstocked inventory supplies in least-cost blends, linear programming contributes substantially to the achievement and maintenance of ideal inventory levels.

MODEL FORMULATION — SINGLE VAT

A linear programming model for ice cream production is a mathematical representation, in the form of linear equations or inequalities, of all known and estimated factors relevant to the production of the specified blend. In order to demonstrate the method of formulating such a model, we postulate a problem: the production of 100 lbs. of a specific ice cream from a number of ingredients. We shall use a limited number of ingredients with only minor availability problems. In actual practice, however, the necessity for taking into account a greater number of available materials and special conditions such as limited availability or overstocked inventory (which complicate manual calculations) greatly increases the utility of LP. Such additional factors can be introduced easily into the LP model without increasing its complexity.

Input Data Requirements

The following basic information is required to formulate the LP model:

1. Blend specifications
2. Quantity of blend required
3. Composition analysis of all ingredients
4. Per-pound cost of all raw ingredients
5. Inventory levels of all ingredients

Most of this information is available from purchasing, cost accounting, inventory accounting, or other sources, and is probably used in existing systems for calculating mixes. Where exact information cannot be readily obtained, estimates should be made since it is an easy matter to change the input data and re-solve the problem once an optimal solution has been obtained. Indeed, the rapid calculation of the effect of changes in the input is a prime advantage of the linear programming approach.

Example Problem

One hundred pounds of ice cream is to be produced with the recipe shown in Figure 1. A combination of available ingredients, which are analyzed and priced as shown in Figure 2, will be used to produce the blend.

The first column under "Identification" in Figure 2 contains symbols (I1, I2, I3, etc.) that will identify each ingredient in the columns of the model matrix to be constructed.

Requirement	Minimum (lbs.)	Maximum (lbs.)
Butterfat	10	16
Milk solids (nonfat)	10.5	13
Total milk solids	20.5	25
Sweetness	11	17
Total solids	37.5	41.5
Water	58.5	62.5
Whey solids	0	4
Corn syrup solids	0	6
Stabilizer	0.37	0.37
Emulsifier	0.10	0.10

Figure 1. Recipe

	Identification	% B. F.	MSNF	Total M. S.	Sugar	Total Solids	Water	Cost
I1	Cream (40%)	40.0	5.4	45.4		45.4	54.6	27.9
I2	Cream (38%)	38.0	5.6	43.6		43.6	56.4	26.3
I3	Milk (3.2%)	3.2	8.7	11.9		11.9	88.1	3.2
I4	Milk (3.4%)	3.4	8.7	12.1		12.1	87.9	3.2
I5	Milk (3.5%)	3.5	8.7	12.2		12.2	87.8	3.3
I6	Milk (3.6%)	3.6	8.7	12.3		12.3	87.7	3.3
I7	Milk (3.7%)	3.7	8.7	12.4		12.4	87.6	3.4
I8	Milk (3.8%)	3.8	8.7	12.5		12.5	87.5	3.5
I9	Milk (3.9%)	3.9	8.6	12.5		12.5	87.5	3.5
I10	Milk (4.0%)	4.0	8.6	12.6		12.6	87.4	3.6
I11	Milk (4.2%)	4.2	8.6	12.8		12.8	87.2	3.7
I12	Skim Milk		9.0	9.0		9.0	91.0	1.8
I13	Condensed Whole Milk	8.0	20.0	28.0		28.0	72.0	7.6
I14	Condensed Skim Milk (28%)		28.0	28.0		28.0	72.0	3.9
I15	Condensed Skim Milk (30%)		30.0	30.0		30.0	70.0	4.9
I16	Condensed Skim Milk (32%)		32.0	32.0		32.0	68.0	4.5
I17	Dry Skim Milk	1.0	96.0	97.0		97.0	3.0	14.8
I18	Dry Buttermilk	5.0	92.0	97.0		97.0	3.0	15.0
I19	Dry Whey Solids		95.0	95.0		95.0	5.0	10.7
I20	Dry Sucrose				100.0	100.0		10.2
I21	Cane Syrup				67.0	67.0	33.0	9.9
I22	Corn Sgr. Solids (50% Sweetness)				50.0	100.0		7.0
I23	Corn Sgr. Solids (45% Sweetness)				45.0	100.0		9.0
I24	Corn Syrup				40.0	100.0		6.6
I25	Stabilizer					80.0	20.0	55.0
I26	Emulsifier							78.0
I27	Water						100.0	0

Figure 2. Ingredient composition table

We shall assume that the availability of 40% butterfat cream (I1) is limited to 10 lbs. and that we must use at least 40 lbs. of 3.6% milk (I6) inventory. Similar limitations can be imposed, depending on market conditions and current inventory, on any of the ingredients for any specific mix formulation.

