

TO: Distribution List

FROM: J. Uskavitch

SUBJECT: MTC Subroutine Library

Group 22 Internal Memorandum No. 22-SA-56

Abstract: Five general-utility subroutines are presented in coded form for use on MTC. Information is also given about average operation time, accuracy, storage requirements and operating details pertinent to each subroutine. Explanatory notes are also included with each one for the aid of programmers.

The subroutines included are:

- (a) Addition, With Scale Factor .....Page 3
- (b) Decimal Printout .....Page 7
- (c) Point Display .....Page 11
- (d) Axes-Grid Display .....Page 17
- (e) Special Square Root.....Page 19

A diagram at the end of the memorandum shows one set of storage registers for eight subroutines for which tapes are available.

General Remarks:

All of the subroutines given in this memorandum have been tested on MTC and are believed to be free of errors. However, if mistakes are found in any cases, the author will sincerely appreciate having his attention called to them.

In developing these subroutines, the writer has attempted to

make them as easy to use as possible. Consequently, time and space have sometimes been sacrificed to increase the ease of programming; moreover, whenever possible, space has been saved in preference to a few microseconds of operating time. However, the writer will sincerely appreciate receiving any information concerning excessive use of either space or time in these subroutines.

As will be noted in the following pages, all addresses have been written as octal numbers, thus minimizing the work needed for transcription into the normally used tapes. However, in all other respects, the notations used are identical with those of Memorandum 6M-3497 including the re-use of the letters a, b, c, d, and e to denote the individual subroutines. The writer has reused these denotations because he did not know what other additions were being made to the subroutine library and consequently did not know just where these subroutines would fit in the library. In any case, when the number of MTC subroutines become large enough, he hopes that Group 62 will re-issue all of them with identical formats in one memorandum.

The duration time given for each subroutine must be considered as only a rough approximation. All operations have been assumed to occur within core memory, and individual execution times were obtained from Drawing B-62366, which follows page 30 of 6M-2527-2. However, in all of the subroutines, the exact operating time depends on the input, and only an approximate averaging was performed over the possible inputs.

The accuracies associated with these subroutines are all derived from round-off errors and were calculated in the same manner as described in 6M-3497. The one exception to this statement is found in subroutine (e), the accuracy of that output being determined in part by that of SRL #7 of 6M-3497.

The general notation used in this memorandum may be summarized as follows:

- pN = relative address of a register in the operating part of a program, where p is a lower-case letter designating a particular subroutine and N is an octal integer.
- KpN = relative address of a register containing a constant used in the program or being used for temporary storage where p and N are as above.
- k = the field in which the particular subroutine operates.
- m = the field in which the main program operates.

Included at the end of this memorandum is a diagram which depicts the storage locations utilized by available tapes of several of the MTC subroutines. The writer gladly offers to lend either the flexo or the converted tape if a reproduction is desired by anyone.

(a) ADDITION, WITH SCALE FACTOR

Input : Store the number ( $N \leq 2^{11}$ ) of the numbers ( $x_i$ ) to be added in Ka3 and the extended address of the first number ( $x_1$ ) in Ka4. Enter at a0 after ra al30 with m. Content of AC is immaterial. (All  $x_i$  must have been previously stored in consecutive registers in a single field.)

Output:  $2^{-n} \sum_{i=1}^n x_i$  (the 15 most significant binary digits of  $\sum_{i=1}^n x_i$ ) in AC, with the scale factor n left in Ka5. (The remaining n digits of the sum will be left in the first n digit positions of BR; i.e., positions 0 through n-1.) If the result is zero, it will be a negative zero, unless all  $x_i = +0$ .

Other Subroutines Needed: None.

Duration: Approximately  $915(N+2)$  microseconds.

Accuracy: Maximum error in  $\sum_{i=1}^n x_i$  is  $2^n$  for  $n > 0$ . (For  $n = 0$ , no error exists.) The answer obtained is exactly that which one would get by just taking the 15 most significant binary digits of the sum without first rounding off. Therefore, the results in the AC is always smaller than the true value, within this tolerance. No error, if the n digits in the BR are also utilized.

Storage: 90 program registers plus 9 constants or temporary storage.

Notes:

- (1) The original numbers ( $x_i$ ) are not destroyed by the addition process.
- (2) If  $x_1$  is in register 376 of field 3, the extended address should be stored as 0.30376.
- (3) The scale factor used is always the smallest one possible. For example, as long as the sum requires no more than 15 digits, no scale factor is used ( $n = 0$ ).

