# MACHINE METHODS OF COMPUTATION
and
# NUMERICAL ANALYSIS

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and

# PROJECT WHIRLWIND

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## FOREWORD

This is a combined report for the two projects at the Massachusetts Institute of Technology which are sponsored by the Office of Naval Research under Contract N5ori60.

### Project on Machine Methods of Computation and Numerical Analysis

This Project is an outgrowth of the activities of the Institute Committee on Machine Methods of Computation, established in November 1950. The purpose of the Project is (1) to integrate the efforts of all the departments and groups at M.I.T. who are working with modern computing machines and their applications, and (2) to train men in the use of these machines for computation and numerical analysis.

People from several departments of the Institute are taking part in the project. In the Appendix will be found a list of the personnel active in this program.

### Project Whirlwind

This Project makes use of the facilities of the Digital Computer Laboratory. The principal objective of the Project is the application of an electronic digital computer of large capacity and very high speed (Whirlwind I) to problems in mathematics, science, engineering, simulation, and control.

### The Whirlwind I Computer

Whirlwind I is of the high-speed electronic digital type, in which quantities are represented as discrete numbers, and complex problems are solved by the repeated use of fundamental arithmetic and logical (i.e., control or selection) operations. Computations are executed by fractional-microsecond pulses in electronic circuits, of which the principal ones are (1) the flip-flop, a circuit containing two vacuum tubes so connected that one tube or the other is conducting, but not both; (2) the gate or coincidence circuit; (3) the magnetic-core memory, in which binary digits are stored as one of two directions of magnetic flux within ferro-magnetic cores.

Whirlwind I uses numbers of 16 binary digits (equivalent to about 5 decimal digits). This length was selected to limit the machine to a practical size, but it permits the computation of many simulation problems. Calculations requiring greater number length are handled by the use of multiple-length numbers. Rapid-access magnetic-core memory has a capacity of 32,768 binary digits. Present speed of the computer is 40,000 single-address operations per second, equivalent to about 20,000 multiplications per second.



# PART I

## Machine Methods of Computation and Numerical Analysis

### 1. GENERAL COMMENTS

**Since the last report, arrangements have been made with the International Business Machines Corp. (IBM) for the installation of a type 704 machine at the Institute in the spring of 1957 and for the organization of the educational and general scientific use of computing equipment at MIT into a Computation Center, with quarters in the new Compton Laboratory. Heretofore, Whirlwind I has provided most of the computational support for this program. Whirlwind will eventually be turned back to Lincoln Laboratory about July 1, 1957, and the 704 and other equipment in the Computation Center will make possible an expanded program of research and training in the use of electronic computers.**

**This expanded program will involve the participation of the other colleges and universities in New England, and a Planning Committee, with representatives from 23 institutions, has been set up to work out details of the joint program. IBM has provided funds for the appointment of part-time research assistants and associates, from MIT and also from the other participating colleges, to assist in carrying out the research. It is hoped that the National Science Foundation (NSF) will provide funds for full-time research staff.**

**The coming academic year, therefore, will be a transitional one, changing from the ONR-supported project which has been reported in the previous Quarterly Reports, to the new program, supported by IBM and NSF and participated in by other New England colleges. The first change is the cessation of the part called Project on Machine Methods of Computation; by September 1956 all ONR part-time research assistantships will be terminated and the new IBM awards will take their place. The Digital Computer Laboratory and the use of Whirlwind I will continue during the next academic year, but the staff of Project Whirlwind will be gradually transferred to the payroll of the Computation Center so that, by July 1, 1957, ONR-supported work will have ceased.**

**The scientific research, reported in this and the preceding Reports, will not cease, however. It will continue at an increased pace, and the only difference will be in the source of financial support. In line with this continuity of research into the utilization of computing equipment in science and engineering, we propose to continue reporting the activities of both the older Project Whirlwind and the newer Computation Center in the one Quarterly Report. This issue will have the same form as before, with reports from the Project on Machine Methods of Computation and from Project Whirlwind. Next quarter, and the succeeding four quarters, will have reports from Project Whirlwind and also from the MIT Computation Center; after that the reports will be solely from the Computation Center. Distribution of the Reports will, of course, change as the new activities develop.**

**Philip M. Morse**
**Director, MIT Computation Center**

# 2. GRADUATE SCHOOL RESEARCH

## 2.1 Index to Reports


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## 2.2 Progress Reports

### DETERMINISTIC PROCESSES AND STOCHASTIC PROCESSES

In this report we summarize work that has been in progress during most of the year concerning deterministic processes and probability. This is an area of research that begins in statistical mechanics and is directed toward the interpretation and use of probability theory. The results could be useful for computation techniques because of what they imply about pseudo-random numbers and Monte Carlo techniques. The details are contained in definition and theorem form and will be published elsewhere.

A dynamics in the physical sense can be represented mathematically by a one-parameter group of transformations

$$\left\{ T^t, \ -\infty < t < \infty \right\}$$

defined on and to a space of points $\Omega$. If a point w in $\Omega$ represents the state of a physical system at time t=0, then $T^t w$ represents the state of the system at time t. In statistical mechanics we take $\Omega$ to be phase space and T to be the Hamiltonian dynamics. Liouville's theorem says that any set of points in $\Omega$ are transformed by T in such a way that the volume measure of the set in phase space is preserved in time. Such a property for T is given the mathematical name "measure-preserving." For definiteness we will call a one-parameter group of measure-preserving transformations a deterministic process. The knowledge of the state of a physical system at any fixed time together with the knowledge of a deterministic process operating on the system will determine all future and past states.

Contrast the idea of a deterministic process with the idea of a sequence of independent and identically distributed random variables. For simplicity let a random variable correspond to each integer point in time. There is no apparent dynamics that links the various sample values of these random variables. On the other hand, we will show that a deterministic process with special properties can be incorporated into the definition of random variables so that almost no generality is lost and yet a deterministic interpretation can be given. Both probability and statistical independence become asymptotic properties of deterministic processes. We thus conclude that abstract probability theory may be useful solely because of the relative orders of magnitude of things measured and things measurable and because of the order of magnitude of the time between measurements relative to characteristic parameters of a deterministic process.

In order to gain more insight into these ideas it is necessary to use two basic theorems in statistical mechanics that were postulated by physicists and later proved (in modified form) by mathematicians. The mathematical condition that it has been found necessary to impose on a group of transformations $T^t$ in order that both of these (modified) theorems hold is called "mixing". It is a group of transformations satisfying this condition that we wish to incorporate into a definition of random variables. The two theorems, themselves, are designated "ergodic" and "mixing" and are roughly described as follows. In the ergodic case, we consider a single trajectory or "orbit" of a point in phase space obtained by operating on this point by a group of measure-preserving transformations. The ergodic theorem asserts that if a measurable set of points in phase space is specified then all trajectories except those beginning in a set of measure zero ultimately enter the measurable set infinitely many times and spend, all told, an average amount of time there that is equal to the measure of the set. In the mixing case, we must consider a collection of trajectories. The weak mixing theorem states that for any measurable collection of trajectories the sets of points that the trajectories occupy at time t=0 and at large time t are such that the measure of their intersection is approximately equal to the product of their measures. This result requires that large t is chosen outside of a temporal set of density zero. We see in the first of these two theorems the germ of an interpretation for probability, for probability can be defined as the measure of a set in phase space. In the second of these theorems we see an interpretation for statistical independence.

Any definition of random variables requires a basic space of events $\Omega$, a Borel field, B, of sets on this space and a probability measure P defined for all sets in B. We may choose for these three entities the phase space, the measurable sets in phase space, and the Lebesque measure. We can show that there exists a countably dense field F of sets in B in the sense that the number of sets in F is countable and every set in B can be approximated arbitrarily closely (symmetric differences being small) by sets in F. Since F is countable, the ergodic theorem, stated with the same exceptional set of measure zero, applies to all sets in F. Thus if the dynamics T satisfies the mixing condition so that the ergodic theorem is true, there exists a trajectory that enters all sets of F and spends in each set an average amount of time that is equal to the measure of the set. We can then consider the intersection of this trajectory with phase space. For each set in F there is a set within the trajectory and we can arbitrarily transfer the measure from one to the other. With this device we can then view the trajectory $\Phi$ as a new space of points, the sets F' so manufactured as composing a field of sets within $\Phi$, and the new measure P' as defined on F'. If necessary we can generate the smallest Borel field BF' containing F' and extend the measure P' to this field, so that the three entities are available for defining random variables.

Instead of defining a random variable on the measure space ($\Omega$, B, P) as is customary, we define it on the measurable group ($\Phi$, BF', P'). We speak of a measurable group because $\Phi$ is now isomorphic to the transformation group representing the dynamics. A random variable X is simply defined as a measurable function on $\Phi$, distinct values of X mapping inversely into sets of BF'. If we write

$$\Phi = \{T^t, -\infty < t < \infty\}$$

then for any point $T^t$ in the sample space $\Phi$ the random variable X takes on the value $X(T^t)$. If we define the $\tau$-translate of X by requiring that $X_\tau(T^t) = X(T^{t+\tau})$ then it can be seen that the mixing property implies for a wide class of random variables that $X_\tau$ and X become statistically independent with increasing $\tau$.

We conclude with a brief theorem suggestive of the type of theorem that can now be obtained.

THEOREM

Let X and Y be random variables defined on the measurable group ($\Phi$, BF', P') such that the smallest Borel fields induced by X and Y in $\Phi$ belong to F'. Assume that F' is invariant under all rational transformations $T^r \in \Phi$, r rational.

1) For any real Borel set $B_1$, any rational r, and for all t

$$P'[X \in B_1] = \lim_{N \to \infty} \frac{1}{N+1} \sum_{k=0}^{N} \psi_{B_1}[X_{kr}(T^t)] \quad ,$$

where $$\psi_{B_1}(X) = \begin{cases} 1 & X \in B_1 \\ 0 & \text{otherwise} . \end{cases}$$





2) For any $\varepsilon > 0$ and any real Borel sets $B_1$, $B_2$ there exists a rational r such that

$$\left| P'[X \in B_1, Y_r \in B_2] - P'[X \in B_1]\, P'[Y_r \in B_2] \right| < \varepsilon$$

Bayard Rankin

A CALCULATION OF THE ENERGY BANDS OF THE GRAPHITE CRYSTAL BY MEANS OF THE TIGHT-BINDING METHOD

The computation of energy bands in a two-dimensional model of graphite by means of the tight-binding method has been completed. Current effort is being devoted to the thesis write-up of this work. Thus the following report is only a brief resumé of the basis of the calculation and the major results which were obtained.

