MACHINE METHODS OF COMPUTATION
and
NUMERICAL ANALYSIS

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FOREWORD

This is a combined report for the two projects at the Massachusetts Institute of Technology which are sponsored by the Office of Naval Research under Contract N5ori60.

Project on Machine Methods of Computation and Numerical Analysis

This Project is an outgrowth of the activities of the Institute Committee on Machine Methods of Computation, established in November 1950. The purpose of the Project is (1) to integrate the efforts of all the departments and groups at M.I.T. who are working with modern computing machines and their applications, and (2) to train men in the use of these machines for computation and numerical analysis.

People from several departments of the Institute are taking part in the project. In the Appendix will be found a list of the personnel active in this program.

Project Whirlwind

This Project makes use of the facilities of the Digital Computer Laboratory. The principal objective of the Project is the application of an electronic digital computer of large capacity and very high speed (Whirlwind I) to problems in mathematics, science, engineering, simulation, and control.

The Whirlwind I Computer

Whirlwind I is of the high-speed electronic digital type, in which quantities are represented as discrete numbers, and complex problems are solved by the repeated use of fundamental arithmetic and logical (i.e., control or selection) operations. Computations are executed by fractional-microsecond pulses in electronic circuits, of which the principal ones are (1) the flip-flop, a circuit containing two vacuum tubes so connected that one tube or the other is conducting, but not both; (2) the gate or coincidence circuit; (3) the magnetic-core memory, in which binary digits are stored as one of two directions of magnetic flux within ferro-magnetic cores.

Whirlwind I uses numbers of 16 binary digits (equivalent to about 5 decimal digits). This length was selected to limit the machine to a practical size, but it permits the computation of many simulation problems. Calculations requiring greater number length are handled by the use of multiple-length numbers. Rapid-access magnetic-core memory has a capacity of 32,768 binary digits. Present speed of the computer is 40,000 single-address operations per second, equivalent to about 20,000 multiplications per second.

PART I

Machine Methods of Computation and Numerical Analysis

1. GENERAL COMMENTS

Two new studies originating from the mathematics members are reported here. Both concern mathematical techniques in the study of deflections of shallow shells. In one case the convergence of iteration procedures to the solution of a differential equation is the subject. In the other case the objective is to obtain integral representations, asymptotic behavior and recursion relations for certain functions in order to solve boundary value problems.

A project that has been completed is the evaluation of multi-center integrals. This work that will be reported fully elsewhere is used in a calculation of energy bands for a graphite crystal model.

In the way of theory of programming and general use of a digital computer, a complete report on the programming algebra introduced last time is given. Though this algebra is a by-product of a general approach to minimizing computing machine time, it serves some practical purposes independently of the general theory. The place of this algebra in reference to the mathematical definitions of computing machines is also discussed.

The Monte Carlo technique for implying diffusion behavior from the study of individual neutrons has been tested throughout a number of component subroutines and is close to being used for reactor studies. A detailed outline of the logical design for this Monte Carlo method is included in these reports.

Six other projects that have been reported before in their early stages are progressing. These will bring results in the fields of physics, geology, civil and mechanical engineering and concern Coulomb wave functions, earth resistivity, dynamic loading on buildings, crack growth in bars, and vapor condensation.

2. GRADUATE SCHOOL RESEARCH

2.1 Index to Reports

2.2 Progress Reports

THE BASIC PROBLEM OF NUMERICAL ANALYSIS EXPRESSED IN THE LANGUAGE OF COMPUTING MACHINES

The first two reports on this subject [1,2] led to a programming algebra and in the second of these reports we stated that the algebra was complete except for decision making. The remaining essential point we intend to discuss this time.

DECISION MAKING

It is largely the decision making that gives to computer programming a unique aspect not known in other branches of mathematics. In 1947 von Neumann and Goldstine [3] associated the words "dynamic" and "macroscopic" to this part of the programming. In their words, "Since coding is not a static process of translation, but rather the technique of providing a dynamic background to control the automatic evolution of a meaning, it has to be viewed as a logical problem and one that represents a new branch of formal logics." Using this terminology we can say that the past progress report was concerned largely with the static translation of a problem. The presence of the modification instruction T_{mnp} and the postbracket with a raised integer $(\ )^n$ served only to condense a program from a direct transliteration of a problem to an array of subroutines. The machine made no choices; all decisions about the values of the subscripts on T and the raised integer after ( ) had to be made at the time of the programming. The conventions of the algebra are constructed, however, to permit these subscripts and raised integers to be equated to the outputs of other programs; that is, instructions in one program can be modified by the outputs of other programs and this is the first step toward containing a decision logic within the algebra of programming itself.

In the present report we will generalize the possibility of modifying instructions. At the same time we will introduce the basic decision instruction. From the mathematical point of view the decision instruction plays the part of a function that is 1 over a set and zero elsewhere; it combines with $(\ )^n$ to play the part of a conditional transfer in conventional programming.

First, let us review+ the instructions already available and at the same time use a more convenient notation. Throughout this report O( ) will stand for the contents of the last register referred to in the sequence of instructions contained in ( ), unless O is a vector in which case we will write O_N.

a:	The identity. Replace contents of register a by itself.
a_{b+c}:	Add O(b) + O(c). Store in a.
a_{b-c}:	Subtract O(b) - O(c). Store in a.
$a_{b \cdot c}$:	Multiply O(b) . O(c). Store in a.
$a_{b/c}$:	Divide O(b) / O(c). Store in a.
T_{mnp}:	If preceded by a_{b+c}, a_{b-c}, etc., replace b by b+m, c by c+n, a by a+p.
$(\ )^n$:	Repeat bracketed sequence n times.

The generalization of the modification instruction T_{mnp} operating on a_{b+c} is a set of equations as follows:

$$X_{Y+Z}$$
$$X = O(\ _1\ ) \qquad Y = O(\ _2\ ) \qquad Z = O(\ _3\ )$$

: Perform the three sequences of instructions contained in brackets 1,2,3 in that order and then perform X_{Y+Z}.

A similar definition holds for the modification of other instructions. (Technically, this is not a strict generalization of the simpler modification instruction, because in the simpler case the instruction a_{b+c} was followed by modification rather than preceded by it. The difference is not important.)

The generalization of the repeat instruction is a set of equations as follows:

+ Note: Some notational errors were present in the definitions given last time.

$$(\ _1 \ )^N \qquad N = O(\ _2)$$: Perform bracketed sequence 2. Then carry out instruction $(\ _1 \ )^N$.

The decision instruction is defined as follows. We will assume throughout that $O(0) = 0$, $O(1) = 1$.

$$a_{b \leq c} = \begin{cases} a_{0+0} & \text{if } O(b) \leq O(c) \\ a_{0+1} & \text{if } O(b) > O(c) \end{cases}$$

We will also use the symbol $\overline{O}$ as the inverse of O:

$$\overline{O}(a) = \text{register containing } a.$$

Thus, if $X = O(a)$, $a = \overline{O}(X)$.

A final instruction introduced to complete the algebra is

$$a_{b \cdot 2^{-c}}$$: Take the coefficient of $2^{-O(c)}$ in the binary representation of $O(b)$ and store it in a.

The generalized modification instructions (instructions made up of variable registers) and the generalized repeat instructions (brackets with variable raised integers) may appear in rather complicated combinations. For instance, there may be several brackets within one another all accompanied by different variable raised integers and within all or some of these brackets there may appear instructions with variable registers or variable contents of registers. Unless a convention is established, it will be ambiguous with respect to which brackets the instructions are variable and with respect to which they are constant. For example the program

$$\left[(X_{3_{X_1+X_2}})^N \right]^M$$

may mean that the X_1, X_2, X_3 are to be evaluated each of the M times the outer bracket is entered as well as each of the N times the inner bracket is entered. Alternatively, it may mean that the compound instruction

$$(X_{3_{X_1+X_2}})^N$$

appearing inside the instruction $(\ )^M$ is to be considered constant and

$$X_{3_{X_1+X_2}}$$

is variable. Thus, the convention will be as follows: Beneath the equation for computing any raised integer (and if the integer is constant, we enter the equation as $N = N$) will be listed in order all of the equations for registers and contents of registers that are variable with respect to the corresponding bracket and raised integer. (See examples.) If a variable with respect to one bracket is constant with respect to a second, the equation for its evaluation will not appear in association with the raised integer of the second bracket. The order of the equations that do appear in conjunction with a bracket should be consistent with the order of appearance of the variables within the corresponding bracket.

A program written in this algebra will appear as a sequence of instructions together with as many sets of equations for evaluating the variables as there are brackets. By following the above rules the temporal sequence for performing instructions will not be lost, even though, for convenience, certain parts of the program are written as separate equations. The rules for combining the primary sequence of instructions with the accompanying equations and the converting to the instructions of an ordinary computing machine will be discussed after some examples.

DISCUSSION

Some remarks on the generality of this algebra and its relation to the definitions of computing machines given in these reports [1,2] and elsewhere will be appropriate. The mathematical definition of a computing machine M that we have used is $M \equiv (PXI, O_N, T)$ where PXI is a space of finite+ sequences of parameters and instructions, and O_N and T are real

+Note: We have not previously specified that the sequences in PXI are finite, but this restriction will be assumed from now on.

mappings (vector and single valued, respectively) defined on this space. For each point (p,i) in PXI, $O_N(p,i)$ and $T(p,i)$ represent the output and time. This definition differs from that given by Turing [4] in a number of respects. First of all, each point (p,i) in PXI corresponds to a Turing machine for which $O_N(p,i) = (Q_1, Q_2, \ldots Q_N)$ is a vector representation of the associated Turing computable function. (Strictly speaking, since for each (p,i) there is a finite collection of numbers $Q_1, Q_2, \ldots Q_N$ defined only for the finite sequence of integers 1,2,...N, each computable function is partial. The fact that Q_i are rational whereas the outputs of a Turing machine are integers is inessential.) Second, since for each point (p,i), $O_N(p,i)$ and $T(p,i)$ are defined (and thus finite) the machine M can never become involved in an infinite loop. In logical terminology, the machine M can never become involved in an undecidable proposition. Thus in defining M we exclude from the start the possibility of handling problems that cannot be resolved in a finite amount of time.

There is another important distinction. The basic problem that we have studies in these reports is that of finding the most efficient program for coming within a certain distance of a desired output. In formulating this problem we have had to assume that at least one program is known (efficient or inefficient) whose output characteristics are desirable. In other words, we have bypassed the problem of greatest interest to the logicians; that is, the problem of what are the computable functions, or what are the problems that can be programmed and solved on a machine in a finite time. Before we can approach a solution to either of these problems in the context of the present definition of M, it is necessary to have a realization of M. Such a realization is, in fact, supplied by the algebra of instructions just presented. It is clear that the conventions of the algebra yield all finite sequences, i, of instructions and this collection can be called I. The convention that all elements of the sequence p are to be the successive contents of the register numbers 3,4,5... appearing in the algebra completes the realization of PXI. If the components of the vector $O_N(p,i)$ are defined as the final contents of any finite collection of registers depending on (p,i), then $O_N(p,i)$ is determined by (p,i). T is a free variable and may be specified independently. The algebra together with a specification of T thus brings us a realization of M. At the same time we have a tool for programming that reflects the mathematical structure of the problems being solved.

A complete solution to the problem of minimizing performance time awaits a fuller investigation of the laws of the algebra, of which one or two were discussed last time. The first solution to this problem will come when the rules of the algebra can be used to explore completely the region in PXI that is algebraically equivalent to an arbitrary point (p_o, i_o). From then on, it will be necessary to select out of each algebraically equivalent region the point or points that minimize T. A solution to the logically interesting problem of characterizing the class of problems computable on M also await further investigation of the instructions that appear in the algebra.

EXAMPLES

As examples of how the algebra might be useful as a programming tool, irrespective of the general problems discussed above, we consider the following that arose in the electron-photon cascade problem [5] .

Example 1. <u>It is required to generate the array of numbers II from the array I and put them in the registers displayed in the array III. The program should be suitable for similar arrays of arbitrary size.</u>

I				II				III			
							14				101
		.	.			4	14			108	102
	.	.	.		1	4	10		114	109	103
0	.	.	.	0	1	3	6	119	115	110	104
0	.	.	.	0	1	2	3	120	116	111	105
0	1	.	.	0	1	1	1	121	117	112	106
0	0	0	0	0	0	0	0	122	118	113	107

It is seen that the array II is generated by addition in an obvious way. The problem involves one basic addition instruction

$$X_{3_{X_1+X_2}}$$

that will be used a variable N number of times for each column. The cycle corresponding to each column must be repeated a fixed number M times, where M+1 is the number of columns. There will be a special program for changing the size of N.

There will be two sets of equations for computing the variable values of X_1, X_2, X_3 because these variables change between rows and between columns according to different rules. As it happens, the variation between rows affects all three and the variation between columns affects only one of the X_1, X_2, X_3 . Thus there are four auxiliary subroutines to the program or bracket for rows, one to compute N and three to compute the variable registers X_1, X_2, X_3. There is one auxiliary subroutine to the bracket for columns. We will use the symbols a1,a2,a3,a4 as floating addresses (that can later be given determined values) for the registers containing the current values of the four variables. The initial values of the four variables must be determined by the boundary conditions of the problem.

The program can be written as follows:

$$\left[\left(X_{3\,X_1+X_2} \right)^{N} \right]^{M}$$

$$M = M \qquad\qquad N = O(a1_{a1+1})$$

$$X_3 = O(a4_{a4-1}) \qquad\qquad X_3 = O(a4_{a4-1})$$

$$X_1 = O(a2_{a2-1})$$

$$X_2 = O(a3_{a4+1})$$

In reference to arrays I, II, III, the initial contents of registers 117, a1, a2 and a4 should be 1, 2, 121, 118. The initial contents of a3 can be anything. M should be set M = 3.

Example II. Let 16 binary fractions of 16 digit accuracy each be stored in registers 101,... 116. Let us examine the first M_1 digits to the right of the decimal in the first number, in the second number, etc., recording the total number of digits 1 found. Stop after examining all 16 numbers or after examining enough numbers to make the total number of 1's greater than or equal to 2.

The basic program here is to search and store. There will be a bracket for selecting a new digit, a bracket of selecting a new number, and a bracket for stopping the process after sixteen numbers have been examined. The digits selected will be variables and the registers containing the numbers will be variables. Since the digit selected must be re-set to zero (a re-set parameter) after a new number has been chosen, the digit number appears as a variable with respect to two brackets. One of the raised integers must, of course, depend on the relative number of digits 1 found. The program is as follows:

$$\left\{ \left[\left(a1_{X_1 . 2^{-O(X_2)}} \; a3_{a3+a1} \right)^{M_1} \right]^{N} \right\}^{M_2}$$

$$M_2 = M_2 \qquad N = O(a6_{a4 \le a3}) \qquad M_1 = M_1$$

$$X_1 = O(a2_{a2+1}) \qquad X_2 = O(a5_{a5+1})$$

$$X_2 = O(a5_{0+0})$$

To satisfy the boundary conditions, the initial contents of registers a2,a3,a4 should be 100, 0,2. The initial contents of a1,a5,a6 may be arbitrary M_2 should be set $M_2 = 16$. The result will be stored in a3.

CONVERSION OF THE ALGEBRA

The first step toward converting the algebra to any traditional form of programming is the elimination of all the variables except the raised integers. This is done by (1) inserting all the programs for computing the variables in order of appearance of the variable into the left-hand side of the corresponding bracket and (2) replacing all the variables by the symbol O(a), where a is the register containing the variable. At this stage in Example 1 the program will appear as follows:

$$\left[a4_{a4-1} \left(a4_{a4-1} \; a2_{a2-1} \; a3_{a4+1} \; O(a4)_{O(a2)+O(a3)} \right)^{N} \right]^{M}$$

$$M = M, \qquad N = O(a1_{a1+1})$$

The second step is the elimination of the variable raised integers. This is done by (1) inserting the program that computes the variable immediately <u>before</u> the corresponding bracket and (2) replacing the variable by O(a), where a is the register containing the variable. In the above example we would then insert $a1_{a1+1}$ immediately after $a4_{a4-1}$ and before the inner bracket. N would be replaced by O(a1).

This much done, the third step, the translation of the linear array of instructions of the algebra into a linear array of given instructions remains. The details of this step

will not be discussed because each machine has its own special instruction. However, a few remarks may be helpful. Instructions involving the symbol O(a) must be translated with the aid of a modification instruction such as "transfer the contents of register a to the address part of instruction ______." Both the decision instruction and the repeat instruction will involve conditional and unconditional transfers of control. The repeat instruction will certainly involve a counter of some sort. The instruction for examining a digit will usually be translated with the aid of a cycling instruction.

It is clear that one to one correspondences can be made between the instructions of this algebra and sequences of instructions for any high-speed digital machine. Once these correspondences have been established, the conversion of the algebra to any machine language is purely mechanical.

Bayard Rankin
Raymond F. Stora

References:

[1] Bayard Rankin, Machine Methods of Computation and Numerical Analysis, Quarterly Progress Report No. 15, March 15 (1955) p. 7.

[2] ibid. Report No. 18, December 18(1955) p.7.

[3] John von Neumann and H. H. Goldstine, "Planning and Coding of Problems for an Electronic Computing Instrument," Institute for Advanced Study, Pt. 2 Vol. 1-3 (1947-48).

