PROJECT WHIRLWIND

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FOREWORD

Project Whirlwind

Project Whirlwind at the Massachusetts Institute of Technology Digital Computer Laboratory is sponsored by the Office of Naval Research under Contract N5ori60. The objectives of the Project are (1) the application of an electronic digital computer of large capacity and very high speed (Whirlwind I) to problems in mathematics, science, engineering, simulation, and control, and (2) the study and development of component reliability in Whirlwind I.

The Whirlwind I Computer

Whirlwind I is of the high-speed electronic digital type, in which quantities are represented as discrete numbers, and complex problems are solved by the repeated use of fundamental arithmetic and logical (i.e., control or selection) operations. Computations are executed by fractional-microsecond pulses in electronic circuits, of which the principal ones are (1) the flip-flop, a circuit containing two vacuum tubes so connected that one tube or the other is conducting, but not both; (2) the gate or coincidence circuit; (3) the magnetic-core memory, in which binary digits are stored as one of two directions of magnetic flux within ferro-magnetic cores.

Whirlwind I uses numbers of 16 binary digits (equivalent to about 5 decimal digits). This length was selected to limit the machine to a practical size, but it permits the computation of many simulation problems. Calculations requiring greater number length are handled by the use of multiple-length numbers. Rapid-access magnetic-core memory has a capacity of 32,768 binary digits. Present speed of the computer is 40,000 single-address operations per second, equivalent to about 20,000 multiplications per second. This speed is higher than general scientific computation demands at the present state of the art, but is needed for control and simulation studies.

1. QUARTERLY REVIEW AND ABSTRACT

During the past quarter 52 problems made use of the computer time allotted to the Scientific and Engineering Computations (S&EC) Group. The computer logging described previously is now being utilized to produce a biweekly report of time used by each problem; the processing is done entirely by WWI. New routines for displaying numerical results on the oscilloscope were introduced into the comprehensive system of service routines.

Greater emphasis is now being placed on reliability and maintenance of WWI to provide the best possible performance. Increased reliability resulted in 95% usable applications time compared with an average of 93% since September 1953. To this end a biweekly analysis is being made of the types of interruptions occurring most frequently and those resulting in greatest loss of time.

There were 32 students representing 12 MIT groups enrolled in the DCL programming course given this quarter. Topics included were relative addresses, temporary storage, floating addresses, preset parameters, programmed arithmetic, cycle counters, buffer storage, automatic output, post mortems, and multipass conversion. The course on machine computation, Machine-Aided Analysis, had an enrollment of 59, primarily seniors in Electrical Engineering. Practice problems were done by the students on a REAC and on WWI (simulating the three-address computer developed for the 1954 summer session).

2. MATHEMATICS, CODING, AND APPLICATIONS

2.1 Introduction

During the period covered by this report 52 problems made use of the computer time allotted to the Scientific and Engineering Computation (S&EC) Group. Progress reports as submitted by the various programmers are presented in numerical order in Section 2.2. Of these, 21 (189, 191, 203, 210-218, 220-222, 225, and 228-232) represent new problems that are being described for the first time. Fourteen problems (107, 159, 166, 184, 190, 200, 208, 210, 211, 212, 214, 215, 220, and 222) have been completed. The results of 24 problems will be included in academic theses (one S.B., two S.M., 18 Ph.D. or Sc.D). The results of 11 problems have been or will be submitted for publication in technical journals.

The computer logging described in Summary Report No. 37 (page 12) is now being utilized to produce a biweekly report of time used by each problem with the processing being done entirely by WWI.

New routines for displaying numerical results on the oscilloscope (for photographic output) have been introduced into the comprehensive system of service routines. A description of the routines will also be found under Problem 100.

2.2 Problems Being Solved

100 COMPREHENSIVE SYSTEM OF SERVICE ROUTINES

The comprehensive system of service routines has been developed by the Scientific and Engineering Computation (S&EC) Group to simplify the process of coding for WWI. The system now in use, called CS II, was described in Summary Reports No. 36, 37, 38, and 39.

Since the reader will find references in some of the reports below to the number system used in CS II, the following brief description is included here for the reader's convenience.

(m, n) numbers shall mean numbers which are of the form $z = x \cdot 2^y$ where x is an m-binary-digit number and y is an n-binary-digit number. For example, (24, 6) signifies a two-register floating-point system dealing with numbers of 24 significant binary digits (roughly seven decimal digits) with magnitudes between 2^{63} and 2^{-64}.

Arithmetic involving these (m, n) numbers is carried out by means of (m, n) interpretive subroutines. These subroutines enable the programmer to write coded programs using (m, n) numbers as easily as, or even more easily than, he might write programs in the single-length fixed-point (15, 0) number system which is built into Whirlwind I.

Automatic Biweekly

During each computer session a logging tape is automatically punched out under the control of the computer. This logging tape contains the following information: (1) the time one problem began and ended, (2) whether the operator erased before read-in, (3) the number of the tape read in (this number also identifies the problem, programmer, and kind of tape), (4) whether a tape was converted, (5) whether a postmortem was given, (6) time used for testing terminal equipment, and (7) the computer's "down" time. The time is either in hours and minutes or standard time.

A routine has been written which will read in these logging tapes and (1) sum up all the time used by each individual problem, (2) compute how many problems were run, (3) compute how many programs were run, (4) compute the time used for drum testing, magnetic-tape testing, calibration of the scopes, photoelectric-tape-reader check, etc., (5) compute the lost time, (6) sum up the over-all time used, and (7) compute the percentage of usable computer time.

These results are combined with special information, stored on the auxiliary drum, about the problem titles and prescribed biweekly report formal to produce a "delayed" punched paper tape. This paper tape is then used to produce on Multilith the S&EC Group's biweekly report.

Even though the present system is working satisfactorily, further development will continue to make the system faster and more flexible.

Automatic Oscilloscope Output

Automatic-output instructions for displaying numerical results on the scope have been added to the CS II vocabulary. The instructions SOA and iSOA (which stand for Scope Output Alphanumerics, the i indicating that an interpretive mode has been selected) are used exactly like TOA and iTOA. Thus iTOA+123.45s means "type the contents of the MRA on the Flexowriter as a signed number having three digits to the left of the decimal point and two to the right and follow it by a space" while iSOA+123.45s will display the same number on the scope followed by a space. The "machine functions," carriage return, tab, and space, have much the same meaning on the scope as on the Flexowriter. A frame has room for 36 lines of numbers, and there are four tab positions and room for 63 characters across the face of the scope.

Some instructions are peculiar to the scope. The FRAME (iFRAME) instruction advances the film in the camera, and, if SOA (iSOA) is used, it resets the display coordinates to the upper left-hand corner of the scope. The carriage return also does this if the number of lines exceeds 36. The COLUMN (iCOLUMN) instruction moves the "left hand margin

Field	Description	Problem Number	% WWI Time	Source
Aeronautical Engineering	Construction and testing of a delta-wing flutter model	166 C.	0.36	MIT
	Transient temperature of a box-type beam	179 C.	0.18	MIT
	Blast response of aircraft	183 D.	7.28	MIT
	Helicopter rotor stability	222 C.	0.54	MIT
Chemical Engineering	Transient effects in distillation	*167 D.	9.64	MIT
	Reactor runaway prevention	*231 C.	0.55	MIT
Civil Engineering	Response of a five-story frame building under dynamic loading	203 C.	0.11	MIT
	Dynamic analysis of bridges	230 C.	0.33	MIT
Electrical Eng.	Crosscorrelation of blast-furnace input-output data	180 B.	0.24	MIT
Geology and Geophysics	Geophysical data analysis	106 C.	1.63	MIT
	Interpretation of earth-surface resistivity measurements	*133 C.	5.10	MIT
	Earthquake epicenter location by Geiger's method	191 B.	0.12	MIT
	Dispersion curves for seismic waves	*212 C.	0.57	MIT
Instrumentation Laboratory	An interpretive program to accept mathematical symbols	108 C.	1.11	MIT
	Servo response to a cosine pulse	211 C.	1.70	MIT
Lincoln Laboratory	A study of recurrent events	200 C.	0.34	MIT
Mechanical Engineering	Flow of compressible fluids (aerothermopressor)	120 D.	4.47	MIT
	Laminar boundary layer of a steady, compressible flow in the entrance region of a tube	199 C.	2.53	MIT
	Industrial process control studies	213 D.	0.58	MIT
	Evaluation of difference diffusion equation	228 A.	0.13	MIT
	Rotating contact seal	229 D.	0.17	MIT
Meteorology	Synoptic climatology	155 D.	11.37	MIT
	Distribution of gustiness in the free atmosphere	189 C.	0.06	MIT
Physics	Coulomb wave functions	*122 B.	1.08	MIT
	Self-consistent molecular orbitals	144 C.	0.75	MIT
	Use of water storage in a hydroelectric system to minimize the expected operating cost	159 D.	3.39	MIT
	Determination of phase shifts from experimental cros. sections	162 C.	0.14	MIT
	Overlap integrals of molecular and crystal physics	*172 B.	0.69	MIT
	Scattering electrons from hydrogen	*184 D.	7.32	MIT
	Zeeman and Stark effect in positronium	190 D.	0.25	MIT
	Ar augmented plane-wave method as applied to sodium	194 B.	0.30	MIT
	Study of the ammonia molecule	201 C.	0.12	MIT
	Exchange integrals between real Slater orbitals	204 C.	1.49	Univ. of Chicago
	Variation-perturbation of atomic wave function and energies	217 A.	0.33	MIT
	Transformation of integrals for diatomic molecules	218 C.	0.14	MIT
	Neutron-deuteron scattering	*225 B.	0.14	MIT
Servomechanisms Laboratory	Autocorrelation and Fourier transform calculation	107 C.	0.11	MIT
	Subroutine for the numerically controlled milling machine	132 C.	0.17	MIT
	Data-reduction program; polynomial fitting	126 C.	4.95	MIT
Dynamic Analysis & Control Laboratory	Interceptor flight-control problem	208 C.	0.27	MIT
Laboratory for Nuclear Science	Energy levels in a spheroidal potential	232 B.	0.45	MIT
Research Laboratory for Electronics	Plant surveys for control systems by statistical methods	215 B.	0.31	MIT
Miscellaneous	Comprehensive system of service routines	100	21.42	MIT
	Special problems (staff training, etc.)	131	0.35	MIT
	Library of subroutines	141	1.01	MIT
	Intestinal motility	195 C.	0.71	Mass. Mem. Hospitals
	Numerical solution of homogeneous linear differential equations with quadratic polynomial coefficients	209 A.	0.12	Camb. Univ. England
	Residue indices and primitive roots	210 A.	0.24	Camb. Univ. England
	Interval distribution	214 A.	0.17	Nat. Phys. Lab England
	Ultrasonic delay lines	216 C.	0.26	Arenberg Ultrasonic Lab-Lincoln Lab
	Problem arising from an algebra	220 A.	0.44	Nat. Phys. Lab England
	Course 6.25, machine-aided analysis	221	1.79	MIT

Table 2-I. Current Problems Arranged According to Field of Application
(* MIT Project on Machine Methods of Computation)

Mathematical Problem	Procedure	Problem Number
1. Matrix algebra and equations		
Matrix multiplication, addition, diagonalization	Basic operations	144 C.
Solution of a matrix equation	Iteration using Crout's method and matrix algebra	166 C.
Eigenvalues	Diagonalization	190 D.
Eigenvalues	Iteration and diagonalization	201 C.
Zeros of a matrix equation	Interpolation	*212 C.
Minimize an analytic equation	Basic arithmetic operation	*217 A.
Orthonormalization	Schmidt process	218 C.
A combinatorial algebra	Direct evaluation of matrix elements	220 A.
Set of 15 algebraic equations	Crout's method	229 D.
Eigenvalues	Diagonalization	232 B.
2. Ordinary differential equations		
General system	Gill's modified fourth-order Runge-Kutta	108 C.
Fourteen nonlinear first order	Gill's method	208 C.
Linear with quadratic polynomial coefficients	Taylor series	209 A.
Seven nonlinear first order	Fourth-order Runge-Kutta	120 D.
Set of nonlinear first-order equations	Second-order Runge-Kutta	*167 D.
Two second order	Extrapolation of differences	183 D.
System	Gill's method	199 C.
Five nonlinear second order	Difference equation	*203 C.
Second-order linear	Power series	211 C.
Second-order nonlinear	Rectangular integration	221
Two linear second order with periodic coefficients	Fourth-order Runge-Kutta	222 C.
Second-order linear	Difference equation	*230 C.
3. Partial differential equations		
Schroedinger's equation	Summation of series	*122 B.
Second-order parabolic	Explicit finite difference approximation	179 C.
Diffusion equation	Recurrence formula	*228 A.
4. Integration		
Auto-, crosscorrelation and Fourier transform	Simpson's rule	107 C.
Integral evaluation	Trapezoidal rule	*123 C.
Numerical integration	Trapezoidal rule	159 D.
Overlap integrals	Evaluation of analytic forms	*172 B.
Fourier transform of the crosscorrelation of two functions	Simpson's rule	180 B.
Two-dimensional integrals	Simpson's rule	*184 D.
Autocorrelation and Fourier transform	Simpson's rule	189 C.
Autocorrelation and Fourier transform	Simpson's rule	195 C.
Integral evaluation	Gaussian quadratic	204 C.
Auto- and crosscorrelation	Simpson's rule	213 D.
Auto- and crosscorrelation	Simpson's rule	215 B.
Stationary point of a variational	Simpson's rule	*225 B.
5. Statistics		
Multiple time series	Prediction by linear operators	106 C.
Calculation of the coefficients of a multiple regression system	Inner products	155 D.
Study of recurrent events	Success run and density test	200 C.
Interval distribution	Monte Carlo	214 A.
6. Transcendental equations		
Curve fitting	Iteration using least squares	162 C.
Implicit system	Iteration using Geiger's and Crout's methods	191 B.
System of 47 equations	Step-by-step	*231 C.
7. Data-reduction program	Polynomial fitting, etc.	126 C.
8. Number theory		
Residue-indices	Trial and error	210 A.
9. Linear inequalities	Iteration	216 C.
10. Group theory		
Generation of projection operators	Machine generation	194 B.