A schematic of the LP model matrix is given in Figure 3 to outline the organization of an ice cream blending matrix. The detailed model matrix is shown in Figure 4.

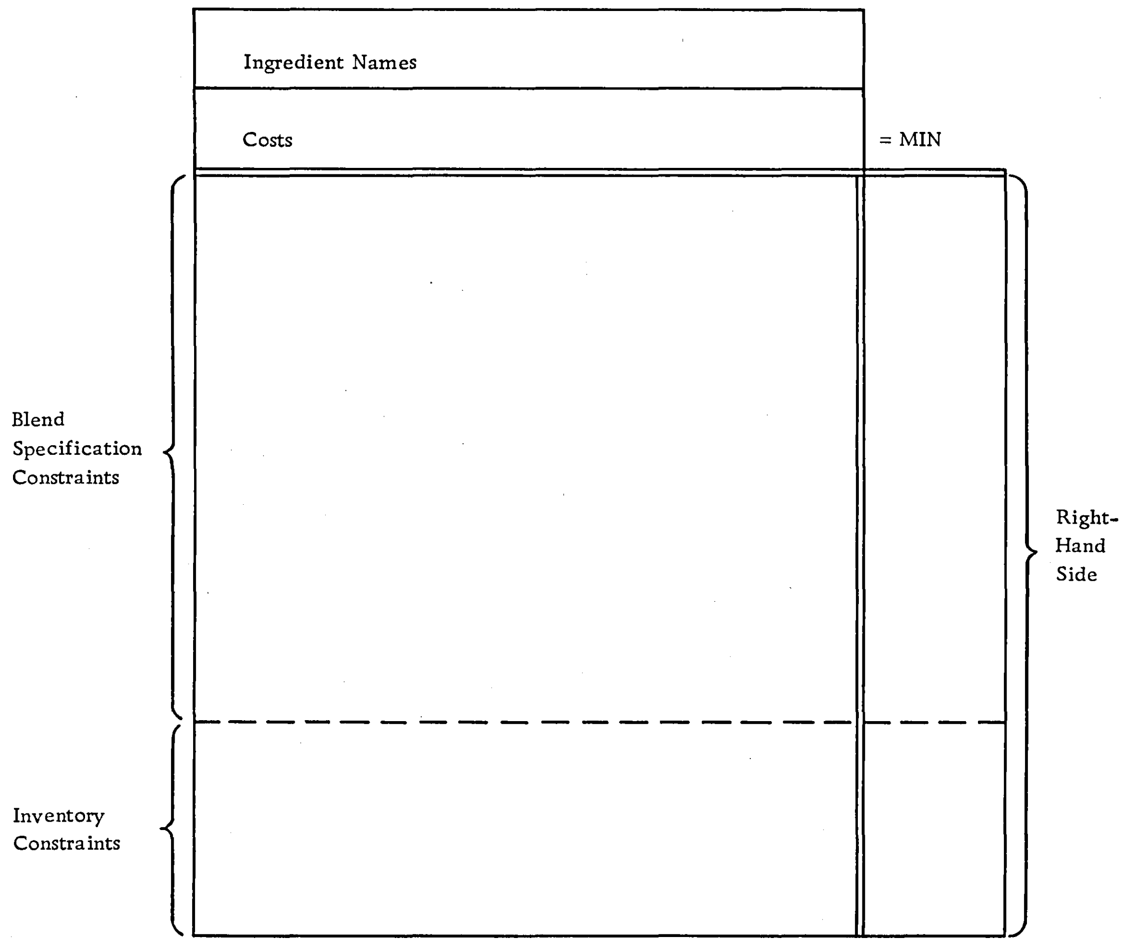

Figure 3. Schematic of matrix — single vat model

Every ingredient that provides a source of the various recipe elements (that is, butterfat, milk solids, sweeteners, etc.) appears at the head of a matrix column which is called a problem activity. Cost and maximum and minimum specifications appear at the end of the matrix rows, which are called problem constraints. Thus we have, in effect, transferred the data given in the tables of Figures 1 and 2 to the matrix of Figure 4. Each of the ingredients heads a column and is symbolically named as indicated in Figure 2.