The graphite energy band calculations were carried out assuming a structure of an infinite two-dimensional hexagonal lattice of Carbon atoms. The atomic orbitals used to make up the one-electron Bloch waves were the Hartree-Fock atomic orbitals of the ground state neutral Carbon $^3P$ configuration calculated by Jucys[1]. The 1s and 2p orbitals were each fitted by a linear combination of three corresponding analytic Slater atomic orbitals, and the noded 2s orbital was fitted by a sum of three 1s and three 2s Slater atomic orbitals[2]. The fits were of high accuracy and were made for ease in the calculation of the many integrals. In the sense that the energy band calculation made was a one-electron approximation, it was necessary to choose an effective potential for the crystal. The crystal potential was taken to be a super-position of spherically symmetric atomic potentials. The atomic potential in turn was assumed to be the Coulombic potential arising from the effective nuclear charge function, $Z_p(r)$, centered on the atomic nucleus. The $Z_p$ function for the $^3P$ configuration has been calculated by Freeman[3], and it was because of this convenience that the particular configuration of Carbon chosen was used since it was not felt that the current knowledge of energy band solutions gave any other conclusive choice. For computational convenience, the $Z_p$ function was also fitted, a linear combination of four exponentials being used.

All the one-electron, two-center integrals (overlap, kinetic energy and potential) were done by the use of the usual prolate spheroidal coordinate analytic integration techniques suitable for Slater atomic orbitals[4]. Thus these integrals required the evaluation of auxiliary functions[5,6] followed by combinations of many terms. All the one-electron, three-center potential integrals (and also the two-center potential integrals) were evaluated by the spherical coordinate expansion-about-another-center technique, similar to that used by Löwdin[7], and Barnett and Coulson[8], which was described extensively in a previous report[9].

It is perhaps illuminating to give some of the computational perspective involved in the over-all calculation. Inasmuch as the magnitude of numerical work involved was such that accuracy would have been difficult to maintain in any hand calculation, most of the computational work was done on the high-speed electronic computer Whirlwind I. The nearly total mechanization of the problem, although eliminating almost entirely any possible random mistakes, had the disadvantage of tending to obscure possible systematic mistakes. The latter shift of emphasis is one of the principle reasons that make the programming of a computer a non-trivial affair. A consequence is that logical simplicity of the computational procedure becomes a goal which is often in opposition to computational efficiency. There is also a great deal of difficulty in devising adequate test procedures for computer programs, since of necessity they must be tailored to the program itself. The ability of the programmer to cope with these computer problems develops mostly with experience. A large fraction of the time spent on the present calculation was thus used learning how to obtain the full potentiality of a high-speed computer.

In carrying out the calculation, the work fell into stages for each of which special computer programs were written. These were: a program for the semi-automatic fitting of the atomic orbitals and the $Z_p$ function[2]; a program for generating the two-center integral auxiliary functions and then automatically combining terms to give the integrals between Hartree-Fock orbitals[5]; a program for generating the atomic orbital expansion functions necessary for the three-center potential integrals[9]; a program for performing the basic numerical quadratures of the three-center potential integrals[9]; and a requantization and summation program for forming the appropriate three-center integrals from the basic numerical quadratures[9]. The foregoing computer programs were sufficient machinery to prepare the basic two-center Hamiltonian and overlap integrals which served as input for the final master program which performed the energy band calculation. Explicitly, the basic two-center integrals were of the form

$$M_{ij}(\vec{P}_a) = \int \Psi_i^*(\vec{r}-\vec{P}_a)\, M(\vec{r})\, \Psi_j(\vec{r})\, d\tau \qquad (1)$$

where the $\Psi$ are atomic orbitals. $\vec{P}_a$ is the neighbor-vector, and

$$M(\vec{r}) = \begin{cases} S(\vec{r}) = 1 & \text{, (Overlap)} \\ H(\vec{r}) = -1/2\,\nabla^2 - \sum_{\vec{P}_b} \dfrac{Z_p(\vec{r})-\vec{P}_b)}{|\vec{r}-\vec{P}_b|} & \text{, (Hamiltonian)} \end{cases} \qquad (2)$$

the sum in the potential term being over all neighbor-vectors. In the present calculation, the three-center potential terms were neglected when $|\vec{P}_b|$ or $|\vec{P}_b-\vec{P}_a|$ exceeded the fourth neighbor-distance in the sum over $\vec{P}_b$.

The operation of the master energy band program then proceeded as follows. For a given value of the reduced wave vector, $\vec{k}$, the program computed the Hamiltonian and overlap matrix elements arising from the Bloch waves constructed from the atomic orbitals. These matrix elements were made real by taking judicious linear combinations of the Bloch waves as basis states, and had the form

$$M_{ij}(\vec{k}) = \sum_{\vec{P}_a} \pm M_{ij}(\vec{P}_a) \begin{cases} \cos\,(\vec{k}\cdot\vec{P}_a) \\ \sin\,(\vec{k}\cdot\vec{P}_a) \end{cases} \qquad (3)$$

where the terms in which $\vec{P}_a$ exceeded the ninth neighbor-distance were neglected. The program next solved the usual variationally-derived secular equation of the form

$$\sum_j H_{ij}(\vec{k})V_{jk}(\vec{k}) = E_i(\vec{k}) \sum_j S_{ij}(\vec{k})V_{jk}(\vec{k}) \qquad (4)$$

and stored for later use the eigenvalues $E_i(\vec{k})$. A new value of the wave vector was then selected and the generation and solution of the secular equation repeated until the preset values of the wave vector were exhausted. Finally, for convenience, the program displayed graphically on a photographic oscilloscope cross-sectional views of the energy bands, $E_i(\vec{k})$ vs. $\vec{k}$, for values of the wave vector along the edges of a basic non-repeating $30^\circ$ -$60^\circ$ -$90^\circ$ triangle of the first Brillouin zone.





As has been implied, the secular equation which was considered had as basis states linear combinations of the ten one-electron Bloch waves formed from the 1s, 2s, and 2p atomic orbitals on each of the two atomic sites in the spatial unit cell. Because of the reflection symmetry in the Hamiltonian operator of the two-dimensional graphite lattice, the secular equation one obtains from these ten states immediately factors into two independent equations, one of order eight arising from reflection symmetric Bloch waves (sigma states), and the other of order two from reflection antisymmetric Bloch waves (pi states). Thus the master computer program was arranged to independently calculate the energy band solutions arising from each of these secular equations, but the final results were graphically superimposed.

The major physical significance in a two-dimensional energy band calculation of graphite is the size of the gap between the five lowest (occupied) and the higher (excited) sigma bands, and in particular whether or not the two lower pi bands (valance and conduction), which are degenerate at one value of the wave vector, have their point of degeneracy in the sigma band gap. If such a sigma band gap is large enough to include all of the pi bands, then it follows that a reasonable approximation for computing the conduction properties of a three-dimensional graphite would be to ignore the sigma states; (in the three-dimensional crystal the sigma and pi are no longer "good" symmetry designations since the sigma and pi Bloch waves of alternate graphite layers interact, but the terminology is still used.) Starting from the original work of Wallace[10], this approximation has been the basis of all graphite energy band calculations with the exception of that of Lomer who has also considered a two-dimensional graphite with sigma bands[11]. As will be seen later, the numerical work of Lomer is believed to have a very serious approximation.

The energy band solution of the present calculation is shown in fig. 2 where the horizontal dimensions are the edges of the basic triangle of the first Brillouin zone as shown in fig. 1. Energy values of interior points of the triangle were also found, but for brevity these values are not given here, since it was found that the interior energy bands vary smoothly from the edge band values. Furthermore the two lowest sigma bands arising almost entirely from the 1s Bloch waves, are omitted from fig. 2 since the bands are nearly independent of the wave vector and at an average value of -15.75 Rydbergs. It is observed that the highest occupied point in the sigma bands occurs at pt. 0, ($\vec{k}$=0), with a value of +.430 Rydbergs. The lowest excited sigma band has roughly a constant minimum value for wave vector values forming an approximate circle about pt. 0, the minimum value being about +1.158 Rydbergs. The degeneracy point of the pi bands falls at +.570 Rydbergs, a value representing the Fermi level for zero temperature. Thus it is clear that these results support to some extent the usual approximation of neglecting the sigma states in calculations of graphite conduction properties. Moreover this calculation shows by virtue of the overlapping of the sigma and pi bands that for a cohesive energy calculation, which depends on energy values for all wave vectors of the Brillouis zone, the sigma states must be included.

Consideration can also be made of the potential function used in the present calculation. The potential, which was formed by superposition of the $Z_p$ function, clearly omits exchange effects. Slater has given a procedure for introducing an approximate exchange potential correction within the framework of the one-electron approximation[12]. This exchange potential correction, if applied, would be proportional to the cube root of the charge density of the occupied crystal wave functions. Consequently, one would expect the occupied bands to be lowered more in energy than the unoccupied bands since the unoccupied wave functions are orthogonal to the occupied wave functions. Thus it is plausible that a more careful consideration of exchange effects in the present calculation would only broaden the sigma band energy gap and would leave the present results qualitatively unchanged.

One of the more striking features of the present results is the smoothness of the energy bands. In fact these bands are similar to those obtained from the Slater and Koster interpolation procedure wherein the tight-binding method is used with neglect of all but a few nearest-neighbor integrals[13]. It is therefore of interest to examine the stability of the present results in view of the several possible simplifying approximations.

Omission of the 1s Bloch waves was found to have an over-all lowering and warping effect on the sigma bands in such a way that the sigma band gap was roughly reduced by half. Thus the often-recognized importance of orthogonality is again emphasized.

A second possible approximation made was the omission of all the three-center potential integrals. This left the pi bands nearly the same but made a very pronounced change in the sigma bands, again closing the sigma band gap down to about half but also lowering the gap so far that the pi band Fermi level no longer was included.

Finally the solutions were examined with respect to the approximation of omitting the higher neighbor integrals. The stable solution, which included up to ninth neighbor integrals, was found to be only slightly warped when only up to fifth neighbor integrals were included in the sigma bands and when only up to third neighbor integrals were included in the pi bands. However, further truncation of the neighbor integrals of the sigma bands caused violent changes. (It is for this reason that the calculation made by Lomer which included only first neighbor integrals, is not believed to be valid.) In addition, the effects of truncating only Hamiltonian or only overlap neighbor integrals were investigated. It was found that the solution was sensitive to both Hamiltonian and overlap neighbor truncation to roughly the same extent and that the two effects were essentially additive.

Thus the results of all the approximation tests indicate clearly that the tight-binding method when used in a non-empirical way must be carried out with considerable mathematical rigor in order that a meaningful solution will be obtained.