Table 2-II. Current Problems Arranged According to the Mathematics Involved
(* MIT Project on Machine Methods of Computation)

stop" to the next tab position to the right and resets the display position to the top line. If the next tab position is off the right-hand side of the scope, it acts like FRAME.

106 MIT SEISMIC PROJECT

As discussed in previous Summary Reports, Problem 106 is concerned with the investigation of the use of statistical-analysis techniques in seismic-record interpretations, and in particular in the separation of "reflections" from background interference on these records. More complete descriptions of the problem and the approaches used are contained in "Detection of Reflections on Seismic Records by Linear Operators" (Wadsworth, Robinson, Bryan, and Hurley--Geophysics, Vol. 18, No. 3, July 1953). This work is being carried out by the MIT Geophysical Analysis Group under the direction of Dr. S. M. Simpson.

In Summary Report No. 39 a list was presented of the various categories of computation in which Problem 106 has been interested. During the past 3 months emphasis has been put on three of these categories:

1. Computations designed to test techniques for probing the statistical and frequency behavior of a given input seismogram;

2. Computations aiding the development of the relationships between discrete linear operators and linear electric filters;

3. Computations designed to simulate and study various statistical noise models.

Under 1 we have programmed a finely spaced, multiterm correlation cosine transform program, using a tailormade programmed arithmetic to solve the speed-accuracy problem, and we have completed computation yielding a table for use in estimating signal-to-noise ratios as functions of frequency from multitrace signal noise mixtures.

Under 2 we have programmed the computation of inverse linear operators, the computation of linear operators from the location of their poles in the "z" plane, and the computation of linear operators from their real frequency power spectra. This last program, which is a means of bypassing least-squares methods by spectral factorization, is not yet completed. In addition, we have computed tables of linear operators with special characteristics: one set with high pass filter characteristics and another with narrow band pass characteristics.

Under 3 we have computed a 1200-term stationary series by moving summation from a wavelet and a random-number table. From this series, estimates of variance, correlation, and power spectra have been obtained for various sample lengths as an experimental determination of variability, the results to be compared with observed seismic variability in these quantities.

The most important results obtained during this period have been: verification of a new concept of noise on actual seismograms; programs to exploit this concept; the tabulated operators mentioned above; experimental information about seismic-wavelet stability; and experimental information about variability in estimates.

For the future we plan to pursue these topics and to complete and use the spectrum-factorization program.

107 (a) AUTOCORRELATION AND (b) FOURIER TRANSFORM INTEGRAL EVALUATION

Routines developed under this problem by D. T. Ross of the MIT Servomechanisms Laboratory have been used by F. Raichlen of the MIT Hydrodynamics Laboratory in the investigation of turbulent-velocity fluctuations in open-channel flow measured by means of a Pitot-tube pressure-cell combination. This project is sponsored by the Office of Naval Research under Contract No. N5-ori-07874. In the future, this project will be reported on under Problem 223.

During the past quarter it was found necessary to delete the secondary diaphragm of the transducer to increase the mechanical stability of the system. This removal necessitated an increase in thickness of the primary diaphragm of the gage to bring the natural frequency of the system back to an acceptable value. A number of runs in a high-velocity flume were then made to investigate the effect of this change on the previous results obtained over the past year from the Digital Computer Laboratory.

As before, the correlation curve obtained showed superimposed periodicities of approximately 180-cps frequency. These curves, however, were much cleaner in appearance as far as low-frequency components were concerned. This made possible a different approach to the computation of the macroscale of turbulence L_x. By the application of a three-point smoothing process to the results obtained from the Digital Computer Laboratory the high-frequency periodicities were averaged out. The low-frequency component was removed from the curve by subtracting a cosine curve which had been fitted to this predominant periodicity. A theoretical curve of $R_\tau = e^{-k\tau}$ was then fitted to the smoothed experimental correlation curve. From this the value of L_x was obtained and found to be approximately the same as that computed previously by more laborious methods.

108 AN INTERPRETIVE PROGRAM

During the last quarter an effort has been started by Dr. J. H. Laning of the MIT Instrumentation Laboratory to rewrite the algebraic interpretive program in its entirety. The object of this effort is to increase both the speed and versatility of the algebraic routines. It is expected that an interim revised program will be available shortly in a form suited to future development and extensions.

Among the anticipated features of the new program are the following:

1. The operation speed will be increased by an estimated factor of 5 to 10;

2. The usable computer storage capacity will be considerably enlarged;

3. The basic vocabulary of the program will be considerably increased: for example, function subroutines will now be designated by familiar words making the programming task somewhat easier. In addition, a number of new features will be included such as the automatic evaluation of sums, products, and integrals.

At present the main logical structure of the program is about 75% complete. It is expected that this will be operating by the end of the present quarterly period. Following this, future revisions will be added as time and manpower permit.

120 THERMODYNAMIC AND DYNAMIC EFFECTS OF WATER INJECTION INTO HIGH-TEMPERATURE, HIGH-VELOCITY GAS STREAMS

This problem is connected with the development of a potential gas-turbine component, called an "aerothermopressor," in which a net rise in stagnation pressure of a hot gas stream is brought about by the evaporation of liquid water injected into a high-velocity region of the flow. The concepts underlying the operation of the aerothermopressor are an outgrowth of comparatively recent work in the field of gas dynamics, and its intended function in the gas-turbine cycle is analogous to that of the condenser in a steam power plant.

The device consists of a converging nozzle which accelerates the exhaust gases from the turbine into a circular duct of varying diameter terminated by a conventional conical diffuser, which recovers the kinetic energy of the flow before discharging it to the atmosphere. At the entrance of the duct (evaporation section), special injectors deliver minute jets of water which are in turn atomized by the rapidly moving gas stream. The changes in state within the aerothermopressor are brought about by the simultaneous thermodynamic and dynamic effects of (1) evaporation of the liquid water, (2) momentum and energy interactions between the phases, (3) friction, and (4) variations in cross-sectional area of the duct. Under proper circumstances, these effects bring about a net rise in stagnation pressure across the device. Further descriptions of this device may be found in earlier reports, beginning with Summary Report No. 32, Fourth Quarter 1952.

The role of Whirlwind I in the successful development of the aerothermopressor is intimately connected with the determination of performance characteristics of the device under all conditions of operation by means of a comprehensive one-dimensional analysis of the process. The principal objectives of this analytical study, which began late in 1952, are (1) to establish a firm theoretical foundation for an understanding of the process and (2) to obtain design data for experimental work.

During the past quarter, most of the effort was directed toward the planning and preparation of a completely revised Whirlwind program. This revision, which is currently about one-half completed, was motivated by obsolescence of the original program, a natural consequence of both expanded knowledge concerning the aerothermopressor and changes in the Whirlwind computer. The basic objective of the new program is to provide a rapid and convenient scheme of analysis, and the essential changes in the program include:

1. Use of the fourth-order Runge-Kutta method for numerical solution of the differential equations, rather than the elementary Euler method;

2. Change of independent variable from fraction evaporated (humidity) to axial distance along duct;

3. Extensive use of the auxiliary magnetic drum for both computations and program organization, the latter in an attempt to avoid a plethora of punched tapes;

4. Elimination of all interpretive instructions except those mandatory for arithmetical operations;

5. Careful revision of all portions of the program involving iteration, for example, the subroutine used for determining roots; (The revised program is more than twice as fast as the original.)

6. Elimination of necessity for hand computation of a portion of the initial data by incorporation into program;

7. Provision for the optional use of a concentric circular plug within the evaporation section of the aerothermopressor;

8. Revised treatment of the liquid temperature and liquid velocity to avoid instability toward the end of the calculation.

Following completion of this program, it is planned to resume computations on a broad scale. Considerable experimental data from the medium-scale test facility at the MIT Gas Turbine Laboratory is being accumulated and awaits corroboration. Further, several procedures for calculating optimum performance of the aerothermopressor await exploitation.

The aerothermopressor development program is being carried out at MIT under the sponsorship of the Office of Naval Research and is directed by Professor Ascher H. Shapiro of the Department of Mechanical Engineering. The theoretical aspects of the problem treated by Whirlwind I are being carried out by Dr. Bruce D. Gavril.

122 COULOMB WAVE FUNCTIONS

A program has been written by J. Uretsky and M. Rotenberg of the MIT Physics Department to generate Coulomb wave functions by the method of Breit, Yost, and Wheeler.[1]

1. Physical Review 49, 175 (1936)

Runs are now being made to determine the region of usefulness for this method.

The program has proven to be logically correct. However, there is some doubt as to the convergence of the formulae being used.

123 EARTH RESISTIVITY INTERPRETATION

This problem is concerned with a means of analyzing earth-resistivity data. The procedure is to assume a fixed number of homogeneous horizontal layers, together with the thicknesses and conductivities of these layers, and then to relax the Slichter kernel calculated therefrom to the observed kernel by systematically changing the assumed parameters (see Summary Report No. 37).

To investigate the behavior of the solution, several kinds of data have been run. Kernels were calculated theoretically for cases in which the second layer was much thinner than the first or in which the conductivity contrast between successive layers was very small. These were rounded off to the accuracy of field data (larger random errors were added to some) before analysis. Kernels calculated for a number of layers greater than assumed in the analysis were also analyzed. In the statement of the problem, it is assumed that the conductivities of the first and last layers are known, as is the case. Data in which an incorrect value of bottom-layer conductivity was used were analyzed. Also, an actual observed kernel, from an area in which drilling information was available for comparison, was run.

At the same time the program was sped up so that an analysis takes about 4 minutes instead of 15 minutes, and several features were incorporated to eliminate the unstable oscillation of the solutions which sometimes results from an increase in the parameter-change multiplier.

The behavior of the analyses in general seems satisfactory. However, if the initially assumed values of the parameters being sought are too far wrong, no solution will be obtained, because one of the denominators in the Crout calculation of the parameter changes is the result of subtraction of two nearly equal numbers. A change in the number system used from (24, 6) to (25, 5) or (26, 4) might help in some such cases. Provided that such is not the case, the analysis yields solutions which seem reasonable intuitively and whose kernels match the initial kernels to a degree depending on the situation. Three-layer kernels fit to better than the accuracy of the data. Four-layer kernels, in the three-layer analysis, yield a "smoothed" solution. The field case fitted the drill information extremely well. A few more difficult cases and three more field cases are being run.

The kernels for the field cases were integrated by Whirlwind from field-resistivity data taken in mining areas in the Southwest.

This work is being carried out by K. Vozoff of the Department of Geology and Geophysics under the sponsorship of the Project for Machine Methods of Computation and Numerical Analysis.

126 A DATA-REDUCTION PROGRAM

Problem 126 is a very large data-reduction program for use in the Servomechanisms Laboratory. The over-all problem is composed of many component sections which will be developed separately and then combined at a later date. Early efforts were focused on the development of utility-type programs. These programs, which have been described in previous Quarterly Reports, include a fully automatic program to fit polynomials to arbitrary empirical functions, a general-purpose Lagrange interpolation program, and a flexible and fairly elaborate post-mortem routine. The programs are being developed by Douglas T. Ross and William M. Wolf with the assistance of Miss Dorothy A. Hamilton, Servomechanisms Laboratory staff members.

The basic data-reduction program has been completely checked using the mistake diagnosis routine (MDR) to extract over 30 intermediate results for accuracy and logical checks. Work on this program has been dormant pending the development of the manual-intervention program (MIV). The MIV program is a general routine for the interpretation of manual interventions into the computer via insertion registers and activate buttons. It has facilities for several types of alpha-numeric and graphical scope outputs, sounding of audible alarms, and elaborate program modification. The MIV program will operate in conjunction with the basic data-reduction program (or any similar program), and the intent is to have the flexibility normally associated only with analog computers available for instantaneous modification and monitoring of this class of digital-computer programs.

The mistake diagnosis routine has been modified so that it now will operate from and on any drum group or combination of groups. The method of specifying parameters for break points has also been simplified and made more foolproof. A slight modification operates in conjunction with the MIV program to give scope outputs of MDR results as they occur.

A "scope input" routine has been written which gives a type of two-dimensional analog input to the Whirlwind computer. The principle of the program is similar to the "flying spot scanners" used on analog computers. The equipment used is a 16-inch oscilloscope under the control of the computer with a photocell mounted so that its field of view is the entire scope face. Then by programming a flying spot and asking whether or not the photocell "saw" the spot, the program can be made to follow an opaque pointer as it is moved in a random fashion over the face of the scope. Since the program displays the spot by digital coordinates, the tracking of the pointer constitutes analog input to the computer.

Present work includes testing and use of the MIV program with the present basic data-reduction program and also extensive modification of the basic program to include several new equations.

131 SPECIAL PROBLEMS (STAFF TRAINING, DEMONSTRATIONS, ETC.)

A payroll program has been prepared for use in demonstrations. It computes hours worked, regular earnings, premium earnings, gross earnings, social security tax, withholding tax, and net pay.

There are two inputs to the program. One is a file of employee records containing employee name, numoer, rate of pay, and number of dependents. The other input, which the demonstration audiences will be invited to prepare, represents the weekly time cards.

The direct printer is used to prepare payroll checks and deduction slips on standard forms.

During the past 3 months, eight groups visited the Laboratory. The affiliations of these groups are given in Section 5.3.

132 SUBROUTINES FOR THE NUMERICALLY CONTROLLED MILLING MACHINE

Use of the computer during this quarter by J. H. Runyon of the MIT Servomechanisms Laboratory has been restricted mainly to the checking of milling-machine tapes. Two short subroutines for use with a feedrate-control routine were written and tested. A test program for this routine is being written. Programming for milling-machine tape preparation for a two-dimensional cam has begun.

141 S&EC SUBROUTINE STUDY

A subroutine for solving simultaneous equations by Crout's method has been included in the library under the title LSR MA 6. This subroutine will compute $A^{-1}B$, where B is a matrix consisting of one or several columns. Check columns are provided to indicate the accuracy of the results.

A new sine-cosine evaluation subroutine has also been included in the library. This routine, LSR FU 4b, differs from the older LSR FU 4 sin-cos routine in that the reduction of the given angle to the appropriate range is accomplished more rapidly, thereby reducing computing time in cases where many large angles are involved.

An improved version of LSR MA 5, Symmetric Matrix Inversion and Square-Root Inversion, has been written, tested, and placed in the library.