	I1	I2	I3	I4	I5	I6	I7	I8	I9	I10	I11	I12	I13	I14	I15	I16	I17	I18	I19	I20	I21	I22	I23	I24	I25	I26	I27	RHS	
COST	27.9	26.3	3.2	3.2	3.3	3.3	3.4	3.5	3.5	3.6	3.7	1.8	7.6	3.9	4.9	4.5	14.8	15.0	10.7	10.2	9.9	7.0	9.0	6.6	55.0	78.0	0	MIN	1
MIN. BF	.40	.38	.032	.034	.035	.036	.037	.038	.039	.040	.042		.08				.01	.05										≥ 10	2
MAX. BF	.40	.38	.032	.034	.035	.036	.037	.038	.039	.040	.042		.08				.01	.05										≤ 16	3
MIN. MSNF	.054	.056	.087	.087	.087	.087	.087	.087	.086	.086	.086	.09	.2	.28	.3	.32	.96	.92	.95									≥ 10.5	4
MAX. MSNF	.054	.056	.087	.087	.087	.087	.087	.087	.086	.086	.086	.09	.2	.28	.3	.32	.96	.92	.95									≤ 13	5
MIN. TMS	.454	.436	.119	.121	.122	.123	.124	.125	.125	.126	.128	.09	.28	.28	.3	.32	.97	.97	.95									≥ 20.5	6
MAX. TMS	.454	.436	.119	.121	.122	.123	.124	.125	.125	.126	.128	.09	.28	.28	.3	.32	.97	.97	.95									≤ 25	7
MIN. SUG																				1.0	.67	.5	.45	.4				≥ 11	8
MAX. SUG																				1.0	.67	.5	.45	.4				≤ 17	9
CSS																						1.0	1.0	.8				≤ 6	10
MIN. TS	.454	.436	.119	.121	.122	.123	.124	.125	.125	.126	.128	.09	.28	.28	.3	.32	.97	.97	.95	1.0	.67	1.0	1.0	.8				≥ 37.5	11
MAX. TS	.454	.436	.119	.121	.122	.123	.124	.125	.125	.126	.128	.09	.28	.28	.3	.32	.97	.97	.95	1.0	.67	1.0	1.0	.8				≤ 41.5	12
MIN. H20	.546	.564	.881	.879	.879	.877	.876	.875	.875	.874	.872	.91	.72	.72	.7	.68	.03	.03	.05		.33			.2			1.0	≥ 58.5	13
MAX. H20	.546	.564	.881	.879	.879	.877	.876	.875	.875	.874	.872	.91	.72	.72	.7	.68	.03	.03	.05		.33			.2			1.0	≤ 62.5	14
STAB																									1.0			= .37	15
EMUL																										1.0		= .01	16
YIELD	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	= 100	17
	≥ 10					≤ 40													≥ 4										

Figure 4. Model matrix

The rows are symbolically named as indicated in Figure 5. Essentially, the columns (ingredients) are the variables and the rows are equations or inequalities formed by the summing of variables multiplied by appropriate coefficients.

Row Mnemonic	Row Identification
BF	Butterfat
MSNF	Milk solids (nonfat)
TMS	Total milk solids
SUG	Sweetness
CSS	Corn syrup solids
TS	Total solids
H20	Water
STAB	Stabilizer
EMUL	Emulsifier
YIELD	Amount to be made

Figure 5. Row mnemonic table

COST CONSTRAINT (OBJECTIVE FUNCTION)

The first problem constraint, row 1, incorporates the cost per pound of each of the ingredients. Hence, the cost of the blend may be expressed as a linear equation:

$$27.9\ I1 + 26.3\ I2 + 3.2\ I3 + 3.2\ I4 + 3.3\ I5 \ldots = \text{MINIMUM COST},$$

where the symbols represent the quantities, in pounds, of each ingredient used in the mix, and each coefficient is the price of that ingredient in cents per pound. The solution will give the quantity (activity level) of each ingredient required to produce the specified ice cream at minimum cost.