(a) ADDITION , WITH SCALE FACTOR

Program:

(enter)	a0	rf	a131	set up to leave subroutine	
	1	ca	Ka0	+0	} initial conditions.
	2	st	Ka5	n	
	3	st	Ka6	$\sum n$	
	4	st	Ka7	$\leq r$	
	5	ca	a14	srl4	
-----					
(a53) →	6	ra	a45		} prepare for extraction of proper digits
	7	ca	a14	srl4	
	a10	su	a45	sr(14,11,6,3,0)	
	11	ra	a23		
	12	ca	Ka4	extended address of $x_1$	
	13	ra	a21	ca(RCx <sub>1</sub> )	
	14	sr	14		
	15	ra	a20	sof( )	
	16	cs	Ka3	N	
(a37) →	17	st	Ka10	counter	
	a20	sof--			} obtain $x_i$ in AC.
	21	ca --	$x_i$		
	22	sof k			} extract the sign plus three digits, beginning with the 3 most insignificant
	23	sr --	(0,3,6,11,14)		
	24	et Ka2	1.00007		
	25	tn a27			} is the sign negative? if no, transfer to add. if yes, correct extracted number to provide true value of digits.
	26	tr a30			
	27	su Ka2	1.00007		} add to sum of same digits obtained from preceding x's and store.
	a30	ad Ka6	$\sum n$		
	31	st Ka6	$\sum n$		} prepare to extract from $x_{i+1}$ .
	32	ca a21			
	33	ad Ka1	+1		} add +1 to counter.
	34	ra a21			
	35	ca Ka1	+1		} have these digits been obtained from all $x_i$ ? if no, return to pick up $x_{i+1}$
	36	sm Ka10			
	37	tn a17			
-----					
	a40	ca Ka6	$\sum n$		} if yes, shift 3 most insignificant digits of the sum into the BR. Store those left, for addition of next 3 extracted digits.
	41	sr 3			
	42	st Ka6	$\sum n$		

(a) ADDITION, WITH SCALE FACTOR

Program:

a43	sr	15		} obtain the remainder in the AC in correct position for addition to preceding remainders.
44	cr	403		
45	sr	--	(14,11,6,3,0)	
46	ad	Ka7	$\Sigma r$	} add to preceding remainders and store.
47	st	Ka7	$\Sigma r$	
a50	ca	a45	sr(14,11,6,3,0)	} prepare to change the position in the AC of the next remainder.
51	su	a41	sr 3	
52	tn	a54		} have all digits been added? if yes, transfer to combine the two sums $\Sigma n$ and $\Sigma r$ . If no, return to reset initial conditions, and to add next 3 digits.
53	tr	a6		
54	ca	Ka0	+0	} if yes, 15 most insignificant digits are now in $\Sigma r$ , with the larger ones being in $\Sigma n$ . is $ \Sigma n  = 0$ ? if no, transfer to continue.
55	sm	Ka6	$\Sigma n$	
56	tn	a61		
57	ca	Ka7	$\Sigma r$	} if yes, put $\Sigma r$ in the AC, and transfer out.
a60	tr	a130		
61	ca	Ka0	+0	} if $ \Sigma n  \neq 0$ , is $ \Sigma r  = 0$ ? if no, transfer to continue.
62	sm	Ka7	$\Sigma r$	
63	tn	a67		
64	ca	Ka6	$\Sigma n$	} if yes, make $\Sigma r$ the same sign as $\Sigma n$ . (this particular amount of shifting was picked to also provide a needed constant.)
65	sr	21		
66	st	Ka7	$\Sigma r$	
67	ca	Ka6	$\Sigma n$	} are the signs of $\Sigma n$ and $\Sigma r$ the same if yes, transfer to find n, and to combine.
a70	mh	Ka7	$\Sigma r$	
71	tn	a106		
72	ca	Ka6	$\Sigma n$	} if no, proceed to prepare the two sums for proper combining. decrease the magnitude of $\Sigma n$ by 1.
73	tn	a76		
74	su	Ka1	+1	
75	tr	a77		
76	ad	Ka1	+1	
77	st	Ka6	$\Sigma n$	
a100	ca	Ka7	$\Sigma r$	} decrease the magnitude of $\Sigma r$ by 1.
101	tn	a104		
102	su	Ka1	+1	
103	tr	a105		
104	ad	a1	+1	
105	st	Ka7	$\Sigma r$	

(a) ADDITION, WITH SCALE FACTOR

Program:

(a71)→106	ca	Ka6	$\Sigma n$	} to find n, first obtain $ \Sigma n $ .
107	sm	Ka00	+0	
(a110)→a110	st	Ka10	$ \Sigma n $	
111	su	Ka1	+1	
112	tn	a121		} is $ \Sigma n $ , or subsequent remainder =0? if yes, transfer to combine to $\Sigma n$ and $\Sigma r$ .
113	ca	Ka5		
114	ad	Ka1	+1	} if no, add +1 to the scale factor n.
115	st	Ka5		
116	ca	Ka10	$ \Sigma n $	} shift 1 digit out of $ \Sigma n $
117	sr	1		
a120	tr	a110		transfer back to store the remainder and to ask if it is zero.
<hr/>				
(a112)→121	ca	Ka5	n	} prepare to obtain the 15 most significant digits in the AC.
122	ad	a65	sr21	
123	ra	a127		
124	ca	Ka7	$\Sigma r$	} put $\Sigma r$ in digits 0-15 of BR.
125	sr	17		
126	ca	Ka6	$\Sigma n$	put $\Sigma n$ in the AC.
127	cr	--	(21+N)	obtain the 15 most significant digits in AC.
<hr/>				
(a60)→a130	sof	--		} leave subroutine.
131	tro	--		

Ka0	+0	
1	+1	
2	1.00007	extractor
3	--	N = no. of numbers to be added
4	--	extended address of $x_1$
5	--	n = scale factor
6	--	$\Sigma n$
7	--	$\Sigma r$
Ka10	--	counter; $ \Sigma n  \rightarrow 0$

} temporary storage

(b) DECIMAL PRINTOUT

Input: Store the position (n) of the binary point in Kb30: n = +0 for an integer,  $+0 < n < +15$  for a non-integer, n = +15 for a fraction less than one. After ra b53 with m, enter with x in AC either at b0 (if sign is wanted) or at b10 (if only the magnitude is desired).

Output: Prints + or - sign, if wanted. Then, if x is an integer, prints five decimal places, with initial zeros being replaced by spaces; if x is a fraction, then prints a decimal point followed by four decimal digits; if x is a non-integer  $> 1$ , prints the integral part followed immediately by the decimal point and fractional part (each as described in preceding cases). Two spaces are printed after every number.

Other Subroutines Needed: None.

Duration: Approximately 0.125 second for the sign plus an average of 0.9 sec. for an integer or fraction less than one, and 1.8 seconds for a non-integer greater than one.

Accuracy: No error for an integer. A maximum error of 0.0001 (decimal) for both of the other cases. The fraction printed is exactly that which one would obtain by taking only four decimal places of the true value without rounding off; therefore the fraction printed is always less than its true value, within this tolerance.

Storage: 77 (decimal) program registers plus 27 constants or temporary storage.

- Notes:
1. If the integral part of x is known to have less than five significant figures, time and space can be saved by ra Kb24 with (or st Kb24) the address of the register containing +1000, +100, +10, or +1 depending on the number of significant figures.
  2. For those more used to the conventional scale factor, it may be well to note that the position of the binary point is essentially the same as (+15 - n), when the scale factor is  $2^{-n}$ .

(b) DECIMAL PRINTOUT

Program

(enter if sign is wanted.)	→ b0	rf	b54		set up to leave subroutine.
	1	st	Kb31	x	
	2	tn	b5	+	} find sign of x and print.
	3	ca	Kb17	+13(+)	
	4	tr	b6		
	5	ca	Kb20	+29(-)	
	6	pr	400		
	7	tr	b12		
<hr/>					
(enter if only mag. is wanted)	→ b10	rf	b54		
	11	st	Kb31	x	} obtain and store magnitude of x.
	12	cs	Kb31	x	
	13	tn	b15		
	14	st	Kb31	x	} does n = 15? if yes, transfer to print a fraction.
	15	cs	Kb30	n	
	16	ad	Kb7	+15	
	17	tn	b37		
<hr/>					
	b20	su	Kb7	+15	} if no, does n = 0? if n ≠ 0, transfer to print non-integer.
	21	tn	b24		
<hr/>					
	22	tr	b55		} if n=0, transfer to print an integer. return, and transfer to leave subroutine.
(b102) →	23	tr	b50		
<hr/>					
	24	cr	2000		clear BR
	25	ca	Kb30	n	} obtain and store for printing the integral part of x.
	26	ra	b30	sr(n)	
	27	ca	Kb31	x	
	b30	sr	(n)		
	31	st	Kb31	x →  x	} obtain and store temporarily the fractional part of x. transfer to print an integer
	32	cr	420		
	33	st	Kb32	.x	
	34	tr	b55		
<hr/>					
(b102) →	35	ca	Kb32	.x	} return, and prepare to print fraction.
	36	st	Kb31	.x →  x	
(b17) →	37	ca	Kb21	+17(.)	print decimal point.
	b40	pr	400		
	41	ca	Kb31	x	} find 10,000 times the fraction and prepare to print as integer
	42	mh	Kb0		
	43	st	Kb31	x	} prepare to start with a subtraction of +1000.
	44	ca	Kb23	suKb1	
	45	ra	b65		