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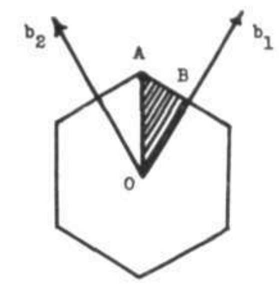
Fernando J. Corbato

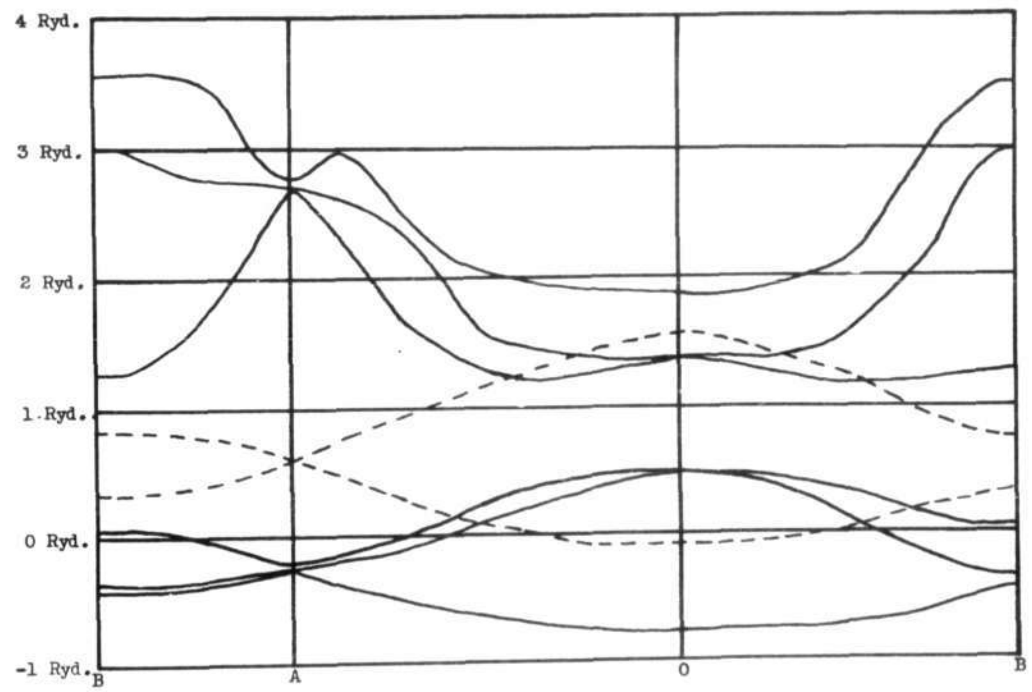







FIRST BRILLOUIN ZONE

FIG. 1




ENERGY BANDS OF GRAPHITE

Sigma Bands

Pi Bands

FIG. 2

## THE GROWTH OF FATIGUE CRACKS

An exact solution to the problem of determining the rate of propagation of a crack under cyclic loading is exceedingly difficult, and analytical solutions may be obtained only by approximating the exact formulation, even for simple loading modes and specimen configurations. The specific case chosen for study here is the torsion of a long cylindrical bar of rectangular cross section when the crack is advancing on a plane through the axis of the cylinder. The following assumptions are made:

1) The material is perfectly plastic, homogeneous, and isotropic;

2) The bar has been strained to an extent that plastic flow has occurred over virtually the entire cross section;

3) The crack propagates in the following manner:

a) The bar is twisted until the material a distance, Xs, in front of the crack reaches the fracture strain, $\gamma_s$.

b) The crack length, $\xi$, then increases by the distance $d\xi$.

c) This advance relieves stresses and the angle of twist, $\theta$, must be increased by $d\theta$ before the fracture condition is again obtained.

Within the limitations of these assumptions, the rate of crack growth, $\phi$, when the crack is of length X, may be expressed as the Volterra integral equation,

$$\phi(X) = \frac{X_s^2}{1+X_s^2}\left[1 + \frac{1}{(X+X_s)^2}\right] + \frac{X_s}{1-X_s^2}\int_0^X \left[1 + \frac{1}{[\xi-(X+X_s)]^2}\right]\phi(\xi)\,d\xi$$

where the variables have been non-dimensionalized.

A steady-state solution to this equation may be obtained.

$$\phi_{ss} = \frac{2}{\ln X_s^2 - (1-X_s^2)}$$

Using this result, the problem may be re-formulated as

$$\Phi(X) = \frac{\phi(X) - \phi_{ss}}{1-\phi_{ss}} = \frac{1}{1-\phi_{ss}}\left[\left(\frac{X_s}{X+X_s}\right)^2 - \phi_{ss}\left(\frac{X_s}{X+X_s}\right)\right]$$

$$+ X_s\int_0^X \left[1 + \frac{1}{[\xi-(X+X_s)]^2}\right]\Phi(\xi)\,d\xi$$





when $\Phi(X)$ goes from 1 to 0. Terms of the order $X_s^2$ are small compared with one and have been discarded. A trial solution of the form

$$\phi(X) = \left(\frac{CX_s}{CX_s+X}\right)^2$$

inserted with the equation and iterated once to evaluate the constant C yields the result that C has a value approximately between two and four. Thus we can expect $\Phi(X)$ to decay to 96% of steady state value after the crack has progressed about $8X_s$ to $16X_s$.

To evaluate the integral equation numerically, the equation is written in finite difference form with spacing, h, where

$$X = nh$$
$$\xi = vh$$

If the integral is approximated by a straight line between intervals, the equation takes the form

$$\Phi(nh) = \frac{1}{1-\phi_{ss}}\left[\left(\frac{X_s}{nh+X_s}\right)^2 - \phi_{ss}\left(\frac{X_s}{nh+X_s}\right)\right] + X_s\sum_{v=0}^{n} A(n,v)\phi(vh)$$

where

$$A(n,0) = \frac{h}{2}K(n,0),$$

$$(A(n,v) = h\,K(n,v) \qquad v\neq 0,\ v\neq n,$$

$$A(n,n) = \frac{h}{2}K(n,n),$$

and

$$K(\ell,m) = 1 + \left(\frac{1}{\ell h - mh + X_s}\right)^2$$

When the summation is evaluated at v=0 and v=n, the difference equation becomes for $n > 1$,

$$\Phi(nh) = \frac{2X_s}{h+2X_s}\left\{\left[\left(\frac{X_s}{nh+X_s}\right)^2 + X_s^2\left(\frac{nh}{nh+X_s}\right)\right]\left[1 - \frac{\phi_{ss}}{1-\phi_{ss}}\ \frac{nh}{X_s}\right]\right.$$

$$\left. + \frac{h}{2X_s}\left(\frac{X_s}{nh+X_s}\right)^2\right\} + 2X_s^2\ \frac{h}{h+2X_s}\sum_{v=1}^{n-1} K(n-v,0)\ \phi(vh)$$

A calculation carried out by hand using $X_s/h=3$ and $X_s=.05$ shows close agreement with previously calculated approximate solution. The equation in the form above has been programmed for Whirlwind. The first results which agree with the hand calculated curve were recently obtained.

Joseph B. Walsh

## COULOMB WAVE FUNCTIONS

The programs for both the regular and irregular coulomb wave functions for L=0 are now complete and an output routine is now being written. Since this is the last formal progress report for the Project, we will summarize in some detail the nature of the computations involved, reviewing some of the work previously reported.

### RANGE OF PARAMETERS

The ranges of parameters $\rho$, $\eta$ and $L$ will be

$$L = 0$$

$$0 \leq \eta \leq 12.5 \qquad (1)$$

$$0 \leq \rho \leq 25$$

with intervals

$$\Delta\rho = \Delta\eta = 0.2$$

$\Delta\eta$ might possibly be changed to 0.1 but this involves only a trivial modification of the present program.

### METHODS OF COMPUTATION

a) Regular Function

For the interval $0 \leq \rho \leq 1$, the power series is used

$$F_o(\eta,\rho) = C_o(\eta)\, \rho \sum_{n=0}^{\infty} B_n$$

$$C_o^2(\eta) = \frac{2\pi\eta}{e^{2\pi\eta}-1} \qquad (2)$$

$$B_o = 1, \; B_1 = \eta\rho$$

$$(n+1)(n+2)B_{n+1} = 2\eta\rho B_n - \rho^2 B_{n-1} \quad ; \; n \geq 1.$$

About twenty terms are needed to give eight digit accuracy for the range of $\rho$ and $\eta$ given above. In the interval $1 < \rho \leq 25$, we use numerical integration of the differential equation

$$F'' + \left(\frac{1-2\eta}{\rho}\right) F = 0$$





in the following sense[1]. Knowing the value of $F(\eta,\rho)$ and $F'(\eta,\rho)$ at $\rho_i$, we find the values of F and F' at $\rho_i + \Delta\rho$ by using the Taylor expansion

$$F(\rho_i + \Delta\rho) = F(\rho_i) + \sum_{n=1}^{\infty} \frac{1}{n!} F^n(\rho_i)(\Delta\rho)^n$$

For machine computation, it is more convenient to use the notation

$$\sigma^n(\rho_i, \Delta\rho) = \frac{1}{n!} F^n(\rho_i)(\Delta\rho)^n$$

$$F(\rho_i + \Delta\rho) = \sum_{n=0}^{\infty} \sigma^{(n)} \; ; \; F'(\rho_i + \Delta\rho) = \frac{1}{\Delta\rho} \sum_{n=0}^{\infty} n\,\sigma^{(n)}$$

$$\sigma^{(0)} = F(\rho_i) \qquad (3)$$

$$\sigma^{(1)} = F'(\rho_i)$$

$$\sigma^{(2)} = -\frac{1}{2}\left(1 - \frac{2\eta}{\rho}\right) F(\rho_i)(\Delta\rho)^2$$

$$\rho\, n(n+1)\,\sigma^{(n+1)} + \delta(n^2-n)\,\sigma^{(n)} + (\rho - 2\eta)\,\delta^2\,\sigma^{(n-1)}$$
$$+ \delta^3\,\sigma^{(n-2)} = 0; \quad n \geq 2.$$

Enough terms in (3) are taken so the number of significant figures in the new F and F' is the same as that in the old F and F'. Of course, near a zero of F or F', merely taking a lot of terms is not sufficient to preserve the number of significant digits. To show this we write,

$$F(\rho_i + \Delta\rho, \eta) = A(\rho_i, \Delta\rho, \eta)\, F(\rho_i, \eta)$$
$$+ B(\rho_i, \Delta\rho, \eta)\, F'(\rho_i, \eta) \qquad (4)$$

$$A = 1 - \frac{1}{2}(\Delta\rho)^2 \left(1 - \frac{2\eta}{\rho} + \cdots\right) \; ; \; B = (\Delta\rho) + \ldots$$

where A and B are infinite series determined by (3). Let us assume that A and B are known exactly and that F and F' are related to the true values of the functions, $F_t$ and $F_t'$, by

$$F = F_t\,(1 + \epsilon_1)$$

$$F' = F_t'\,(1 + \epsilon_2)$$

$$|\epsilon_1| = |\epsilon_2| = \epsilon$$

We have therefore

$$F_c(\rho_i + \Delta\rho, \eta) = F_t(\rho_i + \Delta\rho_i, \eta) + \epsilon \left| A\, F_t(\rho_i) \pm B F_t'(\rho_i) \right| \qquad (5)$$

where $F_c$ stands for the calculated value of F. Since A $F_t$ and B $F_t'$ are of the order of unity, it is possible for the error term in (5) to be of the order of $\epsilon$. But if $F_t(\rho_i + \Delta\rho)$ is considerably smaller in magnitude than $AF(\rho_i)$ or $BF'(\rho_i)$ because of strong subtraction, we will evidently lose some significant figures. In the computations performed so far, the strongest zeros encountered were of the order of $5(10)^{-3}$. At these points, we might expect only five significant figures if seven are carried at other points, as is the case in the present computations. The machine time for calculating $F_o$ and $F_o'$ for a single $\rho$ varies between 400 and 800 milliseconds depending on the number of terms used. For most of the range in $\rho$ $(2 \le \rho \le 25)$ it is sufficient to take terms up to n = 8 in (3) with a corresponding machine time of about 500 milliseconds.

