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TO:

All Fortran Users

FROM:

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SUBJECT: Subroutines in Fortran

INTRODUCTION

This memorandum discusses at some length the five Fortran subroutine types. Included are descriptions of the essential differences between them, their purpose, definition, and proper usage. Special problems associated with the use of subprograms are discussed in detail.

This is essentially a coordination and expansion of material gathered from the Fortran manuals published by IBM.

GENERAL DESCRIPTION

A subroutine may be considered any pre-defined sequence of instructions which performs a specific operation and which may be executed repeatedly, at various points within a program.

Fortran provides two kinds of subroutines. One kind is the set of <u>functions</u>, of which there are four types. The other kind are known as Subroutine subprograms. The functions bear a close resemblance in many ways to "functions" in the mathematical sense; Subroutine subprograms may also represent mathematical functions, but they lack certain resemblances and play in fact a much more general role.

In mathematics, a function is a quantity whose value depends upon the values of variables (arguments of the function) in a fixed relationship. The function

$$f(x) = Ax^2 + Bx \tag{1}$$

implies a series of operations to be carried out on x, regardless of the value of x; however, f(x) itself is a quantity, having a different value for each value of x. Thus. one may write

$$g(x,y) = \frac{f(x) - f(y)}{x - y} , \qquad (2)$$

illustrating the status of f(x) as quantity, as well as the interchangeability of arguments. Lastly, one might define a variable z, such that

$$z = g(x,y) f(x).$$
 (3)

In Fortran, the various functions have much the same status. Equation (1) is in effect the definition of the function; this could be translated in Fortran (as one type of function) as follows:

FIRSTF(X) = A*X**2 + B*X.

Equation (2) illustrates the use of the function, or its appearance as a variable, in an arithmetic statement, (which is incidentally another function definition). In Fortran, this might be written

$$SECØNDF(X,Y) = (FIRSTF(X) - FIRSTF(Y))/(X-Y)$$

while equation (3) would become

 $Z = SEC \emptyset NDF(X,Y) * FIRSTF(X).$

It becomes clear then that a function in Fortran has the following properties:

- 1) Its definition is distinct from its use, or call. It is called within an arithmetic statement where the function value is an operand in the statement.
- 2) Hence, the <u>name</u> of a function is the name of its <u>value</u>. During execution the instruction sequence that comprises the subroutine is carried out, and the result replaces the function name in the arithmetic statement.
- 3) The arguments given in the definition of a function are replaced, when the function is referenced in an arithmetic statement, by the arguments required at that time. Hence, the arguments in the definition are in effect dummy arguments.

The only major restriction on Fortran functions which is not generally applied to mathematical functions is that Fortran functions are always single-valued.

In Fortran, functions, then, are subroutines which represent single-valued functions of single or multiple arguments, called within an arithmetic statement, where the name of the subroutine is the name of the function value.

A Subroutine subprogram is by nature a more general type of subroutine. It may, in fact, be just like a function

in its definition, in the status of its arguments, and in its mathematical significance. It may be quite different, however, in all these respects, and is <u>always</u> different in the following:

- 1) A Subroutine subprogram is called by a separate special statement which initiates the execution of the instruction sequence which it comprises.
- 2) Hence, the <u>name</u> of a Subroutine subprogram corresponds not to a value but to the instruction sequence itself.

In addition, a Subroutine subprogram may produce multiple outputs. Its "arguments" may be not only arguments in the strict mathematical sense but may be input of any sort from the calling program to the Subroutine subprogram; in particular, the argument list may include names to be assigned values computed by the subroutine.

FUNCTIONS - DEFINITION

There are four types of functions in Fortran. The first two are used, but not in general defined, by the Fortran programmer. The second two are used <u>and</u> usually defined by the programmer.

- 1. <u>Built-in Functions</u>. These exist in the Fortran processor. They differ from all the other subprogram types in being open; that is, the instruction sequence is generated by Fortran and incorporated into the program, during compilation, at each occurrence in the source program of the function name.
- 2. <u>Library Functions</u>. These are independent subprograms which exist in a program library, either as a card deck or on a library tape. They are originally coded in machine language according to definite conventions.
- 3. Arithmetic Statement Functions. These are defined by an arithmetic statement in the calling program. Following their definition they are used, or called, by other arithmetic statements in the same program.
- 4. Function Subprograms (also called Fortran Functions). These are independent subprograms defined by separate compilation. The first statement in the definition program has the form

FUNCTIØN name(arg₁, arg₂, ... arg_n)

which identifies the subprogram as being of function form and implies the identification of the value of the function with the specified name.