RECIPE SPECIFICATION CONSTRAINTS

All of the remaining constraint rows, except the last, introduce the recipe specifications (Figure 1) into the model. The second and third rows, for instance, establish the amount of butterfat in the blend. They are linear summations of the amount of butterfat in each ingredient. Since 40% cream (I1) contains 0.40 pound of butterfat per pound, 0.40 appears as the coefficient for I1 in the butterfat rows. Similarly, 38% cream (I2) contains 0.38 pound of butterfat per pound, 3.2% milk (I3) contains 0.032 pound of butterfat per pound, and so on. We can consequently express the total butterfat in the blend as a linear equation in which we sum the percentage of butterfat in each ingredient multiplied by the weight of each ingredient:

$$0.40I1 + 0.38I2 + 0.032I3 + 0.034I4 + 0.035I5 + 0.036I6 + \ldots = \text{TOTAL BUTTERFAT}.$$

To specify that the total butterfat shall not be less than 10 lbs., we simply set the minimum butterfat (row 2) equal to or greater than 10:

$$0.40I1 + 0.38I2 + 0.032I3 + \ldots \geq 10 \text{ lbs}.$$

To constrain the total butterfat to a maximum of 16 lbs., we establish another linear summation of all butterfat (row 3) and set it equal to or less than 16. Thus, the second and third rows of the matrix establish the butterfat specification in the recipe:

(butterfat from all sources) ≥ 10
(butterfat from all sources) ≤ 16.

In precisely the same way, we establish the nonfat milk solids specification with a pair of linear inequalities:

(nonfat milk solids from all sources) ≥ 10.5
(nonfat milk solids from all sources) ≤ 13.0,

and so on for total milk solids, sweetness, corn syrup solids, total solids, water, stabilizer, and emulsifier.

To illustrate graphically the method for constraining element specifications between upper and lower bounds, we established one row for each bound constraint in our model matrix. In practice, however, most IBM linear programming systems now permit the expression of range constraints as one row with a double right-hand side (RHS), establishing both upper and lower bounds. This feature conserves the number of rows employed by a model matrix and results in faster solutions and better utilization of machine capacity.

YIELD CONSTRAINT

The problem calls for the production of 100 lbs. of mix. Hence, we establish a constraint (row 17, labeled YIELD) that sets equal to 100 lbs. the sum of all the ingredients used. This particular weight is useful because it may also be read as "100%", and if some other total quantity is required, the levels of the various ingredients may be determined easily by multiplying the optimal levels by a constant. (If, for example, 575 lbs. of mix are required, we simply multiply the optimal weights for a 100-lb. solution by 5.75.)

In actual practice, however, imposition of availability constraints may make 100 lbs. an infeasible yield total. For instance, some essential materials may be in short supply so that one or more specifications cannot be met in a 100-lb. batch. The method of formulating a matrix for such conditions is discussed later in the section on the capacity constraint.

AVAILABILITY CONSTRAINTS

Frequently the producer is confronted with either shortages or surpluses of inventory stocks. The model matrix may quite easily be made responsive to such conditions. The symbol at the foot of column I1, for example, signifies that a maximum of 10 lbs. of 40% cream may be employed in the blend. The symbol at the foot of column I6 signifies that at least 40 lbs. of 3.6% milk must be used in the blend. Further, the recipe specification limiting whey solids (I19) to a maximum of 4 lbs. is also introduced directly into the activity column.

The ability to introduce bounded variables without using constraint rows is now a feature of most IBM linear programming systems. This feature, along with single row range constraints, conserves computer time and increases effective computer capacity.

Any restriction, either limiting the amount of specific ingredients in short supply or forcing minimum amounts of overstocked ingredients, can be incorporated into the matrix without difficulty. However, the producer must be careful not to force the use of quantities which violate the specifications. For instance, if 45 lbs. of 40% cream (I1) were forced, the maximum butterfat specification constraint would be violated, since 45 lbs. x (0.40 butterfat) = 18 lbs. butterfat, 2 pounds more than specifications allow. Infeasibility of this sort can usually be detected by a visual inspection of the model matrix.

MODEL FORMULATION — MULTIPLE VAT

After a basic matrix for the production of any specified ice cream blend has been designed, it is easy to elaborate this into a multiproduct model. Such a model allows the producer to compute least-cost blends for a number of mixing vats used simultaneously (or consecutive blends for the same vat), even though different products are specified for each vat. As Figure 6 indicates, a multiproduct model consists of a set of submatrices, each of which has the appropriate constraint rows to meet a set of blend specifications, and each of which has a unique designation for its column activities. That is, the raw material mnemonics for Vat 1 may be prefixed with a numeral 1; those for Vat 2, with a numeral 2; and so on. Thus, in a single computer run the producer may determine how much of each ingredient to use in each vat.

Ideally, the model should be solved with no inventory constraints, in order to determine optimal solutions; such constraints can then be introduced, either into a submatrix or into the multiproduct model (reflecting irremedial inventory limitations). The model will then be suitable for computing the optimum distribution of available ingredients for least-cost overall production.