[4] A. M. Turing, "On Computable Numbers, with an Application to the Entscheidungsproblem," <u>Proc</u>. <u>Lond</u>. <u>Math</u>. <u>Soc</u>., Series 2, <u>24</u> (1936) p. 230-265.

[5] Raymond Stora, Same as [1] , Report No. 18, December 15 (1955), p. 12.

FINITE BENDING OF THIN, SHALLOW SPHERICAL SHELLS

1. The problem under consideration is that of small finite deflections of thin shallow spherical shells of constant thickness h subjected to a uniform radial load ρ. The differential equations of this problem have been derived by E. Reissner [1]. They can be written in the following form [2]:

$$(1) \quad M[F,G] = F'' + \frac{1}{x} F' - \frac{1}{x^2} F + \mu^2 G + 2x - \gamma \frac{FG}{x}$$

$$N[F,G] = G'' + \frac{1}{x} G' - \frac{1}{x^2} G - \mu^2 F + \gamma \frac{F^2}{2x}$$

where γ is proportional to the load and μ^2 is a geometrical parameter which is zero for flat plates. Let Ψ be a stress function [1] and β be the angular deflection of the shell, then F, G, γ, and μ^2 are defined as follows:

$$F = \frac{a \cdot \alpha m^2}{\gamma h} \; ; \; G = \frac{m^4 a \alpha}{E h^3 \gamma} \Psi ; \; \gamma = \frac{m^6 a^4 \alpha^4}{4 E h^4} \rho ; \; \mu^2 = \frac{m^2 a \alpha^2}{h}$$

where

- a = radius of the middle surface of the shell
- 2α = opening angle of the shell
- E = Young's modulus of elasticity
- $m^2 = \sqrt{12(1 - \nu^2)}$
- ν = Poisson's ratio.

We shall consider two cases of boundary conditions [2]:

(2) A simply supported edge: $F'(1) - \nu F(1) = G'(1) - \nu G(1) = 0$

(3) A clamped edge: $F'(1) = G'(1) - \nu G(1) = 0$.

Observing that x = 0 is a singular point of the system (1), we require that F and G be finite in both cases (2) and (3):

(4) $F(0) < \infty \qquad G(0) < \infty$

We attempt to find solutions of equations (1), (2) and (4), or (1), (3) and (4) respectively, in form of power series expansions:

$$F = \sum_{n=0}^{\infty} F_n x^n \qquad G = \sum_{n=0}^{\infty} G_n x^n.$$

Substituting these series into the differential equations and equating the coefficients of like powers of x, one finds the following recursion formulas

(5)
$$F_3 = \tfrac{1}{8}(2 - \mu^2 G_1 + \gamma F_1 G_1)$$

$$F_n = \frac{1}{n^2-1}\left(-\mu^2 G_{n-2} + \gamma \sum_{j=1}^{n-2} F_j G_{n-j-1}\right) \qquad n = 5,7,9,\ldots.$$

$$G_n = \frac{1}{n^2-1}\left(\mu^2 F_{n-2} - \frac{\gamma}{2} \sum_{j=1}^{n-2} F_j F_{n-j-1}\right) \qquad n = 3,5,7,\ldots.$$

$$F_{2k} = G_{2k} = 0 \qquad k = 0,1,2,\ldots.$$

If F_1 and G_1 are determined, the remaining coefficients are given by these recursion formulas. The stresses and displacements may also be expressed in terms of F_n and G_n.

2. To obtain initial estimates for the coefficients F_1 and G_1, we approximate the solutions F and G by

$$F^* = Ax + A'x^3;\ G^* = Cx + C'x^3.$$

In order to determine A, A', C, and C' we shall require that F^* and G^* satisfy the differential equations (1) in the mean;,

$$\int_0^1 M[F^*,G^*]dx = 0 \text{ and } \int_0^1 N[F^*,G^*]dx = 0,$$

and also that F^* and G^* satisfy the boundary conditions (2), or (3) respectively. Using these four conditions to determine A and C and taking

$F_1^{(0)} = A$, $G_1^{(0)} = C$, we obtain an approximate solution $F^{(0)}$, $G^{(0)}$ by

means of the recursion formulas (5). In order to obtain successive improvements of the coefficients F_n and G_n, the following procedure has been proposed by R. Simons [2] . Denote the mth approximation of (F_n,G_n) by $(F_n^{(m)},G_n^{(m)})$ and let

(6)
$$F_n^{(m+1)} = F_n^{(m)} + \Delta F_n^{(m)};\ G_n^{(m+1)} = G_n^{(m)} + \Delta G_n^{(m)}$$

$F_n^{(m+1)}$ and $G_n^{(m+1)}$ are required to satisfy the recursion formulas (5) except for terms of higher order than one in the corrections $\Delta F_k^{(m)}$ and $\Delta G_k^{(m)}$ Hence, one obtains recurrence formulas for $\Delta F_n^{(m)}$ and $\Delta G_n^{(m)}$ which are linear in $\Delta F_k^{(m)}$ and $\Delta G_k^{(m)}$ (k=1,.....n). Therefore $\Delta F_n^{(m)}$ and $\Delta G_n^{(m)}$ can be written as linear combinations of $\Delta F_1^{(m)}$ and $\Delta G_1^{(m)}$.

(7)
$$\Delta F_n^{(m)} = a_n^{(m)}\,\Delta F_1^{(m)} + b_n^{(m)}\,\Delta G_1^{(m)}$$

$$\Delta G_n^{(m)} = c_n^{(m)}\,\Delta F_1^{(m)} + d_n^{(m)}\,\Delta G_1^{(m)}$$

where $a_n^{(m)}$, $b_n^{(m)}$, $c_n^{(m)}$ and $d_n^{(m)}$ are given by recursion formulas similar to (5), involving only the known coefficients $F_n^{(m)}$ and $G_n^{(m)}$. The second requirement on $(F_n^{(m+1)}, G_n^{(m+1)})$ is that

$$\sum_n F^{(m+1)}x^n \text{ and } \sum_n G^{(m+1)}x^n$$

satisfy the boundary conditions (2), or (3) respectively. Using (7) this can be written

(8)
$$\Delta F_1^{(m)}\sum_n(n+\nu)a_n^{(m)} + \Delta G_1^{(m)}\sum_n(n+\nu)b_n^{(m)} = -\sum_n(n+\nu)F_n^{(m)}$$

$$\Delta F_1^{(m)}\sum_n(n-\nu)c_n^{(m)} + \Delta G_1^{(m)}\sum_n(n-\nu)d_n^{(m)} = -\sum_n(n-\nu)G_n^{(m)}$$

It remains to solve the linear system (8) to obtain the (m+1)th approximation $F_1^{(m+1)}$ $G_1^{(m+1)}$ as given by (6). Finally the coefficients $F_i^{(m+1)}$ and $G_i^{(m+1)}$ (i>1) are computed from the recursion formulas (5).

3. Computations are now under way on the Whirlwind I Computer for various values of the parameters γ and μ^2. In particular we wish to determine the range of parameters γ and μ^2 for which the above iteration procedure will converge to the solution of the differential equations satisfying the appropriate boundary conditions. We have obtained solutions for positive and negative values of γ in the range $0 \le |\gamma| \le 10$, taking μ^2 also in the range $0 \le \mu^2 \le 10$. The results show that for a simply supported edge only two or three iterations (m = 2, or 3) are necessary to obtain solutions of an accuracy of four significant digits in the coefficients F_n and G_n. For a clamped edge one, or at most two steps yield the same accuracy, which is sufficient in most applications. Carrying the computation up to m = 6 in a few particular cases shows that an accuracy of six digits can be obtained. We have indication, however, that the convergence is considerably slower for values of γ in the region of the critical load. At the present time, this region is being investigated with the aim of obtaining approximate values for the buckling load for various values of the parameter μ^2.

Hubertus J. Weinitschke

References:

[1] E. Reissner, Sympos. in Appl. Math., Proceedings, Vol. III, 1950.

[2] R. M. Simons, "On the Non-linear Theory of Thin Spherical Shells," M.I.T. Ph.D. Thesis, 1955.

PROPERTIES OF THE SOLUTIONS OF A DIFFERENTIAL EQUATION ARISING IN THE THEORY OF LINEAR ELASTIC DEFORMATION OF SHALLOW SHELLS OF REVOLUTION

It is proposed to develop theory and also the necessary tabular information for use in solving for the deflections of shallow shells of revolution. The shells dealt with here are parabolic, having as the equation of the undeflected midsurface

(1) $Z = H(\frac{r}{a})^s$ or $H\xi^s \quad s > 0$

where r is the radial distance from the axis of symmetry and Z is the height of the midsurface above a reference plane. H and a are characteristic lengths. A criterion of shallowness is that H is small compared to a.

Johnson [1] has shown that a solution for the stresses and transverse deflection in such a shell satisfies the single (complex) differential equation

(2)
$$\nabla^2\nabla^2 T = i\sqrt{\tfrac{c}{D}}\left[Z''\left(\frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}\right) + \frac{Z'}{r}\frac{\partial^2 T}{\partial r^2}\right]$$

where T is a complex dependent variable whose real part represents deflection and whose imaginary part represents an Airy stress function. Primes indicate differentiation with respect to r, while the second independent variable is the polar coordinate θ. The real parameter $\sqrt{c/D}$ depends on the thickness and elastic constant of the shell.

Seeking for simplicity solutions for T of the form

(3) $T = t_n(r)\cos n\theta,$

we obtain an ordinary differential equation for t_n,

(4)
$$\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{n^2}{r^2}\right)^2 t_n - i\sqrt{\tfrac{c}{D}}\left[\frac{Z'}{r}t_n'' + Z''\left(\frac{t_n'}{r} - \frac{n^2}{r^2}t_n\right)\right] = 0$$

Writing $t_n(r) = g_n(p\xi^s)$, $p = +iH\sqrt{c/D}$
and introducing the new independent variable

$$\xi^s = (r/a)^s$$

(4) becomes

(5)
$$\frac{d^4 g_n}{dz^4} + \frac{2(3s-2)}{zs}\frac{d^3 g_n}{dz^3} + \frac{1}{z^2s^2}\left[(s-1)(7s-5) - (1-2n^2) - z\right]\frac{d^2 g_n}{dz^2}$$
$$+ \frac{1}{z^3s^3}\left[(s-2)(s-1)^2 - (s-2)(1+2n^2) - (s-1)z\right]\frac{dg_n}{dz}$$
$$+ \frac{1}{z^4s^4}\left[n^2(n^2-4) + n^2(s-1)z\right]g_n(z) = 0$$

This equation has a regular singular point at the origin. Thus at least one power series solution may be found. The indicial equation is

$$f(\lambda)= s^4\lambda(\lambda-1)(\lambda-2)(\lambda-3)+2s^3(3s-2)\lambda(\lambda-1)(\lambda-2)$$
$$+[(s-1)(7s-5)-(1+2n^2)]\lambda(\lambda-1)s^2$$
$$+s(s-2)[(s-1)^2-(1-2n^2)]\lambda+n^2(n^2-4)=0$$

which has the roots

(6) $$\lambda=\frac{n}{s},\ -\frac{n}{s},\ \frac{2+n}{s},\ \frac{2-n}{s}$$

so that

$$f(\lambda)=(\lambda s-n)(\lambda s+n)(\lambda s-2-n)(\lambda s-2+n)$$

Now, if we write the solution formally

(7) $$g_n(z)=\sum_{k=0}^{\infty} A_k z^{\lambda+k}$$

then the A_k will satisfy a recursion

$$A_{k+1}=A_k\frac{g(\lambda+k)}{f(\lambda+k+1)}$$

where

$$g(\lambda)=s^2\lambda(\lambda-1)+s(s-1)\lambda-n^2(s-1)$$

a closed form may readily be obtained for A_k in terms of gamma functions:

(8) $$A_k=A_0\frac{\Gamma(\lambda+k-\bar{\lambda}_0)\Gamma(\lambda+k-\bar{\lambda}_1)}{s^{2k}\Gamma(\lambda+k+1-\frac{n}{s})\Gamma(\lambda+k+1+\frac{n}{s})\Gamma(\lambda+k+1-\frac{2+n}{s})\Gamma(\lambda+k+1-\frac{2-n}{s})}$$

Here A_0 is not the same as in (7) and λ_0, λ_1 are defined by

$$\bar{\lambda}_0=\frac{s+1}{2s}+\frac{1}{2s}\sqrt{s^2+2s(1-4n^2)+1+4n^2}$$
$$\bar{\lambda}_1=\frac{s+1}{s}-\frac{1}{2s}\sqrt{s^2+2s(1-4n^2+1+4n^2}$$

(We may note that, for real s and real n, λ_0 and λ_1 are also real.) In case none of the indices λ given in (6) differ by an integer, four distinct solutions of type (7) will exist, one for each λ values. In the singular cases (which are often of significant interest), the method of Frobenius will yield further solutions.

Much more information on the characteristics of these functions is necessary in order to solve the boundary value problem. In this project we undertake to find pertinent information such as integral representations, asymptotic behavior, recursion relations, and sufficient tabulations to take care of at least simple boundary value problems.

M. Douglas McIlroy

Reference:

[1] M. Johnson, unpublished paper, M.I.T. Mathematics Department

ASYMPTOTIC SOLUTION OF A DIFFERENTIAL EQUATION

The last quarter was spent in research on a problem proposed by Professor C. C. Lin, which concerns itself with the study of certain special functions required for the asymptotic solution of differential equations with "turning points." In particular, the functions defined by the equations

(1) $u^{iv}+\chi^2(zu''+pu'+qu)=0$ p,q constants (2) $u'''+2u'+pu=0$

are now being investigated.

Tabulation of these functions is necessary for uniformly valid differential equations involving turning points. These were chosen for tabulation since they are the simplest of the type desired and can be treated by the method of Laplace transformation to yield explicit solutions.

Joseph Hershenov

EVALUATION OF MULTI-CENTER INTEGRALS

The evaluation of multi-center integrals for use in a tight-binding energy band calculation of a two-dimensional graphite crystal model is now completed. This task has proved to be very laborious even with the aid of the Whirlwind Computer. One compensation, however, is that the general techniques required are applicable to all the usual molecular integrals and use has been made of programs developed by the writer and by several members of the Solid-State and Molecular Theory Group. These multi-center techniques have been described by the writer in a rather lengthy report which will be published in the Quarterly Progress Report of the Solid-State and Molecular Theory Group, April 15, 1956.

Fernando J. Corbató

COULOMB WAVE FUNCTIONS

The integral representation method of computing the irregular Coulomb Wave Function $g_0(\rho,\eta)$, has given satisfactory results for the region of interest indicated in Fig. 1. The particular representation discussed in the last few progress reports has been discarded in favor of

$$g_0(\rho,\eta)=\int_0^{\infty}d\xi\, e^{-\{\rho\xi-2\eta\tan^{-1}\xi\}}-\int_0^{\infty}d\xi\,\frac{\sin\{\rho\tanh\xi-2\eta\xi\}}{\cosh^2\xi}$$
$$g_0'(\rho,\eta)=-\int_0^{\infty}\xi\, d\xi\, e^{-\{\rho\xi-2\eta\tan^{-1}\xi\}}-\int_0^{\infty}\frac{\tanh\xi}{\cosh^2\xi}d\xi\cos\{\rho\tanh\xi-2\eta\xi\}$$

which does not display the infinite frequency oscillation associated with the former near $\xi=1$. The above representation had been tried previously using a rather clumsy nine-point numerical integration technique. We are now using the more efficient Gaussian quadrature method.

To insure roughly five-place accuracy, the first integral is cut off at $\xi\approx\frac{\pi\eta+15}{\rho}$ and the second is integrated up to $\xi=4$ after which the integrand is replaced by the approximate form

$$4e^{-2\xi}\sin\{0.9996\rho-2\eta\xi\}$$

which can be integrated exactly from $\xi=4$ to $\xi=\infty$. Sixteen point quadratures are used over regions ranging from 0.5 to 1.0 depending on how fast the argument of the sine function is changing.

It is very easy to generate many g_0's simultaneously. If the spacing in the tables is to be $\Delta\rho$ and $\Delta\eta$, we calculate for each value of ξ involved in the quadrature, the functions

$$e^{-(\Delta\rho)\xi};\quad e^{2(\Delta\eta)\tan^{-1}\xi};\quad {\sin \atop \cos}\{\Delta\rho\tanh\xi\};\quad {\sin \atop \cos}\{2\xi\Delta\eta\}.$$

It is then a trivial matter to compute the integrands for the desired mesh of $g_0(\rho,\eta)$ values. In this way the time needed to compute a g_0 and its derivative may be reduced to about two seconds. If a separate integration were needed for each point to be tabulated, the time required would be about 25 seconds.

It is hoped that the full production of the irregular function will start about the middle of April and be completed by the end of May.

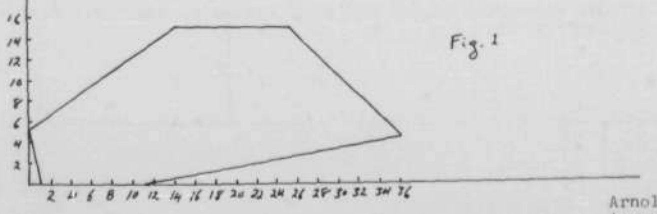

Fig. 1

Arnold Tubis
Aaron Temkin

FIRST APPROXIMATION SOLUTION ON ORE BODY

During the past quarter sufficient progress has been made in the first approximation solution to merit the continuation of this method of attack on the more general problem of the interpretation of field resistivity measurements.