A subroutine, LSR MA 7, has been added to the library. Its function is to generate the augmented matrix of a set of simultaneous equations, the solution of which is the set of coefficients of the polynomial which is the best fit in the least-squares sense to some data given in tabular form. The data is stored initially on the drum. The matrix is left in core

memory in a form such that LSR MA 6 may be used to find the solution. The order of the polynomial, the number of points in the table of data, the location of the data on the drum, and the location of the matrix in core memory are all program parameters. Calculations are done in (30-j, j) arithmetic.

144 SELF-CONSISTENT MOLECULAR ORBITAL

An improved version of the linear combination of atomic orbitals for self-consistent field method has been written by Dr. A. Meckler of the MIT Solid State and Molecular Theory Group. The pattern of the program is much like that indicated in Summary Report No. 35 but with three major modifications:

The new program does not presume the use of an orthonormalized set of orbitals as the basis of the calculation. The equations now have the following form for a closed-shell system:

The molecular orbitals are expressed as

$$\phi_i(x) = \sum_{\mu} v_{\mu}(x)\, c_{\mu i}$$

where $v_{\mu}(x)$ is a prescribed orbital and $c_{\mu i}$ is a coefficient to be determined. In general,

$$\int v_{\mu}^{*}(x) v_{\nu}(x)\,dx = \Delta_{\mu\nu}$$

a nondiagonal but positive-definite matrix. The side condition that the ϕ's be orthonormalized is equivalent to

$$C^{\dagger} \Delta C = 1 .$$

With $\rho = CC^{\dagger}$, the energy expression is

$$E = \mathrm{Tr}\rho F + 1/2 \sum_{\substack{\mu\nu \\ \lambda\sigma}} \rho_{\mu\nu}\, G_{\mu\sigma}^{\nu\lambda}\, \rho_{\sigma\lambda} .$$

F is the one-electron energy matrix and

$$G_{\mu\sigma}^{\nu\lambda} = 2\int \nu^{a}(1)\mu(1) g_{12} \lambda^{a}(2)\sigma(2)\,dx_1 dx_2 - 1/2\, \nu^{a}(1)\sigma(1) g_{12} \lambda^{a}(2)\mu(2)\,dx_1 dx_2 - 1/2 \int \nu^{a}(1) \lambda(1) g_{12} \sigma^{a}(2)\mu(2)\,dx_1 dx_2$$

with the restriction that the integrals be real.

The conditions on ρ are now

$\mathrm{Tr}\rho\Delta = n$ = half the number of electrons

$$(\rho\Delta)^2 = \rho\Delta.$$

The requirement that the energy be stationary with respect to infinitesimal variations of ρ leads to the necessary and sufficient conditions

$$H\rho\Delta = \Delta\rho H$$

where $H_{\lambda\sigma} = F_{\lambda\sigma} + \sum_{\mu\nu} \rho_{\mu\nu} G^{\nu\lambda}_{\mu\sigma}$.

The substitutions
$$\bar{H} = \Delta^{-1/2} H \Delta^{-1/2}$$
$$\bar{\rho} = \Delta^{+1/2} \rho \Delta^{+1/2}$$

yield the equivalent equations

$$\bar{H}\bar{\rho} = \bar{\rho}\bar{H}$$
$$\bar{\rho}^2 = \bar{\rho} \qquad \mathrm{Tr}\,\bar{\rho} = n$$

These are solved as were the set for the old program. That is, $\bar{H}$ is diagonalized, the rectangular matrix $\bar{C}$ is formed from its n lowest eigenvectors, and $\bar{\rho} = \overline{CC}^{\dagger}$. This is followed by a transformation to ρ which is used to form a new H, and so on in the cycle. Thus, except for a preliminary calculation of $\Delta^{-1/2}$ and intermediate transformations, the iteration pattern is like the one previously programmed.

The second improvement in the program is the test for self-consistency. The first program tested the variation of total energy from iteration to iteration. This is numerically bad. The numbers representing the eigenvectors are stabilized long before that number which represents the total energy. In fact, there is enough roundoff error in a computation of the large total energy so that it is impossible to judge the important last decimal places. Self-consistency can be missed.

In the new program, the considered quantity is

$$\mathrm{Tr}\,(\rho' - \rho)^2$$

where ρ is the input matrix and ρ' is the output matrix for one cycle.
Now

$$\mathrm{Tr}(\bar{\rho}' - \bar{\rho})^2 = \mathrm{Tr}(\rho' - \rho)\Delta(\rho' - \rho)\Delta$$
$$\leq \mathrm{Tr}(\rho' - \rho)^2 \,[\mathrm{Tr}\,\Delta]^2$$

the inequality following from the positive-definiteness of Δ. The equations satisfied by any $\bar{\rho}$ imply

$$\mathrm{Tr}(\bar{\rho}' - \bar{\rho})^2 = \mathrm{Tr}\,\delta\bar{\rho} = -\mathrm{Tr}\,\bar{\rho}\delta\bar{\rho} + \delta\bar{\rho}\bar{\rho}.$$

If $\delta\bar{\rho}$ were really a first variation, the idempotency of $\bar{\rho}$ would insist that

$$\delta\bar{\rho} = \bar{\rho}\,\delta\bar{\rho} + \delta\bar{\rho}\bar{\rho}.$$

Every $\bar{\rho}$ has the same trace, so that

$$\mathrm{Tr}\,\delta\bar{\rho} = 0.$$

Therefore, if the initial and final matrices did differ by only a first variation, $\mathrm{Tr}(\bar{\rho}' - \bar{\rho})^2$ would be equal to $-\mathrm{Tr}\,\delta\bar{\rho}$ and would vanish. The smallness of this trace or, in fact, the smallness of $\mathrm{Tr}(\bar{\rho}' - \bar{\rho})^2$ is taken as a test for self-consistency.

The third improvement in the program is an over-all one engendered by machine developments and programming maturity. The use of buffers, for example, should increase the accuracy; the working is more direct. Better use of storage has allowed bigger systems to be handled: if a symmetry type is of order n_i, then

$$\sum_i \frac{n_i(n_i + 1)}{2} \leq 99.$$

155 SYNOPTIC CLIMATOLOGY

Our basic objectives as put forth in Summary Report No. 39 have not changed sufficiently to require a restatement of them here.

Over the past quarter we have made great strides in expanding the size matrix that can be solved. This in turn has permitted us to add to our initial set of parameters others which we feel contain a sufficient amount of information with regard to our problem. The present program for solving matrices allows us to go as high as an n x n matrix, where n is restricted only by the amount of storage available. Thus n is restricted to 145, using only the amount of storage ten auxiliary-drum groups afford. The matrix-solution technique which we make use of is that developed by Crout, and the program is only applicable to symmetrical matrices.

Up to the present time we have solved a matrix as high as 84 x 84. This was accomplished in a little over 12 minutes, even though portions of the program had to be done in the interpretive mode. A check was made on the amount of roundoff error accumulated. It appears from this check that the results, with input data containing seven digits, are accurate to seven places.

Our aim during the next quarter will be to incorporate magnetic-tape storage, which should make the solution of any size matrix possible. Our interest in attempting to solve higher and higher matrices is due to the fact that until now we have as yet to reach a point of diminishing returns, insofar as the number of variables used and the number of degrees of freedom lost are concerned.

Programming is being performed by Mr. Herbert J. Brun and Mr. Robert G. Miller under the supervision of Professor Thomas F. Malone of the MIT Meteorology Department.

159 WATER USE IN A HYDROELECTRIC SYSTEM

This problem, undertaken by J.D.C. Little of the MIT Physics Department, has been described in Summary Report No. 37. In the past quarter some final runs have been made. The work on this problem has been reported in an MIT Physics Department doctoral thesis. A paper is now being prepared for submission to the Journal of the Operations Research Society of America.

162 DETERMINATION OF PHASE SHIFTS FROM EXPERIMENTAL CROSS SECTIONS

A phase-shift analysis is being made of the elastic scattering of protons by O^{16} by Dr. F. J. Eppling of the MIT Laboratory for Nuclear Science. The phase shifts required to provide a fit to a number of experimental angular distributions will be determined from a partial wave expansion of the differential cross section.

During the past quarter, a program designed to find for any combinations of phase shifts the best fit to the experimental cross sections has been worked on. Several tapes have been run to test different parts of the program. It is expected that soon the entire program will be ready to be run for any energy desired.

166 CONSTRUCTION AND TESTING OF A DELTA-WING FLUTTER MODEL

The basic procedures employed in this problem for designing a low-aspect-ratio-wing flutter model to simulate a given set of flexibility influence coefficients of a full-scale wing were described in Summary Reports 37, 38, and 39 by S. I. Gravitz and M. M. Chen of the MIT Aero-Elastic and Structures Research Laboratory.

The basic computations involved were to yield solutions for a set of simultaneous nonlinear algebraic equations for the problem considered. The computation of a three-bay static model simulating the flexibility influence coefficients of an actual delta-wing airplane has been completed, and the results show good agreement with experimental data. Such a flutter model is still under construction and will be tested in the near future.

The computation was satisfactorily concluded early in this quarter. The basic computational procedures appeared in an S.M. thesis submitted by S.I. Gravitz to the MIT Aeronautical Engineering Department. A more detailed report concerning the designing, construction, and testing, etc., will be published under Contract Noas53-712-C, Bureau of Aeronautics, Department of the Navy.

167 TRANSIENT EFFECTS IN DISTILLATION

Work has continued on the problems of unsteady-state behavior of distillation systems which have been described in preceding Summary Reports. The major emphasis has been on utilization of results obtained previously using Whirlwind I to understand the behavior of the systems. Additional calculations were made when the analysis justified them, but no new programs were necessary. The bulk of the work was on batch distillation, and it has been quite fruitful. Programs used were those for equilibration of a system with holdup and for product takeoff assuming holdup finite or zero.

First the batch-distillation operation was studied as a whole, considering the equilibration period as well as the takeoff period when holdup was present. Results were studied on the basis of the ratio of product to charge, at constant values of product composition. Calculations were made changing the holdup-to-charge ratio and using various combinations of vapor consumption in equilibration and takeoff. The first conclusion was that to obtain any specified product at any holdup-to-charge ratio there was an optimum combination of vapor consumption in equilibration and takeoff which resulted in minimizing the total vapor consumption. Second, the minimum consumption was less, as the holdup was less. Consequently this conclusion would dictate the minimization of the holdup in a practical system (subject to the necessity of having some holdup).

These results may also be used to predict approximately the optimum vapor consumptions for other operations of distillation systems with holdup. A graphical method for predicting the total vapor consumption has been developed. The optimum vapor consumption in equilibration can also be estimated. In general, it appears that a column should be equilibrated at total reflux at least until the top-vapor composition equals the desired product composition. How much more vapor should be used in equilibration depends on the variables of the system. However, only with very high vapor consumptions during takeoff was more than twice this amount of vapor required for equilibration.

Another phase of the investigation was concerned with studying how the vapor consumption during equilibration affected the top-vapor composition as a function of these system variables: holdup-to-charge ratio, charge composition, relative volatility, and number of theoretical plates. Using Whirlwind, data were obtained over a broad range of these variables. Using these data along with analytical developments based on linear equilibrium relationships, a substantial understanding of this nonlinear system has been obtained. A method has been found for predicting the top-vapor composition approximately, as a function of vapor throughout. This is based primarily on the initial rates of equilibration of various parts of the system. Consequently it can be calculated from the initial conditions. This method may well be applicable in understanding and predicting the transient behavior of continuous distillation systems. These are the major developments. They are being written up by J. F. O'Donnell of the MIT Chemical Engineering Department in his Sc.D. thesis.

172 OVERLAP INTEGRALS OF MOLECULAR AND CRYSTAL PHYSICS

Since the last report it has been necessary to rewrite and extend several of the programs used in the calculations. In particular, production-style programs were written and tested for solving two types of secular equations involving real symmetric matrices. These equations are the ordinary variety (Type I),

$$\sum_{j=1}^{n} H_{ij}Xjm = \sum_{j=1}^{n} X_{jm}E_{mm},$$

and the more general variety (Type II),

$$\sum_{j=1}^{n} H_{ij}X_{jm} = \sum_{j=1}^{n} S_{ij}X_{jm}E_{mm},$$

where n is an integer from 1 to 32 and H and S are the given real symmetric matrices (S positive definite). The results are displayed photographically and consist of the E_{mm}, the eigenvalues, and the X_{jm}, the eigenvectors, subject to the normalizations:

$$\text{Type I, } \sum_{j=1}^{n} X_{ij}{}^{t}X_{jk} = \delta_{ik} \quad \text{and Type II, } \sum_{j=1}^{n} X_{\ell i}{}^{t}S_{ij}X_{jk} = \delta_{\ell k}.$$

These programs are available for the use of any interested persons.

A second program which was written and tested is a subroutine for punching out program-specified sections of the auxiliary magnetic drum as an edited, sum-checked binary tape capable of being immediately read into any program-specified storage location of the computer. The use of such a subroutine is obviously advantageous in a calculation where one has intermediate data which required a long time to compute and which must be reused several times. This routine is being submitted to the library of subroutines.

A third program developed is a subroutine for generating spherical Bessel functions of imaginary argument, namely,

$$i_n(x) \equiv \sqrt{\frac{\pi}{2x}}\; I_{n+1/2}(x) \quad \text{and} \quad k_n(x) \equiv \sqrt{\frac{2}{\pi x}}\; K_{n+1/2}(x) \text{ where } n = 0(1)\,20.$$

A fourth and closely related subroutine essentially generates the functions $A_n(x)$ and $B_n(x)$, n = 0(1)20, which are useful in two-center molecular integral calculations and are only sparsely tabulated. These are defined by

$$A_n(x) = \int_1^{\infty} t^n e^{-xt}\, dt$$

$$B_n(x) = \int_{-1}^{1} t^n e^{-xt}\, dt = 2(-1)^n \frac{d^n}{dx^n}\left[i_0(x)\right]$$

In addition, two more programs have been written and are currently being tested. The first routine consists of a complete rewrite of the previous matrix-element generator program. The second program, which is in production style for other users, is for the computation of single and double integrations (Simpson's rule) of numerically given functions and is a basic part of one procedure of evaluating multicenter integrals.

All of these routines have been developed by F. J. Corbató of the MIT Physics Department.