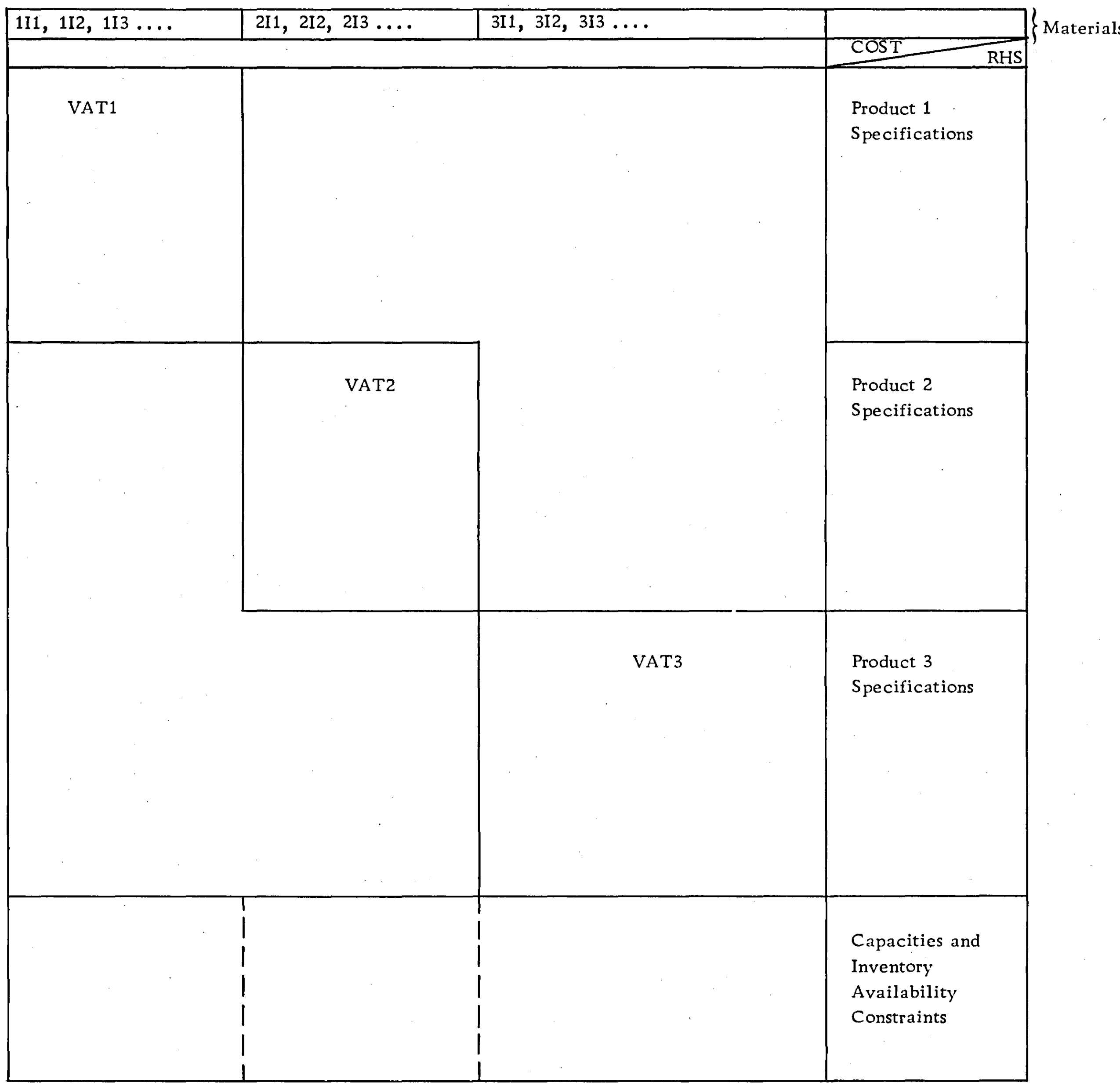

Figure 6. Schematic of matrix — multiple-vat model

The most obvious advantage of the multiproduct LP model is that, in a minimum of computer time, it allocates from all available inventory supplies to all the vats at optimum levels. The result is the least-cost use of both stocks and vats. Further, re-solutions based on output report suggestions will respond to overall considerations of inventory and costs rather than to a single blend problem. Quite conceivably, a multiproduct model solution may indicate that a nonoptimal solution for one blend will allow the best overall use of inventory and vat capacity.