As outlined in the previous report, a solution for the two-dimensional case was to be developed and a comparison was to be made with the results of resistivity modeling. This work is being done in the Department of Geology and Geophysics in conjunction with the Ph.D. thesis of Mr. P. G. Hallof. A program for the theoretical solution was written and run for variations in the depth, shape and resistivity contrast of the ore body which corresponded to those cases already modeled. The theoretical results showed a general difference in character for a vertical and a horizontal tabular ore body, but the comparison with the modeled results was somewhat poor. The character was such that the theoretical results from a horizontal ore body corresponded better with those from the vertical ore body in modeling and vice versa. Numerical agreement was not satisfactory, and further consideration of the two-dimensional case has ceased.

The investigation was then directed towards the case of three-dimensions, in which modeling has also been done by Mr. Hallof. The analysis of this problem is similar in approach to that of the two-dimensional case and will not be given here [1]. As a simplification of the computational procedure, an approximation was made to the actual potential difference measured on the surface of the earth by the receiver electrodes. This was to compute the rate of change of potential with respect to the distance from the sender electrode at a point midway between the receiving positions and then multiply by the spread separation distance in order to obtain the measured potential distance. A program to compute the potential difference as specified above was written and results obtained for variations in the shape, depth and contrast of the ore body. The results of one of the theoretical solutions is plotted in Figure B, the actual value plotted being the percentage difference in the potential measured as caused by the presence of the ore body. The results are plotted in the manner described in the previous report, for the case of a horizontal tabular ore body one unit deep, with the dimensions of unit thickness, four units wide and eight units long. The location of the ore body is shown dotted, and the contouring is done at levels of 0, 10, 20 and 40 per cent.

The results from the modeled case are plotted in Figure A, and it is seen that the comparison between the results plotted in Figure B are quite good. The character of the two is almost identical, and the numerical agreement is relatively good. However, there are a few critical positions of the sender and receivers that give poor results and further work is planned to improve these results. It was expected that some improvement might be possible by the use of discrete receivers in the theoretical solution, rather than the values obtained from the derivative of the potential. Another program to calculate this was developed and the results are plotted in Figure C. The comparison with the modeled values is somewhat poorer in general, but it has not yet been definitely established as to which receiving method is best in general.

The final goal of this immediate investigation is to determine the optimum size and shape of the ore body, that will give the best comparison with the modeled results. With this unit as a building block, it is proposed to consider any large ore deposit as being composed of a number of these basic elements, each with different conductivities. The conductivities determined would be those that would best fit the field measurements. An economic evaluation of the prospect could then be made by the geophysicist with a knowledge of the conductivities, somewhat dependent on the amount of metal present, and of the geology of the area.

Norman F. Ness

Reference:

[1] Norman F. Ness, Machine Methods ofComputation and Numerical Analysis, Quarterly Progress Report No. 18, December 15 (1955) p. 21.

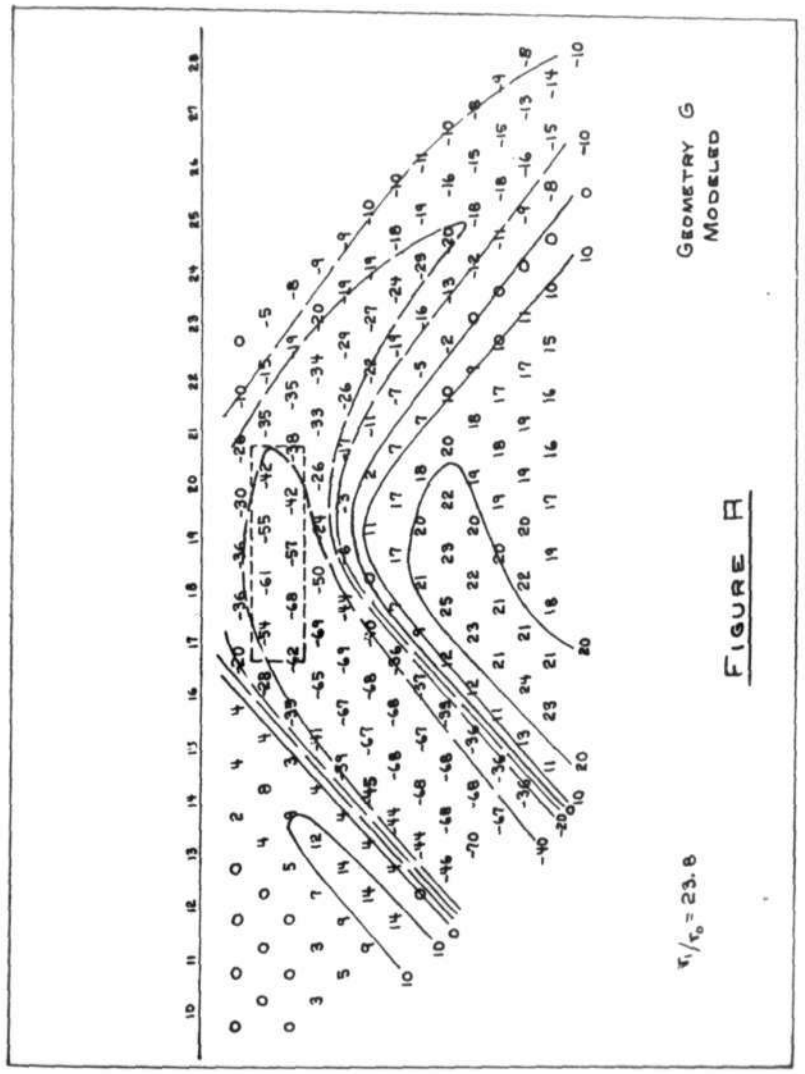

Figure A

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Norman F. Ness

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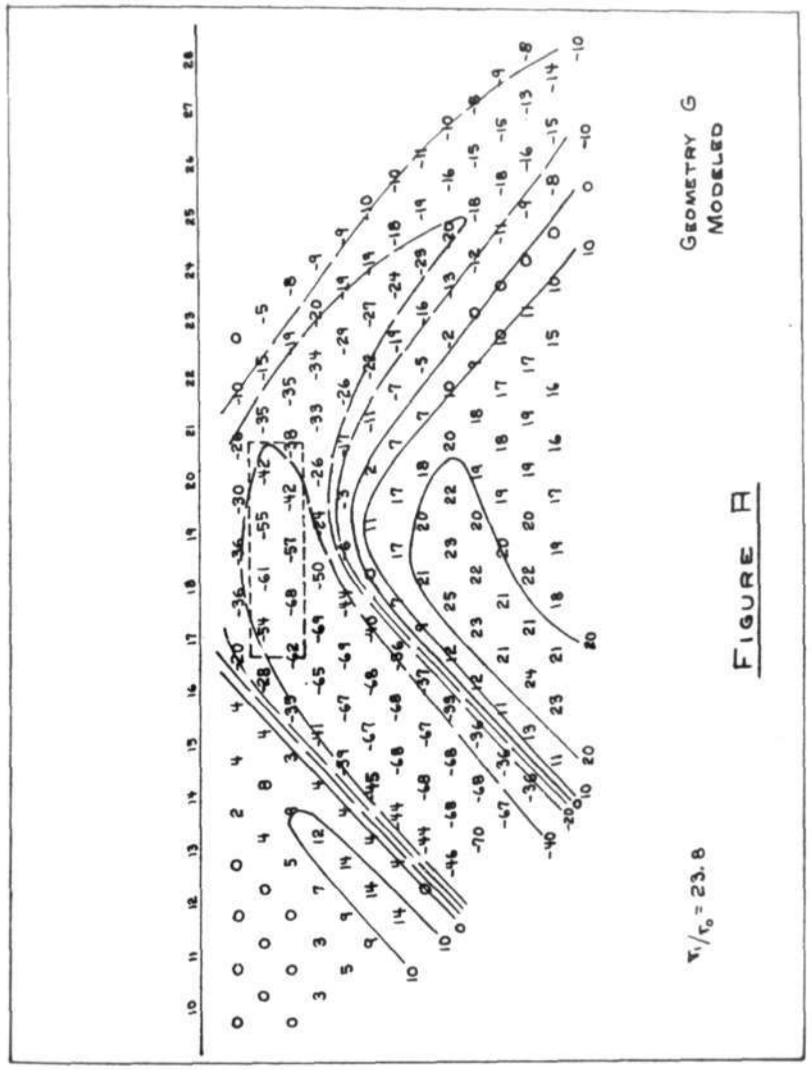
[1] Norman F. Ness, Machine Methods ofComputation and Numerical Analysis, Quarterly Progress Report No. 18, December 15 (1955) p. 21.



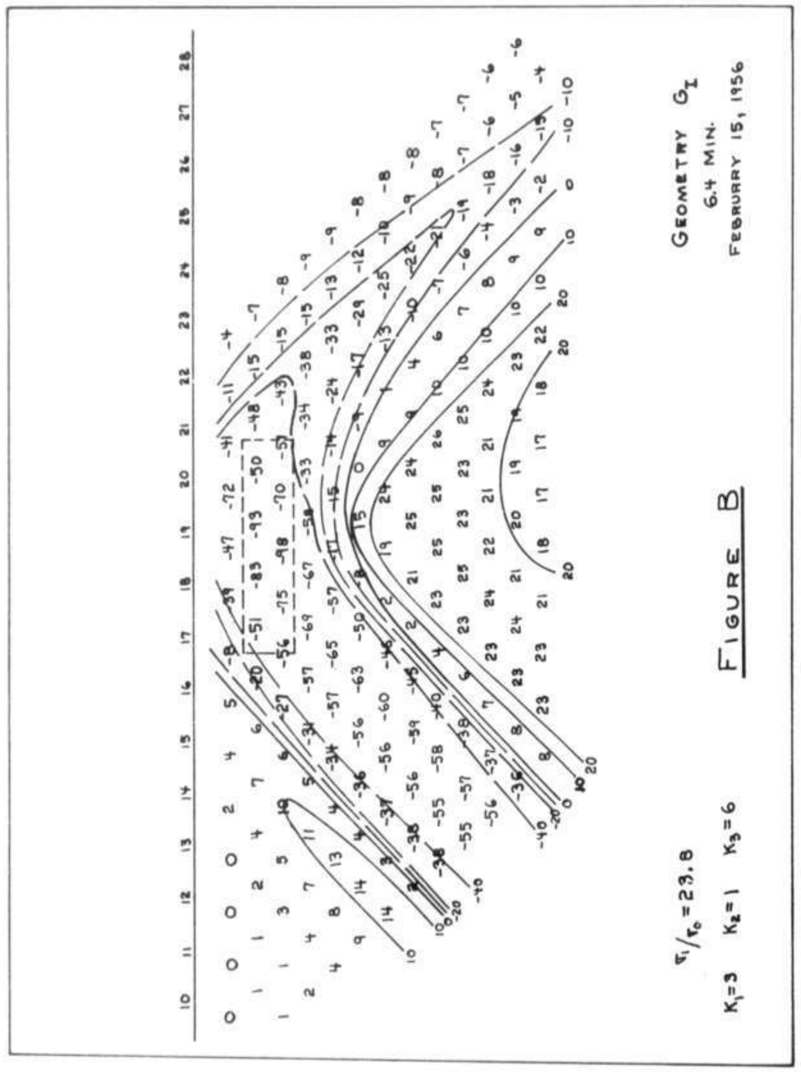


Figure B

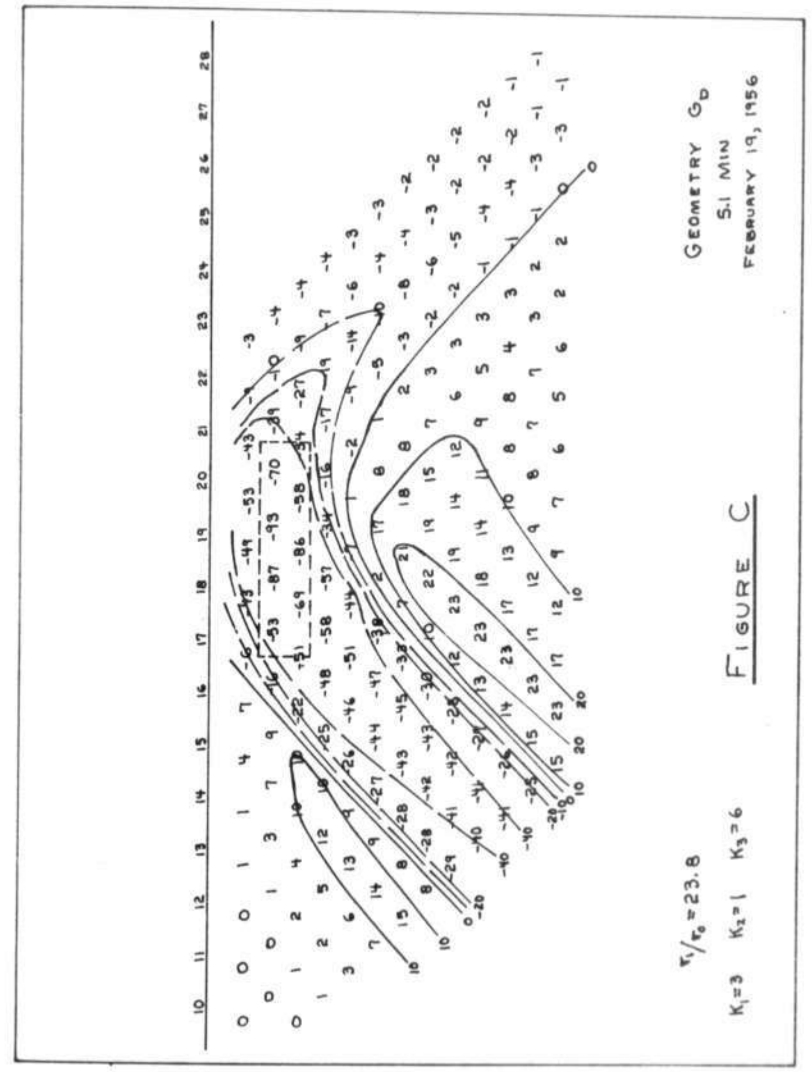


Figure C

AN APPLICATION OF MONTE CARLO METHODS TO NEUTRON DIFFUSION

Introduction

In the last report several methods were described to obtain approximate answers in neutron diffusion problems. The reasons for the selection of a Monte Carlo method were given. This report will give an overall description of the application of this method and an outline of the logical design for a program in which this method is used.

Outline of the Method

The main feature of the Monte Carlo method is that the behavior of individual neutrons is studied. This behavior is then averaged in such a way that the desired results can be obtained; e.g., if we want to know the neutron balance of a reactor, we record the number of neutrons that is absorbed or leaked out, and the number of neutrons produced by fission. More information, like the coordinates of each absorption or the neutron fluxes in the reactor, can be obtained if that is desirable.

The behavior of the individual neutrons is studied in this way: We define a "bank" which is a part of the memory of the computer in which a supply of neutrons with their space coordinates is available. If new neutrons are formed by fission, they are put in this bank, or if neutrons are needed, they are taken from this bank.

We start the problem by taking a neutron from the bank. The following description can be followed on the attached flowsheet. First we determine now in which direction the neutron will fly by calculating from random numbers a set of three direction cosines such that the direction of the resulting vector is random in space. As neutrons are produced at high energies, we set the velocity switch on fast. The use of this switch is explained later on. We choose now a velocity from the fission spectrum and determine in which velocity-group this velocity is. A velocity-group is characterized by a number which the computer uses to determine which constants to use. Each velocity interval between two chosen limits has its own velocity-group number.

Now we determine in which part of the system our neutron is. With this knowledge and the velocity group, we determine which nuclear constants to use. On the basis of these constants and a random number, we calculate how far the neutron will fly. We can now determine if the neutron at the end of its flight was still in the same material. If this is not the case, we set the domain switch on <u>out</u>, advance the neutron to the boundary and proceed to find out the nuclear properties on the other side of the boundary. If the neutron was still in the same material, set the domain switch on <u>in</u>. We let a random number decide if the event at the end of the path was a capture, fission or scattering. For simplicity only these three processes are mentioned.

If the neutron was captured, we record its death coordinates for later print out and take a new neutron from the bank. If the neutron caused a fission, two or three neutrons (av. 2.46) will be put in the bank with the coordinates where the fission took place. We then take a new neutron from the bank. If the neutron is scattered, we determine by which atom it was scattered. With this knowledge we are able to calculate the new direction cosines of the neutron by assuming random scattering in the center of mass system.

If the velocity switch is on fast, we calculate the new velocity, determine the velocity group, and store the constants that belong to this velocity group.

If the new velocity group is the thermal group, set the velocity switch on thermal. This has the effect that from that moment on no velocities are calculated anymore, this saves time.

If the velocity switch was on thermal, we do not calculate the new velocity because we assume that all thermal neutrons have the same average speed. In this case we calculate directly the distance of flight without calculating the velocity.