179 TRANSIENT TEMPERATURE OF A BOX-TYPE BEAM

The transient temperature and stress response of an extruded rectangular cross section exposed to a step function heat-flux input has been calculated by L. A. Schmit of the MIT Aero-Elastic and Structures Research Laboratory making use of an uncoupled finite-difference procedure. Reradiation and convection losses were neglected, and the thermal properties of the structure were assumed constant. The calculations have been compared with laboratory experimental results for the first 10 seconds of exposure, and the agreement is considered satisfactory. The experimental results referred to here were obtained by Lt. F. L. Williams, USAF, and are reported in his Master's thesis for the Department of Aeronautical Engineering. The title of Lt. Williams' thesis is "The Combined Effects of High Intensity Heating and Dynamic Loading on a One Cell Box Beam."

It should be noted that there is a chordwise variation in the heat flux. This is due to laboratory furnace edge effects. This chordwise variation is taken into account in the finite-difference solution carried out on the Whirlwind computer. An approximate analytical solution for the temperature response has been obtained for the special case where the heat flux is uniform with respect to chord and time. In the particular case under investigation the chordwise variation of heat flux is small, and the results obtained from the approximate analytical solution are in substantial agreement with the results of the finite-difference solution.

180 CROSSCORRELATION OF BLAST-FURNACE INPUT-OUTPUT DATA

This problem was described in Summary Report No. 38. The combined program for computing and plotting the correlation function and its Fourier transform has been completed. An estimated 2 hours of computer time will be required to complete test runs, after which a summary of operating instructions will be compiled for the use of programmers wishing to use the program.

This work will successfully complete the project initiated under Problem 108. A further phase of the research on application of advanced statistical techniques to operations research problems is now in its preliminary stages, and a future problem may be initiated to carry out the computational phases of the work.

This problem has been carried out by Richard G. Mills, formerly studying in the Department of Electrical Engineering at MIT.

183 BLAST RESPONSE OF AIRCRAFT

The results of numerical integration of the blast-response problem by Milne's method were found unsatisfactory because of the existence of a discontinuity in acceleration. The formulation was then revised using the Runge-Kutta method. The time history obtained agrees very well with the corresponding case computed by desk calculators according to a closed-form solution for the quasi-steady damping case.

A parametric study of peak response was made, totaling approximately 200 cases. The basic tape was then modified to provide an automatic increase of peak gust ratio to determine the lethal gust for a given set of airplane and operation parameters. The computing by WWI for determining the lethal criterion of two typical airplanes is about to be completed.

This study is being carried out for his doctorate thesis by H. Lin of the MIT Aero-Elastic and Structures Research Laboratory with the assistance of Y. Shulman.

184 SCATTERING OF ELECTRONS FROM HYDROGEN

The calculations described in the previous Summary Reports have been completed. These amounted to the evaluation of various two- and three-dimensional integrals which arose in the calculation of the differential amplitudes for elastic scattering of low-energy electrons from monatomic hydrogen taking into account polarization and exchange. The results have been presented in the thesis of Maurice Newstein which has been submitted to the Physics Department at MIT in partial fulfillment of the requirements for the degree of Ph. D. They will also appear as a final report for the Committee on Machine Methods of Computation at MIT. Also it is planned to submit portions of the thesis to be published as an article in the Physical Review.

189 DISTRIBUTION OF GUSTINESS IN THE FREE ATMOSPHERE

The first probability distribution of gustiness can be obtained from the spectrum of the radar signal returned by clouds or precipitation. These meteorological developments are being used essentially as tracers of the atmospheric motions.

The spectrum was computed from the autocorrelation function through a program devised by Douglas Ross. Six examples of such spectra were obtained, all of which were entirely satisfactory.

This study is being carried out by A. Fleischer of the MIT Project on Weather Radar Research.

190 ZEEMAN AND STARK EFFECT IN POSITRONIUM

The computations in this problem, reported as being completed in Summary Report No. 39, actually extended over into the beginning of this period. A detailed report was given in Summary Report No. 39 by H. W. Kendall of the MIT Radioactivity Group.

191 EARTHQUAKE EPICENTER LOCATION BY GEIGER'S METHOD

Geiger's method of locating an earthquake uses arrival times of elastic waves at three or more observing stations. The method is basically a least-square process of fitting observed data to previously obtained empirical travel-time curves. This method has been mechanized, for an example using eight observing stations, as a part of D. R. Grine's S.M. thesis (Department of Geology and Geophysics, September 1954).

The program iterates Geiger's method until corrections to earthquake location are smaller than inherent errors in the observed data.

A generalization of the program is to be written for publication at a later date.

194 AN AUGMENTED PLANE WAVE METHOD AS APPLIED TO SODIUM

Summary Report No. 38 outlined the problem of reducing the order of the secular equation for energy levels in solids by first making proper linear combinations of the wave functions so that they are broken down into irreducible subspaces.

Mr. M. M. Saffren of the Solid State and Molecular Theory Group at MIT has written and partially tested a program which chooses these linear combinations of wave functions as a function of the reduced wave vector of these wave functions and the irreducible subspace to which these linear combinations are to belong. When wave functions belong to the same order of neighbor in reciprocal space and are made into several linear combinations there is always a possibility that not all these combinations are linearly independent. This is extremely troublesome in the solution of the secular equation so the program has been provided with the facility of disregarding linearly dependent solutions. The program has the additional feature of being able to choose the linear combination with the least number of nonzero coefficients if all that is desired is a linear combination which takes functions into an irreducible subspace without consideration of how basis functions are chosen in that space. This feature is a valuable one when the time for formation of the matrix elements between these linear combinations is strongly dependent on the number of nonzero members in the combinations.

The program operates by picking up a wave vector from magnetic-core memory and then finding the subgroup which belongs to this wave vector. The subgroup is coded in three WWI words. The program then picks up the simple code words which tell into what basis function of which irreducible subspace the linear combination of wave functions is to go. If no basis function is specified the program chooses the minimal linear combination for the subspace. The result is six WWI words which are then fed into a matrix-generation and diagonalization routine (developed by Dr. Howarth under Problem 147). Finally the program picks up a series of reciprocal lattice vectors each of which characterizes a different order of neighbor and generates all the different reciprocal lattice vectors for each order of neighbor. These vectors are then processed so that only reciprocal lattice vectors which cannot be obtained from one another by operations of the original wave-vector group appear. This ensures against a secular equation with rank lower than order. These reciprocal lattice vectors are then fed into the matrix-generation and diagonalization routine and this routine is entered.

It should be mentioned that Mr. Saffren's routine can, at the moment, only be applied to solids which are face- or body-centered crystals. This is because Dr. Howarth's routine applied only to such crystals.

More details regarding the theory of the calculation can be found in the Quarterly Progress Reports, Solid State and Molecular Theory Group, MIT.

195 INTESTINAL MOTILITY

A study of the effect of radiation upon the gastrointestinal tract of rabbits is being carried out for Dr. J. T. Farrar of the Gastroenterological Section of the Evans Memorial Hospital by Miss D. Hamilton of the MIT Servomechanisms Laboratory.

During this period several runs were made, but the main effort has been to program a routine for plotting results and to use techniques which reduce truncation errors in the Fourier transformation. Since difficulties arose with the automatic scope routine, special scope routines are being developed.

The autocorrelations and the Fourier transforms cited above use programs developed under Problem 107. The scope routine for plotting and the techniques for reducing

truncation errors were developed under Problem 171. These problems have been described by Douglas Ross in previous reports under their respective problem numbers.

199 LAMINAR BOUNDARY LAYER OF A STEADY, COMPRESSIBLE FLOW IN THE ENTRANCE REGION OF A TUBE

In connection with research on heat-transfer coefficients, recovery factors, and fraction coefficients for supersonic flow of air in a tube, a theoretical investigation of the characteristics of the laminar boundary layer in the entrance region has been carried out by Dr. T. Y. Toong of the MIT Mechanical Engineering Department. Partial differential equations of continuity, momentum, and energy were developed for the boundary layer. These were then transformed into a series of ordinary differential equations to be solved by WWI for several entrance Mach numbers and thermal conditions at the tube wall.

First, solutions are to be obtained for the case of constant viscosity and thermal conductivity. Then, the effects of temperature dependence of these properties are to be studied.

Solutions of the second set of differential equations for the case of constant viscosity and thermal conductivity have been obtained for six cases of different entrance Mach numbers and thermal conditions at the tube wall. This represents another 15% of the entire job to be done by WWI.

Work is being done to solve the third set of differential equations corresponding to two of the six cases mentioned above.

200 A STUDY OF RECURRENT EVENTS

The investigation of various digital methods for identifying one of two possible Bernouilli sequences has been completed. This work has been done by Miss B. Jensen for Dr. G. P. Dineen, both of the Lincoln Laboratory at MIT.

Although various methods were tried in order to determine the most efficient method, no rigorous optimization scheme was carried out. The use of "success runs"[1] and their recurrence times as a means of distinguishing a Bernouilli sequence with high probability has been investigated using computer-generated sequences. A similar study for moving average detectors was also carried out. The use of Whirlwind I made possible the acquisition of a large amount of numerical data on these detectors.

1. Feller, W., Probability Theory and Its Applications, Wiley, New York, 1950.

201 STUDY OF THE AMMONIA MOLECULE

Since the last report the self-consistent field solutions for the ammonia molecule have been computed. From these, orbitals which represent the ground state of the molecule have been constructed. These, in addition to the excited wave functions for the molecule, will be used to get the best approximation to the wave function of this molecule within the approximations used. The interaction of the lowest state and the excited state will give rise to a matrix of interaction. The diagonalization of this matrix will be carried out on Whirlwind through the use of a program written by Alvin Meckler and improved by F. Corbató of the Solid State and Molecular Theory Group at MIT. The calculation of the matrix of interaction is now in progress.

203 RESPONSE OF A FIVE-STORY FRAME BUILDING UNDER DYNAMIC LOADING

The five-story building has been approximated by a system of five mass points (representing the floors) connected by nonlinear acting springs (simulating the building columns). This representation of a building frame is conventional for frames with stiff floors so that any failure will occur in the columns. For frames up to five stories high it seems to be a good approximation and results in equations which are readily adaptable to numerical calculation.

The equation describing the motion of a mass point is:

$$m_s \ddot{y}_s (t_n) = P(t_n)\, Y_s + R_{s+1} - R_s$$

where

m_s = mass s
y_s = horizontal displacement of mass s
$\ddot{y}_s$ = acceleration of mass s
$P(t_n)$ = load at time t_n (impulse)
Y_s = loading constant for mass s
R_{s+1} = resistance of spring above mass s
R_s = resistance of spring below mass s

The loading is calculated from pressures determined from the actual analysis of a five-story building under dynamic loading.

To find the value of the resistance of any spring s at any time t it is necessary to calculate an assumed elastic value of the resistance and then compare this value with the maximum possible spring resistance at this time. These maximum values of the spring resistance at various times are known in advance and are included in the program as a

series of constants. The resistance calculation is based on the previous value of the resistance and the change in the relative displacements of the two masses adjacent to the spring. Therefore:

$$R_s^{1}(t_n) = R_s(t_{n-1}) + K_{s1}\left[\alpha_{s,\,s-1}(t_n) - \alpha_{s,\,s-1}(t_{n-1})\right]$$

$$\left|R_s^{1}(t_n)\right| < R_{s(max)}(t_n)$$

where

$R_s^{1}(t_n)$ = value of the resistance at time (t_n)
$R_s(t_{n-1})$ = previous value of the resistance
K_{s1} = slope of the equation relating α and R in the linear range (for spring s)
$\alpha_{s,\,s-1}(t_n)$ = relative deflection of mass s with respect to mass s-1 at time (t_n)
$R_{s(max)}(t_n)$ = maximum value of the spring resistance of spring s at time t_n

This procedure allows for increasing or decreasing relative deflections which may result in either positive or negative values of the resistance. One disadvantage of the procedure is that errors in the value of $R_s(t_n)$ tend to accumulate. For this problem this disadvantage will not result in obvious errors.

For the integration of the five nonlinear second-order differential equations, the second difference equation,

$$(\Delta t)^2\, \ddot{y}_s(t_n) = y_s(t_{n+1}) - 2y_s(t_n) + y_s(t_{n-1}),$$

is used.

To check the program, simplifications were introduced in the original program and the response of a three-story frame building was determined. These results checked very well with a hand calculation and also compared favorably with experimental results. The five-story frame has been analyzed under two similar loading conditions and has produced results which agree rather closely with a very elaborate hand calculation.

The program is now being modified slightly to include the effect of vertical load plus dead load. It is hoped that this refinement will help in producing results even closer to those achieved in the hand analysis. The handling of the loading constants is also being modified to smooth out the data, particularly in the neighborhood of sharp changes in slope. The modified program will then be used to determine the response of the five-story building for about 30 different loading conditions.

This study is being carried out by C. W. Johnson of the MIT Department of Civil and Sanitary Engineering.

204 EXCHANGE INTEGRALS BETWEEN REAL SLATER ORBITALS

The purpose of Problem 204 is the computation of exchange integrals between Slater atomic orbitals located on two separate centers. The program designed to accomplish this can, with slight modifications, be generalized to give two-center hybrid and Coulomb integrals as well as exchange integrals. The efficiency with which these extra tasks can be accomplished by use of the program is dubious. However, this generalization has been incorporated. The program has been coded in its entirety. The tapes have been prepared and the testing procedure initiated.

The computation of one of the auxiliary functions,

$$f(n, \beta; x) = \int_x^\infty y^n e^{-\beta y} \, dy,$$

is expected to introduce additional difficulties resulting from cancellation errors in the generation of the functions. Since P. Merryman has discovered that this function can be evaluated (in the desired range) as the quotient of two series of positive terms, this difficulty--if it does appear--is not expected to prove insurmountable.

This work is being carried out by P. Merryman under the direction of Professors Mulliken and Roothaan of the University of Chicago Physics Department.

208 INTERCEPTOR FLIGHT-CONTROL PROBLEM

This problem on interceptor flight control was described in the last Summary Report as a system of 16 nonlinear differential equations and was solved using the Gill method, an adaptation of the Runge-Kutta fourth-order procedure.

The system was varied in this quarter, a new equation substituted, and a constant changed to give a system of 13 differential equations. Two solutions of this system were run on Whirlwind using CS II. The stability of one solution and the instability of the other demonstrated the sensitivity of the whole system.