(b) DECIMAL PRINTOUT

	46	ca	b111	tn b76	set up to print initial zeros.
	47	tr	b60		transfer to print.
-----					
(b23, b102) →	b50	ca	Kb22	+8(space)	} when x has been printed, print two spaces.  } leave subroutine
	51	pr	400		
	52	pr	400		
	53	sof	--		
	54	tro	--		
(b22, b34) →	55	ca	Kb24	su Kb0	} when an integer, prepare to start with subtraction of +10000. prepare to replace initial zeros with spaces.
	56	ra	b65		
	57	ca	Kb25	tn b107	
(b47) →	b60	ra	b66		} set up to return.
	61	rf	b102		
(b106) →	62	ca	Kb26	ca Kb5	} initially set flexo code to print a zero.
	63	ra	b76		
-----					
(b75) →	64	ca	Kb31	x	is  x  or later remainder $\geq$
	65	su	--	(Kb0-Kb4)	10,000 (1000, 100, 10 or 1)?
	66	tn	--	(b76 or b107)	if no, transfer to print the digit or space if initial zero not wanted.
	67	st	Kb31	x	if yes, store the remainder.
	b70	ca	b76		} add 1 to the Flexo-code storage, to be printed. since this digit is $>0$ , prepare to print subsequent zeros. return to subtract again.
	71	ad	Kb4	+1	
	72	ra	b76		
	73	ca	b111	tn b76	
	74	ra	b66		
	75	tr	b64		
(b66, b111) →	76	ca	--	(Kb5-Kb16)	
	77	pr	400		
	b100	ca	Kb27	su Kb3	} is this the last digit? if yes, leave.
	101	su	b65	su Kb0-Kb4	
	102	tn	--	(b23, b35, b50)	
-----					
(b114) →	103	ca	b65		} if no, prepare for subtraction of next smaller power of 10. return to find next digit.
	104	ad	Kb4	+1	
	105	ra	b65		
	106	tr	b62		
-----					
(b66) →	107	ca	Kb27	su Kb3	} if digit is an initial zero, is it the last digit?
	b110	su	b65	su Kb0-Kb4	
	111	tn	b76		} if yes, transfer to print it. if no, print a space.
	112	ca	Kb22	+8(space)	
	113	pr	400		} prepare to find next digit.
	114	tr	b103		

(b) DECIMAL PRINTOUT

Kb0	+10000		
1	+1000		
2	+100		
3	+10		
4	+1		
5	+62	(0)	}
6	+21	(1)	
7	+15	(2)	
Kb10	+7	(3)	
11	+11	(4)	
12	+19	(5)	
13	+27	(6)	
14	+23	(7)	
15	+3	(8)	
16	+54	(9)	
17	+13	(+)	}
Kb20	+29	(-)	
21	+17	(.)	
22	+8	(space)	} Flexo codes
23	su Kb1		
24	su Kb0		
25	tn b107		
26	ca Kb5		
27	su Kb3		
Kb30	--	n ( position of binary point)	
31	--	x,  x  , remainder, etc.	} temporary storage
32	--	.x(fractional part of  x )	

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(c) POINT DISPLAY

Input: Having ra c36 with m, put abscissa (x) in Kc10 and ordinate (y) in Kc11 and enter at c0. Content of AC is immaterial. (The constants required in registers Kc1 - Kc7 and the two cr instructions in c21 and c25 must have previously been stored. See notes below.)

Output: If  $x_{\min} \leq x \leq x_{\max}$  and  $y_{\min} \leq y \leq y_{\max}$ , where the minimum and maximum values are the desired limits for plotting, the point (x,y) will be displayed N times on the oscilloscope with whatever linear scale desired. If either coordinate is outside the set limits, no point is displayed and the program may be halted. (See notes below.)

Other Subroutines Needed: None.

Duration:  $(413 + 76N)$  microseconds.

Accuracy: Each coordinate is displayed with an accuracy of 1 part in  $2^9$ .

Storage: 35 (decimal) program registers, 11 (decimal) constants or temporary storage.