SUBROUTINE SUBPROGRAMS - DEFINITION

These are all of one type. They are independent subprograms defined by separate compilation. The first statement of the definition program has the form

which identifies the subprogram as being of Subroutine subprogram form and may imply the identification of one or more of the argument names with the value(s) computed by the subroutine. The execution of the subroutine is effected by a statement in the calling program of the form

These five types of subroutines may be differentiated on the basis of their form (restrictive or not) and also on the basis of the generality of their application (i.e. distinguishing functions frequently used by most programs from special-purpose routines). One may expect that those most commonly used should be most readily available, or simplest to use.

- 1. The <u>built-in functions</u> are a special case, being open routines. In fact, they look like subroutines from the vantage point of the source program but not from that of the object program. Since the generated coding is reproduced in the object program each time the function is referenced, such routines must be limited to functions which are simple in terms of machine-language coding. There is a decrease in execution time in the object program, though at the expense of memory space. Since these routines are implicit in the Fortran Processor (occupying valuable space and time in it) they must be restricted to functions which are of the <u>most</u> general utility and which become an extension of the Fortran language itself, rather than being germane to its applications. For such basic functions, the open type combines the simplicity of subroutine usage in the source program and the efficiency of machine-language straight line coding in the object program.
- 2. <u>Library functions</u>, as the name implies, are of sufficiently general utility to be permanently maintained by the computer installation. The most commonly used of these are maintained on a special tape which is automatically available to every program running in the system. Others are maintained as binary decks also readily available to the user. Functions made available in this form are likely to include those most often needed by the user of a particular <u>installation</u> -- their generality, in other words, is peculiar to the principal applications of one computer.

Library functions, since they are pre-written and used frequently, are coded in machine language mainly in the interests of economy. They are subprograms with a unique convention for

linkage which demands machine-language coding.

- 3. Arithmetic statement functions are the first and simplest of the subroutines written by the user. They are restricted to those functions which may be expressed in a single Fortran arithmetic statement. They satisfy, however, the basic requirements of a subroutine -- i.e., repeated availability without redefinition, and interchangeability of arguments. The limitations on their definition are counterbalanced by a distinct advantage over subprograms: since they are internal to the calling program, all the variables of the calling program are available to the subroutine as arguments and variables, without the problems of transmission introduced by the use of other subprograms.
- 4. Function subprograms are used for the computation of functions which may not be defined by a single arithmetic statement. They may, therefore, be mathematically as complicated as necessary. (We may note here that at one end of the scale are subroutine types which are highly restrictive but which compute functions of the most common type; on the other end are subroutine types of the most generalized form, used for more special applications.)

Function subprograms are principally used for mathematical operations. This is implicit in the major restriction placed on them, that they compute a single quantity which takes its place in the arithmetic statement which referenced the subroutine. It is possible to extend the application of these programs so that, for example, they may compute many other quantities as well, or may perform additional non-arithmetic operations. Their most usual application, however, and the one for which they are intended, is the computation of a single-valued arithmetic function.

5. <u>Subroutine subprograms</u>, finally, are the most general form of subroutines available. They can be used to do anything a Function subprogram does, but are even less restrictive. Probably their most fitting applications are for the computation of multi-valued functions (although this can also be done indirectly by means of a Function subprogram) and especially for the execution of non-arithmetic operations (e.g. input and output).

FUNCTIONS - NAMING AND CALLING

1. Built-in, Library and Arithmetic Statement functions are all named and called in the same manner.

The name consists of 4 to 7* alphanumeric characters with

Double precision and complex <u>Library</u> and <u>Built-in</u> functions: 4 to 6 characters.

these restrictions: the last character must be F; the first character must be alphabetic, and is X if and only if the value of the function is of fixed point mode.