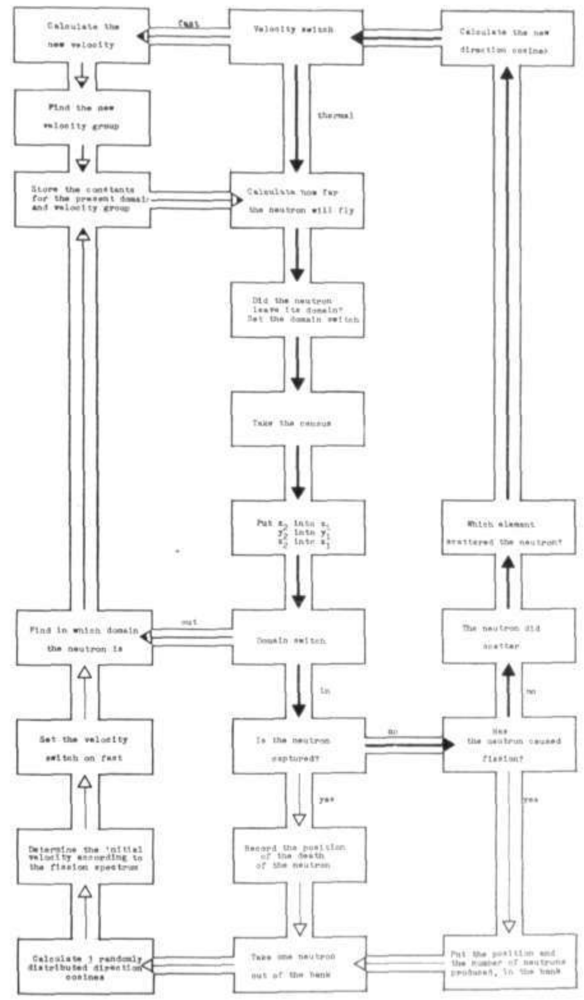
We have now completed the cycle. The neutron will have one collision after the other until it is finally absorbed or leaked out.

The dark arrows in the slowsheet indicate the part of the program that is used after each collision. The semi-dark arrows show the part that is only used several times in the life of a neutron; it is used only after boundaries are crossed or velocity groups are changed. The light arrows indicate the part of the program that is used only once in the life of a neutron, after its absorption or leakage.

Conclusions

All routines outlined in the previous pages have been programmed, coded and tested separately. During the next quarter these routines will be assembled and the complete program will be tested. Then a number of problems of which the analytical solution is known will be run. Only after all these tests have been completed successfully, actual problems will be run.

Marius Troost



RESPONSE OF A SINGLE STORY REINFORCED CONCRETE BUILDING TO DYNAMIC BLAST LOADING

The problem is to study the dynamic response of the roof system of this building. This is a system of two degrees of freedom consisting of a slab supported on relatively stiff beams. The acceleration of the beam has a great effect on the slab acceleration.

The loading portion of the problem also has distinctive characteristics since the slab area is small relative to total roof area and greatly effected by edge vortex action.

The program to solve this problem has three main parts:

(1) The loading portion of the program is written and is about to be "debugged."

(2) The integration portion uses the backward difference technique. It includes calculation of resistances and has been written and "debugged."

(3) The control portion is almost completed.

Ralph G. Gray

RESPONSE OF A FIVE-STORY BUILDING TO DYNAMIC LOADING

This is a program to determine the response of actual multi-story rectangular rigid-frame buildings subjected to blast loads. The program as a whole resolves itself into four parts:

(1) Control Program

This program, currently being written, varies the fundamental parameters of load.

(2) Load Program

Written and "debugged," this program computes the loads due to blast pressures on the inner and outer surfaces of the building as functions of time.

(3) Integration Program

This program has been written. It finds the lateral displacements of the floors by numerical integration, using the "backward difference" technique to determine peak displacements and compares them with allowable values.

(4) Resistance Program

Also written, this program utilizes the displacements determined in the integration program to compute resistances within each story.

This last program solves a matrix composed of the slope-deflection equations to determine joint rotations and uses the rotations at each end of a particular member to obtain a value of elastic moment at each end. This moment is compared with the value at which the member enters the plastic range. If it has entered this range, the matrix is modified accordingly. A member changing from plastic to elastic also causes proper modification of the matrix. The sequence elastic-plastic-elastic may be repeated any number of times, values of residual plastic strain ("kinks") being accumulated during the process.

From the moments existing at the joints, the story shears are computed and treated by the integration program on the next cycle as resistances.

Ralph G. Gray

GROWTH OF FATIGUE CRACKS

Recent work is being directed toward investigating the growth of a crack in an axially cracked bar loaded in torsion. This work is confined to consideration of bars which are perfectly plastic and which are strained to an extent that all stresses are in the plastic region. The assumed mode of crack growth is as follows: The bar is twisted until the strain at a distance x_s in front of the crack reaches the value γ_s; the crack

then advances a distance $d\xi$, relieving the stresses; the angle of twist, θ, is increased until the strain x_s from the crack tip reaches γ_s, and again the crack advances, etc. By combining this criterion for growth with the equation expressing the strain distribution ahead of the crack [1], one may express the inverse of the non-dimensionalized rate of crack growth in a rectangular bar as the following Volterra integral equation:

$$\left(\frac{d\theta}{d\xi}\right)_{\xi=x} = \frac{x_s^2}{1+x_s^2}\left[1+\frac{1}{(x+x_s)^2}\right] + \frac{x_s}{1-x_s^2}\int_0^x \left[1+\frac{1}{(x-\xi+x_s)^2}\right]\left(\frac{d\theta}{d\xi}\right)d\xi$$

Numerical solutions to this equation were carried out manually, using two methods: The integral was approximated by the trapezoidal rule and a stepwise solution obtained [2]. This solution was used as the first approximation for the second method--an iterative solution using Simpson's rule to approximate the integral [2]. One iteration sufficed to show that the two solutions were in sufficiently close agreement. The equation has been programmed for the Whirlwind CS computer, using the first method outlined above.

Joseph B. Walsh

References:

[1] F. A. McClintock, "Growth of Fatigue Cracks under Plastic Torsion," unpublished paper submitted to International Conference on Fatigue of Metals, Institution of Mechanical Engineers, London, (1956), pp. 1-12.

[2] L. Collatz, Numerische Behandlung von Differentialgleichungen, Springer, Berlin, (1951), pp. 411-415.

CONDENSATION OF VAPOR IN A VERTICAL TUBE

The equations given in the past report for the film condensation of a vapor in a vertical tube in the case when the vapor is in turbulent flow and the condensate film in laminar flow have been changed. It was found that the ordinary friction factor equation for turbulent, single phase flow in a tube could not be used for calculating the shear force imposed by the vapor on the condensate at the vapor-condensate interface. Data for the friction factor in a condenser tube have been published by O. P. Bergelin, P. K. Kegel, F. G. Carpenter, and C. Gazley, Jr., in the Journal of the Heat Transfer and Fluid Mechanics Institute, 1949. These data indicate that the friction factor of the gas in the condenser tube can be several times the value as given by the ordinary single phase equation at the corresponding gas Reynolds number. The friction factor in the above publication has been plotted as a function of the gas Reynolds number and a parameter, $S_w \Gamma / s\rho$.

Using these results, the form of the equations governing the laminar condensate-turbulent vapor case did not change, although it was necessary to modify the definition of the dimensionless variables and the parameters, and to introduce one new parameter.

The turbulent condensate-turbulent vapor case is being analyzed in a manner very similar to the laminar condensate-turbulent vapor case. The Nikuradse velocity distribution has been assumed for the condensate film near the wall while there is still some question whether this distribution will be valid near the vapor-condensate interface. The Martinelli result to the heat transfer-momentum transfer analogy is being used for determining the resistance to heat transfer of the turbulent condensate film.

The laminar condensate-turbulent vapor equations are being programmed for the computer.

Jukka A. Lehtinen

Part II

Project Whirlwind

1. REVIEW AND PROBLEM INDEX

This report covers the specific period of December 14, 1955 to March 20, 1956. During this time, 87 problems made use of 405.8 hours of the 545.6 hours of Whirlwind computer time allocated to the Scientific and Engineering Computation (S&EC) Group. The remaining 139.8 hours of the allocated time were used for terminal equipment testing and calibration, demonstrations, and various inter-run operations not logged to specific problems.

The 87 problems run during this quarter cover some 16 fields of applications. The results of 24 of the problems have been or will be included in academic theses. In these 24 problems, there are represented 23 Doctoral theses, 3 Naval Engineer's, 1 Civil Engineer's, and 4 Master's. Thirty-five of the problems have originated from research projects sponsored at MIT by the Office of Naval Research.

Two tables are provided as an index to the problems for which progress reports have been submitted. In the first table the problems are arranged according to the field of application, and the source and amount of time used on WWI is given. In Table 2-II, the problems are listed according to the principal mathematical problem involved in each. In each table, the letter after the problem number indicates whether the problem is for academic credit and whether it is sponsored. The code is explained on page 28.

Field	Description	Problem Number	Min. of WWI Time	Supervisor or Programmer
Aeronautical Engineering	Transient temperature of a box type beam	179 C.	196.6	L. Schmit
	Transient response of aircraft structures to aerodynamic heating	236 C.	129.0	L. Schmit
	Factoring high order polynomials	314 C.	68.9	V.W.Howard
	Horizontal stabilizer analysis	257 C.	381.7	E. Criscione
	Parametric study of modal coupling and damping	334 C.	133.8	K. Wetmore
	Optimization of Ram-air cooling systems	338 C.	128.4	R. Moroney
Chemistry Department	Optical properties of thin metal films	101 N.	4.1	A.L.Loeb
Chemical Engineering	Transients in distillation columns	241 B.N.	273.8	S.H.Davis,Jr.
	Calculations for the MIT reactor	266 A.	343.5	M. Troost
	Critical mass calculations for cylindrical geometry	270 B.	782.2	J.R.Powell
Civil Engineering	Solving simultaneous equations	347 B.	7.6	S.G.Fattal
Dynamic Analysis and Control Lab	Game theory optimization of an interception system	332 C.	237.4	G.R.Welti
Electrical Engineering	Optimization of alternator control systems	264 C.	452.6	J. Dennis
Geology	Fourier synthesis for crystal structures	261 C.	696.0	M.Buerger
Instrumentation Laboratory	Data reduction for X-1 fire control	244 C.	56.1	J. Stark
	Three dimensional aerodynamic lead pursuit study using WWI as a real time computer	318 C.	161.1	J. Stark
Lincoln Laboratory	Eigenvalue problem for propagation of electromagnetic waves	193 L.	337.9	H.B.Dwight
	General raydist solution	272 L.	89.8	G.Sponsler
	Error analysis	312 L.	984.4	R.Bennett
	Prediction analysis	327 L.	631.8	I. Reed
	Tropospheric propagation	300 L.	123.2	H.B.Dwight
Nuclear Engineering	Non-uniform fuel distribution	351 B.	51.8	A. Sutter
Mathematics Department	Buckling of shallow elastic shells	275 B.	1120.9	A. Ralston
	Matrix multiplication	345 B.	85.2	R. Archer
	Wave propagation	348 A.	112.5	L. Roberts
Mechanical Engineering	Flow of compressible fluids (aerothermopressor)	120 B,N.	24.1	A. Erickson
	Rolling bearings	293 C.	656.3	A. Shashaty
	Maximum size of bubbles	322 B.	500.5	P. Griffith
	Boundary-layer characteristics of a steady laminar flow of a combustible mixture over a hot surface	333 A.	325.6	T.Y.Toong
	Beam vibration	339 A.	5.1	S.H.Crandall
Meteorology	Investigation of the vorticity field in the general circulation of the atmosphere	226 D.	134.3	D. Cooley
	Spectral analysis of atmospheric data	306 D.	83.7	B. Saltzman
	Statistical and dynamic methods in forecasting	341 C.	252.6	E. Lorenz
	Weather prediction	343 C.	175.4	J. Austin
Physics	Atomic integrals	234 N.	5.9	R. Nesbet
	Application of the APW method to face- and body-centered iron	253 N.	919.2	J. Wood
	Energy levels of diatomic hydrides	260 N.	405.7	A. Freeman
	Evaluation of two-centered molecular integrals	262 N.	510.0	A.Aghajanian
	Analysis of air shower data	273 N.	191.0	G.Clark
	Energy levels of diatomic hydrides LiH	278 N.	184.1	G.Koster
	Atomic wave functions	288 N.	1028.1	R. Nesbet
	Pure and impure KCl crystal	309 B,N.	431.1	L.P.Howland
	Determination of phase shifts from experimental cross-sections	162 N.	42.3	E. Campbell
	Analysis of cloud chamber photographs	323 N.	8.7	D.O.Caldwell
	Theory of neutron reactions	245 N.	991.1	K. Campbell
	Neutron-deuteron scattering	225 B,N.	76.4	L. Sartori
	Polarizability effects in atoms and molecules	290 N.	653.3	L.C.Allen
	Dynamic programming	344 B.	111.3	R.M.Oliver
	Waiting line-constant holding time	353 A.	86.9	H.Galliher
Servomechanisms Laboratory	Data reduction	126 C.	997.6	D.T.Ross
Miscellaneous	Comparison of simplex and relaxation methods in linear programming	219.	242.8	D. Arden
	Comprehensive system of service routines	100.	980.8	S&EC Group
	S&EC subroutine study	141.	4.1	J. Roseman
	Solution of partial differential equations	349.	459.4	F.M.Verzuh

Table 2-I Current Problems Arranged According to Field of Application

Mathematical Problem	Procedure	Problem Number
1. Matrix algebra and equations		
Root of determinantal equation	Iteration	266 A.
Linear equations	Crout	270 B.
Eigenvalues	Iteration	275 B.
Eigenvalues	Diagonalization	278 B.
Eigenvalues	Diagonalization	288 N.
Inversion	Crout	312 L.
Factorization of polynomials	Hitchcock	314 C.
Equations	Direct evaluation	323 N.
Inversion	Crout	332 C.
Orthonormalization, eigenvalues	Schmidt process, diagonalization	290 N.
Ordinary algebra	Iteration	338 C.
Diagonalization	Successive approximation	341 C.
Inversion	Crout	341 C.
Inversion	Crout	343 C.
Simultaneous algebraic equations	Crout	347 B.
2. Ordinary differential equations		
Seven non-linear first order	Fourth order Kutta-Gill	120 B,N.
Non-linear first order	Second order Kutta-Gill	241 B.
Wave equation	Milne predictor-corrector formula	245 N.
Non-linear	Gill's	297 B.
Different sets of differential equations	Step by step	257 C.
Set of differential equations	Gill's	318 C.
Set of differential equations	Gill's	333 A.
Simultaneous differential equations	Finite difference	334 C.
Non-linear differential equations	Perturbation procedure	345 B.
Second order	Finite difference	351 B.
3. Partial differential equations		
Second order parabolic	Finite difference	179 C.
First order system	Finite difference	226 D.
Second order parabolic	Finite difference	236 C.
Non-linear	Finite difference	322 B.
Fourth order	Finite difference	339 A.
Twenty-six simultaneous partials	Finite difference	349.
4. Integration		
Integral transformation	Algebraic recursion formula	234 N.
Overlap integrals	Evaluation of analytic forms	260 N.
Overlap integrals	Evaluation of analytic forms	262 N.
Integrations	Simpson's rule	278 N.
Integrations	Gauss quadrature	312 L.
Integrations	Barnet and Coulson expansion	309 B.
Stationary point of a variational	Simpson's rule	225 B,N.
Fresnel integral	Conversion power series with a complex argument	300 L.
5. Transcendental equations		
Curve fitting	Least squares	162 N.
Curve fitting	Least squares	273 N.
Non-linear equations	Steepest descent	264 C.
Non-linear equations	Iteration	272 N.
Non-linear equations	Steepest descent	293 C.
6. Complex algebra		
Complex roots and function evaluation	Iteration	193 L.
7. Data reduction		
Miss distribution	Arithmetic operations	244 C.
8. Fourier series		
Fourier synthesis	Direct evaluation	261 C.
Fourier synthesis	Direct evaluation	306 D.
9. Linear Programming		
Linear Programming	Simplex relaxation	219.

Table 2-II Current Problems Arranged According to the Mathematics Involved

2. WHIRLWIND CODING AND APPLICATIONS

2.1 Introduction

Progress reports as submitted by the various programmers are presented in numerical order in Section 2.2. Since this summary report presents the combined efforts of DIC Projects 6345 and 6915, reports on problems undertaken by members of the Machine Methods of Computation (MMC) Group have been omitted from Section 2.2 of Part II to avoid duplication of Part I. For reference purposes, a list of the MMC Group problems appears below.

Letters have been added to the problem numbers to indicate whether the problem is for academic credit and whether it is sponsored. The letters have the following significance.

A implies the problem is NOT for academic credit, is UNsponsored.

B implies the problem IS for academic credit, is UNsponsored.

C implies the problem is NOT for academic credit, IS sponsored.

D implies the problem IS for academic credit, IS sponsored.

N implies the problem is sponsored by the Office of Naval Research.

L implies the problem is sponsored by Lincoln Laboratory.

The absence of a letter indicates that the problem originated within the S&EC Group.

List of Machine Methods of Computation Group Reports

The following problems used Whirlwind I computer time during this quarter, but are reported on in Section 2.2 of Part I.