- Notes:
1. Since the intensity of the displayed point may want to be varied—either to distinguish one set of points from another or to provide for easy distinction from a superimposed grid, the number of times (N) that the point is to be displayed must be stored in Kc1.
  2. When the desired coordinate limits have been determined, the coordinates of the middle point ( $x_m, y_m$ ) must be stored in Kc2 and Kc5, respectively. If this point is displayed, it appears at the center (+0,0) of the display.

Examples:	$x_{\max}$	+0.9999	+200	+20000
	$x_{\min}$	-0.9999	+100	-20000
$\frac{1}{2}(y_{\max} + y_{\min})$ :	Kc2	+0	+150	+0
	$y_{\max}$	+100	-1	+0.9999
	$y_{\min}$	+0	-11	+0.5000
$\frac{1}{2}(y_{\max} + y_{\min})$ :	Kc5	+50	-6	+0.7499

3. Since the limits of the display may be smaller than those of the available data, the desired ranges of the coordinates (with a factor of  $\frac{1}{2}$ ) must be stored in Kc3 and Kc6. The factor of  $\frac{1}{2}$  is included to provide for the times when the range is greater than 0.77777. A continuation of previous examples gives:

(c) POINT DISPLAY

$\frac{1}{2}(x_{\max} - x_{\min})$ : Kc3    +.9999    +50    +20000  
 $\frac{1}{2}(y_{\max} - y_{\min})$ : Kc6    +50    +5    +.2499

4. To provide for a display with the proper scales the two scale factors must be stored in Kc4 and Kc7. To be safe, each scale factor must be such that the product of it and the half-range  $\leq +511$ ; i.e.,  $(Kc3)(Kc4) \leq 511$  and  $(Kc6)(Kc7) \leq 511$ . continuing the previous examples:

x-scale factor: Kc4    +511    +10    +.0255  
y-scale factor: Kc7    +10    +102    +2044

5. The cr instructions to be stored in c21 and c25 are determined by the ranges and scale factors used. If both are  $\neq +1$  and short-cycle is used, 12 (octal) places must be cycled; if either is  $\neq +1$ , 31 (octal) places must be cycled. Concluding the previous examples:

program x:            c21            cr431            cr412            cr431  
program y:            c25            cr412            cr412            cr431

6. If the point to be displayed lies outside the predetermined region, one of several actions may be wanted. As programmed here, ha 0 is given and pushing the restart button changes frames and again halts the program. However, if the point should merely be ignored, then the instruction tr c36 can be put in c41; if one wished to return to a different part of the main program when this happens, an sof and a tro instruction can be stored in c41 and c42 respectively.

(c) POINT DISPLAYProgram:

(enter)	c0	rf	c37	set up to leave subroutine.	
	1	ca	Kc10	x	} obtain the difference between x and the median of the range. if this difference produces an overflow, x lies outside desired range; transfer to halt.
	2	su	Kc2	$x_m = \frac{1}{2}(x_{max} + x_{min})$	
	3	to	c40		
	4	st	Kc10	$x (= x - x_m)$	
	5	ca	Kc3	$x/2 = \frac{1}{2}(x_{max} - x_{min})$	} is $x_{min} \leq x \leq x_{max}$ ? if not, transfer to halt.
	6	sm	Kc10	$ x - x_m $	
	7	tn	c40		
-----					
	c10	ca	Kc11	y	} obtain the difference between y and the median of the range. if this difference produces an overflow, y lies outside desired range; transfer to halt.
	11	su	Kc5	$y_m$	
	12	to	c40		
	13	st	Kc11	$y (= y - y_m)$	} is $y_{min} \leq y \leq y_{max}$ ? if not, transfer to halt.
	14	ca	Kc6	$\Delta y/2$	
	15	sm	Kc11	$ y - y_m $	
	16	tn	c40		
-----					
	17	ca	Kc10	$x - x_m$	} scale the distance from display center in x-direction.
	c20	mh	Kc4	x-scale factor	
	21	cr	( )	put in AC in position to display. (note 5.)	
	22	st	Kc10	$x - x_m$	
-----					
	23	ca	Kc11	$y - y_m$	} scale the distance from display center in y-direction.
	24	mh	Kc7	y-scale factor	
	25	cr	( )	put in AC in position to display. (Note 5.)	
	26	st	Kc11	$y - y_m$	
-----					
	27	cs	Kc1	N	} set up counter.
	c30	st	Kc12	counter	
	31	ca	Kc11	$y - y_m$	} display (x,y).
	32	ds	Kc10	$x - x_m$	
	33	ca	Kc0	$\frac{1}{2}$	} add $\frac{1}{2}$ to counter.
	34	sm	Kc12	counter	
	35	tn	c30	if not displayed N times, go back to display again.	
-----					
	36	sof	--	} leave subroutine.	
	37	tro	--		

(c) POINT DISPLAY

c40 (ha 0)      if x or y is not in desired range, halt.  
 41 (op 0)      } upon RESTART, change frame and halt.  
 42 (ha 0)      } (Note 6.)