The mode of a function and of its arguments is determined by the function definition. Hence, a change in the mode configuration would require a new definition and a new name.**

Examples:

Built-in

I = (M+N/2)*XMØDF(M,N)

X = DELTAX + MAX1F(YA, YB, YC, 10.0)

Library

 $B = A + C \emptyset SF(A)$

M = XDETRMF(NN, N, AMATR, D)

Arithmetic Statement

(Definition) TESTF(ARG1, ARG2) = (A/ARG2)*(ARG1-ARG2)

A = 3.0 * DELTA

GAMMA = ALFA*TESTF(ALFA, BETA)

Function Subprograms. The name consists of 1 to 6 alphanumeric characters with these restrictions: the last character must not be F unless the entire name is less than 4 characters; the first character must be alphabetic and must govern the mode of the function according to the rules for ... naming variables. This name is used to call the function in the same manner as the other functions. No F is attached to the function name when it is used in a statement.

Examples:

(Use)

(Definition) FUNCTION FUNK(A, B, C)

FUNK = P+Q*B*C

RETURN

^{**} There are no fixed-point double precision or complex function names. The names of double-precision and complex <u>Library</u> and <u>Built-in</u> functions are listed with a prefix D or I. This prefix must <u>not</u> be used in function references.

END

(Use): A = FUNK(DELTA, X, 25.) + ALPHA

SUBROUTINE SUBPROGRAMS - NAMING AND CALLING

The name consists of 1 to 6 alphanumeric characters with these restrictions: the first character is alphabetic; the last character must <u>not</u> be F unless the entire name is less than 4 characters. Since the name of the subroutine is not the name of a value, the name does not affect the mode of any of its values.

The names of values computed by the subroutine may be designated in the argument list. These govern the mode of the values they represent. There must be correspondence in the mode of each argument in the subroutine definition with the respective arguments in every call for the subroutine.

EXAMPLES:

(Definition) SUBROUTINE LOVE (ABLE, BAKER, NAN)

. . .

 $NAN = \dots$

RETURN

END

(Use)

X = X + DELX

CALL L \emptyset VE(X,Y,M)

Y = Y**M

DOUBLE-PRECISION AND COMPLEX FUNCTIONS AND SUBROUTINES

An arithmetic statement referring to a double-precision or complex function is prefixed with a D or I in column 1. A D or I prefixing a CALL statement signifies that all floating-point arguments in the statement are double-precision or complex, respectively.

A double-precision or complex Arithmetic Statement function definition must of course have D or I in column 1.

EXAMPLE:

D AREAF(R) = SQRTF (PI*R**2)

USE OF FUNCTION AND SUBROUTINE NAMES AS ARGUMENTS

The argument list of a Function subprogram or a Subroutine

subprogram may include the names of Library functions, other Function subprograms, or other Subroutine subprograms (but not Built-in or Arithmetic Statement functions). As with any other argument, a dummy function or subroutine name appears in the argument list of the subroutine definition. The following rules apply:

1. The name of a Function subprogram, Subroutine subprogram, or Library function used in an argument list must appear also in a pseudo-statement, in the calling program, of the form

F NAME1, NAME2, ... NAMEn (F in column 1).

- 2. In the case of a Library function the terminal F is dropped in the argument list and also in the F pseudo-statement.
- 3. In the case of a double-precision or complex <u>Library</u> function, the name appearing on the F card is prefixed with a D or I.

EXAMPLES:

SUBRØUTINE PRIME(DUMMY, X, A)

. . .

A = DUMMYF(X)

• •

This would allow the selection of any library function when calling PRIME, as

CALL PRIME (SIN, ALPHA, Q)

CALL PRIME(C¢s.ALPHA.P)

F SIN, CØS

Again,

FUNCTION PETER (DUMMY, X, A)

PETER = DUMMY(Y)

. . .

allows the calling program to specify various Function subprograms when calling PETER, as X = Y + PETER(FIRST, ARG, Al)

 $A = PETER(SEC \not OND, ARG, A2)$

• • •

F FIRST, SECØND

Finally,

SUBROUTINE PAUL(DUMMY, X, A)

• • •

CALL DUMMY(X)

. . .

allows the calling program to specify subroutine subprograms when calling PAUL, as

CALL PAUL(SUBR, EX, EA)

• • •

F SUBR

Note that the <u>definitions</u> (though not the argument list) of Arithmetic Statement functions, as well as of Function subprograms and Subroutine subprograms, may involve other functions. An Arithmetic Statement function definition may involve other Arithmetic Statement functions which have been previously defined in the same program.