122 N.	COULOMB WAVE FUNCTIONS	A. Temkin A. Tubis
203 B,N.	RESPONSE OF A MULTI-STORY FRAME BUILDING UNDER DYNAMIC LOADING	R. G. Gray
231 B,N.	REACTOR RUNAWAY PREVENTION	M. Troost
329 N.	FIRST APPROXIMATION SOLUTION ON ORE BODY	N. F. Ness
337 N.	NON-LINEAR SECOND ORDER DIFFERENTIAL EQUATIONS IN THE THEORY OF ELASTIC SHELLS	H. Weinitschke

100 COMPREHENSIVE SYSTEM OF SERVICE ROUTINES

Upon the dissemination of DCL Memo 117 on a proposed post-mortem program which would print out instructions only, but which would give the reference addresses relative to floating addresses and place flad tags in the print-out, the staff decided that the present Generalized Post-Mortem program should be revised to incorporate these features and to add several others to make the GPM more useful as a means of getting output. The goals for the GPM are to give print-outs of storage that are as close as practicable to the input language to facilitate checking and print-outs of data in an easy to use form and in an automatic operation that will not use up very much time.

A new GPM has been coded but has not yet begun to be checked out. It has these facilities:

(1) The programmer may specify up to 7 different (i)STOP instructions as intermediate break points. These "STOP" instructions will not stop the computer or cause any logging to take place, but will transfer control to the GPM which will execute a special request pertaining to the particular break point and return control to the next instruction after the break point. A final break point #0 should be specified. When it is reached another post-mortem will be executed, the STOP will be logged and the computer stopped (or the next director tape word read). If the computer stops in any other way, the operator may assume that there was trouble and he will be able to call in the GPM to execute a special "in case of trouble" post-mortem by setting the right MIR to 1.00000 and pressing the Examine and Read-In buttons. The operator may call for any break point post-mortem by setting the right MIR to 0.00033, the left MIR to 0.0000n, where n is the break point number, and pushing both activate buttons and the Read-In button. (Unless the Erase button is pushed, the left MIR will be considered to be zero.) The operator may obtain scope post-mortems and programmed arithmetic post-mortems (PAPMs) as at present (rm4, ri will get a PAPM on the scope), but for more automatic operation it will be possible to request these on an fp tape.

(2) In general, the new print-outs will have the same form as the old. However, STOP is given for sp25 as a Whirlwind instruction and the interpreted instructions iSTOP, IN, and OUT are printed. In the flad modes automatic in-out requests will appear as $AIORQ^n$ as explained in Memo DCL-117. Whenever flads are requested, flad tags are placed in the body of the print-out. The absolute addresses are always printed in the left-most column, but these will usually be the current addresses (the place in core memory these words would occupy) rather than drum addresses (the actual locations on the drum). The drum address of the first word of the block is always given on the first (subtitle) line, even when data is printed in the format mode where no current addresses are given. There will be 16 modes: gda, gdf, ofa, off, dia, dif, dfa, dff, wwa, wwf, wia, wif, iia, iif, iwa, and iwf, where the last letter calls for absolute or floating address and the first two letters call for numbers in the same way as the old GPM and for instructions the first letter sets the mode and the second sets the mode after IN or OUT, i.e., wi and iw imply switching.* For numbers, a special format word in place of the initial current address will set the number per line and number per block. Except in this mode, large blocks of zeros will be suppressed.

(3) The punch mode may be selected by an fp tape. The new punch mode will give an fb tape complete with title and sp block, but only the 100-octal block length will be retained.

(4) Because flads will be allowed in the addresses appearing in the fp tape and the flad table must be preserved for the post-mortem in many cases, the fp tape must be read in immediately after the fc tape containing those flads. As the read-in of tape does not result in immediate execution of the request, but only in storing the request

on the buffer drum, it will be possible to check for many errors and reject illegal tapes before the program is operated.

(5) A PAPM will not be given unless it is specifically requested at a break point. It will be given automatically in case of trouble.

It is expected that the new GPM will be operating routinely in the next quarter. A new memo and/or supplement to the CS Manual will be published to acquaint those concerned with the new GPM.

M. R. Weinstein
Digital Computer Laboratory

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* Memo DCL-117 Flad Post-Mortems and ff Tapes, M.R.Weinstein, December 19, 1955

101 N. OPTICAL PROPERTIES OF THIN METAL FILMS

One successful run on some samples vastly different from ones previously used was preceded by a few unsuccessful ones to bring performance and tape requests up to date after more than a year's interval.

A. L. Loeb
Lincoln Laboratory

120 B,N. THE AEROTHERMOPRESSOR

The present program was found to be unsatisfactory for certain initial conditions because some iterative procedures did not converge. Small changes should once more make the present program satisfactory.

Future calculations are planned to check the experimental results that are just being obtained in the Gas Turbine Laboratory.

A. J. Erickson
Mechanical Engineering Department

126 C. DATA REDUCTION

Problem 126 is a very large data-reduction program for use in the Servomechanisms Laboratory. The overall problem is composed of many component sections which have been developed separately and are now being combined into complete prototype programs. Descriptions of the various component sections have appeared in past quarterly reports. After the development and testing of the prototype Whirlwind programs is completed, the programs will be re-coded for other, commerically available, large scale computers, (probably the Univac Scientific 1103, IBM 701 and IBM 704 computers), for use by interested agencies for actual data reduction at other locations. The programs are currently being developed by Douglas T. Ross, David F. McAvinn, Walter E. Weissblum, Benson H. Scheff, and Dorothy A. Hamilton, Servomechanisms Laboratory staff members with the assistance of John F. Walsh. This work is sponsored by the Air Force Weapons Guidance Laboratory.

The nature of the problem requires extreme automaticity and efficiency in the actual running of the program, but also requires the presence of human operators in the computation loop for the purpose of decision making and program modification. For this

reason extensive use is made of output oscilloscopes so that the computer can communicate with the human, and manual intervention registers so that the human can communicate with the computer in terms of broad ideas, while the computer is running, and have the computer program translate these ideas into the detailed steps necessary for program modification to conform to the human operator's decision. The program which does this translation and modification is called the Manual Intervention Program (MIV). The most recent version of the prototype data reduction program is called the Basic Evaluation Program.

The basic features of the MIV Program were described in the previous progress report. During the past quarter the Basic Evaluation Program was completely debugged using the Mistake Diagnosis Routine (described earlier) to extract pertinent information at over seventy break points. The nature of the Evaluation Program is such that this debugging process would have been virtually impossible without the use of the MDR program. In addition, an entire new section of the Evaluation Program, which will allow automatic starting of the data reduction process at any point, was incorporated and checked.

The decision to operate all Problem 126 programs under Director Tape Control has materially increased the effectiveness and reliability of these programs. The re-coding of these programs for the Univac Scientific 1103 Computer using the facilities of the WWI-1103 Input Translation Program of Problem 256 has begun in earnest.

The Logging Program section of the MIV Program is now undergoing changes to provide greater flexibility of operation. The function of the Logging Program is to record logical information on a magnetic tape unit, giving all pertinent information concerning all manual interventions, i.e., which buttons have been pushed and what certain parameter values are, and similar information concerning all displays seen by the human operator. The complexity of the manual operations makes such a complete record essential to the successful use of the programs.

The logical magnetic tape output of the Logging Program is to be processed at the end of a run by the Editing Program which will give a delayed printer tabulation of the log in English phrases and numbers of appropriate types. A preliminary version with very restricted capabilities is now in use, and the elaborate complete version is now being written. Mr. Frank C. Helwig of the MIT Digital Computer Laboratory is a major contributor to the Logging and Editing Programs.

A very flexible and elaborate Scope Plot Program is also being written. This program will give axes calibrated with four sizes of pips arranged in a pattern similar to that on a ruler, a faint grid in the background, plotted points connected by faint straight lines, alphanumeric labeling and automatic initiation of new frames whenever any boundaries are exceeded. The program is fed a series of parameters which tell the desired format, scaling, and labeling and then is fully automatic. The program can be initiated either automatically or through the MIV program.

Both the Editing Program and the Scope Plot Program require the use of arbitrary sequences of alphanumeric characters. For use in these and future programs a pair of general programs are being written. The first program accepts arbitrary six-bit characters from Flexo Tape and alphanumeric code words in a special language, and stores them in arbitrary locations with optimum packing. The second program unpacks the code words and characters, and under their direction distributes the arbitrary information to selected in-out units.

The Unpacking Program is being realized by programming a new type of computer which has three modes of operation of its control element. Each instruction in the program to be executed by this computer is tagged whether or not it is to be executed in each of the various modes. A system of jump and remember-jump instructions, which can change both location and mode, gives this type of computer extreme flexibility and compactness, and should prove to be very educational from all standpoints. The future work to be done by this group has a very real need for investigations of this type.

D. T. Ross
Servomechanisms Laboratory

141 S & E C SUBROUTINE STUDY

Subroutines are being checked out for function evaluation, via truncated Chebyshev and Legendre expansions, for root extraction (Barristow and Bernouilli), and for matrix converses (Craig's method and Frame's method).

M. S. Watkins
Digital Computer Laboratory

155 N. SYNOPTIC CLIMATOLOGY

A complete discussion of the research carried on by the Synoptic Climatology Project, under sponsorship of the Office of Naval Research, may be found in the final report, "Studies in Synoptic Climatology" published by the MIT Department of Meteorology, March 15, 1956.

Programs for this problem were written by members of the Synoptic Climatology Project under the supervision of Dr. T. J. Malone, formerly of the MIT Department of Meteorology and Professor E. N. Lorenz, MIT Department of Meteorology.

This problem is now terminated.

E. A. Kelley
Meteorology Department

162 N. ANALYSIS OF A SCATTERING EXPERIMENT IN NUCLEAR PHYSICS

A fit of various combinations of phase shifts has been obtained at discreet points for low energies. Now, a slightly finer mesh of points is being used in the same region to determine the set of phase shift combinations which will give the truest physical picture.

Future computations will be based on this set, as it is planned to project out computations to try to find a fit at middle and high range energies.

Programmers working on this problem are E. Campbell and E. Mack of the Nuclear Science Laboratory.

E. Mack
Nuclear Science Laboratory

179 C. TRANSIENT TEMPERATURE AND STRESS OF A BOX-TYPE BEAM

Closer correlation with experimental temperatures was achieved by improving the value of contact admittance. Future plans are to improve stress correlation.

J.C.Loria
Aeronautical Engineering Department

193 L. EIGENVALUE PROBLEM FOR PROPAGATION OF ELECTROMAGNETIC WAVES

For purposes of comparison with the results already obtained for a bilinear model of the atmosphere at 50 Mc, we plan to carry out similar calculations for an inverse-square model. This work will involve computation of Bessel functions of large complex order and argument.

Calculations of 410 Mc and 3000 Mc have been continued, as described previously. Tapes have been prepared with which eigenvalues, eigenfunctions and mode sums may be calculated at one time, rather than in two parts, as was done formerly. Preliminary tapes have been made for the inverse-square model.

The 410 Mc and 3000 Mc calculations with the bilinear model will be continued until completed. Calculations will be carried out for the inverse-square model at 50 Mc, for comparison with computations at this frequency which were completed during the preceding quarter.

Programmers working on this problem are Professor H.B.Dwight and Dr. R.M.Ring of Lincoln Laboratory.

H. B. Dwight
Lincoln Laboratory

194 B,N. AN AUGMENTED PLANE WAVE METHOD AS APPLIED TO SODIUM

Unfortunately, although the routines mentioned in the last report have been individually tested, the tests of the complete program have not yet been successfully completed.

In the meantime, it has been decided to use the symmetrization routines of the matrix element generation routine in conjunction with the E vs E_0 routine to obtain E vs E_0 curves for symmetrized combinations of augmented plane waves.

Details can be found in the Quarterly Progress Reports of the Solid State and Molecular Theory Group, MIT.

M. M. Saffren
Solid State and Molecular Theory Group

216 C. ULTRASONIC DELAY LINES

This period has been spent in debugging the program and in making a few test runs. The program is very complex, consisting of an iterative loop which contains a linear program. The reason for iteration with a <u>linear</u> program is that the restrictions (inequalities) are <u>not</u> linear, but well-behaved <u>analytic</u> functions in many variables, for which linear approximations consisting of the first terms of Taylor series are made.

The sizes of the problems to be solved vary from 40 restrictions with 20 variables to 90 restrictions with 32 variables. The time needed for one cycle varies correspondingly from 3 minutes to 15 minutes, most of which is taken by the linear program. Usually 3 cycles or less suffice for convergence.

There are about five of the larger problems yet to be solved and several more test runs using the smaller problems will be made to determine the mode of operation preferred.

The programmers working on this problem are Richard Bishop and John Ackley.

R. Bishop
Mathematics Department

219 LINEAR PROGRAMMING - TRANSPORTATION PROBLEM

The Whirlwind program for the solution of the classical transportation problem has been completed and proved on a number of trial problems, and is now available for general use. The program will handle problems with up to 127 plants and up to a total of 401 plants and customers provided the product of the number of plants and the number of customers is less than 7169. Several sets of data for a practical distribution problem with 9 plants and 69 customers have been tried. The average computer time for each solution was about 1.5 minutes.

The Classical Transportation Problem

The transportation problem may be phrased as follows: A company operates m plants producing a commodity, the i^{th} of which can supply S_i units of the commodity. The company sells its production to n customers, the j^{th} of which demands D_j units of the commodity. The costs of manufacturing and transporting a unit of the commodity from plant i to customer j is C_{ij}. It is desired to find the number of units X_{ij} that should be shipped from each plant to each customer, in order that the total cost of the operation be a minimum. Thus the problem may be stated mathematically as:

Minimize

$$C = \sum_{i,j} C_{ij} X_{ij} \qquad (1)$$

subject to the constraints

$$\sum_i X_{ij} = D_j \qquad (2)$$

$$\sum_j X_{ij} = S_i \qquad (3)$$

$$X_{ij} \geq 0 \qquad (4)$$

This is a special case of the general linear programming problem.

The Stepping Stone Method *

A set of X_{ij} which satisfies equations 2, 3 and 4 is called a feasible solution to the problem; if the set also minimizes C, it is an optimum feasible solution containing, in general, exactly m + n - 1 non-zero X_{ij}. The stepping stone method of solution consists of

(1) Generating a feasible solution having exactly m + n - 1 non-zero X_{ij}.

(2) Finding a zero X_{ij} which, if allowed to be positive, would yield a decrease in C. In the process of increasing this X_{ij}, one of the non-zero X_{ij} must go to zero in order that equations 2 and 3 remain satisfied. Thus the new feasible solution will again have exactly m + n - 1 non-zero X_{ij}.

Step 2 is repeated until there are no zero X_{ij} which could be changed so as to reduce C. The set of X_{ij} is then an optimum feasible solution.

The program incorporated a unique means of storing the feasible solution which makes it faster than computer techniques for solving the transportation problem in use elsewhere. It is planned that the basis of our computer technique be published in the Journal of the Association for Computing Machinery.

The programmers working on this problem are J. B. Dennis and W.J. Eccles of the MIT Electrical Engineering Department.

J. B. Dennis
Electrical Engineering Department

* Dantzig, G. B., "Application of the Simplex Method to a Transportation Problem", Activity Analysis of Production and Allocation, T.C.Koopmans, Ed., John Wiley & Sons, New York, 1951, pp 359-373.

Charnes, A., and Cooper, W. W., "The Stepping Stone Method of Explaining Linear Programming Calculations in Transportation Problems", Management Science, Vol. 1, No. 1, October 1954.

225 N. NEUTRON-DEUTERON SCATTERING

The calculation of the neutron-deuteron scattering lengths with the trial function described in previous reports has been completed. * Unique minima were found for the variational expressions for k cot δ in both quartet and doublet spin states. The scattering lengths so determined show little effect of polarization in either spin state. However, it is felt that these results are inconclusive since the following two interpretations are both possible: (a) the physical system is indeed unpolarized, (b) the trial function used is not adequate to describe the physical system.

In an effort to throw further light on the situation, it is planned to continue the calculation using other forms for the trial function. Two such forms are being considered: one contains two additional parameters to describe the no-polarization wave function, while the other contains a more flexible description of the polarization. Both of these new trial functions are similar enough to the original one so that the existing formalism can be taken over with little change. All the required integrals have already been evaluated, and the existing programs can be used with only a small amount of modification. This calculation is now in progress.

L. Sartori
Physics Department

* The results of this calculation have been submitted to the MIT Physics Department as part of a Ph.D. thesis by the author.

225 D. INVESTIGATION OF THE VORTICITY FIELD IN THE GENERAL CIRCULATION OF THE ATMOSPHERE

The following programs have been completed during the past quarter:

a) A 30th order asymmetric matrix inversion program (24,6 input).

b) A matrix transpose program.

c) A relaxation program for $\nabla^2 \phi$ over a 60 point square grid.

d) The program for the computation of $N(h, h', \zeta_m, \zeta_T; \lambda, y)$ in equation (1) [see Summary Report No. 42].

e) The program for the computation of $\dot{P}_m$.