b) Irregular Function

The irregular functions $G_o$ and $G_o'$ are calculated by the technique indicated in (3). The starting points for the procedure are the transition line $\rho = 2\eta$ for $\eta \ge 2$ and the points $\rho = 4, 0 \le \eta < 2$ for the remaining region. For $\rho = 2\eta \; \{2 \le \eta \le 6\}$ and $\rho = 4, 0 \le \eta \le 2$, we use the integral representation for $G_o$ and $G_o'$ [2].

$$G_o(\rho,\eta) = A_c(\eta)\,\rho\, g_o(\rho,\eta)$$

$$G_o'(\rho,\eta) = A_c(\eta)\{\rho\, g_o'(\rho,\eta) + g_o(\rho,\eta)\} \qquad (6)$$

$$A_o^2(\eta) = \frac{1 - e^{-2\pi\eta}}{2\pi\eta}$$

$$g_o(\rho,\eta) = \int_0^\infty \left\{ e^{-(\rho\xi - 2\eta \tan^{-1}\xi)} - \frac{\operatorname{Sin}(\rho \tan h\,\xi - 2\eta\xi)}{\operatorname{Cos} h^2 \xi} \right\} d\xi$$

Gaussian quadrature is used to evaluate the definite integrals with the cut-off in $\xi$ chosen so as to give seven to eight significant figures.

For $\rho = 2\eta$, $\eta \ge 6$ we may use the method of steepest descent to evaluate the integrals and obtain a rapidly converging series in inverse powers of $\eta$.[1] The method of computation is subject to the same limitation discussed in reference to the regular fraction, namely that near the zeros of the functions the number of significant digits accurately retained is (7 - Z) where Z is the number of zeros between the decimal point and the first significant digit.

## SAMPLE COMPUTATIONS

a) Regular Function

$\rho = 10$

$\eta = 5$

$F_o = +0.91794503$ } numerical integration

$F_o' = +0.33103219$ } from $\eta = 5, \rho = 1.$





$F_o = +0.91794506$ } Integral representation

$F_o' = +0.33103212$ } or steepest descent

b) Irregular Function

$\rho = 20$

$\eta = 5$

$G_o = +1.1657166$ — Numerical integration from $\eta = 5$, $\rho = 10$

$G_o = +1.1657160$ — Asymptotic formula

References

1. Abramowitz and Rabinowitz, Phys. Rev. 96, 77 (1954).
2. Newton, Atomic Energy of Canada, Limited, CRT-526 (1952).

Arnold Tubis
Aaron Temkin
Zoltan Fried

## FIRST APPROXIMATION SOLUTION ON ORE BODY

Proceeding from the favorable results reported in the previous Quarterly Progress Report No. 19, the investigation of the first approximation solution was directed towards the determination of the optimum size and shape of the ore body to be used in the inverse problem of interpretation of field data. However, certain difficulties were met which have not been completely solved, specifically: the physically modeled results have been found to be somewhat unreliable, and a paradox has been met in the determination of the size and shape of the fundamental building block (optimum ore body) for the inverse problem. The character of the modeled results is always consistent but the numerical values are not, and hence more accurate experimental work will have to be done before a final decision can be made concerning the feasibility of the first approximation solution. Also the paradox will have to be resolved as regards the optimum ore body size and shape.

The first approximation solution does have its merits and as such will be of valuable assistance in the interpretation of field data and the economic evaluation of a possible mineralized area.

In conjunction with course 6.538 (Electronic Computational Laboratory) a complete theoretical and numerical solution to the three-vertical-layered earth has been done utilizing the IBM Type 650 Magnetic Drum Computer. That is to say, the apparent resistivity profiles for a vertically layered earth have been obtained for both pole and dipole senders for a large number of geometries and resistivity contrasts. The programs for these calculations are available for further work, and the results obtained are presented in the report VERTICAL LAYERING IN GEOPHYSICAL PROSPECTING submitted as a term project in course 6.538. The results obtained have been of assistance in the evaluation of the first approximation solution and also as a tabulation of resistivity profiles for certain idealized geological situations. These can be used to assist in the interpretation of field results before further investigation on high-speed computers is made.

Norman F. Ness

## DYNAMIC RESPONSE OF SHEAR WALLS

A "shear wall" carries loads applied in the plane of the wall parallel to the length of the wall. Walls in actual buildings carry portions of transverse loads (wind, blast, earthquake, etc.) in this manner.

In this investigation the actual wall is replaced by a dynamic model consisting of a number of concentrated masses inter-connected by weightless springs (see figure). The properties of the springs are obtained by equating the deformations of the actual wall and dynamic model under similar stress conditions. With the spring properties known, the equations of motion are obtained for both horizontal and vertical motion of each mass point.

Three different size walls are being studied for two different load conditions and two different sized grids for a total of twelve solutions. The solutions are restricted to the elastic range. The number of equations varies with the wall size and fineness of grid; the minimum number is 15 for a wall 4'-0" square with a 2'-0" square grid, and the maximum number is 77 for a wall 4'-0" by 12'-0" with a 1'-4" grid. The equations of motion are solved by numerical integration. Equations of the form

$$x_{n+1} = 2x_n - x_{n-1} + \ddot{x}_n(\Delta t)^2$$

where $x_{n+1}$ = displacement at n+1 st time interval

$\dot{x}_n$ = velocity at nth time interval

$\ddot{x}$ = acceleration at nth time interval

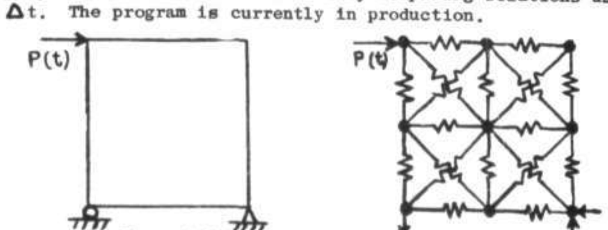
and $\Delta t$ = length of time interval

are used to predict deflection at time $t_{n+1}$. A second equation of the form

$$x_{n+1} = x_n + \dot{x}_n(\Delta t) + \frac{\ddot{x}_n(\Delta t)^2}{3} + \frac{\ddot{x}_{n+1}(\Delta t)^2}{6}$$

is used to correct the previous estimate. An iterative procedure is required to obtain a stable solution since $\ddot{x}_{n+1}$ is involved. After a set of consistent displacements at time $t_{n+1}$ has been obtained, the principal tensile stresses at a discrete number of points on the actual wall are calculated. These are compared with the cracking stress of reinforced concrete. If cracking has not yet occurred, the process is repeated. At the end of each time interval, horizontal and vertical displacements, stresses and reactions are displayed on the scope.

The proper value of $\Delta t$ is obtained by comparing solutions using several different values of $\Delta t$. The program is currently in production.




Shear Wall    Dynamic Model

Ralph G. Gray





## RESPONSE OF A FIVE-STORY FRAME BUILDING TO DYNAMIC LOADING

The object is to determine the response of actual multistory rectangular rigid-frame buildings subjected to blast loads. A more complete description may be found in the March 1956 Progress Report.

The control program is written and being "debugged". The load program is written and debugged, as is the integration program. The resistance program is in two parts. The first part, the solution of a matrix of the slope-deflection equations, is currently being tested in conjunction with the second part, which reviews the matrix solution for changes from elastic-to-plastic or vice-versa and modifies the matrix accordingly. This last part has been successfully tested by itself. On completion of this test, the various programs will be combined and tested for complete solutions.

Ralph G. Gray

## CALCULATIONS FOR SPHEROIDAL NUCLEI

The computations which were originally projected under these problem numbers is now complete. Before submitting a final report, however, two additional investigations will be undertaken.

a) Problem 235, Bound States in a Spheroidal Well. It would be desirable to try to include the effect of a spin-orbit coupling term upon the energy eigenvalues. Such a calculation will be carried out if it does not involve any substantial revision of the existing programs.

b) Problem 319, Scattering from a Spheroidal Square-Well of Zero Energy Neutrons. There is a relatively simple way of approximating the case where the scattering potential is complex. In essence it involves the calculation of the integral of the square of the wave function (for a real potential) over the volume of the scatterer. A program to do this has been written.

Jack L. Uretsky

## FINITE BENDING OF THIN, SHALLOW SPHERICAL SHELLS

Computations of the bending of a uniformly loaded shallow spherical shell have been completed in the case of simply supported edge. In particular, buckling loads for very shallow shells have been obtained. The results show that the convergence of the employed power series method as outlined in the last Progress Report is sufficiently rapid as to give fairly accurate solutions for stable and unstable equilibrium positions in the post buckling region, provided the shell is sufficiently shallow. Calculations for less shallow shells indicate, on the other hand, that solutions can be obtained in this framework only for loads below the buckling load and in some cases also approximate solutions beyond buckling. The power series method seems to be able to reveal equilibrium states close to the buckling region in the latter case. Similar phenomena were observed in the case of clamped edge. Two circumstances may be mentioned here that are responsible for the break-down of the method. In the region of buckling the vertical deflections of the shell show a complicated pattern which cannot be approximated by a polynomial of sufficiently low degree. The second reason is that the initial estimate is not sufficiently close to the actual solution; hence the number of iterations to be carried out to obtain a solution would be extremely high.

The results obtained during this quarter are now being evaluated and analyzed in somewhat more detail with respect to some special solutions that have been obtained hitherto for this problem. A more complete description of the results will be given in the next Progress Report

Hubertus J. Weinitschke

## THE NUMBER DISTRIBUTION OF ELECTRONS AND PHOTONS

The Whirlwind coding of the grid described in Progress Report No. 13, p. 55, has been successfully carried out. At present, a program selecting paths according to the Monte Carlo method described in Progress Report No. 18, p. 12, is being tested in a simple case ($n=m=5$), and will be easily extended for larger n and m.

The next step which does not involve any logical programming will be the computation of the products of transition probabilities and convolutions of exponentials involved in the formula given in Progress Report No. 14, p. 16. It will be partly carried out by Dr. B. Rankin, and continued by myself starting on August 1st.