RULES FOR DIMENSIONING SUBROUTINE ARGUMENTS

The arguments in a SUBROUTINE or FUNCTION statement (the dummy arguments of the subprogram definition) may not be subscripted. No subscripted variables may appear in the statement defining an arithmetic statement function. An array may be an argument; the array name (non-subscripted) appearing in a CALL statement or function subprogram reference must of course be dimensioned in the calling subprogram; the corresponding dummy argument of the subprogram definition must be dimensioned in the subprogram and the dimensions must correspond in both calling and called programs.

In fact, the dimensions of the dummy array in a subprogram need not be exactly the same as those of the corresponding array in the calling program. Variation is permitted as follows:

The dimension of a one-dimensional array may vary;

The second dimension of a two-dimensional array may vary:

The third dimension of a three-dimensional array may vary;

in other words, for an n-dimensional array, the first (n-1) dimensions must correspond exactly between subprograms.

A single element of an array may be used in the argument list of a calling statement. It is, of course, subscripted, and corresponds to a single non-subscripted dummy variable, as:

Calling:

Called:

CALL SUB(A, B(I, J))

SUBROUTINE SUB(X,Y)

On the other hand,

Calling:

Called:

DIMENSION B(15, 10)

SUBRØUTINE SUB(X,Y)

. . .

CALL SUB(A, B)

DIMENSION Y(15, 10)

transmits the whole array B.

An exception to the above occurs in the case of double-precision or complex variables. Given a double-precision or complex array (say ALPHA, dimensioned (n,n)) where it is desired to use a single element ALPHA(I,J) as an argument to a subprogram, then the corresponding dummy argument in the sub-program must be given like dimensions (n,n) in a double-precision or complex DIMENSION statement. Otherwise ALPHA(I,J) must be set equal to a single variable for use in the argument list. For example, the following illustrates both cases:

Calling:

Called:

SUBROUTINE NAME(X,Y)

Y(5,5)

D DIMENSIØN A(10), B(5,5) D DIMENSIØN

D C = A(5) D Y = X + (2.,1.)

D CALL NAME (C, B(1,3)) RETURN

Dimensioning Y in the subroutine enables the FORTRAN processor properly to compute the address of the imaginary or least significant element of the pair represented by B(1,3). Reference is then made in the subprogram to the variable Y, which is in fact the first element, i.e. Y(1,1), of an implicit array. Use of subscripts (other than (1,1)) with Y should be employed only with great care and a thorough understanding of the ordering of arrays in storage.

USE OF COMMON FOR SUBPROGRAM LINKAGE

Variables may be transmitted to a function or subroutine subprogram by the use of CDMD storage. Consider the arithmetic statement function

FIRSTF (X,Y) = (A*X+B)/Y.

The arguments X and Y are dummy arguments and will be replaced when the function is called; A and B, however, are variables which always remain the same (though not necessarily of the same value). This sort of arrangement is possible because the arithmetic statement function is, though closed, contained within the calling program.

In the case of a function or subroutine subprogram, however, where the subroutine is an independent program and all the programs are relocatable, variables like A,B which we may call "function constants", must be transmitted from the calling to the called subprogram. One could write

CALL SUBR (X,Y,A,B)
CALL SUBR (P,Q,A,B)
etc..

but it is also possible to use COMMON storage for the purpose.

The statement

CØMMØN A, B,

assigns the given variables to specific non-relocatable locations in upper core. The first variable is assigned to 774618, the second to 774608, and the rest in order proceeding downward. Successive COMMON statements within a program will continue to store variables below those already encountered. Each subprogram will again assign the variables listed in COMMON statements to upper core starting at 774618. Thus one may write:

Calling:

Called:

DIMENSIØN T(5,5) SUBRØUTINE SUBR(D)
CØMMØN A,B,T,C DIMENSIØN DUMMY(5,5)
... CØMMØN P,Q,DUMMY,R

CALL SUBR(X)

Because of the correspondence in the order of variables in the two COMMON statements, the pairs A and P, B and Q, and T and DUMMY will be identical.

For most purposes, the simplest and safest way to guarantee consistency among subprograms is to make CØMMØN statements, and DIMENSIØN statements for those variables which appear in CØMMØN statements, identical in all subprograms and the main program. This may easily be done and there is, of course, no need, generally, to change variable names between subprograms. (The use in those examples of different names for arguments in subprogram definitions is primarily to stress their status as dummy variables.) As will be seen from what follows, departures from this procedure should be made with caution.