CAPACITY CONSTRAINT

In the preceding discussion we set the yield (row 17) equal to 100 lbs. — that is, the sum of the weights of all the ingredients used in the solution equals 100 lbs. (or 100%). Consequently, the right-hand side of each constraint row reflects the percentage of that element required by the recipe. Thus, for a yield of 100 lbs., the butterfat minimum is 10 lbs. (10%) and the butterfat maximum is 16 lbs. (16%). Despite its convenience, however, such a formulation may introduce difficulties if there are a number of inventory constraints.

Quite conceivably, limitations on the availability of a number of ingredients may make it impossible to blend 100 lbs. of mix. There might not be enough total butterfat, say, or enough sweetness available for the matrix solution to reach 100 lbs. On the other hand, the producer may want to produce more than 100 lbs. — again within limitations generated by inventory availability constraints. It is possible, however, to preserve the convenience of percentage constraints while at the same time solving for the actual capacity which the matrix will allow.

A number of new formulations will permit such a solution. In one of them, we establish a new activity column (QBP, Figure 7) which represents the quantity of blend produced and a new constraint row (QBD) which sets the quantity of blend produced less than or equal to the amount desired.

Now, as Figure 7 indicates, we can solve the total butterfat rows (rows 2 and 3) for zero. Row 2 indicates that butterfat from all sources is equal to or greater than 10 lbs. Hence, butterfat from all sources minus 10 is equal to or greater than zero. Similarly, butterfat from all sources minus 16 is equal to or less than zero. We perform this manipulation for all the right-hand sides down to the yield row (row 17). Here, the total yield minus the quantity of blend produced (QBP) equals zero. We then set the quantity of blend produced equal to or less than the specific amount desired — in this case, 250 lbs. This formulation allows the use of percentage specifications regardless of the total amount desired or the possible impact of inventory constraints.

One additional feature must be incorporated to make the new formulation meaningful. Since the new capacity row is an inequality which provides only an upper bound, and the linear program is designed to minimize costs, the solution will indicate that no blend should be produced because such a solution would cost the least. Consequently, the new activity column (QBP) must be provided with a cost which would enable the LP system to operate meaningfully. Hence, the projected selling price per pound (here taken as 75¢) should be introduced into the quantity-of-blend-produced matrix column as a <u>negative</u> cost.

QBP	RHS	
-10	≥ 0	MIN BF
-16	≤ 0	MAX BF
-10.5	≥ 0	MIN MSNF
-13	≤ 0	MAX MSNF
-20.5	≥ 0	MIN TMS
-25	≤ 0	MAX TMS
-11	≥ 0	MIN SUG
-17	≤ 0	MAX SUG
- 6	≤ 0	CSS
-37.5	≥ 0	MINTS
-41.5	≤ 0	MAXTS
-58.5	≥ 0	MIN H20
-62.5	≤ 0	MAX H20
- .37	= 0	STAB
- .10	= 0	EMUL
- 1	= 0	YIELD
1	≤ 250	QBD (CAPACITY)

Figure 7. Modification of matrix showing new column and row to introduce capacity solution

Since the LP system is designed to minimize cost, the computer will make the new variable (associated with a high negative cost) as large as possible — limited, of course, by the upper bound on the right-hand side. Such a formulation provides, in the output reports, not the cost of the optimal blend, but rather the profit — that is, the difference between the cost and the projected selling price of the mix. Figure 7 illustrates the new column and row — the remainder of the matrix being unchanged.

OUTPUT REPORTS

The linear programming system (for the IBM 1620/1311) employs the input data described previously to compute four basic output reports.

The first of these is called the basis variables (BASIS. VARBLS) report. It provides a list of the quantities of all the ingredients required in an optimal blend.

The second is the check report, which displays graphically the relationship between the specification bounds and the actual solution. Further, it provides the price of the optimal solution.

The third is the DO.D/J report, which is made up of two parts. The first part lists all the ingredients that are employed in the optimal solution at a bound. Often the bound is zero; the report indicates for each such ingredient its current cost and the amount the cost must drop before it reaches a level at which the ingredient may be introduced into the basis — that is, the optimal solution of the problem. When an upper bound restrains the ingredient, the DO.D/J report indicates the highest price at which that material would remain in the optimal solution at its bound.

The second part of the DO.D/J report lists all of the constraint row mnemonics and, for each constraint, indicates the cost of changing the right-hand side of the equation by one unit.