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Kc0	+1	
1	--	N: no. of times each point is to be displayed
2	--	$x_m = \frac{1}{2}(x_{max} + x_{min})$ : median of desired range.
3	--	$\Delta x/2 = \frac{1}{2}(x_{max} - x_{min})$ : half the desired range.
4	--	x-scale factor
5	--	$y_m = \frac{1}{2}(y_{max} + y_{min})$ : median of desired range.
6	--	$\Delta y/2 = \frac{1}{2}(y_{max} - y_{min})$ : half the desired range.
7	--	y-scale factor
Kc10	--	x; $x - x_m$ ; } temporary storage
11	--	y; $y - y_m$ ; }
12	--	counter

(d) AXES - GRID DISPLAY

Input: First ra c36 with m. Then enter at d0 if a new frame is wanted in the camera, otherwise at dl. Content of AC is immaterial. (The constants required in Kd1-Kd7 must have been previously stored. See notes below.)

Output: A set of orthogonal axes displayed N times on the oscilloscope with any predetermined origin. A set of grid lines, with any predetermined spacings, displayed once to provide linear scales measured from the origin. (See notes below.)

Other Subroutines Needed: (c) Point Display.

Duration: To display axes:  

$$\left[ (134N)(\text{no. of points / axis}) + (134)(\text{no. of points / grid line}) \right]$$
 microsec. Maximum  $\approx 137(N+1)$  milliseconds.  
 For each grid line displayed, add:  

$$\left[ 109 + 67(\text{no. of points / grid line}) \right]$$
 microseconds.  
 Maximum  $\approx 68$  milliseconds / line

Accuracy: No inaccuracy.

Storage: 63 (decimal) program register, 8 constants, and 6 registers of subroutine (c).

Notes:

1. Although this subroutine requires only 6 additional registers to make it independent of the preceding routine, the writer cannot foresee an independent use and consequently has saved 6 registers. The changes required to make this an independent subroutine are believed to be obvious.
2. This subroutine may be much longer than actually necessary because of the many variations permitted: not only may the no. of times (N) that the axes are displayed be changed independently of the no. of times that points are displayed in subroutine (c), but also the densities of the points in the axes and in the grid lines may be changed independently. This great variability has been included since large variations in the display equipment and/or photographic equipment and developing have in the past made it impossible to choose optimum values.
3. The densities of the axes and grid lines are determined by the values of  $\Delta_a$  and  $\Delta_g$  put in Kd2 and Kd3. These numbers are easiest stored as positive octal numbers, the first three digits giving the spacing between points and the last two being zeros. (See Note 6 for an example.)

4. The coordinates of the origin are also most easily stored as octal numbers, remembering that the `ds` instruction only uses the sign plus first three digits. If a coordinate is positive, the last two digits should be zeros, as is the case for the densities; if a coordinate is negative the last two octal digits should be made 77. (See Note 6 for example.)
5. Instead of storing the distance between adjacent grid lines, the half-distances are to be put in Kd6 and Kd7. Kd6 provides the spacing of the vertical lines, while Kd7 gives that of the horizontal ones. The half-distance is used to provide for the case when the desired spacing is greater than 0.77700. Each half-distance can be stored as a positive octal constant with a zero in the last digit, the first four digits producing the spacing. (See Note 6 for an example.) In case no grid lines are wanted, merely store 0.77740 in both registers. However, the axes are always further intensified by one pair of superimposed grid lines.
6. Example: Origin to be located in center of left half-plane. The axes are to be displayed 5 times with only 50% of the points being displayed. Only one out of every eight points is to be displayed in each grid line. The vertical grid lines are to be 100 points apart, whereas the horizontal spacing is to be 175 points.