Plans for the next quarter include the writing and checking out of two programs for the Relaxation, over two 107-point graded net grids, of the operator

$$G(y) \nabla^2_s \frac{\partial h'}{\partial t} - H(y) \frac{\partial}{\partial y} \left(\frac{\partial h'}{\partial t} \right) - I(\zeta_m, \zeta_T; y) \frac{\partial h'}{\partial t} = 0$$

The programmers working on this problem are Duane Cooley and Martin Jacobs.

R. L. Pfeffer
Meteorology Department

234 N. ATOMIC INTEGRALS

Subprograms have been written and tested which evaluate the radial integrals arising from operators

$$r^L Y_L^M \quad \text{and} \quad \frac{1}{r^{L+1}} Y_L^M$$

for all values of L. The functions Y_L^M are spherical harmonics. These programs also evaluate radial integrals for the diamagnetic susceptibility operator

$$r^2 Y_0$$

and the contact hyperfine interaction

$$\frac{\delta(r)}{r^2} Y_0.$$

All of these programs are based on simple recurrence relations for matrix elements of these operators between analytic atomic orbitals of the form

$$\eta_a = N_\alpha e^{-\zeta_\alpha r} r^{A+\ell_a} Y_{\ell a}^m$$

where ζ_α is positive and A is a non-negative integer.

R. K. Nesbet
Solid State and Molecular Theory Group

236 C. TRANSIENT RESPONSE OF AIRCRAFT TO HEATING

During this quarter the basic temperature distribution computing program has been modified in such a way as to permit calculation of the transient temperature and stress response of a T-section which is half of a doubly symmetric I-section subjected to any given continuous timewise variation of the boundary conditions, (i.e., heat transfer coefficient h and adiabatic wall temperature T_{aw}).

Eleven satisfactory runs were made. These runs included accelerated dives and level flight decelerations of Aluminum, Titanium, and Inconel X T-sections. The results have been interpreted in terms of structural weight penalties. The results are further reported in a paper entitled "Some Structural Penalties Associated with Thermal Flight" by Professor James W. Mar and Lucien A. Schmit, which was presented at the Spring Conference 1956 of the Aviation Division of the ASME in Los Angeles, California.

A study of thermally unsymmetrical systems is contemplated for the future.

Programmers working on this problem are L. A. Schmit and H. Parechanian.

L. A. Schmit
Aeronautical Engineering Department

241 B. TRANSIENTS IN CONTINUOUS DISTILLATION COLUMNS

A program has been written to determine the importance of the non-linear variation of vapor composition with liquid composition leaving the plates in a distillation column. If a linear variation for each plate is assumed and the usual simplifying assumptions are used, the transient behavior of a column in which a step change has been made in one or more of its operating variables can be described by a set of simultaneous linear first-order differential equations. The solution of any such set of equations can be carried out by standard techniques to find the roots of the determinant of the coefficients of the dependent variables and the constants from the initial and boundary conditions of the problem. The complexity of the solution is such that a high-speed computer is necessary to carry out these calculations.

A comparison of the results obtained in this manner with the results obtained earlier by using a finite difference approximation to solve the more accurate non-linear equations should show how important the non-linearities are in many cases.

S. H. Davis, Jr.
Chemical Engineering Department

244 C. DATA REDUCTION FOR X-1 FIRE CONTROL

This problem is concerned with computing from fire control signals how far fictitious projectiles miss an observed target. The program tape was ready prior to this quarter. Work during this quarter consisted of reducing new target run data by use of the program tape.

Future plans for the problem call for still more data reduction, perhaps also using supplementary programs (yet to be prepared) for clearer presentation of the reduced data.

J. M. Stark
Instrumentation Laboratory

245 N. THEORY OF NEUTRON REACTIONS

Some of the coding has been reversed to increase the accuracy of the results for $\ell = 5$ and 6. Results have been obtained for $\delta = .2$ $\zeta = .15$ for x's up to 1.6, including $x = 0$.

Future plans are to revise the program so that δ is a function of x_0 instead of a constant.

E. Campbell
Nuclear Science Laboratory

253 N. AN AUGMENTED PLANE WAVE METHOD FOR IRON

E vs. E_0 curves have been completed for both the face-centered and body-centered structures. These curves serve to define the augmented plane waves entering into a variational procedure which in turn gives the so-called "E vs. k" curves.

J. Wood
Solid State and Molecular Theory Group

256 C. WHIRLWIND I - UNIVAC SCIENTIFIC 1103 TRANSLATION PROGRAM

The vocabulary of the WWI-1103 input translation program (as described in Summary Report No. 41) is currently being enlarged to include a more generalized form of numbers. Programmers will be permitted to write numbers of the form

$$\pm 12.34 \cdot 2^3 \cdot 10^{-4}$$

with or without as many associated factors of powers of 2 and 10 as desired, wherever they were previously allowed to write integer numbers. In addition, the post-mortem program has been modified to print an indication of all the illegal words detected by the translation program.

Work is expected to be completed on the translation program during the next quarter, and a final report on the program will be issued as a Digital Computer Laboratory memorandum.

Programmers working on this problem are F. C. Helwig and J. M. Frankovich.

J. M. Frankovich
Digital Computer Laboratory

257 C. HORIZONTAL STABILIZER ANALYSIS

During this quarter the program for the solution of the two rigid body equations of motion (vertical translation and pitching) has been checked out and is now ready to be incorporated in the overall analysis of the horizontal stabilizer. The program for the response of a thirteen lumped mass and a one lumped inertia system, representing a typical present-day fighter aircraft, to a step-function input is almost completely checked out in the linear elastic regime of deformation.

In the future, the two programs mentioned above will be incorporated into one program to yield realistic structural response, or deformations, of a present-day fighter to a gust type input. The response of the fighter will be solved for in its elastic

regime of deformation and also beyond, when the structure incurs permanent deformations. The solution of the differential equations of motion of the system involves the use of numerical integration in a step-by-step procedure.

E. Criscione
Aeroelastic and Structures Research Lab

260 N. ELECTRONIC ENERGY OF THE OH MOLECULE

Most of the work done in this quarter has involved checking the results previously obtained and making use of programs (described in earlier Summary Reports) for repeating some of the calculations involved in calculating the binding energy of the molecule at new internuclear distances.

A. Freeman
Solid State and Molecular Theory Group

261 C. FOURIER SYNTHESIS FOR CRYSTAL STRUCTURES

The crystal structure of pectolite mentioned in earlier reports was finished during the last month, while wollastonite is nearing completion.

The refinement of diglycine hydrochloride, also explained in earlier reports, is finished as far as projections are concerned. Only one more refinement in three dimensions remains to be done.

The isomorphic diglycine hydrobromide is under investigation, and refinement is carried on in its main projections by difference syntheses.

Further progress toward the completion of the structure of jamesonite has been made.

Programmers working on this problem are Professor M. J. Buerger, T. Hahn and N. Niizeki of the Geology Department, and J. Roseman of the Digital Computer Laboratory.

T. Hahn
Geology Department

262 N. EVALUATION OF TWO-CENTER INTEGRALS

The exchange integrals evaluated by Merryman's program, was not found to be sufficiently accurate at internuclear separations of one and two atomic units. These integrals are now being evaluated by Corbató's program.

H. A. Aghajanian
Solid State and Molecular Theory Group

264 C. OPTIMIZATION OF AIRCRAFT ALTERNATOR REGULATING SYSTEM

The method for minimizing a function of n constraints in the presence of equality constraints by means of the method of steepest descent has been proven for two simple mathematical problems.

Work is being done on improving the above program, adapting it to large scale problems, and modifying it for inequality constraints.

Programmers working on this problem are R. R. Brown and J. B. Dennis of the MIT Electrical Engineering Department.

R. R. Brown
Energy Conversion Laboratory

266 A. CALCULATIONS FOR THE MIT REACTOR

During this quarter, work continued on calculations for the MIT Reactor using parameters that are closer to the actual reactor design. The same basic two-group, four region model is being used. Other models are being developed and calculations using these models will be tried in the future.

The reactor power transient studies, carried on with Whirlwind I and the Dynamic Analysis and Control Laboratory analog computer, have been temporarily suspended.

M. Troost of the MIT Chemical Engineering Department is doing the programming for this problem.

T. Cantwell
Nuclear Engineering Group

270 B. CRITICAL MASSES IN D_2O MODERATED REACTORS

During this quarter tapes have been prepared so that a large number of cases may be run. The following is a list of all methods now in working order:

Spherical Geometry

1. Serber Wilson one-group diffusion.

2. Three-group Fourier Expansion.

Cylindrical Geometry

1. Two-group diffusion.

2. Three-group Fourier Expansion - both approximate and exact expansion.

The only remaining program to be debugged is the spherical multi-group transport. This should require only a few weeks.

Future plans involve testing the worth of these methods by comparing them with each other, and with what experimental data is available.

If there is time, a completely new method involving expanding the flux in N groups by Fourier analysis will be tried. This should be free of certain restrictions that hamper the use of the three-group Fourier technique.

J. R. Powell
Nuclear Engineering

272 L. GENERAL RAYDIST SOLUTION

A new method of solution has been tried during this quarter. This method is purely algebraic and solves for a set of coefficients for an eight order polynomial from the coefficients of the original three second-order non-linear equations.

This method proves the problem to be well conditioned and shows the prospect of solution to the desired accuracy. The program at present is in a debugging stage.

A. Zabludowsky
Digital Computer Laboratory

273 N. ANALYSIS OF AIR SHOWER DATA

The last progress report describes how the method of steepest descent is used to make a least-squares fit of an empirical function f to air shower data. That report mentioned the necessity of speeding up the computation. In order to do this the function has been changed from an exponential form to the following:

$$f = \frac{\alpha}{r + \beta r^2}$$

This function gives an adequate fit to the data and the time per shower is only about 30 seconds. But it has the defect that a good fit usually can be obtained with different values of β. For this reason, we are planning to go back to the exponential function

$$f = \frac{\alpha e^{-r/\beta}}{r}$$

The computation time per shower may still be less than one minute since the program is now capable of making good estimates of the initial values of the variables. Also, we plan to develop a rapid exponential subroutine.

The problems connected with the empirical function have not prevented us from analyzing some data. So far, we have results on about 50 showers. This work will continue along with the above mentioned problem and other testing of the program.

Programmers working on this problem are Professor G. W. Clark and F. Scherb of the MIT Physics Department.

F. Scherb
Physics Department

275 B. BUCKLING OF SHALLOW ELASTIC SHELLS

The work on the problem of the buckling of a compressed shallow cylindrical shell is virtually complete. The results of this problem and the problem of the buckling of a hyperbolic paraboloidal shell loaded by its own weight will appear shortly in a doctoral thesis for the MIT Mathematics Department.

A. Ralston
Mathematics Department

278 N. ENERGY LEVELS OF DIATOMIC MOLECULES (LiH)

The Ground Electronic State

The ground electronic state as determined spectroscopically occurs at an equilibrium internuclear separation of 3.0141 atomic units. The spectroscopic value of the vibrational constant, ω_e, is 1.406 x 10^3 cm^{-1}, and the experimental value of the dissociation energy is 2.5 ± 0.2 electron volts.

The configuration interaction problem has been carried out by considering only configurations of proper ground state symmetry which occur when the 1s shell is kept filled. The other two electrons may occupy the lithium 2s and 2p orbitals and the hydrogen 1s orbital. Six configurations of appropriate symmetry occur.

The value of the binding energy has been determined for seven internuclear distances between 2.0 and 6.0 atomic units. The potential energy curve which results has been approximated by a Morse curve, from which we obtain a value of 1.212 x 10^3 cm^{-1} for the vibrational constant, 3.245 atomic units for the equilibrium internuclear distance, and 1.669 electron volts for the binding energy.

The First Excited Singlet State

The first excited electronic configuration has the same symmetry as the ground state ($^1\Sigma^+$). It occurs at an equilibrium separation of 4.9052 atomic units, and lies 3.287 electron volts above the ground state. The vibrational constant, determined spectroscopically, is 234.4 cm^{-1}. Since the symmetry is unchanged in going from the ground state to the first excited state, it is reasonable to assume that a determination of the next to lowest eigenvalue of the secular determinant might be expected to give a fair approximation to the first excited state.

The calculated potential energy curve is very broad as expected, with a minimum very close to the observed value and about 0.29 electron volts too high. A power series form,

$$U(r) = a_0 \xi^2 (1 + a_1 \xi + a_2 \xi^2 + \ldots) \quad \text{where} \quad \xi = \frac{r - r_e}{r_e}$$

has been used to represent the potential energy near the minimum. The vibrational constant, found from a_0, was in good agreement with the experimental value.

SCF LCAO-MO Calculation

We intend to compare the results of the configuration interaction calculation with a self-consistent field molecular orbital treatment. Adequate programs have been developed by Meckler and Nesbet for the Whirlwind computer, and are available for evaluating the electronic energy of lithium hydride by this procedure.

This phase of the work on LiH is being initiated, and further work is in progress on the electronic binding energy at more extended internuclear distances using the conventional configuration interaction scheme.

A. M. Karo
Solid State and Molecular Theory Group

285 N. AUGMENTED PLANE WAVE METHOD AS APPLIED TO CHROMIUM CRYSTAL

The display routine mentioned in the last report has been tested and will be used to examine the discontinuity of slope in augmented plane waves.

E vs. E_0 curves which are being calculated for a slight change in the chromium potential indicate the sensitivity of these curves to potential, at least in the transition metals. The potential used was obtained in a way not usual for energy band calculations. However, other members of the Solid State and Molecular Theory Group using Whirlwind I will soon furnish Hartree-Fock wave functions for the transition metals. From these it is hoped that potentials suitable for an energy band calculation can be obtained.

Further calculations await the final testing of routines mentioned under Problem 194.

Details can be found in the Quarterly Progress Reports of the MIT Solid State and Molecular Theory Group.

M. M. Saffren
Solid State and Molecular Theory Group

288 N. ATOMIC WAVE FUNCTIONS

Approximate atomic wave functions, expressed as sums of determinantal functions constructed from orbitals which are linear combinations of basic exponential one-center orbitals, have been obtained for He, Li, B, and Be.

The calculations on Li and B are primarily concerned with evaluating the electronic contribution to the hyperfine structure, making use of auxiliary programs described under problem 234.

A series of calculations on the isoelectronic systems Be, LiH, He_2 and HLi have been carried out by the one-center method (see Quarterly Progress Report, SS&MT Group, April 15, 1956), at a fixed internuclear separation of 3.0 atomic units. The difference in total electronic energy of these systems can be obtained from the mean value

$$\left\langle \sum_{i=1}^{4} \frac{1}{r_i} \right\rangle_Z$$

taken about the nucleus with charge Z in this isoelectronic system (the charge on the other nucleus is 4 - Z). These calculations are intended to explore the feasibility of calculating molecular binding energies by this indirect method.

R. K. Nesbet
Solid State and Molecular Theory Group

Approximate atomic wave functions are being obtained for the following transition elements:

$$C_u^+, \; K^+, \; C_v^{++}$$

These calculations are being done in an attempt to obtain criteria concerning the choice of one-center orbitals and the initial approximation for the density matrix for transition element calculations.

R. Watson
Solid State and Molecular Theory Group

290 N. POLARIZABILITY EFFECTS IN ATOMS AND MOLECULES

To describe the distortion of atoms in a uniform field, we have set up the perturbed Hartree-Fock equations for a general external perturbing field of arbitrary order and symmetry. This atomic polarizability problem corresponds to a first order perturbing potential of the form r cos θ. As the first step in solving the self-consistent Hartree-Fock problem and also to get functions suitable for describing the distortion of an atom in an ionic molecule we are using an equation derived by Sternheimer.[1] The derivation of the perturbed Hartree-Fock equations, a discussion of the physical assumptions and of Sternheimer's equation has been given in recent issues of the Solid State and Molecular Theory Group Quarterly Progress Report.

Sternheimer's equation is:

$$\left(-\frac{d^2}{dr^2} + V_{\ell\to\ell'}\right) U_{1\ell\to\ell'} = rU_{0\ell}$$

where

$$V_{\ell\to\ell'} = \frac{\ell'(\ell'+1)}{r^2} - \frac{\ell(\ell+1)}{r^2} + \frac{1}{U_{0\ell}}\frac{d^2U_{0\ell}}{dr^2}$$

$U_{1\ell\to\ell'}$ = The desired perturbed function characterized by ℓ and ℓ'.

$U_{0\ell}$ = The known unperturbed atomic function characterized by ℓ and given as a sum of terms each of which is of the form $r^n e^{-ar}$.

With A. Temkin (Morse Group) we have set up the solution of this equation on Whirlwind. At present we have obtained solutions for the $\ell' = 2$, $\ell = 1$ case for the F^-, N_ℓ, A^{+3}_ℓ, and N_a^+ ions (the various ions being characterized by the different U_0's) by an inward integration technique using the Whirlwind Library Subroutine DE 2 Kutta-Gill.

Besides this case there are also important solutions corresponding to $\ell' = 0$, $\ell = 1$ and $\ell' = 1$, $\ell = 0$. For these we have used an outward integration also with the Kutta-Gill subroutine but so far we have not been able to get satisfactory results either with respect to the form or magnitude of the functions. In particular, the number of modes for the case $\ell' = 0$, $\ell = 1$ has changed as we have changed our mesh size. Unfortunately, in contrast to the case of the homogenous ordinary differential equation, there appears to be no general theory[2] relating to our unhomogenous equation. The equation possesses great sensitivity to the mesh size since we have a very rapidly varying one at large r. This has necessitated several revisions in both the number and distribution of our mesh points and at present we are investigating a procedure employing 160 points and 19 different mesh intervals.