The fourth report is the cost range (COST.R) report, which indicates for each ingredient employed in the optimal solution the following data: current cost, highest cost before the quantity in the optimal solution changes, what ingredient would enter the optimal solution if that highest cost were exceeded, the lowest cost before the quantity in the optimal solution changes, what ingredient would enter the optimal solution if the ingredient dropped below that lowest cost.

Each of these reports is discussed and illustrated below.

Basis Variables Report

The basis variables report (Figure 8) produced by the LP system shows the optimal variables and activity levels — that is, the solution — of the problem. Each ingredient to be used, along with the quantity required, is listed. This report, if it is to be implemented without change, can be used as a production order in the plant and, as a record of ingredients expended, by the inventory accounting department.

VARBLS	NAME	ACTIVITY LEVEL
	I2	20.285
	I6	53.814
	I13	4.432
	I19	4.000
	I20	11.000
	I22	6.000
	I25	.370
	I26	.100

Figure 8. Basis variables report — optimal variables and activity levels

Check Report

The check report (Figure 9) indicates, for each element in the blend recipe, the relationship between the quantity required in the optimal solution and any bounds that have been imposed by either specification or inventory constraint. In our problem, butterfat (BF), nonfat milk solids (MSNF), total milk solids (TMS), and total solids (TS) are present in the optimal solution at their lower bounds, while corn syrup solids (CSS) is present at an upper bound. The report is a blend analysis that provides a graphic display of the product's composition and relative quality.

If some ingredient composition analysis were based on an estimate and the report indicated its use at an upper or lower limit, errors in the estimate might result in a violation of recipe specifications. Thus, the appearance of an estimated-composition ingredient in the check report with activity at a constraint limit signals the need for specific estimate verification. If, for instance, the butterfat content of some materials were overestimated, an optimal solution with butterfat at a lower bound might violate the butterfat specification. Such a condition could be corrected by raising the butterfat minimum constraint and re-solving.

ROW NAME	UPPER LIMIT	SOLUTION VALUE	LOWER LIMIT
BF	16.00	10.00	10.00
MSNF	13.00	10.50	10.50
TMS	25.00	20.50	20.50
SUG	17.00	14.00	11.00
CSS	6.00	6.00	
TS	41.50	37.50	37.50
H20	62.50	62.03	58.50
YIELD	100.00	100.00	
COST		9.69	

Figure 9. Check report

DO.D/J Report

The DO.D/J report, often called the reduced costs report, consists of two parts (Figures 10 and 11, respectively). The first part may be thought of as listing all the problem activities (ingredients) which enter the solution at a bound. Most often that bound is zero, an indication that the ingredient, at its specific price, is not used in the optimal solution. The first line in Figure 10, for example, indicates that no 40% cream (I1) is present in the optimal solution, because its current price, 27.90¢ per pound, is too high. If the price of 40% cream were to drop by more than 0.263¢ to less than 27.637¢ per pound, this ingredient would then enter the optimal solution and some other variable would drop out. Similarly, if 3.2% milk (I3) were to drop from 3.2¢ per pound to less than 3.028¢, then this ingredient would enter the optimal solution; and so on down the list of nonoptimal variables.

REDUCED COSTS DO.D/J

VARIABLE TYPE	NAME	CURRENT COST	REDUCED COST	BASIS VALUE
	I1	27.900	.263	27.637
	I3	3.200	.172	3.028
	I4	3.200	.036	3.164
	I5	3.300	.068	3.232
	I7	3.400	.032	3.368
	I8	3.500	.064	3.436
	I9	3.500	.008	3.492
	I10	3.600	.040	3.560
	I11	3.700	.004	3.696
	I12	1.800	.910	.890
	I14	3.900	.804	3.096
	I15	4.900	1.572	3.328
	I16	4.500	.940	3.560
	I17	14.800	3.707	11.093
	I18	15.00	1.079	13.921
	I19	10.700	.174	10.874
	I21	9.900	3.117	6.783
	I23	9.000	2.000	7.000
	I24	6.600	1.031	5.569
	I25	55.000	55.155	.155
	I26	78.000	78.155	.155

Figure 10. DO. D/J report — nonoptimal variables

In this example, however, we also established upper constraint limits on the use of a number of ingredients, and several — notably, dry whey solids (I19), stabilizer (I25), and emulsifier (I26) — are present in the optimal solution at an upper bound. The report indicates that the limitations on these materials are forcing the price of the final product up and that, for example, even if the cost of dry whey solids (I19) were 0.174¢ higher, for a total cost of 10.874¢ per pound, additional whey solids would result in a lower total cost for the end product.