This represents the end of the digital computer phase of this problem. The solution was carried out by M. Merwin of the MIT Dynamic Analysis and Control Laboratory.

209 NUMERICAL SOLUTION OF HOMOGENEOUS LINEAR DIFFERENTIAL EQUATIONS WITH QUADRATIC POLYNOMIAL COEFFICIENTS

This problem was selected by Dr. J. C. P. Miller of Cambridge University, Cambridge, England, as one that not only would provide a generally useful and interesting routine but could be completed during his brief visit at the Digital Computer Laboratory.

The method of solution makes use of a recurrence relation for terms in the Taylor expansion. A detailed discussion may be found in Summary Report No. 39.

During the quarter a few more test runs were completed, and the problem has terminated.

210 RESIDUE INDICES AND PRIMITIVE ROOTS

This problem was also selected by Dr. J. C. P. Miller of Cambridge University, Cambridge, England, to be coded on WWI during his brief visit. The problem is a familiar one: given a prime p and a small integer a, determine the least e such that $a^e - 1$ is a multiple of p.

The numerical procedure selected was to determine from $r_i = a^i \bmod p$, $r_{i+1} = a r_i$. This was then divided by p to give $|r_{i+1}| \leq \frac{1}{2} p$. This was tested for ± 1 and the process repeated, counting the number of repetitions.

Dr. Miller plans to code this problem for five or six computers as part of a comparison test of speed and capabilities. The coding on WWI was completed before Dr. Miller's return to England.

211 SERVO RESPONSE TO A COSINE PULSE

This problem was concerned with extending the curves in Figure 25-16, p. 705 of Instrument Engineering, Volume II, by Draper, McKay, and Lees. Work on this problem has been completed. The results have been plotted and are being considered for inclusion in Volume III of the above-mentioned book. The original problem was to find numerically the maximum response x(t) for some parameter pairs $(\zeta, T_p/T_n)$, $0.01 \leq \zeta \leq 5$, $0.1 \leq T_p/T_n \leq 10$ in the equation

$$(1/w_n^2)\ddot{x} + (2\zeta/w_n)\dot{x} + x = G(t),$$

where $T_n = 2\pi/w_n$, $\dot{x} = dx/dt$, $\ddot{x} = d^2x/dt^2$, and where $G(t) = 1/2\left[1 - \cos(2\pi t/T_p)\right]$ for $0 \leq t \leq T_p$, and $G(t) = 0$ for $t > T_p$. As work progressed on this problem it was decided also to treat in the same manner the similar problem corresponding to

$$G(t) = 1/2 \left[1 - \cos(2\pi t/T_p) + 2\zeta(T_n/T_p)\sin w_{pt}\right]$$

for $0 \leq t \leq T_p$, and $G(t) = 0$ for $t > T_p$.

Coding of the problem using the CS II system was accomplished by Dr. J. M. Stark of the MIT Instrumentation Laboratory. Application of the curves obtained is described in the above-mentioned reference.

212 DISPERSION CURVES FOR SEISMIC WAVES: MULTILAYERED MEDIA

A program has been written which calculates dispersion curves for surface seismic (Rayleigh) waves on multilayered media. The calculation follows that outlined by Haskell ("The Dispersion of Surface Waves on Multilayered Media," N. A. Haskell, Bul. Seismological Society of America, V. 43, No. 1, January 1955, pp 17-34). The only difference is that, rather than assuming a phase velocity and iterating to find a wave number, the reverse procedure is followed to avoid difficulties which would arise at a phase-velocity minimum. The program has been written for as many as five layers. When the main body of the program is working properly, a routine will be added to differentiate the results numerically in order to obtain the group velocities.

This work is being carried out by K. Vozoff of the Department of Geology and Geophysics in partial fulfillment of the requirements for the degree of Ph. D.

213 INDUSTRIAL PROCESS CONTROL STUDIES

The Machine Design Division of the Mechanical Engineering Department is engaged in determining the characteristics of a physical system or process from the random fluctuations in the input and output records during the normal course of operation of the system or process. The idea has been developed by T. P. Goodman, C. M. Chang, and J. B. Reswick from work done by N. Wiener, A. Tustin, and Y. W. Lee and has been found to be promising in its applications both to linear systems and to some nonlinear systems under actual operation in industry. It is believed to have certain advantages over the conventional approaches (e.g., step and frequency responses).

It may be shown that certain statistical functions of the random data (the auto- and crosscorrelations) bear the same relationship with each other as the random input and output themselves. Since these statistical functions possess many desirable properties that the random functions themselves do not have, they are used for the determination of the system characteristics. The computation of these statistical functions is hence a most essential step in the current investigation. Such computations involve a great number of successive multiplications and averaging and can be done most readily on an automatic computer.

In the past 3 months, some exploratory work on the new technique was done. Statistical functions on data taken from a real flow system and some artificial systems were computed. These functions, after analysis, gave system characteristics which compared very favorably with the system characteristics obtained by the conventional approach. The essential results are reported by C. M. Chang in his Mechanical Engineer's thesis, entitled "A New Technique of Determining System Characteristics from Normal Random Operating

Records."

Efforts will be continued in much the same direction in the near future. Arrangements have been made with some industrial concerns so that data on actual industrial systems can be taken in their plants. The statistical functions can then be computed and analyzed and the system characteristics determined. It is hoped that with further experience and study many of the present difficulties can be solved and new light will be shed on the characteristics of industrial processes,

214 INTERVAL DISTRIBUTION

This problem arose in the textile industry. A suitable distribution for the "slubs" or thickenings on an artificial yarn was needed. The one suggested, which could be produced by a simple mechanism, was this: Take a series of events occurring at regular intervals a and delay each event in a random manner, the delays being distributed about a certain mean in a gaussian way.

The problem is to calculate the distribution of the intervals between successive events produced in this way. The method was the Monte Carlo one of making a long series of events in the prescribed manner and calculating the interval distribution. There was no loss in generality in taking the mean delay as zero (since it is irrelevant) and the standard deviation of the delays as 1. The only parameter is then a, the original interval.

For large a, the interval distribution itself is normal, because the sequence of events is almost never disturbed by the random deviation. For small a, the distribution is like that of a Poisson distribution, except that for large intervals it decays as a gaussian distribution does. The intervals are not uncorrelated, however. The interesting region to investigate is that around a=1 in which the distribution changes from one kind to the other.

To produce the random normal deviates, uniformly distributed random numbers were added. The difference equation $a_{n+1} = a_n + a_{n-1}$ (mod 1) produces such numbers. These series were calculated double length to reduce the dangers of a periodic solution.

Twelve successive values were added, and, after subtracting the mean, which is 6, a reasonably good approximation to a normal deviate with standard deviation 1 should be obtained. This distribution was tested by distributing into cells of width 1/4 in the range -2 to +2 four runs of 10,000 cases. A count of cases falling outside the range was kept as a check. The results were satisfactory.

Having the spacing a and random deviates v_1 v_2 v_3, the intervals, if the sequence were preserved, would be $a + v_2 - v_1$, $a + v_3 - v_2$, etc. However, some of these would appear negative, because the sequence is altered. We therefore keep the last ten intervals in case the next event falls somewhere in this region.

Suppose three successive intervals are i_1, i_2, i_3 and i_2 is negative. Then transposing the two events of which i_2 is the interval gives the new intervals $i_1 + i_2$, $-i_2$, $i_3 + i_2$.

If the ten intervals kept have been "sorted" and are positive, when the next interval is found, if it is negative, the transposition process, carried down the series of intervals, will eventually make them positive. If, however, the last interval stored were to turn out negative, the process would have failed because more than ten should have been stored.

As a new interval is added to the series of ten, the oldest member of the series is added to the distribution and discarded.

For this problem, cells of width 1/4 between the ranges 0 and 8 were used, and, as a check, a count was kept of intervals greater than 8 and apparently negative intervals, though in fact none were found.

Two runs for each of the values a = 1.0, a = 1.5, a = 2.0 were done, with 10,000 intervals in each. Before beginning to accumulate the distribution, 20 intervals were calculated to fill the store of intervals properly.

The results were treated like experimental results and when plotted gave smooth curves.

This study was carried out by D. W. Davies of the National Physical Laboratory, Teddington, England.

215 PLANT SURVEYS BY STATISTICAL METHODS

The application of automatic feedback control to existing industrial heat transfer processes requires knowledge of the transfer function of the process. Such transfer functions are ordinarily determined by observing the response of the process to a sinusoidal variation of its input variable. The aim of the present study by S. Margolis of MIT's Research Laboratory of Electronics is to perfect methods for determining transfer functions from formal operating records without introducing any variations in the input variable other than the random fluctuations already existing.

A small heat exchanger installed in the Process Control Laboratory of the Electrical Engineering Department has been used for experimental studies. Random fluctuations of the input and output variables were recorded on a Sanborn Two-Channel Recorder. This data was reduced manually to numerical form. The autocorrelation function of the input and the crosscorrelation between input and output were computed by Whirlwind I for two separate sets of data, using routines developed by D. T. Ross of the Servomechanisms Laboratory. The transfer functions determined from the correlation functions agree with those determined by subjecting the system to sinusoidal variations.

216 ULTRASONIC DELAY LINES

This problem was undertaken in cooperation with the Arenberg Ultrasonic Laboratory because of the importance of the results for MIT's Lincoln Laboratory. The problem is concerned with the optimum design of an ultrasonic delay line.

For the optimum packing of microseconds delay per cubic inch, radial or "asteriated" symmetry paths are used. This design was developed by the workers of the Bureau of Aeronautics of the U.S. Navy. These designs have given way to MS or "Multiple Symmetry" designs where many of the regular facets are tilted.

A routine has been developed for WWI that will select by iteration an optimum set of radii for such designs.

217 VARIATION-PERTURBATION OF ATOMIC WAVE FUNCTIONS AND ENERGIES

The wave functions and energies for low-lying stationary states of helium through neon and their isoelectric series have been calculated approximately from a variational method.[1] The method consists of minimizing the integral

$$E = \int \psi \, H \, \psi \, dt \qquad (1)$$

where H is the Hamiltonian of the system and t includes all spatial and spin coordinates. ψ, the trial wave function, is a properly normalized linear combination of determinantal functions whose components are hydrogen-like. The nuclear charge in the hydrogen-like functions is replaced by variational parameters.

The integration in (1) may be carried out analytically leaving E as an algebraic function of the variational parameters and reducing the problem to one of finding the minimum of a rather complex expression.

A program for Whirlwind I, which computes the best wave functions for any atom containing 1s, 2p, and 2s orbitals, has almost been completed. It will probably be extended shortly to include the 3s and sp orbitals. The extremum for E is found by expanding it in a Taylor's series to second order about a point assumed to lie near the extreme point, differentiating the series with respect to the parameter increments, and solving the resulting series of linear inhomogeneous equations for the coordinates of the extreme point relative to the zero point of the Taylor expansion.

After the initial variational treatment, the wave functions obtained can be improved by adding to them a linear combination of correction terms similar in structure but corresponding to higher excited states.[2]

$$\psi_{corrected} = \psi_{variational} + \sum_i c_i \phi_i$$

1. Morse, P.M., Young, L.A., Haurwitz, E.S., Physical Review 48, 948 (1935)
2. Yilmaz, H., Ph.D. Thesis (unpublished), MIT Physics Department (1954)

The c_i's are determined by first-order perturbation theory. In cases where the ϕ_i contain parameters not already fixed by the first variation, a second variation involving the unfixed parameters is carried out.

Several of these perturbation corrections involving the $(1s^2)$'s and $(1s2p)\,^3p$, ^1p configurations of helium have been carried out on Whirlwind by Arnold Tubis of the MIT Physics Department. The results are discussed in Quarterly Progress Report No. 14 of the Project on Machine Methods of Computation and Numerical Analysis, MIT, December 15, 1954. The work on these perturbation corrections is being continued at present. It has been found that both the variation and perturbation calculations may be broken down into well defined subroutines which greatly facilitate the programming.

218 TRANSFORMATION OF INTEGRALS FOR DIATOMIC MOLECULES

Eigenvalue equations are frequently encountered of the form

$$[H]X = \lambda [S] X, \tag{1}$$

where $[H]$ is Hermitian and $[S]$ is real, symmetric, and positive definite. Standard methods based upon successive orthogonal or unitary transformations can be applied to solve equations

$$[H'] Y = \lambda I Y, \tag{2}$$

where I is the unit matrix. To convert equations (1) into this form by the Schmidt process, a lower triangular matrix $[M]$ is constructed which has the property

$$[M] [S] [\tilde{M}] = I. \tag{3}$$

Here a tilde (~) denotes a transposed matrix. Applying transformation (3), equations (1) are reduced to the form (2) with

$$[H'] = [M][H][\tilde{M}] \tag{4}$$

and

$$X = [\tilde{M}] Y. \tag{5}$$

Whirlwind programs have been coded which construct $[M]$ from a given positive-definite matrix $[S]$ of arbitrary dimension n x n and carry out the transformations (4) and (5) for any real symmetric n x n matrix $[H]$ and n-vector Y.

Given $[S]$, the Schmidt matrix $[M]$ is constructed as a product of triangular matrices

$$[M] = [B^n] \dots [B^1]$$

The matrix

$$[S^m] = [B^{m-1}] \dots [B^1] [S] [\tilde{B}^1] \dots [\tilde{B}^{m-1}]$$

has the unit matrix I^{m-1} as its upper left-hand submatrix of order $(m - 1)^2$. The remaining elements are in general nonzero. The well-known Schmidt orthonormalization process, applied to the basis vectors of $[S^m]$, normalizes the vector of index m and makes it orthogonal to all vectors with indices 1, ..., m - 1. Vectors of index lower than m are not affected by $[B^m]$, which is defined by

$$B_{ij}^{\ m} = \delta_{ij} \qquad ; i, j < m \text{ or } i, j > m$$

$$B_{im}^{\ m} = 0 \qquad ; i < m$$

$$B_{mj}^{\ m} = -S_{mj}^{\ m} B_{mm}^{\ m} \quad ; j < m$$

$$B_{mm}^{\ m} = \left[S_{mm}^{\ m} - \sum_{j=1}^{m-1} (S_{mj}^{\ m})^2 \right]^{-1/2}$$

The independent elements of the symmetric matrix S are stored in lower triangle form, i.e., in the order $S_{11}\ S_{21}\ S_{22}\ S_{31}\ S_{32}\ S_{33}$, etc. The program which constructs $[M]$ is arranged to replace $[S]$ by $[M]$ as the latter is computed, and only four additional number registers are needed. The programs for transformations (4) and (5) have similar properties.