L. C. Allen
Solid State and Molecular Theory Group

References

1. Sternheimer, R. M., Phys. Rev. 96, 951 (1954).
2. As indicated by a survey of the literature and discussion with Professor N.Levinson of the MIT Mathematics Department.

293 C. ROLLING BEARINGS

Past results were unsatisfactory. Some research is presently being done on a new approach to the solution of the non-linear simultaneous equations.

Future attempts may be made if a sufficiently promising method is developed.

A. Shashaty
Lubrication Laboratory

297 B. DIFFUSION BOUNDARY LAYER

A general description of the problem was given in Summary Report No. 43. The computations completed during the past quarter were concerned with a revised form of the energy equation. Previous results from iteration programs for the concentration and momentum equations were employed to obtain the additional solutions. The program will be concluded during the next quarter.

J. Baron
Naval Supersonics Laboratory

300 L. TROPOSPHERIC PROPAGATION

Values of the Fresnel integrals for complex arguments have been computed using convergent power series. Satisfactory values have been obtained for constant imaginary part of the argument as a function of the real part over ranges in which the convergence is not too slow.

J. F. Roche
Lincoln Laboratory

306 D. SPECTRAL ANALYSIS OF ATMOSPHERIC DATA

During the past quarter the writing of the program was successfully completed. It is expected that, in the near future, extensive computations of the type outlined in Summary Report No. 43 will be made using this program.

B. Saltzman
Meteorology Department

309 B,N. PURE AND IMPURE POTASSIUM CHLORIDE CRYSTAL

The calculation of molecular KCl integrals outlined in previous reports has been completed, and the combination of these integrals into matrix elements is nearing completion. Since the last report, however, it has been found that second neighbor Cl-Cl matrix elements must also be included. This correction is now being made. It is expected that the main objectives of the calculation will be attained during the next quarter.

L. P. Howland
Solid State and Molecular Theory Group

312 L. ERROR ANALYSIS

This problem, which was described in Summary Report No. 43, has been in working order for several months now, and is being used frequently for production runs. Modifications are being prepared to make the program represent more accurately the real physical situation.

L. Peterson
E. Hutcheson
Lincoln Laboratory

314 C. HIGH ORDER POLYNOMIAL FACTORIZATION

The present program has been extended to factor a series of polynomials on one tape. This resulted in a substantial saving of total problem time. Thus, the tapes fc 141-227-79 and fc 141-227-1 were replaced by the binary tape fb 314-88-4. A typical result is: factoring ten 10^{th} order polynomials in 2.5 minutes. This permits economical use of Whirlwind in computing root loci for systems stability analysis.

Y. Shulman of the Aeroelastic and Structures Research Laboratory has been the programmer for this section of the problem.

V. W. Howard
Y. Shulman
Aeroelastic and Structures Research Lab

317 C. EXTRACTION OF STABILITY DERIVATIVES FROM FLIGHT TEST DATA

The aerodynamic terms of the equations of motion of a high-speed aircraft have been calculated using the method reported in NACA TN3288 (December 1954). It has been determined that the type of simulated response used in the study to date adversely affected the accuracy of the results. A program has been written, therefore, which will provide simulated responses to controlled inputs. During the next quarter, use will be made of this program to get successively more complex responses, which will then be used in the further study of stability derivative extraction.

The programmers working on this problem are L. L. Mazzola and K. E. Kavanagh.

L. L. Mazzola
Aerophysics Research Group

318 C. THREE DIMENSIONAL AERODYNAMIC LEAD PURSUIT STUDY USING WHIRLWIND I AS A REAL TIME COMPUTER

Coding of the problem is being carried out in three parts: aerodynamic, geometric, and display. The first two of these parts have been completed in the sense that they give correct results for certain inputs. Currently, work is being continued on the display routine, trying to image correctly on the scope a given set of points representing an outline of the target. After the desired images are obtained on the scope, the immediate future plans are to program the computation of points outlining the target. After the various parts of the problem are checked out, the parts will be incorporated into the whole.

Programmers working on this problem are B. Romberg and J. M. Stark of the MIT Instrumentation Laboratory.

J. M. Stark
Instrumentation Laboratory

322 B. MAXIMUM BUBBLE SIZE

The calculations are completed and will appear in the Sc. D. thesis of Peter Griffith, due to be finished June 1956. Later the results will form part of a technical report for DIC Project 6627.

The radius time curves are wanted to aid in the correlation of boiling heat flux and maximum heat flux data. Using part of these computer results, a correlation has been completed for the maximum heat flux data for pool boiling of a variety of fluids and the correlation is satisfactory. It remains still to complete a correlation for forced convection, subcooled boiling. Details of these results will be presented in the thesis.

P. Griffith
Mechanical Engineering Department

323 N. ANALYSIS OF CLOUD CHAMBER PHOTOGRAPHS

During this quarter, a few additional cases were run, completing the first stage of the work intended for this program. Some time in the near future, the program will be enlarged to handle some other types of problems.

The programmer working on this problem is F. Abrams.

D. O. Caldwell
Physics Department

327 L. PREDICTION ANALYSIS

This problem deals with the prediction of certain events when information giving rise to the prediction has been garbled by Gaussian noise. The solution is primarily geometric and algebraic, with a statistical analysis of the predictions appended. The predictions themselves are plotted on the oscilloscope.

The program is in working order. It is being used with alternative methods of solution to compare results with previously obtained hand-computed cases.

L. Peterson
E. Hutcheson
Lincoln Laboratory

332 C. GAME-THEORY OPTIMIZATION OF INTERCEPTION SYSTEM

The problem described in the previous Whirlwind Summary Report was completed during the quarter. Entries for the 46-by-46 payoff matrix of the reduced game were computed. A study of the matrix indicated that the reduced-game solution would consist of mixed strategies for both players and that both mixed strategies would contain a large number of pure strategies. These indications led to two conclusions: First, an exact solution of the reduced game would require computer time on the order of years;

second, because the defender's optional strategy was known to be a pure strategy, the reduced game was an inadequate representation of the actual game.

The decision was made, therefore, to attempt an approximate solution of the exact game rather than an exact solution of the approximate (reduced) game. A method of successive approximations, called the method of fictitious play, was adopted. The procedure consisted of choosing an alternating succession of strategies for the enemy and the defender, each choice being based on the past strategies used by the opponent. The method converged rapidly; after 22 iterations an approximate solution was obtained, and the true game value was determined within 1 per cent. The total computing time for this run was 29 minutes.

A final program, lasting 6 minutes, was run to compute some additional results. This run yielded data specifying (a) the optimum response function of the cascaded director and interceptor and (b) the rms interceptor acceleration of the optimum system.

The programmer for this problem was K. Kavanagh.

G. R. Welti
Dynamic Analysis and Control Laboratory

333 A. BOUNDARY-LAYER CHARACTERISTICS OF A STEADY LAMINAR FLOW OF A COMBUSTIBLE MIXTURE OVER A HOT SURFACE

During the past few years, the study of ignition and flame stabilization in a steady flow of a combustible mixture by means of a hot surface has received increasing attention in the combustion field. If a usual bluff-body flame holder is replaced by a hot surface, it has been found experimentally that the combustion intensity can be increased at least twenty times. However, a basic study of the problem is as yet not available.

A program, both theoretical and experimental, has been set up recently to study the basic characteristics of this problem. Four theses are now involved in the experimental program and three term projects are engaged in the theoretical study. Preliminary work shows that it is essential to investigate the boundary-layer characteristics of the flow of a combustible mixture over a hot surface. This investigation will permit one to understand not only the problem of surface ignition in particular but also the effects of reaction rate and transfer of momentum, mass, and heat on combustion processes in general.

Partial differential equations of continuity, momentum, and energy are first developed for a laminar boundary layer, and then transformed into a series of ordinary differential equations. Solutions are to be obtained with the aid of the Whirlwind I computer for specific reactions and temperatures of the hot surface.

Solutions of the first five sets of the differential equations were obtained for two values of the temperature of the hot surface and for a second-order reaction of the Arrhenius type.

A study of the profiles of the velocity, temperature, concentration, and reaction rate in the boundary layer next to the hot surface indicates that few more solutions of the boundary-layer equations are necessary to study the effect of surface temperature on incipient ignition.

The programmers working on this problem are Professor T. Y. Toong and A. Shashaty of the Mechanical Engineering Department.

T. Y. Toong
Mechanical Engineering Department

334 C. PARAMETRIC STUDY OF MODAL COUPLING AND DAMPING

During this quarter, the work has been concentrated on the solution of a single vibratory differential equation and the simultaneous solution of both two and three coupled vibratory equations. A satisfactory solution has been obtained for each of the above three cases based on one set of initial conditions. To obtain a solution to the problem for a different set of initial conditions, the program must be rerun with a modification tape for the constants. Thus a parametric study at the present time involves a series of runs with a new modification tape for each run.

Future work on the problem will entail the alteration of the set-up to produce a parametric study of the problem in only a single run instead of a series of runs.

K. Wetmore
Aeroelastic and Structures Research Lab

338 C. SELECTION OF AIRCRAFT GENERATOR COOLING SYSTEMS

With increasing flight speeds it has become impossible to cool aircraft generators with unprocessed ram air. Systems of turbines, compressors, and heat exchangers must be provided to cool the air to the point where it can remove generator losses without requiring excessive generator temperatures. Use of such systems imposes a penalty on aircraft performance due to the weight of components, and in addition a much larger penalty due to drag created by taking air aboard at high velocity and exhausting it at low velocity. Equations exist relating this drag to system parameters, and it is desired to find the minimum-drag system for an aircraft flying at Mach 3, 70,000 feet.

A selected range of discreet values of system variables are given to the computer together with a program for calculating drag. Another program causes the computer to try all combinations of variables and record the drag caused by each. A selection of minimum-penalty system may then be made by checking weight variations of low-drag systems.

Five hundred and sixty calculations of drag are required, each calculation requiring approximately 200 numerical computations.

The original problem as described above has been extended to include optimization of cooling systems for all Mach numbers up to 4 and all altitudes up to 100,000 feet, as well as investigation of off-design performance of cooling systems. Two types of generators have been considered instead of the original one. Some generator design work is also planned for programming.

The project is substantially completed with the exception of Mach 4 systems.

R. Moroney
Energy Conversion Group
Electrical Engineering Department

339 A. BEAM VIBRATION

An earlier study of the equation

$$\frac{\partial^4 \psi}{\partial x^4} + \frac{\partial^2 \psi}{\partial t^2} = 0$$

was reported on in "Numerical Treatment of a Fourth Order Parabolic Partial Differential Equation" by Stephen H. Crandall, J. Assoc. Comp. Mach., 1,111-118 (1954). An improved finite difference recurrence formula has now been developed and it is desired to test this in actual computations.

This problem was used as an exercise for the students of MIT Course 2.215, Methods of Engineering Analysis, to illustrate the fundamentals of digital computer programming. A program was prepared, run and partially debugged by interested students.

I expect to complete the program debugging and make a few runs during the next quarter.

S. H. Crandall
Mechanical Engineering Department

341 C. STATISTICAL AND DYNAMIC FORECASTING METHODS

This project is the start of an extended research program utilizing various mathematical procedures for the establishment of empirical equations for weather prediction.

All of the mathematical procedures to be used cannot be stated at present, as many are dependent upon initial results. The initial procedure is the computing of the elements of covariance matrices. There are several of these matrices to be computed, one for each weather element of interest. Each of these matrices involves input data of approximately 10,000 numbers.

The method of operating on these covariance matrices is in unpublished form, and because of its extent it is not feasible to include it here. However, in the not too distant future it will be included in a project Scientific Report.

During the current quarter the major effort has been devoted toward the systematization of a method for multiple linear prediction. In this method a large number of predictors is originally chosen. The matrix of the covariances of the predictors is obtained. This matrix is then diagonalized. Those linear combinations of the original predictors whose variances are the large elements on the diagonalized matrix are then used as new predictors. In this way the total number of predictors is reduced. Whirlwind I has been used to obtain the necessary cross-products for the covariance matrix. The raw data amounts to 10,500 numbers and the elements for a matrix of the 75th order are the results. This programming has been under the direction of Miss Elizabeth A. Kelley.

One phase of the Project is concerned with an extensive study of the extended-range forecasting possibilities of the methods developed by the Synoptic Climatology Project, now defunct. The sea-level pressure and 700 mb height anomaly charts for each March from 1946 through 1955 have been expressed in terms of 26 coefficients of Tschebyscheff orthogonal polynomials. All maps cover a 135 point grid bounded by 25°N and 65°N and by 45°W and 185°W. The points have been taken at each 10° of longitude and each 5° of latitude. The Whirlwind computer has been utilized in order to determine the coefficients. This work has been under the supervision of Mr. William D. Sellers.

Work is being continued in numerical experiments to investigate the "climatic" statistics of very simplified dynamic models of the atmosphere. The dynamic models are defined by a series of non-linear difference equations, which express the change with respect to time of the amplitude of certain harmonic components of a flow pattern. Integration is carried forward by short time steps. The programming on this phase of the project has been done by Mr. Kirk Bryan.

All of the work done on the project has been supervised by Professor Edward N. Lorenz, MIT Department of Meteorology.

E. A. Kelley
Meteorology Department

343 C. WEATHER PREDICTION

The success of the research conducted by the Synoptic Climatology Group at MIT (Whirlwind Problem No. 155 N.) has indicated that a statistical approach to the weather forecast problem should be fully explored. It is planned to incorporate meteorological knowledge and theory into the statistical procedure. The research results of our project have indicated fruitful lines of approach.

There are two aspects of the problem: (1) It is planned to derive linear prediction equations utilizing the circulation parameters developed by Wadsworth and Bryan and used by the Synoptic Climatology Group. Theory and observation have demonstrated that the vertical structure of sea-level pressure systems have a direct influence on their behavior and, therefore, on the future weather. For this reason a non-linear approach to the prediction problem will be made through the classification of back data into various classes of atmospheric structure. Linear prediction equations will be developed for each class of situation instead of attempting to derive a universal linear prediction equation as has been attempted in the past.

The linear prediction equations are

$$E = a_o + \sum_{i=1}^{14} a_i Z_{-1,i} + \sum_{i=1}^{14} b_i Z_{0,i} + c\,\bar{P}_{-1} + d\bar{P}_0 + eS_1 + fS_0$$

where E is the weather element, Z's, $\bar{P}$'s and S's are the known variables and a's-----f are the coefficients to be obtained.

The values of E, Z's S's and $\bar{P}$'s are already computed. Whirlwind will solve for the parameters a_0, a's---f.

The method of obtaining the coefficients is Least Squares Analysis of the Multiple Linear Regression Equation. The solution of the resulting matrix is by the method described by Crout. It is possible to obtain both the coefficients and significant statistical parameters from this method.

Once the coefficients are obtained and the resulting statistical parameters are studied, the equation will be evaluated substituting variables independent of those used in determining the coefficients. These results will be verified with observed weather conditions.

(2) The second aspect of the problem is the development of new predictors to replace the Z's in the above equation. These predictors will be so chosen to represent the maximum amount of variance of a field with a minimum number of parameters. Whirlwind will perform standard arithmetic operations to obtain the latent roots and latent vectors of symmetric matrices. The elements of these matrices will be the correlation coefficients or covariances between the Z's.

During the past quarter, some preliminary work on a desk calculator was undertaken to classify the data for aspect (1) of the problem. Plans are now under way to make an initial computation on this non-linear approach to prediction.

A study was undertaken to determine sets of orthogonal functions to describe patterns of atmospheric variables. Essentially, the technique requires diagonalization of a covariance matrix of such variables. Use was made of Whirlwind library subroutines in the initial programming for this problem.

The future work on prediction will utilize these new orthogonal functions. The programmers working on this problem are E. Kelley and B. Shorr.

J. M. Austin
Meteorology Department

344 B. AN INVENTORY AND PRODUCTION CONTROL MODEL

Initial study has been centered in solving numerically an inventory and production model where three distinct quadratic (by quadratic we mean the form ax^2+bx+c) costs are considered: (1) The cost of production levels, (2) The cost of inventory levels, and (3) The cost of changes in the production rate.

If we have,

$f_n(x_n;\ z_{n-1})$	= the minimum cost of operating for a given number of periods when one starts the nth period with inventory, x_n, and the production rate of the previous period is s_{n-1}.
y_n	= demand of nth period; a random variable
$E(\ \ )$	= expected value of ( ) with respect to y_n
$g_n(x_n;\ z_n;\ z_{n-1})$	= cost during the nth period of production and inventories. In general, this function will be a sum of the costs of type (1), (2), and (3) mentioned above.
a	= discount rate

and, if we were to study the business for only one period, a minimum cost production rate for the nth period could be found by minimizing the expected value of g_n with respect to z_n. Hence,

$$f_n(x_n;\ z_{n-1}) = \underset{z_n}{\text{Min}}\ E\left[g_n(x_n;\ z_n;\ z_{n-1})\right]$$

The right hand side of the equation is, of course, independent of z_n. A common practise in business is that one of considering the effect of future periods, n+1, n+2, and so on.