In the example above, should it be desired to call another subprogram which involves the variable C but not the others, it would be necessary to include in a COMMON statement in the subprogram, dummy variables of proper dimension to assign COMMON storage properly, thus:

SUBRØUTINE NAME(ARG)

DIMENSIØN DUMMY(5,5) (1)

CØMMØN P,Q,DUMMY,R

This is necessary to force correspondence between all CDMMDN assignments.

Additional care must be taken with double-precision or complex variables assigned to COMMON storage, where several subprograms refer to COMMON storage. Consider the example of a main program where A and B are complex variables and T is a complex array:

I DIMENSIÓN T(2,4) CÓMMÓN A,B,T

I ...

I X = A**2+B**2

I CALL ABLE(X)

. . .

Since complex variables consist of two parts, they are stored like arrays. In this example, CØMMØN storage would look like this:

77461 : A (real part)

60 : A (imaginary part)

57 : B (real part)

56 : B (imaginary part)

55 : T(1,1) (real part)

54 : T(2,1) (real part)

53 : T(1,2)

• • • •

77445 : T(1,1) (imaginary part)

The appearance of A and B in complex arithmetic statements, and of T in a complex dimension statement, guarantees that double storage will be assigned. Suppose now that the subprogram ABLE is to operate on the array T, but does not involve A or B. If we write

SUBRØUTINE ABLE(D)
DIMENSIØN DUMMY(2,4)
CØMMØN P,Q,DUMMY

. . .

Ι

then P and Q are for the purpose of "spacing down" to the location of the array in upper core. Since we have assumed, however, that P and Q never appear again in ABLE, there is within this independent subprogram no indication that P and Q are complex variables. Hence COMMON storage will look like this:

77461 : P 77460 : Q

77457 : DUMMY (1,1) (real part) 77456 : DUMMY (2,1) (real part)

etc.

and the desired correspondence between the arrays T and DUMMY is not accomplished.

The simplest way to circumvent this problem is to invlude all complex (double-precision) variables assigned to COMMON in a complex (double-precision) DIMENSION statement. Thus, in the example above, if we write

SUBRØUTINE ABLE(D)

I DIMENSIØN DUMMY(2,4), P(1), Q(1) CØMMØN P,Q,DUMMY

The dimensions of P and Q are not really changed but their status as complex variables is established, and each will be assigned two locations.

Clearly, the use of CØMMØN assignments may be applicable not only to what we have called function constants, but to arguments as well. Arguments which are transmitted to a subprogram by being placed in CØMMØN storage are implicit arguments, as opposed to those explicitly mentioned in the subprogram argument list. Again, it is only necessary to create correspondence in upper core between each argument and its dummy correspondent in the subprogram.

It must be noted that a Function subprogram must have at least one argument. Thus, implicit arguments may be used for all arguments of a Subroutine subprogram, but at least one explicit argument (a dummy, if so desired) must be given for a Function subprogram.

RELATION BETWEEN COMMON AND EQUIVALENCE STATEMENTS

The normal order of COMMON storage assignment, described above, is changed in the case where variables appear in both COMMON and EQUIVALENCE statements. Any COMMON variables appearing in EQUIVALENCE statements will be assigned upper core values above those not in EQUIVALENCE statements regardless of the order of the COMMON statements. For example,

CØMMØN A, B, C, D

EQUIVALENCE (C,G), (E,B)

will cause the following assignments:

77461₈ C and G 60 B and E 57 A 56 D

Note that the variables are assigned in the order in which they appear in the EQUIVALENCE, and then in the order of the C \emptyset MM \emptyset N statement. (Note also that a variable made equivalent to a C \emptyset MM \emptyset N variable is assigned to C \emptyset MM \emptyset N storage.)

Consequently, if equivalences are established involving variables in COMMON storage, similar equivalences should be set up in all subprograms which refer to COMMON in order to force proper correspondence between subprograms.

In the case of double-precision or complex variables, a D or I prefixing a COMMON or EQUIVALENCE statement is ignored, but the double nature of the variables is preserved when they are so defined in other statements within the program. Generally, single-precision variables should not be made equivalent to double-precision or complex variables, nor should they be given the same location in COMMON storage.