Raymond F. Stora

## ASYMPTOTIC SOLUTION OF A DIFFERENTIAL EQUATION

Research has been continued on the solution of the differential equation with "turning points" described in the last report. The third order equation

$$u''' + \lambda^2 z u^1 + 3\mu\lambda^2 u = 0$$

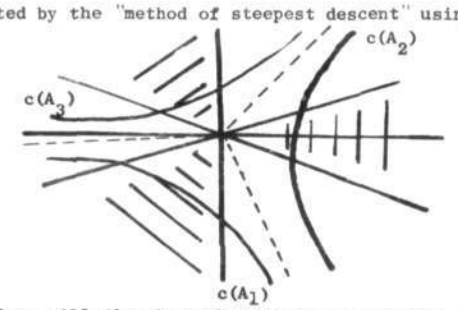
X large

$\mu$ constant, complex

which is a special case of the previous one, is now being solved by use of the Laplace integral. In particular, the asymptotic expansion of the solution

$$u = \int_c t^{3\mu-1} \exp \left(1/3\ \lambda^{-2}\ t^3 + Zt\right)$$

is being investigated by the "method of steepest descent" using the paths $C(A_k)$ $k=1,2,3$ below.




A comparison will then be made with the asymptotic series expansion obtained by Rudolph E. Langer with solutions computed numerically.

Joseph Hershenov

## AN APPLICATION OF MONTE CARLO METHODS TO NEUTRON DIFFUSION

During the past quarter all routines have been rewritten and retested. Several errors were found. The first assembly of the individual routines has been run successfully. Tests of problems, for which the analytical solution is known, are continued. This problem is programmed and coded by M. Troost, Research Assistant in Chemical Engineering.

Marius Troost





## CONDENSATION IN TUBES SUBJECT TO THE EFFECTS OF VARIABLE VAPOR VELOCITY

Two runs at different conditions of entrance vapor flow but with the same vapor and condensate properties (those of water at 212°F) appear to give reasonable results. However, there has not yet been time to check these results with existing data.

The stopping criterion in these cases was the transition of the condensate from laminar to turbulent flow. The program for the turbulent vapor - turbulent condensate case is being completed in order to continue the laminar condensate runs to the condition where all the vapor in the tube has condensed.

It appears that the turbulent vapor-turbulent condensate case analysis cannot be simplified to contain only three dimensionless parameters describing the geometry, initial conditions and vapor and condensate properties as was possible with the turbulent vapor-laminar condensate case.

Jukka A. Lehtinen

## RESPONSE OF A SINGLE STORY REINFORCED CONCRETE BUILDING TO DYNAMIC BLAST LOADING

The object is to study the dynamic response of the roof of a single story reinforced concrete building for various loading functions. The roof is a system of infinite degrees of freedom consisting of a one-way slab supported on relatively stiff beams. This is resolved into a two-degree system by assuming deflection shapes for the one-way slab and its supporting beams and using LaGrange's equation to obtain two dynamic equations for acceleration of the beam and slab. These are integrated for deflections by an "acceleration impulse method" which utilizes backward differences.

The program is currently in production.

Ralph G. Gray

Part II

Project Whirlwind

# 1. REVIEW AND PROBLEM INDEX

This report covers the specific period of March 20, 1956 to June 10, 1956. During this time, 95 problems made use of 398.03 hours of the 514.25 hours of Whirlwind computer time allocated to the Scientific and Engineering Computation (S&EC) Group. Of the 514.25 allocated hours of computer time, .2% was down time due to computer malfunctions. The remaining 104.62 hours of the allocated time were used for terminal equipment testing and calibration, demonstrations, and various inter-run operations not logged to specific problems.

The 95 problems run during this quarter cover some 20 fields of applications. The results of 32 of the problems have been or will be included in academic theses. In these 32 problems, there are represented 25 Doctoral theses, 3 Naval Engineer's, 5 Master's, and 4 Bachelor's. Twenty-six of the problems have originated from research projects sponsored at MIT by the Office of Naval Research.

Two tables are provided as an index to the problems for which progress reports have been submitted. In the first table, the problems are arranged according to the field of application, and the source and amount of time used on WWI are given. In Table 2-II, the problems are listed according to the principal mathematical problem involved in each. In each table, the letter after the problem number indicates whether the problem is for academic credit and whether it is sponsored. The code is explained in section 2.1, Introduction.

| Field | Description | Problem Number | Minutes of WWI Time | Supervisor or Programmer |
|---|---|---|---|---|
| Aeronautical Engineering | Transient temperature of a box-type beam | 179 C. | 93.3 | J.C.Loria |
| | Transient response of aircraft structures to aerodynamic heating | 236 C. | 13.2 | L.C.Schmit |
| | Postfailure response of aircraft structures subjected to blast loading | 330 C. | 170.2 | R.D'Amato |
| | Parametric study of coupling and damping | 334 C. | 81.2 | K.Wetmore |
| | Vertical tail loads due to rolling pull-up | 358 B. | 23.2 | H.Parechanian |
| | Blast response of rotor blades | 364 C. | 90.8 | K. Foss |
| Aerophysics Research Group | Extraction of stability derivatives from flight test data | 317 C. | 262.9 | L.L.Mazzola |
| Chemical Engineering | Transients in distillation columns | 241 B. | 105.0 | S.H.Davis, Jr. |
| | Critical mass calculations for cylindrical geometry | 270 B. | 996.9 | J.R.Powell |
| Civil Engineering | Response of a single story concrete building | 354 D. | 116.4 | B.Landry |
| | Design of spherical shell segments | 372 B. | 3.0 | E.Traum |
| Electrical Engineering | Linear programming | 219 | 557.1 | D.Arden |
| | Production for transportation problem | 326 C. | 69.3 | J.Dennis |
| | Quantization error | 355 B. | 30.6 | H.Pople |
| | Propagation of round-off error | 357 B. | 80.0 | A.I.Green |
| Energy Conversion Lab. | Optimization of alternator control system | 264 C. | 159.5 | J.Dennis |
| | Optimization of ram-air cooling system | 338 C. | 159.7 | R.Moroney |
| Geology | Fourier synthesis for crystal structures | 261 C. | 296.2 | M.J.Buerger |
| Instrumentation Laboratory | Data reduction for X-1 fire control | 244 C. | 68.3 | J.Stark |
| Lincoln Laboratory | Eigenvalue problem for propagation of electromagnetic waves | 193 L. | 547.2 | H.B.Dwight |
| | General raydist solution | 272 L. | 175.8 | G.C.Sponsler |
| | Tropospheric propagation | 300 L. | 229.2 | H.B.Dwight |
| | Error analysis | 312 L. | 257.1 | L.Peterson |
| | Prediction analysis | 327 L. | 508.7 | L.Peterson |
| | Atmospheric propagation of radio waves | 371 L. | 237.4 | W.Mason |
| | Coverage analysis | 377 L. | 30.4 | L.Peterson |
| Mechanical Engineering | Thermodynamic and dynamic effects of water injection into high temperature high velocity gas streams | 120 D,N. | 167.7 | A.Erickson |
| | Laminar boundary layer of a steady, compressible flow in the entrance region of a tube | 199 N. | 60.8 | T.Y.Toong |
| | Rolling bearings | 293 C. | 148.0 | A.Shashaty |
| | Flame stabilization on a heated plate | 333 A. | 74.4 | T.Y.Toong |
| | Beam vibration | 339 A. | 66.8 | S.H.Crandall |
| | Transient heat flow in solids | 342 B. | 189.3 | R.Gelman |
| | Matrix multiplication | 345 B. | 70.0 | R.R.Archer |
| | Asymptotic integration of equations concerning toroidal shell | 363 A. | 14.3 | S.H.Crandall |
| Meteorology | Investigation of the vorticity field in the general circulation of the atmosphere | 226 D. | 76.8 | D.Cooley |
| | Spectral analysis of atmospheric data | 306 D. | 217.8 | B.Saltzman |
| | Statistical and dynamic methods in forecasting | 341 C. | 604.0 | E.Kelley |
| | Weather prediction | 343 C. | 653.4 | J.Austin |
| | Computation of variances and covariances | 350 D. | 202.9 | D.Gilman |
| Naval Architecture | Solution of transverse web frame | 359 B. | 95.4 | C.Brandt |
| Naval Supersonic Laboratory | Diffusion boundary layer | 297 B. | 85.2 | J.Baron |
| Nuclear Engineering | Calculations for the MIT reactor | 266 A. | 75.3 | T.Cantwell |
| | Non-uniform fuel distribution | 351 B. | 126.7 | A.Sutter |
| | Determination of critical mass | 367 B. | 123.5 | J.Barnett |
| | Flux leveling in homogeneous reactor - Part I | 373 B. | 17.9 | R.Kennedy |
| Office of Statistical Services | Finding eigenvalues of an asymmetric matrix | 365. | 134.5 | F.M.Verzuh |
| | Temperature distribution in a beam | 369. | 298.8 | F.M.Verzuh |
| Physics | Determination of phase shifts from experimental cross-sections | 162 N. | 38.4 | F.Eppling |
| | Atomic integrals | 234 N. | 2.5 | R.K.Nesbet |
| | Theory of neutron reactions | 245 N. | 2959.9 | H.Feshbach |
| | Scattering from oxygen | 246 B,N. | 279.3 | A.Temkin |
| | APW as applied to face- and body-centered iron | 253 N. | 127.1 | J.Wood |
| | Energy levels of diatomic hydrides | 260 N. | 349.1 | A.Freeman |
| | Evaluation of two-center molecular integrals | 262 N. | 700.1 | A.Aghajanian |
| | Analysis of air shower data | 273 N. | 243.9 | F.Scherb |
| | Energy levels in diatomic hydrides LiH | 278 N. | 675.1 | A.Karo |
| | Atomic wave functions | 288 N. | 1670.8 | R.K.Nesbet |
| | Polarizability effects in atoms and molecules | 290 N. | 667.6 | L.C.Allen |
| | Pure and impure potassium chloride crystal | 309 B,N. | 276.9 | L.P.Howland |
| Servomechanisms Laboratory | Data reduction | 126 C. | 670.1 | D.T.Ross |
| Spectroscopy Lab | Complex spectrum analysis | 346 B. | 123.3 | J.Lindner |