Kd1	+5	Kd5	0.00000
2	0.00200	6	0.06200
3	0.01000	7	0.12740
4	1.37777 (or 1.40077)		

(d) AXES - GRID DISPLAYProgram:

(enter, if frame to be changed) d0 op 0 change frame.  
 if not, 1 rf c37 set up to return to main program.  
 enter) 2 cs Kd1 N } set up counter,  
 (d25) → 3 st Kc12 counter }  
 4 ca Kd4  $x_0$   
 5 st Kc10  $x$   
 6 ca Kd0 1.00077 } display  
 7 ds Kc10  $x$  } y-axis  
 d10 ad Kd2  $\Delta_A$ ; producing  $y + \Delta_A$ .  
 11 to d13  
 12 tr d7

---

13 ca Kd0 1.00077 } display  
 14 st Kc10  $x \rightarrow 0.77700$  } x-axis  
 15 ca Kd5  $y_0$   
 16 ds Kc10  $x \rightarrow 0.77700$   
 17 ca Kc10  $x \rightarrow 0.77700$   
 d20 ad Kd2  $\Delta_A$   
 21 to d23  
 22 tr d14

---

23 ca Kc0 +1 } add +1 to counter.  
 24 sm Kc12 counter }  
 25 tn d3 if not displayed N times go back to display again.

---

(d34) → 26 ca Kd4  $x_0$   
 27 st Kc10  $x \rightarrow x$ -position of first grid line. } find coordinate of first vertical grid line,  
 d30 su Kd6  $\Delta x/2$  } position has been found when overflow occurs.  
 31 to d35  
 32 su Kd6  $\Delta x/2$   
 33 to d35  
 34 tr d27

---

(d50) → 35 ca Kd0 1.00077 } display vertical grid line.  
 36 ds Kc10  $x =$  coordinate of grid line  
 37 ad Kd3  $\Delta_g$ ; producing  $y + \Delta_g$   
 d40 to d42  
 41 tr d36

(d) AXES - GRID DISPLAY

Program:

(d40) → 42 ca Kc10 x  
 43 ad Kd6  $\Delta x/2$   
 44 to d51  
 45 ad Kd6  $\Delta x/2$   
 46 to d51  
 47 st Kc10  $x + \Delta x$   
 d50 tr d35

find coordinate of next vertical grid line. all vertical lines have been displayed when overflow occurs. if vertical grid incomplete, return to display next line.

→ 51 ca Kd5  $y_0$   
 52 st Kc11  $y \rightarrow$  y-position of first grid line  
 53 su Kd7  $\Delta y/2$   
 54 to d60  
 55 su Kd7  $\Delta y/2$   
 56 to d60  
 57 tr d52

find coordinate of first horizontal grid line. position has been found when overflow occurs.

(d54, d56, d76)

→ d60 ca Kd0 1.00077  
 61 st Kc10  $x \rightarrow 0.77700$   
 62 ca Kc11  $y =$  coordinate of grid line.  
 63 ds Kc10  $x \rightarrow 0.77700$   
 64 ca Kc10  $x \rightarrow 0.77700$   
 65 ad Kd3  $\Delta g$   
 66 to d70  
 67 tr d61

display horizontal grid line.

d70 ca Kc11 y  
 71 ad Kd7  $\Delta y/2$   
 72 to c36  
 73 ad Kd7  $\Delta y/2$   
 74 to c36  
 75 st Kc11  $y + \Delta y$   
 76 tr d60

find coordinate of next horizontal grid line. all horizontal lines have been displayed when overflow occurs; at that time, transfer to leave. if horizontal grid incomplete, return to display next line.

Kd0	1.00077	
1	--	N; no. of times to display axes.
2	0.----00	$\Delta_A$ : distance between points in an axis.
3	0.----00	$\Delta_g$ : distance between points in a grid line.
4	--	$x_0$ } coordinates of origin.
5	--	$y_0$ }
6	0.-----0	$\Delta x/2$ } half spacings between grid lines.
7	0.-----0	$\Delta y/2$ }

(e) SPECIAL SQUARE ROOT

Input: Enter at e0 with the output of the SQUARE ROOT subroutine (SRL #7 of memorandum 6M-3497) in AC, having previously stored the scale factor (n) of x in Ke0. (See Notes.)

Output:  $+\sqrt{x} \cdot 2^{-n}$  in AC, when x is stored as  $x \cdot 2^{-n}$ . (See Notes)

Other Subroutines Needed: SRL#7, assumed to be stored in field k.

Duration: 159 microseconds, including the time spent after re-entering SRL #7.

Accuracy: Maximum error in  $\sqrt{x}$  is  $0.00090 \cdot 2^{n/2}$  (decimal), including that possible in the original output of SRL #7 ---- this error requiring a scale factor of  $2^{-n}$ , the same as the output.