TRANSMISSION OF VARIABLE SUBSCRIPTS BETWEEN SUBPROGRAMS

A variable subscript has a somewhat ambiguous status because of the way FORTRAN performs indexing. If a DØ loop (or an input list) has an index I which appears only as index and subscript, there is no storage location set aside for I: the indexing and the subscript reference are taken care of in another way. If I also appears as a variable, either inside or outside of the loop, then there is a storage location assigned to I, and this location is referred to when necessary.

As a result, the storage location "I" may sometimes have a value different from the indexing value of I. For example:

I = N/2 A(I) = X $D \emptyset 1 I = 1, N$ B(I) = X A(I)

1 CONTINUE

When the loop is completed, i.e., when the index I is equal to N+1, the storage location "I" will still contain the value N/2, and any reference to I as a variable or as a subscript will use this value until a new value is assigned or a new loop is initiated. Suppose we continue the above program, following statement 1 by:

NU = I**2 (equivalent to NU = (N/2)**2) DØ 2 I = 1, N ... 2 L = 2*I-1 (equivalent to L = 1,3, ... 2*N-1)

Following statement 2, the storage location of I (i.e., I in the role of <u>variable</u>) will be equal to N+1, the current value of the <u>index</u> I. This is because I is used as a variable within the loop.

Because of the ambiguous nature of subscripts, care must be exercised in transmitting them between subprograms. (N.B.: A variable subscript which is not being used as an index is called, in the FORTRAN manual, a "relative constant". For further details about variable subscripts, see Fortran Reference Manual, pp. 82-85.)

1. In an Argument List

Transmission of variable subscripts via an argument list is straightforward.

I = N/2 SUBROUTINE SUBR(A, N, Y) CALL SUBR(X, I, B) DIMENSION Q(10) Y = Q(N)

The value of I will be transmitted properly. Other examples of correct linkage are:

 $D\emptyset$ 1 I = 1,10 CALL SUBR(X,I,B)

1 ...

where although I is not explicitly used as a <u>variable</u> its appearance in the argument list allows for proper transmission,

and,

CALL SUPER (I, Y) SUBROUTINE SUPER (K, A) K = XFIXF(A**2-X**2)RETURN

where I will be properly transmitted from the calling to the called program and back again. Note, however, that if the CALL statement in this example were within the range of a D \emptyset with index I, the change in I effected by SUPER would not hold, and I would be reset, upon return, to the proper indexing value.

It should be observed that in this last example, the argument K is used both as an input and an output parameter. This is in general a dangerous procedure; any change in the value of input variables may cause difficulties, because it requires considerable understanding of FORTRAN operation to be sure of exactly how this change will be effected.

2. Transmission of Variable Subscripts via COMMON Statements

Consider the following example:

CØMMØN K (1) SUBRØUTINE ABC(X) (6)

$$K = K + 1$$
 (2)
 $A(K) = B(K)**2$ (3) CØMMØN N (7)
CALL ABC(A(K)) (4) ...
TERM = B(K) (5) N = N + 2 (8)

When ABC is called, K has its proper value in COMMON storage and thus N has the same value. Statement (8) changes the value of K in COMMON storage. At execution of statement (5) however, the value of K in COMMON is not used to find the appropriate term in the array B; because of the way FORTRAN is set up the subscript K is determined by a different storage method, and on return from a subprogram which recalculates the variable K, the subscript K will not have been changed.

The statement

$$K = K \quad (4a),$$

inserted between (4) and (5) in the example, will cause the subscript K to be reset by the value of the variable in COMMON.

Now consider this example.

CØMMØN K (1) SUBRØUTINE ABC(X) (4)

... CØMMØN N (5)

DØ 1 K = 1,J (2)

CALL ABC(A(K)) (3)

Throughout the loop on K, the subscript K will be advanced in accordance with the DØ specification; the value of K in CØMMØN storage, however, as transmitted to the subprogram ABC, will remain at whatever value it had before entering the loop. (If statement (2) is the first appearance of K following statement (1), moreover, K in CØMMØN storage will have had no value assigned to it at all.) A statement

$$K = K (2a)$$

inserted within the range of the DØ, or any appropriate statement with K on the left of an = sign, will reset K in CØMMØN storage (i.e., K in its role as <u>variable</u>) in accordance with the values of the index.