This study is being carried out by R. K. Nesbet of the MIT Solid State and Molecular Theory Group.

220 PROBLEM ARISING FROM AN ALGEBRA

As a sample problem to help in his study of WWI, D. W. Davies of the National Physical Laboratory, Teddington, England, selected the study of the algebra whose basic elements consist of two associated patterns which can be combined as follows.

Consider the pattern (a) representing one way of joining in pairs all of six spots. For six spots there are 15 such patterns. Taking two patterns, (a) and (b), if they are superimposed on one set of spots a number of loops are formed. This number will be called $\ell(a, b)$, and $1 \le \ell \le 3$.

In general there can be 2N spots and $(2N-1)(2N-3) \dots (3)(1)$ ways of joining them into patterns. For any two patterns a, b, $1 \le \ell(a, b) \le N$.

Suppose we take two fixed patterns a, b and a variable pattern x. The numbers $\ell(a, x)$ and $\ell(b, x)$ can be used as coordinates to establish the pattern x in a cell of an N x N matrix. Writing in each cell the number of elements with these coordinates, we have a distribution of all possible elements according to their values $\ell(a, x)\, \ell(b, x)$.

It turns out that the numbers in the matrix depend only on $\ell(a, b)$ and not on the choice of a and b. For this reason, the matrices are symmetric, and there are N of them for the patterns of degree N, corresponding to $\ell(a, b) = 1, 2, \ldots N$.

The work involved in computing these matrices by hand becomes large after N = 3. Cases N = 4, 5 are done quickly on Whirlwind, but the six matrices for N = 6 took 45 minutes to compute. Case N = 7 was therefore not attempted.

The computed matrices showed a pattern of zeros, which could be expressed by the two inequalities

$$\ell(a, x) + \ell(b, x) \leq N + \ell(a, b)$$

$$\ell(a, x) - \ell(b, x) \leq N - \ell(a, b)$$

A proof of these relations (which are really identical) was obtained.

By examining the matrices carefully, a relation between them was discovered. This relation enabled the matrix of order N for $\ell(a, b) = n$ to generate the matrix of order N + 1 for $\ell(a, b) = n + 1$. A proof of this relation was obtained.

With this relation, all the matrices of order N + 1 can be obtained except the one $\ell(a, b) = 1$. The work of calculating the matrices would have been greatly reduced if it had been known.

The symmetry of the matrices $\ell(a, b) = 1$ and their invariance to the choice of a and b can easily be shown. This, however, does not prove the invariance for the other matrices, and no proof has been found for it.

221 COURSE 6.25, 1954

Fifty-nine students enrolled in course 6.25, entitled Machine-Aided Analysis, which was given in three sections during first semester, 1954, by Professors W. K. Linvill, R. C. Booton, and C. W. Adams. Machines which aid in the solution of mathematical equations encountered in engineering were studied. TAC, the three-address summer session computer, was used by the students as an exercise in digital-computer programming. A detailed description of TAC may be found in Summary Report No. 39.

The following problem was solved:

"Consider the Van der Pol Equation:

$$\ddot{y} + \epsilon (y^2 - 1)\, \dot{y} + y = 0, \text{ where}$$

$$\dot{y}(0) = a$$
$$y(0) = b$$

For this equation assumed values of ϵ, a, and b are chosen so that $\ddot{y}_{max} < 10$, $\dot{y}_{max} < 10$, and $y_{max} < 10$. Solve the equation using intervals, h = 0.1. Print out the values of y for integer values of t and carry out the solution until the value of y has passed its second positive peak. Scale factor the solution so that all of TAC's 8 digits are used. Use the rectangular rule of integration."

Each student prepared his own Flexowriter tape and was present at the running of his problem. This problem took approximately 3 minutes of WWI computer time to solve.

222 HELICOPTER ROTOR STABILITY

The equations of motion of a flexible helicopter blade with constant geometric and mass properties in forward flight were developed. This analysis considered two degrees of freedom of the blade: one, rigid flapping, the other, elastic bending. Previous analyses made of stiff blades have shown that the transient solution decreases in stability with increasing forward speed to tip speed ratio. The forced solution has always been assumed to be a Fourier series expansion in ψ, the azimuth position of the blade.

A preliminary program, conducted as a Master's thesis, has shown that inclusion of the bending degree of freedom results in a prediction of decrease in transient stability which is in much closer agreement with experimental results than was obtained earlier.

The proposed program consisted of two parts:

1. To supplement the results already obtained in the preliminary program by desk calculators;

2. To integrate the equations of motion directly, thus to gain an insight into the behavior of the forced as well as the transient solution. The equations of motion are given by a pair of second-order ordinary differential equations.

In part 1 the problem was to expand an 8 x 8 determinant whose elements are polynomials in σ, the exponential factor of the transient solution. The expansion results in a 16th degree polynomial in σ, whose most positive root determines the degree of stability of the solution. This was done for various sets of the parameters of the problem.

The numerical method makes use of the polynomial-transformation theorem and the Routh-Hurwitz criterion for positive real roots. (Ref. Draper, McKay, and Lees - Instrument Engineering, McGraw-Hill, 1953, Chap. 21.)

The results of the first part checked with the hand results, and additional results served to throw more light on the behavior, stability-wise, of rotors with high moments of inertia at high speeds.

Part 2 of this program consisted of a numerical integration, using the fourth-order Runge-Kutta method, of the two simultaneous, ordinary linear differential equations (second-order) with periodic coefficients.

The second part, whose purpose was to provide visual results of the flapping motion of helicopter rotors with different inertia properties at various speeds, will be completed during the next quarter. Twenty-five cases will be run.

Conclusions that may be drawn from this program are: (1) Higher harmonics should be included in the assumed solution; and (2) Whirlwind can be used to advantage in solution of problems of this nature.

This study is being carried out by Y. Shulman of the MIT Aeronautical Engineering Department.

225 NEUTRON-DEUTERON SCATTERING

The problem consists of the calculation of S wave phase shifts for neutron-deuteron scattering by means of a variational principle. Stationary expressions are obtained for the quartet ($J = 3/2$) and doublet ($J = 1/2$) phase shifts, which fully determine the low-energy scattering. These expressions involve only known functions and a so-called "inside wave function" y, which is the difference between the actual wave function and its asymptotic behavior. Any trial function for y which goes to zero when either neutron is far away will automatically satisfy the boundary conditions of the problem. The advantage of this formulation is that it enables us to take account of polarization, i.e., the deformation of the deuteron while all three particles are interacting.

If we make an expansion of our stationary expression in powers on the energy, the first two terms will give the scattering lengths and effective ranges, in complete analogy to the two-body scattering. Results of interest will be the dependence of the scattering lengths and ranges on the exchange nature of the forces and their comparison with experimental values.

The Gaussian well is used for the two-body potentials, and tensor forces are not included. The trial function used is essentially Gaussian in the interparticle distances, with three variational parameters, λ, μ, η. The numerical work consists of calculating a large number of algebraic expressions and a few numerical integrations as functions of λ, μ, and η and looking for a stationary point of the variational expressions. Unfortunately, one cannot tell beforehand whether this will take the form of a maximum, minimum, or inflection point.

A program to calculate the quartet scattering using CS II has been written, and the checking of its various parts is almost complete. The calculation will begin with λ and μ fixed at their "nonpolarization" values, and the variational expression will be plotted as a function of η. λ and μ will then be varied in the neighborhood of the optimum value of η. It appears that a single curve will take between 5 and 10 minutes of machine time.

This study is begin carried out by L. Sartori of the MIT Physics Department.

228 EVALUATION OF DIFFERENCE DIFFUSION EQUATION

The subject involves the one-dimension diffusion equation in the simplest form

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{\partial \psi}{\partial t}$$

applied to a region $0 < x < 1$ with the following conditions:

Initial condition: $\psi = 0$

Boundary condition: $\frac{\partial \psi}{\partial x} = 0$ at $x = 1$

$$\left.\begin{array}{l} 1.\ \psi = 1 \text{ and } \sin wt \\ 2.\ \dfrac{\partial \psi}{\partial x} = -1 \\ 3.\ \psi - Q\dfrac{\partial \psi}{\partial x} = 1 \end{array}\right\} \text{ at } x = 0$$

Explicit and implicit recurrence formulas of the coarse-grid difference approximation are used to derive the transient solution to this equation at the above conditions.

The general recurrence formula can be represented as

$$\psi_{j,k+1} + r\theta(\psi_{j+1,k-1} - 2\psi_{j,k+1} + \psi_{j-1,k+1}) = \psi_{j,k} + r(1-\theta)(\psi_{j+1,k} - 2\psi_{j,k} + \psi_{j-1,k}).$$

By comparing the exact solutions of the continuous equation and the difference approximation, a method is derived to optimize the conformation of the approximate transient solution to the true solution. The problem is to evaluate the merits of such a method over the conventional method. A rational way to compare these methods is to carry out complete solutions by digital computer for a period of time and to compare them with the continuous solution over the same period of time.

With the exception of the interpolation process, the explicit case of boundary conditions 1 and 2, above, are completed. The explicit case of boundary condition 3 has been started. It is expected that the computation for the explicit case will be completed during the next quarter.

This project is being carried out by A. T. Ling under the supervision of Professor S. H. Crandall of the MIT Mechanical Engineering Department.

229 ROTATING CONTACT SEAL

This problem seeks to evaluate the rubbing contact temperature in various designs of a rotating contact seal for a gas turbine. The heat input at any part of the boundary in question is expressed as a linear function of the temperature at all other parts of the boundary, leading to a set of simultaneous algebraic equations.

Two routines were written, and each ran successfully. Each routine generated 18 matrices of order 15 x 15 and solved corresponding sets of simultaneous equations by Crout's method.

This problem is being carried out by J. M. Bonneville of the MIT Mechanical Engineering Department, and the results will be included in his Sc. D. thesis.

230 BRIDGE ANALYSIS

Saul Namyet of the MIT Civil and Sanitary Engineering Department is analyzing the dynamic response of a large number of bridges which vary in span and type. The bridges are represented by a one-mass system. This system is acted upon by a combination of time-variant external forces and a resistance function which is considered to vary with displacement in a predetermined manner. It is required to obtain the magnitude of a parameter (which determines the magnitude and time duration of the external forces) for which a prescribed displacement is achieved. The complete problem will consist of the analysis of the whole set of bridges for four different resistance functions.

The solution of the problem consists of the solution of the following differential equation:

$$m\ddot{x}(t_n) = P_n - R_n,$$

$\ddot{x}(t_n)$ = the acceleration at time (t_n)

P_n = load at time (t_n)

R_n = spring resistance at time (t_n).

The equation is solved by use of the second difference equation:

$$(\Delta t)^2 \ddot{x}(t_n) = x(t_{n+1}) - 2x(t_n) + x(t_{n-1}).$$

The program has been checked by comparing a limited number of response calculations with hand solutions and graphical computations. Since the results have shown satisfactory agreement, the response is now being calculated for the complete set of bridges utilizing the first resistance function.

231 REACTOR RUNAWAY PREVENTION

In this problem, M. Troost of the MIT Chemical Engineering Department has been attempting to determine the safest reactor type and size for MIT's proposed nuclear reactor.

The analysis involves a step-by-step algebraic calculation with an iterative correcting routine. Many interesting results have already been obtained.

232 ENERGY LEVELS IN A SPHEROIDAL POTENTIAL

An investigation of the structure of heavy nuclei based on the Bohr-Mottelson model[1] is being carried out[2] by Kurt Gottfried of the MIT Laboratory for Nuclear Science.

The first step in this program is the determination of energy levels and eigenfunctions associated with the motion of particles in a force field whose equipotentials are ellipsoids of revolution. Most of the required matrices, whose ranks range from four to fourteen, have now been diagonalized using F. J. Corbató's routine developed under Problem 172.

1. Dan. Mat. Fys. Medd. 27, No. 16 (1953).
2. Gottfried, K. and Weisskopf, V. F., Quarterly Progress Report Laboratory for Nuclear Science, November 1954.

3. OPERATION OF WHIRLWIND I

3.1 Summary

Greater emphasis is now being placed on reliability and maintenance in order to provide the best possible performance.

Reliability of the WWI computer system was slightly higher this quarter than in previous quarters. Applications time as reported by computer operators was 95% usable as compared with an average of 93% since September 1953. An account of the assignment of computer time to the various major activities is given in Fig. 3-1.

	Hours Per Week													
	October				November					December				
Activity	1-7	8-14	15-21	22-28	29-4	5-11	12-18	19-25	26-2	3-9	10-16	17-23	24-30	Total Hours
Marginal Checking	4	4	5	5	5	6	7	8	6	7	6	8	4	75
Installation	7	0	7	7	7	0	7	0	7	0	7	0	0	49
Maintenance	25	9	18	22	19	11	19	18	24	21	30	13	10	239
Terminal Equipment Testing	34	36	19	21	22	21	17	13	20	31	15	22	11	282
Technician Instruction	4	4	4	6	4	0	3	0	4	3	4	0	4	40
Scientific and Engineering Computation	45	37	41	33	33	33	24	28	30	30	29	42	21	426
Other Applications	27	34	59	56	59	64	73	74	56	56	64	55	41	718
Total Hours	146	124	153	150	149	135	150	141	147	148	155	140	91	1829

Fig. 3-1. Allocation of Computer Time

3.2 Computer and Terminal Equipment

During the past 6 months studies have been undertaken not only to improve system performance but also to train new personnel and to compensate for the recent transfer of experienced engineers to other areas. Specific phases of the present program are described below:

1. A biweekly analysis of all equipment failures was initiated to point out the types of interruptions that occur most frequently and those that result in greatest loss of time.

Periodic discussions of these records and of unique system-engineering problems have helped to develop the single-system concept and make all personnel aware of the relationships of the various independent efforts to system performance as a whole.

2. As in the past, design weaknesses that show up are being corrected where practicable. Such weaknesses have already been eliminated in the core-memory sense amplifiers, in some of the buffer-drum circuits, in the marginal-checking control, and in the equipment-cooling system.