Since a minimum is desired for the sum of all discounted costs and since an optimal (minimum cost) production rate will be chosen at each period, we have,

$$\begin{aligned} f_n &= \underset{z_n}{\text{Min}}\ E\left(g_n + a \underset{z_{n+1}}{\text{Min}}\ E\left[g_{n+1} + a \underset{z_{n+2}}{\text{Min}}\ E(g_{n+2} + \ldots\ldots\ldots\ldots)\right]\right) \\ &= \underset{z_n}{\text{Min}}\ E\left(g_n(x_n;\ z_n;\ z_{n-1}) + a\ E\left[f_{n+1}(x_n + z_n - y_n;\ z_n)\right]\right) \\ &= \underset{z_n}{\text{Min}}\left(g_n(x_n;\ z_n;\ z_{n-1}) + E\left[af_{n+1}(x_{n+1};\ z_n)\right]\right) \end{aligned}$$

The last equation is found by virtue of the first-order difference relation:

$$\Delta x_n = x_{n+1} - x_n = z_n - y_n$$

which says that the difference in inventory between the (n+1)st and the nth period is the excess of nth period production over nth period demand.

In the event that f_n considers the expected costs of many future periods and that g_n is quadratic in form, one finds an analytic expression for f_n such that,

$$f_n(x_n;\ z_{n-1}) = A\ x_n^2 + B\ z_{n-1}^2 + C\ x_n z_{n-1} + D\ x_n + E\ z_{n-1} + F,$$

where the solutions to A, B, C, D, E, F, are given by six non-linear equations of the form,

$$A^2 + f_1(B,\ldots F)\ A + g_1(B,\ldots F) = 0$$

$$B^2 + f_2(A,C,\ldots F)\ B + g_2(A,B,\ldots F) = 0$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$F^2 + f_6(A,\ldots E)\ F + g_6(A,B,\ldots E) = 0$$

The f's and g's are non-linear functions of the indicated variables.

Whirlwind I programs are being used for the calculation of the cost, f_n, and the optimal decision for production, z_n, for the quadratic model we have outlined. One hopes that the answers to this first phase of the problem will allow us to make more general assumptions about the cost function, g_n. An obvious case is the necessity of non-symmetrical costs for positive and negative inventories as well as costs which are more disproportionately expensive than quadratic costs. An example is the region of high production rate changes where one feels a need for an exponential-type cost.

R. M. Oliver
Operations Research Group

345 B. MATRIX MULTIPLICATION

The problem of the buckling of a clamped shallow spherical thin elastic shell subject to a uniform external pressure can be studied by means of the boundary value problem. (See references 1. and 2.)

$$\frac{1}{k^2}\left(f'' + \frac{1}{x} f' - \frac{1}{x^2} f\right) + g = Px + \frac{fg}{x}$$

$$\frac{1}{k^2}\left(g'' + \frac{1}{x} g' - \frac{1}{x^2} g\right) - f = -1/2\ f^2/x$$

$x = 0$: f, g regular

$x = 1$: $f = 0,\ g' - ng = 0$

where k is a geometric parameter, and P a load parameter. A convenient method for the solution of the problem involves assuming f and g in the form of series of complete orthogonal functions and by means of a conversion of the non-linear problem to a sequence of linear problems to get the coefficients in the series. This leads to sets of equations of the form,

$$-\left(\frac{k_n}{k}\right)^2 a_n^{(p)} + b_n^{(p)} = P_n + C_n^{ij}\, a_i^{(p-1)}\, b_j^{(p-1)}$$

$$a_n^{(p)} + \left(\frac{k_n}{k}\right)^2 b_n^{(p)} = C_n^{ij}\, a_i^{(p-1)}\, a_j^{(p-1)}$$

The matrix calculations in the right hand member can be conveniently programmed and used over again and again at each stage of the perturbation and also for different values of the parameter k that are of interest.

Solutions have been obtained over a certain range of the parameter k during this quarter. It is planned to complete the range of k corresponding to important physical applications.

R. Archer
Mechanical Engineering Department

References

1. Reissner, E., Proc. of Symp. in Appl. Math., Amer. Math. Soc., Vol. 3 (1950)

2. Simons, R. M., Doctoral Thesis MIT (Math), 1955.

347 B. SOLUTION OF SETS OF TEN LINEAR SIMULTANEOUS EQUATIONS

The problem involves the solution of twelve sets of linear algebraic simultaneous equations by the Crout Reduction method.

Each of these sets of equations will solve for the ultimate settlements of the footings of a rigid structure, (which in this case are the settlements of the footings of the John Dorrance Laboratory) for a soil having a particular compressibility

and a superstructure having a particular rigidity.

The object of my thesis is to investigate a structure of different rigidities and a soil of different compressibilities and solve for the differential settlements for each of these combinations and the secondary stresses which these settlements will cause in the members of the structure.

This problem was successfully completed in the first run.

S. G. Fattal
Civil Engineering Department

348 A. UNSTEADY SUPERSONIC FLOWS WITH SPHERICAL SYMMETRY

The equations which govern the spherically symmetric motion of a compressible fluid are:

$$\rho_t + (\rho u)_r + \frac{2}{r}\rho u = 0 \qquad (1)$$

$$(\rho u)_t + (p + \rho u^2)_r + \frac{2}{r}\rho u^2 = 0 \qquad (2)$$

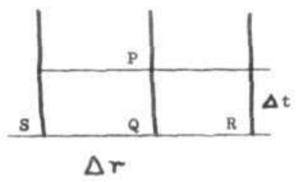
$$\left(\frac{p}{\gamma-1} + \frac{\rho u^2}{2}\right)_t + \left(\frac{\gamma}{\gamma-1} pu + \frac{\rho u^3}{2}\right)_r + \frac{2}{r}\left(\frac{\gamma}{\gamma-1} pu + \frac{\rho u^3}{2}\right) = 0 \qquad (3)$$

where ρ, u, and p are the density, velocity, and pressure respectively at a point distant r from the centre of the gas at time t; subscripts denote differentiation.

The equations 1, 2 and 3 are replaced by finite difference equations by the following substitutions:

$$\frac{\partial f}{\partial t} = f_t \qquad \frac{f_P - \frac{1}{2}(f_R + f_S)}{\Delta t}$$

$$\frac{\partial g}{\partial r} = g_r \qquad \frac{\frac{1}{2}(g_R - g_S)}{\Delta r}$$



where f_P, f_R, f_S are the values of f at point P, R and S.

This form of the forward derivative (f_t) introduces an artificial viscosity term in the difference equations so that discontinuities in the variables (which may arise due to the presence of shocks) are smoothed over.

The resulting system of three finite difference equations, was solved numerically using Whirlwind I.

Starting from an initial distribution (at t=0) of $\rho = 1 + 5e^{-4r^2}$, $p = (1+5e^{-4r^2})^{\gamma}$ u = 0, values of ρ, p and u were found on a rectangular mesh of $\Delta r = .05$ $\Delta t = .0125$ from r=0 to 4 and t=0 to 3.

This requires a total of 19,200 repetitions of the basic cycle.

During the last quarter, the problem has been coded and preliminary results have been obtained for two types of initial conditions,

$\rho = 1 + 5e^{-4r^2}$ and $\rho = 1 + 2e^{-4r^2}$, at $t = 0$. Numerical results show that the initial outgoing wave leaves a region of overexpansion at the centre which causes an ingoing shock wave to occur. This shock wave is reflected from the centre as an outgoing wave which finally subsides.

During the coming quarter it is intended to develop a method whereby certain errors, introduced by the numerical procedure, may be eliminated.

L. Roberts
Mathematics Department

349. SOLUTION OF PARTIAL DIFFERENTIAL EQUATIONS

This problem is concerned with the solution of a set of 23 non-linear partial differential equations which arose in the study of a transient heat transfer problem. First-order finite difference approximation equations were used to solve this problem.

Since the basic program contained 1773 instructions, it was handled by the core memory, and a solution time of 29 seconds per time level was realized. Since there were 400 time levels, a single solution required about 196 minutes (3.3 hours).

As part of this study, this same problem was solved on the IBM Type 650 Magnetic Drum Calculator. The main 650 program contained 1743 instructions and solution for a single time level was obtained in 29 minutes. The 650 program was not optimum programmed; if it had been, the solution time would have been reduced to seven minutes for a single time level.

The results obtained on Whirlwind and the Type 650 Calculator agree to six decimal digits--which is quite good considering the basic differences in the machine structure, etc.

Programmers working on this problem were Dr. F. M. Verzuh and M. Hermann of the Office of Statistical Services, and A. Zabludowsky of the Digital Computer Laboratory.

F. M. Verzuh
Office of Statistical Services

351 B. NON-UNIFORM FUEL DISTRIBUTION

Non-Uniform Fuel Loading in Reactors

Reactors loaded in a uniform pattern throughout the core allow only a partial exploit of the outer core regions, as maximum heat generation at core center is limited by heat transfer and metallurgical properties.

A deliberate redistribution of fuel, moderator, neutron absorbers and coolant will modify the neutron flux and equalize maximum temperatures and stresses within the reactor.

The Age-Diffusion equations are solved for i energy groups, N core-zones and M reflector-zones. Material constants vary for each group i and each region N. The Age-Diffusion equations are approximated by finite differences. The Laplacian in the leakage term becomes a second derivative when introducing modified fluxes $F^i = r\,\phi^i$ in spherical geometry.

Boundary conditions impose F^i to vanish at the center of the core and at the extrapolated boundary. Neutron currents must be equal at core reflector interface. By successive iterations the distribution of fuel concentration in the core producing an arbitrary thermal flux can be approximated.

Age-Diffusion equations in finite differences: (solved by Whirlwind I)

$$F^i_{n+1} - k^i_n F^i_n + F^i_{n-1} + I^i_n = 0 \qquad (n = 0, \ldots N+M)$$

The first three terms include leakage and absorption. The last term describes the slowing down and production of neutrons. The number of iterations is a function of the accepted error.

The problem is divided into two major programs: space criterion and criticality criterion.

Space Criterion

From an arbitrary assumed thermal flux the correct spatial flux distribution is approximated by successive iterations (regardless of criticality conditions).

The program for the space criterion is being fulfilled for a 2-group, 2-region reactor with 18 core zones and 20 reflector zones. Print-out of single iterations shows rapid convergence. A program for any number of boundaries will be developed.

Criticality Criterion

Material properties of the reactor are changed to acquire prescribed criticality. The changes are fuel concentration, fuel distribution, reflector thickness, reflector composition, and critical size.

The routine for criticality criterion will be programmed according to the various changes in material properties as mentioned above.

The programmers working on this problem are A. Sutter and M. Troost.

A. Sutter
Nuclear Engineering

352 B. CRITICAL WHIRLING FREQUENCIES OF PROPELLER SHAFTS

The whirling or lateral vibrations of a propeller shafting induced by the blade frequency of the propeller will be calculated by numerical iteration. The method involves a numerical solution to a fourth degree differential equation. Frequencies will be assumed and the residual moment at the free end of the shafting will be plotted versus these frequencies. The point at which the end moment equals zero is a resonant point or natural whirling frequency of the shafting system. This method of output was chosen to enable comparison with existing data. The numerical calculations are:

$$\frac{\partial^4 y}{\partial x^4} - \lambda^4 g = 0 \qquad \lambda^4 = \frac{M\omega^2}{EI}$$

$$y = C_1 \sin\lambda x + C_2 \cos\lambda x + C_3 \sinh\lambda x + C_4 \cosh\lambda x$$

Using end conditions to solve for constants the following parameters are obtained. These constants may be considered lumped parameters.

$$\varphi_\omega = \frac{W_P L_0^3 E_1 I_1}{W_1 L_1^3 E_0 I_0} \pi^4 \left(\frac{\beta}{180}\right)^4$$

$$\varphi_J = \frac{J L_0 E_1 I_1 \left(\frac{1}{2} \pm \frac{1}{8}\right)}{W_1 L_1^3 E_0 I_o} \pi^4 \left(\frac{\beta}{180}\right)^4$$

$$C_{d1} = \frac{E_1 I_1}{W_1 L_1^3} \qquad C_{a1} = \frac{E_1 I_1}{L_1}$$

$$f = \left(\frac{\beta}{180}\right)^2 \frac{\pi}{2} \sqrt{g C_{d1}}$$

$$S_1 = -\frac{\cos\beta \sinh\beta - \sin\beta \cosh\beta}{\sinh\beta - \sin\beta}$$

$$S_2 = -\frac{\cos\beta \cosh\beta - 1}{\beta(\sinh\beta - \sin\beta)}$$

$$S_3 = \frac{2\beta \sin\beta \sinh\beta}{\sinh\beta - \sin\beta}$$

$$M_{01} = -\frac{E_0 I_0}{L_0} \left\{ \frac{\varphi_\omega + \varphi_J - \frac{\varphi_\omega \varphi_J}{3}}{1 - \frac{\varphi_\omega}{3} - \varphi_J + \frac{\varphi_\omega \varphi_J}{12}} \right\}$$

$$M_{N,N+1} = -\left\{ (S_1)_N M_{N-1,N} + C_{aN} (S_3)_N \theta_{N-1,N} \right\}$$

$$\theta_{N,N+1} = -\left\{ S_{a1} \theta_{N-1,N} + \frac{M_{N-1,N} (S_1)_N}{C_{aN}} \right\}$$

During the past quarter a program has been written for this problem and is in the process of being tested.

Programmers working on this problem are Lt. C. R. Brandt, USN, Lt. J. C. Snyder, USN, and Lt. C. R. Thompson, USCG, associated with the MIT Department of Naval Architecture.

C. R. Brandt
Naval Architecture

353 A. WAITING LINES - CONSTANT HOLDING TIME

The problem is as follows:

Given: $x = mu, \quad a(n,x) = \frac{x^n e^{-x}}{n!} \qquad n = 0, 1, \ldots\ldots$

to compute: $\lim_{i \to \infty} P(n,i)$ for $n = 0, 1, \ldots\ldots$

where $P(n,0) = 1$

for all n and $P(n,i+1) = \sum_{j=0}^{n} P(m+j,i)\, a(n-j)$

It is known that for $u < 1$ the limits exist. Computations are required for $m = 1, 2, 5, 10, 20$ and $u = .2, .4, .6, .7, .8, .85, .90, .95, .98$, and $.99$.

The limiting probabilities here are the steady state probabilities for the number of units waiting or being served in a queue with m servicing channels, Poisson arrivals, and constant holding (servicing) time. u is the ratio of mean arrival rate to total mean servicing rate.

The feasibility has been verified and seven problems have been successfully run.

In the future, it is planned to run up to 43 remaining problems.

H. P. Galliher
Operations Research Group

1. SYSTEMS ENGINEERING

Performance Records for the Whirlwind Computer System

The WWI computer system has been performing a 168 hour week since 1 June 1955. This 7 day per week, 24 hours per day schedule is expected to be continued for at least one year. The amelioration of computer system reliability noted in the two most recent quarterly periods was apparently a direct result of the refinements made to several of the equipment areas during the preceding 6 months.

The following table illustrates how the equipment has performed during the most recent 18 months.

	28 Sept.'54 to 6 Oct.'55	7 Oct.'55 to 29 Dec.'55	30 Dec.'55 to 8 Mar.'56
Total Computer Operating Time In Hours	7537	1876	1584
Total Time Lost in Hours	230	48	36.3
Percentage Operating Time Usable	97.0	97.4	97.7
Average Uninterrupted Operating Time Between Failure Incidents In Hours	12	17.8	22.2
Total Number of Failure Incidents	605	103	70
Failure Incidents per 24-Hour Day	1.92	1.32	1.06
Average Lost Time Per Incident In Minutes	36.6	28	22.6
Average Preventive Maintenance Time Per Day In Hours	1.55	1.7	1.45

3. VISITORS

Tours of the Whirlwind I installation include a showing of the film, "Making Electrons Count", a computer demonstration, and an informal discussion of the major computer components. During the past quarter, the following 10 groups totalling 143 people visited the computer installation:

January 4	MIT Aeroelastic Laboratory
January 6	MIT Course in "Fundamentals in Electrical Engineering"
January 10	Lexington High School
January 11	MIT Course in "Pulsed-Data Systems"
January 16	Harvard Business School
January 19	MIT Air Force Cadets
January 30	General Electric Co.
January 30	"Frontiers of Science" Group
March 7	Dr. Heinz Krekeler, German Ambassador to the United States, and members of his staff
March 15	Mills College (New York)

The procedure of holding Open House at the Digital Computer Laboratory on the first and third Tuesday of each month has continued, with the exception of March 20th when the Institute was closed because of a severe blizzard. A total of 112 people attended the five Open House tours during the quarter, representing members and friends of the MIT Community, Children's Medical Center, Medfield State Hospital, and Harvard University.

During the last quarter, there were also 45 individuals who made brief tours of the computer installation at different times. Represented by the individuals were: MIT, Melpar, Lynn Mutual Insurance Co., A.O.Smith Corp., Barnard College, International Business Machines Corp., the United States Navy, Burroughs Corporation, Princeton University, Metal Box Co., Ltd., Continental Can Co., Harvard University, Chance Vought Aircraft, Tohoka University, Oberlin College, and Babson Institute.

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Herman Feshbach	Physics
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Bayard Rankin	Mathematics

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Raymond F. Stora	Physics
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PROJECT WHIRLWIND

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