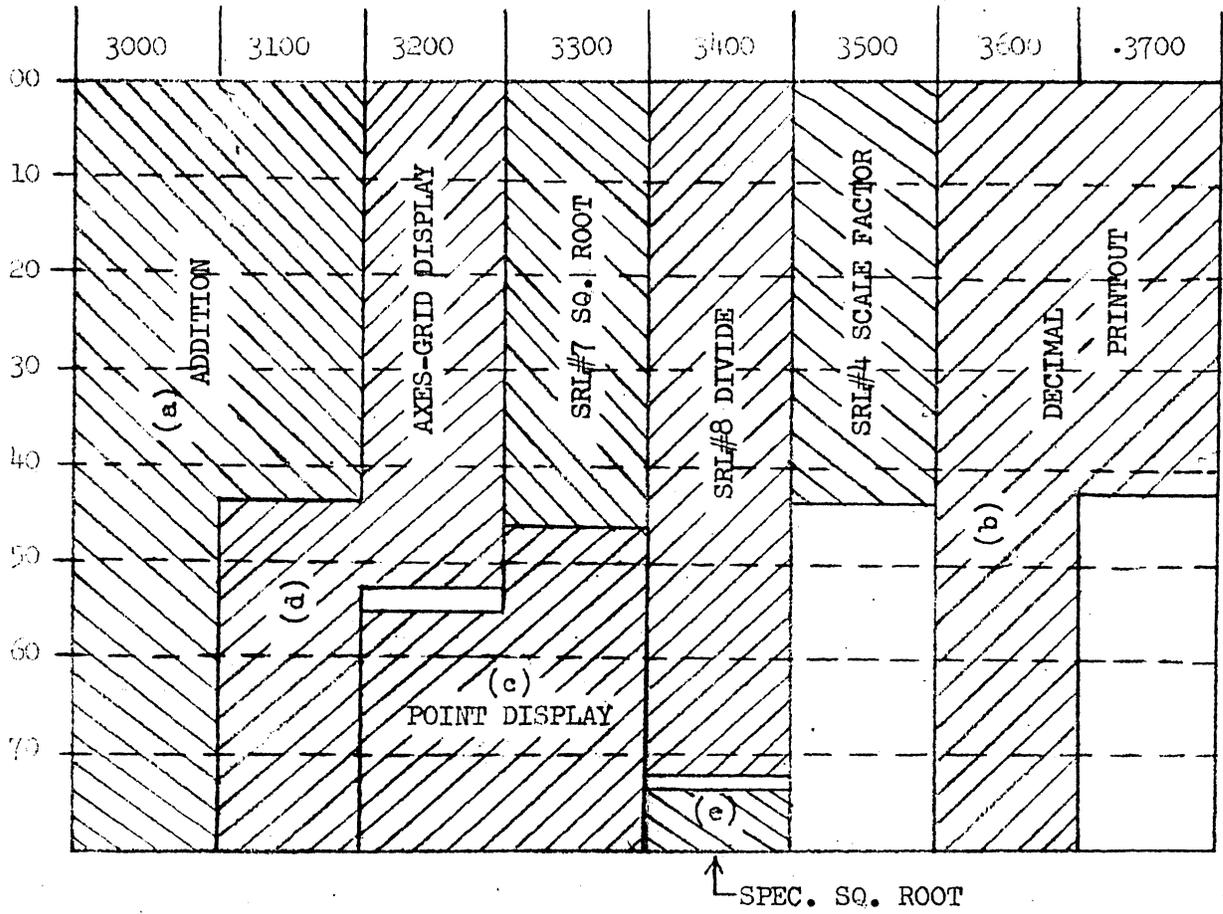
Storage: 4 program registers, 1 storage, plus 14 (decimal) registers of SRL #7.

- Notes:
1. If  $n \neq 0$ , the result obtained from use of the SCALE FACTOR subroutine (SRL #7 of memorandum 6M-3497) possesses a different scale factor than that of x. The subroutine described here takes the results of SRL #7 and produces  $\sqrt{x}$  with the original scale factor of x. For example:  $\sqrt{+4}$  may be desired to appear as 0.00002; whereas SRL #7 gives 0.00552, which is  $\sqrt{2^{-13}}$ .
  2. As used here,  $n = +15$  indicates a scale factor of  $2^{-15}$ , or, in other words, that the binary point is considered to be at the right-hand end of the register.
  3. Each relative address denoted with a g in the following program refers to the decimal address as given in SRL #7 of memorandum 6M-3497.

Program:

e0	rf	g28	prepare to return to main program.
1	st	Kg1	$\sqrt{x}$ , as obtained from SRL #7.
2	ca	Ke0	n
3	tr	gl7	transfer to last part of SRL #7.
Ke0	--		n: original scale factor of x.

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