3. New computer test programs have been written to facilitate routine marginal checking of some of the newly installed terminal equipment. However, power distribution for marginal checking must still be evaluated along with the test programs, and both may require changes in order to obtain satisfactory checking procedures.

4. An extended training course in computer-system details is being conducted for new engineers and technicians for about 3 hours each week. Experience has shown that a thorough understanding of system logic is essential in order to locate trouble effectively. The course is supplemented by actual practice in locating simulated computer failures.

Over-all system reliability improved according to the daily estimates of computer operators. Their records show that 95% of the assigned applications time was usable, compared with the 93% average since September 1953.

The biweekly interrupting-failure analyses initiated about 25 September provide preliminary averages to supplement the figures above:

1. Since 25 September, the average time between interrupting failures has been about 9.5 hours. (All system failures except those attributable to new installations or modifications are included whether or not applications time was lost.)

2. During the 3-month period, the single category of failures causing most frequent stoppages was transient computer alarms which could not be immediately associated with faulty components or other system damage or deterioration. These alarms, over half of which are not yet explained, accounted for 34% of the interrupting failures.

3. During each biweekly period a few isolated failures, generally nonrepeating, caused major loss of time. Over the past 3 months, 10% of the interrupting failures have accounted for about 50% of the total time lost.

It seems reasonable to expect some reduction in lost time as maintenance procedures are improved and operating margins are widened in the newer sections of the system. However, a large increase in the average time between failures seems doubtful, since detection and elimination of causes of transient errors are difficult.

4. CIRCUITS AND COMPONENTS

4.1 Vacuum Tubes

4.1.1 Vacuum-Tube Life

During the fourth quarter of 1954 the WWI computer operated for 1760 hours. During 1954 total hours of operation were 6900, about 79% of the maximum possible hours in the year based on 365 twenty-four-hour days. This over-all utility factor is almost identical to that for the year 1953.

The tube complement of WWI, including the computer proper, magnetic-core memory, magnetic drums, and miscellaneous input-output equipment, is now approximately 12,500.

Vacuum-tube life has been calculated for the six most numerous types in WWI. These six types comprise about 80% of the total tube complement. A summary of this information together with previous data is shown below.

FAILURE RATE, PER CENT PER 1000 HOURS

Tube Type	First Quarter 1954	Second Quarter 1954	Third Quarter 1954	Fourth Quarter 1954	Quantity Now in Service
7AD7/SR1407/6145	1.75	1.6	1.4	1.3	4237
7AK7	0.5	0.6	0.3	0.35	3340
6080/6080WA/6AS7G	1.1	1.5	1.4	0.40	718
5965	0.4	0.3	0.3	0.26	878
6BL7GT	0.3	0.5	0.5	0.35	494

ANNUAL AVERAGE FAILURE RATE, PER CENT PER 1000 HOURS

Tube Type	1952	1953	1954
7AD7/SR1407/6145	2.0	3.3	1.5
7AK7	0.26	0.43	0.44
6080/6080WA/6AS7G			1.1
5965			0.32
6BL7GT			0.41

The failure rates for the 7AD7/SR1407/6145 and 7AK7 continue to be essentially constant. The failure rates for the 6080/6080WA, 5965, and 6BL7GT, which have been in

service in quantity only about 15 months, continue to be encouragingly low, especially the latter two.

In Figure 4-1 are given the vacuum-tube failures for the past quarter. A word of explanation is in order on several of the tube types. The excessively high number of early-life failures of the 2D21 thyratron is due to improper circuit design. In a relay driving application, the control and shield grid voltages were excessively negative during conduction. This resulted in a high ion current to these electrodes, causing sputtering and rapid tube deterioration. Subsequent to a change in circuitry to reduce the magnitude of these voltages, no difficulty has been experienced.

The 5963 is now the sixth most numerous tube in WWI, its quantity being only slightly less than that of the 6BL7GT. The failure rate for 5963's continues to be quite high, on the order of 4% per 1000 hours. Practically all of the failures have been electrical and of the nature of a change in characteristics, notably a decrease in plate current at zero bias. A detailed investigation has shown cathode interface impedance to be the cause of this low plate current in a considerable percentage of the failures. Previous life tests conducted here had indicated that this problem would develop (see Summary Report No. 33). Discussions with the manufacturer indicate that steps have been taken to eliminate the problem.

In Figure 4-2 are shown vacuum-tube failures for the year 1954.

Failure rates for the years 1948 through 1954 for the 7AD7/SR1407/6145 and 7AK7 are shown in Figure 4-3. The points plotted are for each quarter of the years 1952 through 1954 and the average of the years 1949 through 1951. The relatively high failure rates for both types during the first quarter of 1953 are a result of an intensive maintenance program undergone by WWI at that time. The sharp decrease in the 7AD7 failure rate in the first quarter of 1952 was due to a change in the marginal checking of flip-flops. The failure rate for the 7AK7 appears to be fairly constant at about 0.4% per 1000 hours. That of the 7AD7/SR1407/6145 appears to be tapering off between 1.0 and 1.5% per 1000 hours.

In Figure 4-4 is shown operation time at failure for the 7AK7 and 7AD7/SR1407/6145 for the years 1949 through 1954 normalized to the number of failures during the first 1000 hours of operation. In both cases the number of tubes failing during a given 1000-hour period after about 10,000 hours of operation is only about 10-20% of the quantity failing during the first 1000 hours of operation. That the curve of the 7AD7/SR1407/6145 is smoother than that of the 7AK7 may be due to the fact that the total failures during the first 1000 hours of operation are 249 and 33, respectively, and a small statistical variation represents a much larger percentage of the first 1000-hour quantity in the case of the 7AK7.

4.1.2 Vacuum-Tube Research

The life test of tungsten-nickel A31 Cathaloy Z-2177's has reached 2800 hours with one section conducting and the other cut off. There are no major changes from the last

			Reason for failure; number failed					
Type	Total in Service	Hours of Operation	Change in Characteristics	Shorts, Opens	Breakage	Burn-out	Gassy	Preventive Maintenance
C16J	12	8000 - 9000	1					
		9000 - 10000	1					
OA2	76	0 - 1000			1			
		6000 - 7000	1					
OB2	15	3000 - 4000			1			
2D21	269	0 - 1000	16	1				1
		1000 - 2000	4					
		3000 - 4000	3					
		4000 - 5000	6					
		5000 - 6000	3					
		6000 - 7000	1				1	
		15000 - 16000	1					
		18000 - 19000	1					
		22000 - 23000	1					
		27000 - 28000					1	
3E29	141	3000 - 4000	1					
		4000 - 5000	1					
		19000 - 20000	1					
		27000 - 28000	1					
5U4G	19	0 - 1000			1	1		
		12000 - 13000					1	
		15000 - 16000		1			1	
5Y3	6	6000 - 7000			1		1	
		7000 - 8000					1	
6AN5	102	0 - 1000			1			
		1000 - 2000		1				
		4000 - 5000		4				
		5000 - 6000	1					
		11000 - 12000	1					
6080/6080WA	718	0 - 1000		1				
		4000 - 5000		1				
		5000 - 6000		1				
		8000 - 9000		1				
		18000 - 19000		1				
6AU6/6136	314							
6136		1000 - 2000		1				
6AU6		2000 - 3000	1	1				
		7000 - 8000		1				
		10000 - 11000	2	2				
6BL7	494	4000 - 5000		1				
		8000 - 9000	1	1				
6J5	8	10000 - 11000			1			
6L6G/5881	105							
5881		0 - 1000		1				
		1000 - 2000		1				
		2000 - 3000	1					
		5000 - 6000	2					
		6000 - 7000	1					
6SH7	3	0 - 1000	1					
		2000 - 3000	1					
		4000 - 5000		1				
		5000 - 6000		1				
6SN7	307	27000 - 28000		3				
		28000 - 29000	1					
6V6	30	8000 - 9000		1				
6X5	25	28000 - 29000			1			

Fig. 4-1. WWI Tube Failures

			Reason for failure; number failed					
Type	Total in Service	Hours of Operation	Change in Characteristics	Shorts, Opens	Breakage	Burn-out	Gassy	Preventive Maintenance
6Y6	323	14000 - 15000		2				
		15000 - 16000		1				
		22000 - 23000	2					
		23000 - 24000		1				
		26000 - 27000	4	4				
		27000 - 28000	1					
		28000 - 29000	1	1				
7AD7/6145/5B4407	4237							
7AD7		1000 - 2000	1					1
		6000 - 7000	2					
		7000 - 8000	1					
		8000 - 9000		2		1		
		10000 - 11000		1				
		12000 - 13000	1					
		16000 - 17000	2					
		20000 - 21000	2					
		21000 - 22000	2					
		22000 - 23000	5					
		23000 - 24000	1					
		24000 - 25000	2					
		26000 - 27000	1	1				
		27000 - 28000	2	1				
		28000 - 29000	1					
6145		0 - 1000	3	4	3			
		1000 - 2000	1					2
		2000 - 3000	1	15				
		3000 - 4000	1	5				
		4000 - 5000	3	2				
		5000 - 6000	1	3				
		6000 - 7000	3	12				
		7000 - 8000	2	5				
		11000 - 12000		2				
5B4407		12000 - 13000		1				
7AK7	3340	2000 - 3000		5				
		3000 - 4000	2	1				
		4000 - 5000	1	1				
		5000 - 6000	3					
		7000 - 8000	1	1				
		8000 - 9000		1				
		9000 - 10000	1					
		10000 - 11000		1				
		18000 - 19000		1				
		27000 - 28000			1			
12AU7/5963	463							
5963		0 - 1000	1					
		1000 - 2000	1					
		2000 - 3000	3					
		3000 - 4000	3					
		4000 - 5000	8	1				
		5000 - 6000	1					
		6000 - 7000	1					
		7000 - 8000	2					
		9000 - 10000	4					
		10000 - 11000	9					
5651	23	10000 - 11000			1			
5687	86	0 - 1000		2	1			
		1000 - 2000		2				
		2000 - 3000		1				
		5000 - 6000		1				
		8000 - 9000		1				
		15000 - 16000		3				
5965	878	0 - 1000		1				
		1000 - 2000		1		1		
		9000 - 10000		1				
6293	20	0 - 1000					1	
		4000 - 5000		1				
715C	124	hours not kept	2	1	1			

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			Reason for failure, number failed					
Type	Total in Service	Hours of Operation	Change in Characteristics	Shorts, Opens	Breakage	Burn-out	Gassy	Preventive Maintenance
C3J	18	0 - 1000	1					
C6J	4	0 - 1000	1					
		19000 - 20000	1					
		22000 - 23000	2					
C16J	12	2000 - 3000			1			
		4000 - 5000	1					
		8000 - 9000	1					
		9000 - 10000	1					
OA2	76	hours not kept		1				
		0 - 1000			1			
		6000 - 7000	1					
OA3/VR-75	4							
VR-75		20000 - 21000	1					
OB2	15	3000 - 4000			1			
OC3	6	3000 - 4000	1					
OD3/VR-150	23							
OD3		2000 - 3000	1					
VR-150		0 - 1000			1			
		1000 - 2000	1					
		2000 - 3000	1					
		15000 - 16000	1					
2C51/5670	46							
5670		9000 - 10000	2					
		11000 - 12000	1					
		13000 - 14000		1				
2D21/5727	269							
2D21		no clock hours	4	1				
		0 - 1000	19	1				1
		1000 - 2000	14					
		2000 - 3000	3					
		3000 - 4000	7					
		4000 - 5000	8					
		5000 - 6000	4					
		6000 - 7000	1					
		7000 - 8000	1				1	
		13000 - 14000	1					
		15000 - 16000	1					
		18000 - 19000	1					
		22000 - 23000	1					
		27000 - 28000					1	
3E29/829B	141							
3E29		hours not kept	1					
		2000 - 3000	1					
		3000 - 4000	1					
		4000 - 5000	5					
		6000 - 7000	1					
		10000 - 11000	1					
		13000 - 14000	1	1				
		15000 - 16000	1					
		19000 - 20000	1					
		23000 - 24000	1	1				
		24000 - 25000		1				
		26000 - 27000	1					
		27000 - 28000	1					
829B		5000 - 6000	1					
5U4G	19	0 - 1000		1	1	1		
		5000 - 6000				1		
		9000 - 10000		1				
		12000 - 13000					1	
		14000 - 15000					1	
		15000 - 16000		1			1	
5Y3	6	0 - 1000	1					
		3000 - 4000		1				
		6000 - 7000			1		1	
		7000 - 8000					1	
		11000 - 12000				1		

Fig. 4-2. WWI Tube Failures

			Reason for failure, number failed					
Type	Total in Service	Hours of Operation	Change in Characteristics	Shorts, Opens	Breakage	Burn-out	Gassy	Preventive Maintenance
6AC7	9	2000 - 3000		1				
		3000 - 4000		1				
		10000 - 11000					1	
6AG7	81	0 - 1000		1				
		3000 - 4000		1				
		13000 - 14000	1					
		22000 - 23000		1				
		24000 - 25000		1				
6AK5/5654	13							
6AK5		2000 - 3000	1					
5654		0 - 1000			1			
6AL5	6	7000 - 8000	1					
6AN5	102	0 - 1000		1	4			
		1000 - 2000		1				
		2000 - 3000		1	4			
		3000 - 4000		1	1			
		4000 - 5000		4				
		5000 - 6000	1					
		6000 - 7000			1			
		11000 - 12000	1					
6AS7/6080	718							
6AS7		16000 - 17000	1					
		20000 - 21000		1			1	
		21000 - 22000		1				
6080		0 - 1000	2	4				
		1000 - 2000	4	2			2	
		2000 - 3000		5	1		1	
		3000 - 4000	3	10			3	
		4000 - 5000	2	3		1		
		5000 - 6000	2	1				
		18000 - 19000		1				
6AU6/6136	314							
6AU6		0 - 1000	1	7				
		1000 - 2000	2	7	3		1	
		2000 - 3000	1	4	1			
		5000 - 6000		3				
		6000 - 7000	1	1				
		7000 - 8000		1				
		8000 - 9000	3	5				
		10000 - 11000	2	2				
6136		0 - 1000		2	8			
		1000 - 2000		1	1			
		2000 - 3000		5			1	
6BL7	494	0 - 1000		2				
		1000 - 2000		1				
		2000 - 3000	2	1				
		3000 - 4000	1					
		4000 - 5000		1				
		6000 - 7000		1				
		7000 - 8000		2				
		8000 - 9000	1	1				
6J5	8	4000 - 5000		1				
		6000 - 7000			1			
		10000 - 11000			1			
6L6/5881	105							
6L6		4000 - 5000	1					
		5000 - 6000	1					
		7000 - 8000	1					
		8000 - 9000	3					
		10000 - 11000	4					
		11000 - 12000	2					
		22000 - 23000	1	1				
		23000 - 24000	1					
5881		0 - 1000	3	3				
		1000 - 2000	2	2				
		2000 - 3000	4	1				
		3000 - 4000	1					
		4000 - 5000		1				
		5000 - 6000	2					
		6000 - 7000	1					