Table 2-I Current Problems Arranged According to Field of Application





| Mathematical Problem | Procedure | Problem Number |
|---|---|---|
| 1. Matrix algebra and equations | | |
| Root of determinantal equation | Iteration | 266 A. |
| Linear equations | Crout | 270 B. |
| Eigenvalues | Diagonalization | 278 N. |
| Eigenvalues | Diagonalization | 288 N. |
| Orthonormalization, eigenvalues | Schmidt process, diagonalization | 290 N. |
| Inversion | Crout | 312 L. |
| Linear equations | Crout | 317 C. |
| Equations | Matrix iteration | 330 C. |
| Ordinary algebra | Iteration | 338 C. |
| Diagonalization, inversion | Successive approximations, Crout | 341 C. |
| Inversion | Crout | 343 C. |
| Arithmetic operations | Direct | 346 B. |
| Solution of 5 simultaneous first degree equations | Crout | 359 B. |
| Inversion, eigenvalues | Crout, iteration | 364 C. |
| Eigenvalues, factoring polynomials | Frame, Hitchcock | 365. |
| Arithmetic operations | Direct | 377 L. |
| 2. Ordinary differential equations | | |
| Seven non-linear first order | Fourth order Kutta-Gill | 120 D,N. |
| System | Gill | 199 N. |
| Non-linear first order | Second order Kutta-Gill | 241 B. |
| Wave equation | Milne predictor-corrector formula | 245 N. |
| Non-linear | Gill | 297 B. |
| Set of differential equations | Gill | 333 A. |
| Simultaneous differential equations | Finite difference | 334 C. |
| Second order | Power series | 342 B. |
| Non-linear differential equations | Perturbation procedure | 345 B. |
| Second order | Finite difference | 351 B. |
| Three simultaneous second order | Trapezoidal rule | 358 B. |
| Pair of simultaneous second order | Finite difference | 363 A. |
| Ordinary differential equations | Finite difference | 364 C. |
| Set of differential equations | Kutta-Gill | 371 L. |
| Second order | Finite difference | 374 B. |
| 3. Partial differential equations | | |
| Second order parabolic | Finite difference | 179 C. |
| First order system | Finite difference | 226 D. |
| Second order parabolic | Finite difference | 236 C. |
| Fourth order | Finite difference | 339 A. |
| Second order, variable coefficients | Finite difference, trapezoidal integration | 373 B. |
| 4. Integration | | |
| Integral transformation | Algebraic recursion formula | 234 N. |
| Evaluation of integrals | Approximation by algebraic functions | 246 B,N. |
| Overlap integrals | Evaluation of analytic forms | 260 N. |
| Overlap integrals | Evaluation of analytic forms | 262 N. |
| Integrations | Simpson's rule | 278 N. |
| Fresnel integral | Conversion power series with a complex argument | 300 L. |
| Integrations | Barnet and Coulson expansion | 309 B. |
| Integrations | Gauss quadrature | 312 L. |
| Integration | Finite difference | 354 D. |
| Integration | Trapezoidal rule | 359 B. |
| Integrations | Series substitution | 372 B. |
| 5. Transcendental equations | | |
| Curve fitting | Least squares | 162 N. |
| Non-linear equations | Steepest descent | 264 C. |
| Non-linear equations | Iteration | 272 N. |
| Curve fitting | Least squares | 273 N. |
| Non-linear equations | Steepest descent | 293 C. |
| 6. Complex algebra | | |
| Complex roots and function evaluation | Iteration | 193 L. |
| 7. Data reduction | | |
| Data reduction | Polynomial fitting, etc. | 126 C. |
| Miss distribution | Arithmetic operations | 244 C. |
| 8. Fourier series | | |
| Fourier synthesis | Direct evaluation | 261 C. |
| Fourier synthesis | Direct evaluation | 306 D. |
| 9. Linear programming | | |
| Linear programming | Simplex relaxation | 219. |
| 10. Statistics | | |
| Evaluation of covariance and variance | Direct | 350 D. |
| Autocorrelation | Direct | 355 B. |
| Mean and second moment | Direct | 357 B. |

Table 2-II Current Problems Arranged According to the Mathematics Involved

# 2. WHIRLWIND CODING AND APPLICATIONS

## 2.1 Introduction

Progress reports as submitted by the various programmers are presented in numerical order in Section 2.2. Since this summary report presents the combined efforts of DIC Projects 6345 and 6915, reports on problems undertaken by members of the Machine Methods of Computation (MMC) Group have been omitted from Section 2.2 of Part II to avoid duplication of Part I. For reference purposes, a list of the MMC Group problems appears below.

Letters have been added to the problem numbers to indicate whether the problem is for academic credit and whether it is sponsored. The letters have the following significance.

A implies the problem is NOT for academic credit, is UNsponsored.

B implies the problem IS for academic credit, is UNsponsored.

C implies the problem is NOT for academic credit, IS sponsored.

D implies the problem IS for academic credit, IS sponsored.

N implies the problem is sponsored by the Office of Naval Research.

L implies the problem is sponsored by Lincoln Laboratory.

The absence of a letter indicates that the problem originated within the S&EC Group.

### List of Machine Methods of Computation Group Reports

The following problems used Whirlwind I computer time during this quarter, but are reported on in Section 2.2 of Part I.

| | | |
|---|---|---|
| 122 N. | Coulomb Wave Functions | A. Temkin<br>A. Tubis<br>Z. Fried |
| 172 B,N. | Energy Bands in Graphite | F.J.Corbato |
| 203 C. | Response of a Multi-Story Frame Building Under Dynamic Loading | R.G.Gray |
| 231 B,N. | Reactor Runaway Prevention | M. Troost |
| 240 A. | Number Distribution of Electrons and Photons in Cascade | B. Rankin |
| 319 B,N. | Zero Energy Scattering Cross-Section of a Spheroidal Well | J. Uretsky |
| 329 N. | First Approximation Solution on Ore Body | N.F.Ness |
| 337 N. | Non-linear Second Order Differential Equations in the Theory of Elastic Shells | H. Weinitschke |
| 361 B,N. | Growth of Fatigue Cracks | J.B.Walsh |
| 368 B,N. | Condensation in a Vertical Tube | J. Lehtinen |





### 100. COMPREHENSIVE SYSTEM OF SERVICE ROUTINES

Several minor and obscure errors have been detected in the CS conversion program. For the most important of these, the necessary corrections already have been incorporated in the program. This corrects the situation in which the current address indicator could be set "definite" even though it was set by a word containing the syllable "r" when r is "indefinite" (see C.S.Manual, Chapter XIV). The conversion post-mortem has been printing out the wrong register as the address for "Program too long at " when a ditto block runs beyond the beginning of the PA. The necessary changes have been discovered and checked out, but due to the difficulty of re-recording magnetic tape unit 0, the correction has not yet been made permanently. A logical error in the program which gives the wrong result only in a peculiar and quasi-legal situation, a word like "ts5r-" after an "LSR" in which the minus sign is taken as preceeding the "r", has been found and a correction has been checked out.

The manual modes of the new Generalized Post-Mortem (see Summary Report #45) are being checked out. Most of the errors have been discovered and corrected. The new program will display memory on the scope at a rate comparable to that of the scope post-mortem in the Utility Control Program, a considerable advantage in speed over the present program. Besides, not only zeros, but any block of identical lines is suppressed and only the first and last lines are given.

The fp tape reading program and the flad modes will be checked out after the manual modes are operating satisfactorily. As the Utility Control Program must be modified to accommodate the new Generalized Post-Mortem and some other changes seem desirable to take advantage of new in-out equipment, etc., a large number of revisions seem necessary. The new system will be working in the near future.

M. R. Weinstein
Digital Computer Laboratory

### 120 B,N. THE AEROTHERMOPRESSOR

During the past quarter, computations were carried out to check with the experimental results obtained from the Aerothermopressor in the Gas Turbine Laboratory. The agreement is very good. More experiments and theoretical computations are to be carried out in the next quarter.

A. Erickson
Mechanical Engineering

### 126 C. DATA REDUCTION

Problem 126 is a very large data reduction program for use in the Servomechanisms Laboratory. The overall problem is composed of many component sections which have been developed separately and are now being combined into complete prototype programs. Descriptions of the various component sections have appeared in past quarterly reports. After the development and testing of the prototype Whirlwind programs is completed, the programs will be re-coded for other, commercially available, large scale computers, (probably the ERA 1103, IBM 701 and IBM 704 computers), for use by interested agencies for actual data reduction at other locations. The programs are currently being developed by Douglas T. Ross, David F. McAvinn, Walter E. Weissblum, Benson H. Scheff, and Dorothy A. Hamilton, Servomechanisms Laboratory staff members with the assistance of John F. Walsh. This work is sponsored by the Air Force Weapons Guidance Laboratory through DIC Project 7138.

The nature of the problem requires extreme automaticity and efficiency in the actual running of the program, but also requires the presence of human operators in the computation loop for the purpose of decision making and program modification. For this reason extensive use is made of output oscilloscopes so that the computer can communicate with the human, and manual intervention registers so that the human can communicate with the computer in terms of broad ideas, while the computer is running, and have the computer program translate these ideas into the detailed steps necessary for program modification to conform to the human operator's decision. The program which does this translation and modification is called the Manual Intervention Program (MIV). The most recent version of the prototype data reduction program is called the Basic Evaluation Program.

During the past quarter work has continued on the Evaluation Program and the MIV Systems. A number of changes and revisions were made and the complete system has been used to make several test runs for comparison against previously computed values. The elaborate scope plot which was described in the past quarterly report has been written and incorporated into the manual intervention system so that all pertinent functions within the evaluation program may be plotted for any range specified by throwing appropriate switches.

As mentioned in previous quarterly reports, the programs which are being developed on the Whirlwind Computer are prototype programs which are to serve as models for production Univac-Scientific 1103 computer programs. Several meetings have been held with personnel of the Air Force Armament Center, Eglin Air Force Base, Florida, for the purpose of coordinating these efforts with their 1103 installation. Besides the design of these programs, procedures have been initiated for the procurement of additional input-output equipment for the 1103 computer for the use of this and other problems. In particular, a Charactron display tube for visual and photographic facilities, and a complete manual intervention system have been ordered for the 1103.

A proposal has been made and accepted for the addition of a Flexowriter keyboard input facility to the Whirlwind Computer as a result of the requirements of problem 126. This facility will prove highly useful to other users of the Whirlwind Computer as well. The modification, which is scheduled for completion in July, consists of assigning new si addresses to the Flexowriter keyboard in test control (and also an auxiliary Flexowriter at the manual intervention console used by problem 126) so that the Flexowriter keyboard may be used in the same way that a photoelectric or mechanical tape reader now operates. In this mode arbitrary alpha-numeric information may be typed directly into the computer through the in-out system, and arbitrary interpretation of this information programmed into the computer. The Comprehensive System will be modified so that this facility may be used for ordinary CS input when the occasion warrants. Due to the low time efficiency and reliability of this mode of input it is expected that its use will be restricted to applications of a manual intervention nature. Applications of interest to the general user, however, will be to operate the Director Tape Program for program manipulations and operating demonstration programs. Of major interest to problem 126 is the fact that individual keys on the typewriter may be treated as separated intervention switches so that this new facility in effect will add a large number of intervention switches to the present system, in addition to the more general flexibility mentioned above.

In the past quarter, a brief study was made of the possibility of incorporating a new instruction into the Whirlwind repertoire to allow a type of microprogramming on the Whirlwind Computer. The proposed instruction, mi, would allow referencing of individual command pulse output (CPO) lines so that special purpose instructions could be manufactured by the programmer. The study was restricted to consider schemes which would require that





a minimum of additional hardware be added to the Whirlwind Computer and that the normal operation of the computer would be unaffected. These conditions required that the new facility be built out of existing CPO units and consist primarily of a rewiring of certain parts of the control element. Since the program register, PR, is still a part of the hardware of the computer although it is not used at present, this register was chosen as the focal point for the mi instruction and an assignment of CPO units to the 48 gate tubes associated with the PR circuitry was made. It was found that a simple, logical circuit could be made to load the program register from arbitrary core memory locations and use the word thus obtained in PR to energize gate tubes to control individual CPO units on selected time pulses. It is also possible to have an arbitrary number of addresses in the mi instruction, the meanings of the addresses being controlled by the selection of the proper CPO units. The design which evolved calls for the execution of one mi instruction to automatically initiate the reading of an arbitrarily long sequence of core memory address locations into PR for execution one at a time. In this way, arbitrarily long and complex instructions can be manufactured.