The second section of the DO.D/J report (Figure 11) provides an analysis of the impact that specifications (constraint rows) have on the price of the end product. The first line reveals that the minimum butterfat (BF) requirement forces the price up. In the neighborhood of the optimal solution, a decrease of one pound of butterfat in the specification would result in a 56.304¢ saving in the total price of the end product. Similarly, a decrease in the minimum total milk solids (TMS) specification would result in a 1.254¢ saving in the total price. On the other hand, an increase of one pound in the maximum corn syrup solids (CSS) allowable would result in a 3.2¢ saving in the total price. These figures graphically reveal the cost of quality and suggest that re-solutions with slightly relaxed specifications may result in significant cost reduction.

RHS ANALYSIS

TYPE	NAME	INCREMENT VALUE	DECREMENT VALUE
+	BF	-	56.304
+	TMS	-	1.254
0	CSS	3.200	-
+	TS	-	10.355

Figure 11. DO. D/J report — requirements

Cost Range Report

The quantity of each ingredient required (given in the basis variables report) will remain unchanged within the cost range indicated by the cost range (COST.R) report (Figure 12). For example, 20.285 lbs. of 38% cream (I2) would still be required in an optimal solution even if it cost 26.466¢ per pound. Similarly, the same amount would be required if the cost dropped to 25.012¢ per pound. If the price of 38% cream exceeded 26.466¢, however, certain changes in the activity levels of the ingredients in the optimal blend would occur, because an ingredient which is bounded would move away from its bound. In this case less whey (I19), which is present in the optimal solution at its upper bound of 4 lbs., would be required — other ingredients would be allocated to fill in the slack left by the reduced amount of whey and the reduced amount of 38% cream if the cost of the cream exceeded the cost range limit. Thus, if the cost does change past the range indicated, the problem should be re-solved.

COSTR

NAME	CURRENT COST	HIGHEST COST	HIGH VARIABLE	LOW VARIABLE	LOWEST COST
I2	26.3	26.466	I19	I18	25.012
I6	3.3	3.304	I11	I18	3.095
I13	7.6	7.751	I18	I19	7.576
I20	10.2	11.454	TMS	TS	.155
I22	7.0	8.289	I24	-	INFINITE

Figure 12. Cost range report

On the other hand, if the cost of 38% cream were to drop below 25.012¢, dry buttermilk (I18) — which is not used in the optimal blend (that is, is bounded at zero) — would move away from its bound and enter the optimal blend to replace some of the cream.

Again, 3.6% milk (I6) would remain at its 53.814-lb. activity level in the optimal blend within a 3.095¢ to 3.304¢ cost range. If the cost exceeded 3.304¢, some of it would be replaced by 4.2% milk (I11); if the cost fell below 3.095¢, some of it would be replaced by dry buttermilk (I18). The cost range report provides a good measure of sensitivity to price changes since it indicates at what prices the optimal solution will change and what ingredients may be used most appropriately to substitute for unavailable or overpriced stock. The producer is alerted to those price changes which, if incorporated into the matrix for re-solution, will affect the composition of a least-cost blend.

SUMMARY

The foregoing discussion has demonstrated the applicability of linear programming techniques in ice cream production. To that end we constructed an LP model designed to solve a typical production problem. Though the problem was simplified by assuming that most of the materials were available in unlimited quantities, the LP model formulated here can be readily adapted to reflect inventory limitations, recipe changes, and basic ingredient changes.

Construction of the basic LP model entails little more than organizing, in a special format, the data historically used in calculating ice cream blends. Once constructed initially and converted to input media for computer processing, the model becomes a master record. It can be updated regularly to account for new conditions such as the addition or deletion of activities, changes in inventory constraints, changes in costs, and changes in specifications. In short, it can be made to respond immediately to every change in market and production conditions.

Further, the various output reports alert the producer not only to the specific optimal solution of the problem at hand, but also to a variety of relationships any one of which may profoundly influence the total cost of the final blend. The computer thus enables the producer to re-solve the same problem rapidly with a number of variations suggested by the output reports. He can, in effect, use the program as a model to aid in the solution of a series of different problems: What if the price of each raw material varies? What if certain inventory purchases are possible at specific prices — can the inventory be forced economically into future production requirements? What if quality controls vary? Because it can answer all these questions, the linear programming system enables the producer to make the most judicious policy decisions in blending, quality control, inventory control, purchasing, and product research. It makes possible continuous management study resulting in decreased costs, increased efficiency, and maximum profits.

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