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			Reason for failure; number failed					
Type	Total in Service	Hours of Operation	Change in Characteristics	Shorts, Opens	Breakage	Burn-out	Gassy	Preventive Maintenance
6BH7	3	0 - 1000	2					
		1000 - 2000	1					
		2000 - 3000	3					
		4000 - 5000		1				
		5000 - 6000		1				
6SN7	307	0 - 1000		1				
		7000 - 8000	1	1				
		11000 - 12000	1					
		17000 - 18000	1					
		22000 - 23000		1				
		23000 - 24000	3					
		24000 - 25000		1				
		25000 - 26000		1				
		26000 - 27000		3				
		27000 - 28000	1	4				
		28000 - 29000	1					
6V6	30	0 - 1000		1				
		2000 - 3000		1				
		8000 - 9000		1				
		23000 - 24000	4		1			
6Y6	323	0 - 1000		1				
		5000 - 6000					1	
		10000 - 11000		2				
		11000 - 12000		1				
		12000 - 13000		3				
		14000 - 15000		2				
		15000 - 16000		1				
		21000 - 22000	1	5			1	
		22000 - 23000	4	1				
		23000 - 24000	2	1				
		24000 - 25000	3	1				
		25000 - 26000	2	1	1			
		26000 - 27000	4	4				
		27000 - 28000	1					
		28000 - 29000	1	1				
6X5	25	6000 - 7000		2				
		9000 - 10000		2				
		18000 - 19000	1					
		28000 - 29000			1			
7AD7/6145/ SR-1407	4237							
7AD7		0 - 1000	1	1				
		1000 - 2000	1	1				1
		2000 - 3000		1				
		3000 - 4000		1				
		6000 - 7000	2	1				
		7500 - 8000	1	2			1	
		8000 - 9000		5		1		
		10000 - 11000	2	1				
		11000 - 12000		4				
		12000 - 13000	1	1				
		14000 - 15000		1	1			
		15000 - 16000		3				
		16000 - 17000	3	12				
		17000 - 18000	1					
		18000 - 19000		4				
		19000 - 20000		1				
		20000 - 21000	4	2				
		21000 - 22000	2	7				
		22000 - 23000	6	10				
		23000 - 24000	4	14				
		24000 - 25000	13	8	2			
		25000 - 26000	8	9				
		26000 - 27000	16	1				
		27000 - 28000	9	1				
		28000 - 29000	1					
6145		hours not kept		1				
		0 - 1000	9	42	3	1		
		1000 - 2000	6	21			1	2
		2000 - 3000	4	32				
		3000 - 4000	4	17				
		4000 - 5000	4	9				
		5000 - 6000	4	17				
		6000 - 7000	3	19				
		7000 - 8000	2	7				
		8000 - 9000		1				
		9000 - 10000		1				
		11000 - 12000		2				
SR-1407		6000 - 7000	1					
		7000 - 8000	2	1	1			
		10000 - 11000		1				
		12000 - 13000		1				

Fig. 4-2. WWI Tube Failures

			Reason for failure; number failed					
Type	Total in Service	Hours of Operation	Change in Characteristics	Shorts, Opens	Breakage	Burn-out	Gassy	Preventive Maintenance
7AK7	1340	0 - 1000	2	1	1	2		
		1000 - 2000	1	5			1	
		2000 - 3000	7	12				
		3000 - 4000	6	5				
		4000 - 5000	1	1				
		5000 - 6000	3					
		7000 - 8000	3	3				
		8000 - 9000		4				
		9000 - 10000	1		2			
		10000 - 11000		1				
		11000 - 12000		1				
		14000 - 15000		1				
		16000 - 17000		1				
		18000 - 19000		1				
		20000 - 21000	1	1				
		21000 - 22000	2	4				
		22000 - 23000		5				
		23000 - 24000	2	2				
		24000 - 25000			1			
		27000 - 28000			1			
12AU7/5963	453							
12AU7		2000 - 3000	2	1				
		8000 - 9000	1	1				
5963		0 - 1000	1	5			1	
		1000 - 2000	4	5				
		2000 - 3000	15	7				
		3000 - 4000	7					
		4000 - 5000	9	2				
		5000 - 6000	3	1				
		6000 - 7000	1					
		7000 - 8000	13			1		
		8000 - 9000	8	1				
		9000 - 10000	12	2				
		10000 - 11000	11	2				
		12000 - 13000	2					
5651	23	0 - 1000	1					
		4000 - 5000			2			
		10000 - 11000			1			
		22000 - 23000	1					
5687	86	0 - 1000		6	1			
		1000 - 2000		5				
		2000 - 3000	1	3				
		3000 - 4000	1	1	1			
		5000 - 6000	1	1				
		7000 - 8000		2	2			
		8000 - 9000	2	2				
		9000 - 10000	2	2				
		10000 - 11000		5				
		15000 - 16000		3				
		16000 - 17000	1					
5965	878	no clock hours	1	1				
		0 - 1000		2				
		1000 - 2000		3		1		
		3000 - 4000		1				
		4000 - 5000		1				
		5000 - 6000		1				
		7000 - 8000		1				
		9000 - 10000		1				
5998	33	0 - 1000		3				
		2000 - 3000		2				
		4000 - 5000	1					
		7000 - 8000		1				
715C	124	hours not kept	13	2	2		1	
5855	6	3000 - 4000	1					
6072	31	2000 - 3000	1					
		8000 - 9000	1					
		10000 - 11000	1					
6293	20	0 - 1000			1		1	
		4000 - 5000		1				
8008	30	4000 - 5000	1					

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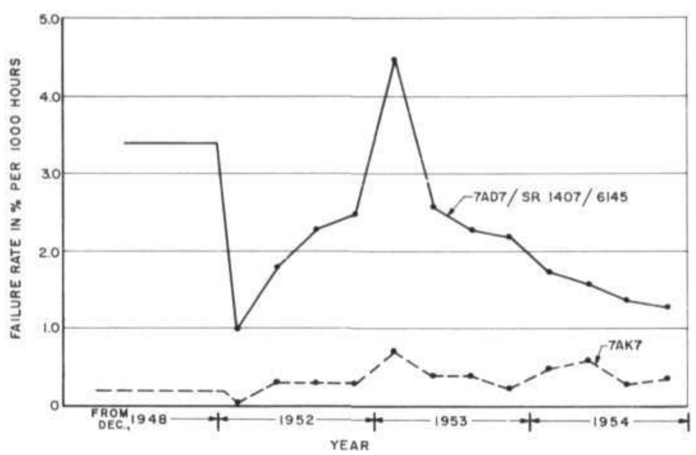

Fig. 4-3. Failure Rate in WWI
Tube Types 7AD7/SR-1407/6145 and 7AK7
1948-1954

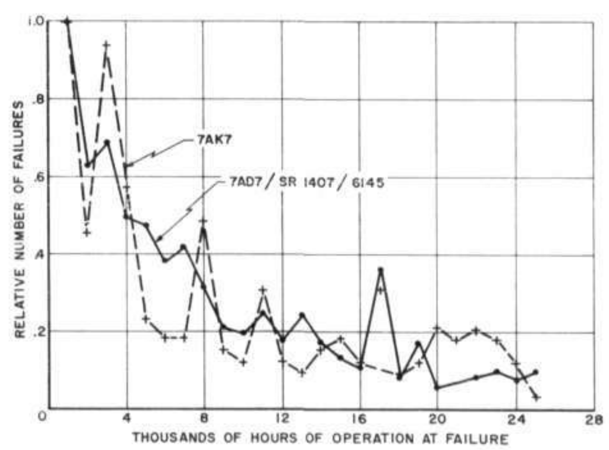

Fig. 4-4. Hours of Operation at Failure
Tube Types 7AD7/SR-1407/6145 and 7AK7
1949-1954

reading, considerable interface impedance remaining present on the conducting sections and much less on the cutoff sections.

The life test of tungsten-nickel A31 Cathaloy 5687's has reached 4200 hours with one section conducting and the other cut off. Grid emission on the cutoff sections continues to increase; the grid currents on the conducting sections remain low. Only the same three of the ten conducting sections previously noted (Summary Report No. 39) show any cathode interface impedance; there is none on the cutoff sections.

As a result of interface-impedance difficulties encountered on the 5963 as mentioned in Section 4.1.1, a life test of new tubes was instituted during the past quarter. At 800 hours the data is inconclusive.

The 5965 life test has reached 6200 hours. Plate currents are holding up well; there is, however, considerable grid emission on the cutoff sections.

A vacuum-tube pulse-characteristic tester is under construction for the tube lab. It will replace the existing breadboarded model which is much less versatile.

4.2 Component Replacements

Figure 4-5 lists the replacements of components other than tubes during the fourth quarter of 1954.

Component	Type	Total in Service	No. of Failures	Hours of Operation	Comments
Capacitors	0.01-μf mica 10%	5622	1	2000 - 3000	Shorted
	0.01-μf fixed ceramic disc	5740	1	0 - 1000	Shorted
Crystal Diodes	1N34A	17781	6	0 - 1000	Low R_b
			2	2000 - 3000	Low R_b
			9	3000 - 4000	4 low R_b; 5 shorted
			6	4000 - 5000	Low R_b
			12	6000 - 7000	11 low R_b; 1 shorted
			8	8000 - 9000	Low R_b
			1	9000 - 10000	High R_f
			2	7000 - 8000	1 shorted, 1 low R_b
	1N92	61	1	2000 - 3000	Low R_b
			2	3000 - 4000	Low R_b
	1N38A	5961	1	1000 - 2000	High R_f
			1	6000 - 7000	High R_f
Fuse Holder	3AG (Buss)		1	9000 - 10000	Intermittent
Resistors	6300-ohm 8-watt ± 5%	72	1	0 - 1000	Open
	9000-ohm 1/2-watt ± 1%	950	3	6000 - 7000	Out of tolerance
			1	8000 - 9000	Out of tolerance
			1	9000 - 10000	Out of tolerance
			4	18000 - 19000	Out of tolerance
	0.45-megohm 1/2-watt ± 1%	950	1	1000 - 2000	Out of tolerance
			1	5000 - 6000	Out of tolerance
			3	6000 - 7000	Out of tolerance
			6	18000 - 19000	Out of tolerance
	0.125-megohm 1/2-watt ± 1%	950	1	1000 - 2000	Out of tolerance
			1	5000 - 6000	Out of tolerance
			3	6000 - 7000	Out of tolerance
			4	18000 - 19000	Out of tolerance
	2500-ohm 1/2-watt ± 1% (Nobleloy)	950	3	6000 - 7000	Out of tolerance
			1	9000 - 10000	Out of tolerance
			2	18000 - 19000	Out of tolerance
Switch	Sensitive (Microswitch)	92	1	0 - 1000	Switch jammed
Transformer	5:1 pulse, hipersil core	238	1	19000 - 20000	Shorted turns in primary
Tube Socket	7-pin miniature		1	3000 - 4000	Faulty pin

Fig. 4-5. WWI Component Failures October 1 - December 31, 1954

5. ACADEMIC PROGRAM

5.1 Programming Course

The DCL CS II programming course was given once during this quarter. The course includes the following topics: relative addresses, temporary storage, floating addresses, preset parameters, programmed arithmetic, cycle counters, buffer storage, automatic output, post mortems, and multipass conversion. The text for the course is a programmer's manual written by staff members of the S&EC Group.

The 32 students enrolled during this quarter represented the following groups: Aero-Elastic and Structures Research Laboratory, Instrumentation Laboratory, Geophysical Analysis Group, Servomechanisms Laboratory, Meteorology Department, Physics Department, Electrical Engineering Department, Chemical Engineering Department, Civil Engineering Department, Mathematics Department, the School of Industrial Management, and Lincoln Laboratory.

5.2 MIT Course

The principal course on machine computation being offered at MIT in the fall of 1954 was 6.25, Machine-Aided Analysis, a survey of computing techniques aimed largely at seniors in Electrical Engineering. This subject, first offered in the spring of 1954, had a fall-term enrollment of 59 students. It consequently was divided into three 20-man sections taught separately by Professors Linvill, Booton, and Adams. Practice problems were planned to allow each student to use both a REAC and the Whirlwind I computer (simulating the hypothetical TAC which had been developed for the 1954 summer session). The description of the problem done on TAC will be found in the report on Problem 221.

5.3 Visitors

Tours of the WWI installation include a showing of the film "Making Electrons Count," a computer demonstration, and an informal discussion of the major computer components. During the past quarter eight groups visited the computer installation. Included in these groups were:

October 1	Industrial Liaison Office Personnel
November 3	Professor Crandall's class (MIT Mechanical Engineering Department
November 30	Emmanuel College Seniors
December 14	Professor D. P. Campbell s class (MIT Electrical Engineering Department)
December 15	AIEE-IRE MIT Student Branch

6. APPENDIX

6.1 Publications

Project Whirlwind technical reports and memoranda are routinely distributed to only a restricted group known to have a particular interest in the Project, and to ASTIA (Armed Services Technical Information Agency) Document Service Center, Knott Building, Dayton, Ohio. Requests for copies of individual reports should be made to ASTIA.

The following is a list of memoranda published by the Scientific and Engineering Computations Group during the past quarter.

No.	Title	Date	Author
SR-39	Summary Report No. 39, Third Quarter 1954		
DCL-17	A Program for Checking the Contents of Test Storage	10-4-54	A. Siegel
DCL-22	Utility Control Program	11-22-54	F. C. Helwig
DCL-23	Memo on S&EC Performance Request	11-19-54	F. C. Helwig
DCL-24	Director Tapes	11-24-54	F. C. Helwig
DCL-25	CS II Instruction STOP	11-24-54	F. C. Helwig