After the design was completed, however, it was found that the flexibility which appears to result from the system is in fact quite illusory due to the very tight design of the Whirlwind logical-arithmetic element. A major drawback seems to be that the only access to the accumulator in the Whirlwind arithmetic element is through the A register, so that the AR is always tied inflexibly to the AC. The net affect of this is to require an inordinately large amount of extraneous shuffling of information in order to use the mi facility so that the system is extremely inefficient. Although one of the basic premises upon which this study was undertaken was that the system would not be required to be efficient time-wise, it turns out that it is almost always easier and faster to write a Whirlwind program in the existing order code, to do a particular job, than to manufacture an mi instruction sequence to do it. For this reason the present design will not be given consideration for an actual proposal.

At this time it is very difficult to estimate how much this picture could be improved by allowing the addition of several new and more general CPO units, but it seems probable that quite a drastic revision of this evaluation of an mi facility could result from the choice of a few appropriate additional CPO units. The details of the present study will appear in a Servomechanisms Laboratory memorandum, and a supplementary study to consider alternatives may be made in the future. It is felt quite strongly that not enough information is known at this time about what a microprogramming facility should consist of, nor how one would be used if available, so that some inexpensive facility should be designed and used before consideration is given to building an actual microprogram computer. It may be, however, that such an experimental facility can best be achieved by simulation on existing computers rather than rewiring of existing computers.

D. T. Ross
Servomechanisms Laboratory

## 162 N. NUCLEAR SCATTERING PHASE SHIFTS

This quarter has seen further work in fitting the phase shift combinations to the experimental curve at the first minimum. It has been necessary to take a much finer mesh in this region because of the sensitivity of the phase shift equation.

Future plans include further work along the same lines to fit the curve beyond the first minimum in such a way that the results can be projected to encompass the values of the entire curve at high energies.

Programmers working on this problem are E. Mack and E. Campbell.

E. Mack
Nuclear Science Laboratory

168 D. INDICIAL DOWNWASH BEHIND A TWO-DIMENSIONAL WING

This problem is now completed. The results will be found in an Sc. D. thesis, entitled "Indicial Downwash and Its Structural Effect on the Horizontal Tail", submitted by N. P. Hobbs to the MIT Aeronautical Engineering Department, June 1956. The results of the problem will also be published as a Wright Air Development Center Technical Report 56-164.

N. P. Hobbs
Aeroelastic and Structures Research Lab.

179 C. TRANSIENT TEMPERATURES AND STRESSES IN A BOX-TYPE BEAM

The programs have been modified to obtain transient temperature and stress responses for the same structure for different thermal inputs.

During the next quarter, the program will be repeated for additional thermal inputs to define the dependence of these responses in terms of input parameters.

J. C. Loria
Aeronautical Engineering

193 L. EIGENVALUE PROBLEM FOR PROPAGATION OF ELECTROMAGNETIC WAVES

Calculations at 410 Mc and 3000 Mc using the bilinear model have been continued, as described previously.

Eigenvalues have been obtained for the inverse-square model at 50 Mc using power series expansions. Work has been done toward similar calculations using asymptotic expansions and the calculation of normalized eigenfunctions by asymptotic and power series methods.

Programmers working on this problem are Professor H. B. Dwight and Dr. R. M. Ring.

R. M. Ring
Lincoln Laboratory

199 N. STEADY LAMINAR FLOW OF A COMPRESSIBLE FLUID IN THE ENTRANCE REGION OF A TUBE

Solutions for the first and second sets of the differential equations were obtained for the case where temperature dependence of the viscosity and thermal conductivity of the compressible fluid is taken into consideration.

More solutions are to be obtained for different entrance Mach numbers and thermal conditions at the tube wall.

T. Y. Toong
Mechanical Engineering

219. TRANSPORTATION PROBLEM

Changes have been made in the program to solve the classical transportation problem by the stepping-stone method, (see Summary Report #45) which will allow the storage of cost data without specifically storing costs for shipping routes which are





known to be impractical or absurd. Also, provision has been made for storing the program and basis table on the buffer drum, thus making the whole auxiliary drum available for storage of cost data. With these improvements, the program has the capacity to handle up to approximately 10,000 significant costs.

In the course of trying to solve a large problem ($m = 60$, $n = 291$), a number of coding errors showed up that had not been detected earlier. These have been tracked down and the 60 x 291 problem has been solved in 30 minutes of computer time.

J. B. Dennis
Electrical Engineering

226 D. CIRCULATION OF THE ATMOSPHERE

During this quarter a program has been written by Martin Jacobs for solving by relaxation the equation

$$G \nabla_s^2 \frac{\partial h'}{\partial t} - H \frac{\partial}{\partial y} \left( \frac{\partial h'}{\partial t} \right) - I \frac{\partial h'}{\partial t} + \alpha = 0$$

where G, H, and I are known functions of the initial conditions and $\alpha$ is a unit impulse function. This program is nearly checked out. With this form of the equation the solution will be a Green's function which can be used in obtaining iterated solutions of the original equations.

Preparations are being made to combine the various parts of the program which have been successfully checked out so that the solutions to the equations can be computed in a continuous iterative process.

Programmers working on this problem are P. Castillo, M. Jacobs and D. Cooley.

D. S. Cooley
Meteorology

234 N. ATOMIC INTEGRALS

A subprogram has been written and tested which evaluates the radial integrals arising from matrix elements of

$$\frac{\partial}{\partial R} \frac{1}{r_x}$$

where $r_x$ is distance measured from a point X itself at distance R from the origin of coordinates. The radial integrals involve products of two normalized atomic orbitals

$$\eta_a(r) = N_a \, e^{-\zeta_a r} \, r^{A+\ell_a}$$

and the radial operator

$$g_L(r,R) = \begin{cases} \dfrac{(L+1)\, r^L}{R^{L+2}} & , \ r < R \\ -\dfrac{L R^{L-1}}{r^{L+1}} & , \ r > R \end{cases}$$

These integrals are evaluated by simple recurrence relations similar to those for the operator $1/r_x$, described elsewhere.*

R. K. Nesbet
Solid State and Molecular Theory Group

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* Quarterly Progress Report, Solid State and Molecular Theory Group, October 15, 1955, p. 9.

236 C. **TRANSIENT RESPONSE OF AIRCRAFT TO HEATING**

During the past quarter the WWI has been primarily used to evaluate characteristic values and characteristic vectors for matrix equations of the form $Ax = \lambda Bx$. The program developed by F. J. Corbato and described in Memorandum DCL-58 has been employed.

The work accomplished is reported in Wright Air Development Center Technical Report 56-287, entitled, "Application of the Variational Method, the Galerkin Technique, and Normal Coordinates in a Transient Temperature Distribution Problem", which is soon to be published.

At this time, it appears that future use of the WWI Computer on this problem will be limited to a) production runs of existing programs; b) determination of characteristic values and vectors for matrix equations; c) inversion and multiplication of matrices.

L. A. Schmit
Aeronautical Engineering

241 B. **TRANSIENTS IN CONTINUOUS DISTILLATION SYSTEMS**

A study of linear approximations in solution of control problems and problems involving the rate of change from one equilibrium state to another has been expanded. Correlations based on these approximations have been developed for determining the approximate of a column as the time after a sudden change in operating variables becomes large.

A final report on this problem should be completed shortly.

S. H. Davis, Jr.
Chemical Engineering

244 C. **DATA REDUCTION FOR X-1 FIRE CONTROL**

This problem is concerned with computing from fire control signals how far fictitious projectiles miss an observed target. The program tape was ready prior to this quarter. Work during this quarter consisted in continuing to reduce new target run data by use of the program tape.

Future plans for the problem again call for still more data reduction, perhaps also using supplementary programs (yet to be prepared) for clearer presentation of the reduced data.

J. M. Stark
Instrumentation Laboratory





245 N. **THEORY OF NEUTRON REACTIONS**

During this period the program was revised to give greater accuracy for $\ell = 5$ and $\ell = 6$, to make the cross sections a function of $\mathcal{X}_0$ instead of $\mathcal{X}_0^2$ and to let

$$\delta = \frac{C}{\mathcal{X}_0}$$

where C is a constant. Several cases with $x = 0$ and C and $\zeta$ as parameters were run. On the basis of these a complete case was done for $C = 1.65$ and $\zeta = .08$. For this same C and $\zeta$ cases were done varying $\mathcal{X}_0$ and letting

$$\frac{\mathcal{X}_0^2}{x^2} = 42.$$

From these, angular distributions were calculated in order to make comparisons with experimental values.

Future plans are to do more cases in the vicinity of the present C and $\zeta$.

E. Campbell
Nuclear Science Laboratory

246 B,N. **SCATTERING FROM OXYGEN**

This problem is now terminated. The results can be found in a Ph.D. thesis entitled "Polarization and Exchange Effects in the Scattering of Electrons From Atomic Oxygen" submitted by Aaron Temkin to the MIT Physics Department, May, 1956.

A. Temkin
Physics Department

253 N. **AN AUGMENTED PLANE WAVE METHOD FOR IRON**

Little machine time has been used during the past quarter. That used has been on a few calculations on the fine structure of atomic iron.

Future plans are to obtain the energy band structure of face- and body-centered iron.

J. H. Wood
Solid State and Molecular Theory Group

256 C. **WHIRLWIND I- UNIVAC SCIENTIFIC 1103 INPUT TRANSLATION PROGRAM**

During the past quarter, work described in Summary Report No. 45 on the WWI-1103 Input Translation Program to enlarge the vocabulary of the system has been completed. Numbers of the general form

$$\pm 12.3456 \cdot 10^5 \cdot 2^{-3}$$

may be written where integer numbers only were previously allowed.

A Digital Computer Laboratory report describing the entire system will be completed during the next quarter.

J. M. Frankovich
Lincoln Laboratory

260 N. ENERGY LEVELS OF DIATOMIC HYDRIDES

Results of the work to date have been submitted as a thesis to the Physics Department at MIT. For details see the Quarterly Progress Report, Solid State and Molecular Theory Group, MIT, July 15, 1956. p. 8.

The problem is being continued with the hope of calculating other molecular quantities, such as dipole moment.

A. J. Freeman
Solid State and Molecular Theory Group

261 C. FOURIER SYNTHESIS FOR CRYSTAL STRUCTURES

During the past quarter, Professor Buerger continued with the refinement of the structure of Wollastonite.

The structures of Diglycinhydrochloride and -bromide are completely solved. A very accurate solution has been obtained for the chloride. Both structure determinations are now prepared for publication.

Programmers working on this problem are Professor M. J. Buerger and T. Hahn.

M. J. Buerger
T. Hahn
Geology

262 N. EVALUATION OF TWO-CENTER INTEGRALS

Some of the exchange and hybrid integrals which were evaluated by Merryman's program were found to be inaccurate. These integrals have been recalculated by F. J. Corbato's routine. The Whirlwind time per integral averages about 80 seconds.

H. A. Aghajanian
Solid State and Molecular Theory Group

264 C. OPTIMIZATION OF AIRCRAFT ALTERNATOR REGULATING SYSTEM

The program for minimizing a function of n variables in the presence of equality constraints by means of the method of steepest descent has been rewritten in order to utilize the secondary storage units of the computer. Thus, it is now possible to solve problems of relatively large magnitude. Changes in the order of computations have been made to reduce the time required to achieve a solution. This new program has been tested and proven capable of handling a 27 variable problem.

In the next quarter, the optimization of an aircraft alternator with respect to weight will be run. In addition, work on improving the methods of computation will continue.

Programmers working on this problem are R. R. Brown and J. B. Dennis.

R. R. Brown
Energy Conversion Laboratory
Electrical Engineering





266 A. CALCULATIONS FOR THE MIT REACTOR

For this past quarter, the most work has been done on improving the input data to refine answers. Future plans are to continue this improvement throughout the summer.

T. Cantwell
Chemical Engineering

270 B. CRITICAL MASSES IN $D_2O$ MODERATED REACTORS

During the past quarter, the following programs were completed and production runs finished: 1) Serber Wilson sphere, 2) Fourier 3-group sphere, 3) Two-group cylinder, 4) Fourier 3-group (approximate) cylinder, 5) Fourier 3-group (exact) cylinder.

The data is consistent with theory for the most part.

Future plans include further debugging of the multi-group transport program for spherical reactors.

J. R. Powell
Chemical Engineering

272 L. GENERAL RAYDIST SOLUTION

This problem was solved satisfactorily by the method described in previous Summary Reports. A full description of the method and program can be found in Lincoln Laboratory Memorandum 2M-0508, "Solution of General Raydist Equations on Whirlwind I".

A. Zabludowsky
Digital Computer Laboratory

273 N. COSMIC RAY AIR SHOWERS

In the last Summary Report we mentioned that we were going to change the empirical function that is used in the least-squares curve-fitting part of the program. This has been done and now there is a unique value of each parameter of the fit for a given fit. The resolution of the method we use will be determined by a Monte Carlo type of calculation in which artificial showers will be made up and Poisson fluctuations will be put in. The program is virtually in its final form now. The computation time per shower is about 45 seconds and any changes in the program will probably be for the purpose of further reduction of the time.

Most of the computer time used by us in the past quarter was devoted to analyzing air shower data. This will be the main effort in the next quarter. There is a backlog of over 1000 showers to be analyzed.

F. Scherb
Cosmic Ray Group
Physics

278 N. ENERGY LEVELS OF DIATOMIC MOLECULES (LiH)

We are continuing with the self-consistent field molecular orbital treatment of the electronic energy. The binding energy for the ground state has been obtained for five internuclear distances. The molecular orbital approach, however, becomes increasingly poor as the internuclear separation increases, and it will be necessary to apply additional configuration interaction to the molecular orbital ground state calculation. Work is continuing at more extended internuclear distances using the conventional configuration interaction scheme discussed in previous reports.

Programmers working on this problem are A. M. Karo and A. R. Olson.

A. M. Karo
Solid State and Molecular Theory Group

288 N. ATOMIC WAVE FUNCTIONS

Calculations have continued on the hyperfine structure of lithium and boron, and on a wave function of high accuracy for the ground state of helium.

Calculations on 4-electron molecules by the one-center method are still in progress.

Approximate wave functions are being obtained for several configurations of Fe and for Mn, $Mn^{++}$, and $Cr^{++}$.

R. K. Nesbet
R. E. Watson
Solid State and Molecular Theory Group

290 N. POLARIZATION EFFECTS IN THE FLUORINE ION

The numerical integration of the equations for the perturbed functions $V_{\ell \to \ell'}$ still show considerable sensitivity to the smoothness of the potential and the mesh size (we are using the Kutta-Gill library subroutine). However, for the uniform field case we have obtained a complete check on the perturbed functions of d-like symmetry and considerable improvement in the p- and s-like ones. Recently we have obtained the V function of d-like symmetry for the dipole term of a point charge perturbing potential at a finite distance, R, from the origin. For R = 1.76 atomic units the V function for Ne has a behavior near the origin very similar to the corresponding uniform field ($R = \infty$) case and a maximum at approximately r = R.

Because of the delays and difficulties in obtaining our distorted functions we have undertaken a calculation for the HF molecule based on a small set of unperturbed functions to describe the $F^-$ ion and including a d-like distorted function similar to the one described above. To carry through this problem we have used R. K. Nesbet's tapes for the atomic integrals, potential integrals, A matrix preparation, Roothaan scheme and integral transformation. These programs have all been described by R. K. Nesbet in previous Whirlwind I Summary Reports.

L. C. Allen
Solid State and Molecular Theory Group

293 C. ROLLING BEARINGS

Progress has been held up while a new routine for non-linear equations is under investigation. If this new routine is proved useful, it will be applied to the bearings equations.

A. J. Shashaty
Lubrication Laboratory
Mechanical Engineering





297 C. DIFFUSION BOUNDARY LAYER

The effects of mass transfer have been studied numerically in connection with an analysis of the binary-mixture boundary layer. A system of equations describing mass, momentum, and energy conservation for the high-speed mixture boundary layer was numerically integrated to yield "similarity" solutions for the case of relatively large and small densities for the injected mixture component. Beneficial alteration of the heat transfer rate, skin friction, and ultimate equilibrium ("recovery") temperatures at supersonic velocities were shown to result upon employing low density "coolants" such as, for example, helium. Primarily, the thermal effect is a consequence of the relatively large thermal capacity for such gases and a major requirement of the diffusion analysis is the determination of the specific heat variation through the layer.

The Gill-Kutta procedure was used for integration purposes. Simultaneous solution of the differential equations (of a total order of 7) with two-point boundary conditions was completed with the aid of a linear influence coefficient routine. Successive integrations were programmed with initial estimates being self-corrected so as to yield proper end-point conditions. For a majority of the cases considered, approximation of the conditions was sufficiently reliable as to result in convergence within a time period on the order of a few computation periods of the results for the exact values.

This problem is now terminated. A more complete description of the results will be found in a doctoral thesis submitted by the author to the MIT Aeronautical Engineering Department, June 1956.

J. R. Baron
Naval Supersonic Laboratory

300 L. TROPOSPHERIC PROPAGATION

The last Summary Report on Problem 300 stated that tables of values of the Fresnel integrals for complex arguments had been computed using a convergent power series. These tables have been extended by using asymptotic series to regions in which convergence of the power series is too slow for machine computation. These tables are to be extended in the coming quarter.

To simplify programming, closed subroutines using program parameters have been written and tested which carry out the common arithmetic operations on complex numbers. The use of these subroutines permits a program to be written in which real and imaginary parts of complex quantities do not need to be considered separately. It is planned to construct interpretive subroutines for complex numbers, using a complex MRA, so that a single operation may be packed into one register.

A closed subroutine has been written which obtains all the ordinates and abscissae of the extreme of a real single-valued function of a real variable throughout a specified range of the independent variable.

The autocorrelation function and power density spectrum of several samples taken from SHF signal strength records have been computed. Examination of the results indicates that improved methods of statistical analysis must be employed.

A number of matrices, ranging in order from six to twenty-five, were solved for their largest Eigenvalue and Eigenvector. The results of these computations are to be used in a report on the design of scatter antennas.

The LSR MA 1 was used to compute these Eigenvalues and vectors. An error in the coding of this LSR was found, and has been corrected to give satisfactory results.

Programmers working on this problem are W. Mason, P. Duffy and J. Skenian.

W. Mason
Lincoln Laboratory

306 D. SPECTRAL ANALYSIS OF ATMOSPHERIC DATA

The program for this problem has been successfully checked out and during the quarter, the routine use of this program was begun.

It is anticipated that during the next quarter the extensive computations for which the program was written will be completed.

B. Saltzman
Meteorology

309 B,N. PERFECT AND ALMOST PERFECT KCl

For the perfect crystal the calculation of matrix elements was completed, and the energy bands were calculated. The energy levels were calculated for twenty independent values of the one-electron propagation constant k. Thus for each of these values of k an 8 x 8 secular equation with overlap terms was solved, using a program written by F. J. Corbato; this order results from inclusion of eight different types of free ion orbitals, i.e., $Cl^-$ 3px, 3py, 3pz, 3ps, and $K^+$ 3px, 3py, 3pz, and 3ps. The resultant $Cl^-$3p band had a minimum at the Brillouin Zone Face in the (111) direction in k space and a maximum of very little curvature at the same point. The matrix elements involved were used to calculate a value for the cohesive energy of KCl; the result compared well with experiments and with previous calculations done in the same general manner.

For the imperfect crystal the calculation of additional matrix elements is almost complete and preliminary secular equations for energy levels have been solved. This work will be completed during the coming quarter.

L. P. Howland
Solid State and Molecular Theory Group

312 L. ERROR ANALYSIS

The following modification has been made on this problem, as first described in Summary Report No. 43. A three dimensional system in space rather than a two dimensional system on a surface leads to the solution of a 3 x 3 matrix (rather than a 2 x 2) for the eigenvalues and one eigenvector as final results. There will be other modifications and changes with the goal of studying different systems.

Programmers working on this problem are L. Peterson and E. Hutcheson.

L. Peterson
Lincoln Laboratory





317 C. EXTRACTION OF STABILITY DERIVATIVES FROM FLIGHT TEST DATA

The non-linear pitching moment and lift coefficients have been successfully extracted from simulated flight records over an angle of attack range from -0.1 radian to +0.25 radians. A three second response and a time increment of .1 second between data points were used. Programs are now being written which will permit the extraction of the derivatives over all angles of attack of interest and also for the extraction of the lateral stability derivatives.

A program written by Miss Katherine Kavanagh of the MIT Dynamic Analysis and Control Laboratory has been used to obtain the simulated responses for the above studies and also has been used to verify analytic studies of the cross coupling between the lateral and longitudinal modes of a high-speed fighter aircraft.

Programmers working on this problem are T. M. Carney, L. L. Mazzola, M. N. Springer, and L. E. Wilkie.

L. L. Mazzola
Aerophysics Research Group

326 C. PRODUCTION FOR TRANSPORTATION PROBLEM

Work done under problem 326 during the past quarter has been reported on in problem 219.

W. S. Jewell
Electrical Engineering

327 L. PREDICTION ANALYSIS

The problem , as described in Summary Report No. 45, is in working order. There was one modification completed, with respect to alternate methods of computing certain functions. During the past quarter the major activity has been production runs. For the future, the main body of this problem will be incorporated in a much larger systems study.

Programmers working on this problem are L. Peterson and E. Hutcheson.

L. Peterson
Lincoln Laboratory

330 C. POSTFAILURE RESPONSE OF AIRCRAFT STRUCTURES

Numerical solutions for the postfailure response of a beam represented by four lumped masses have been obtained using a finite difference solution of the equations of motion. Several simplifications have been explored and have been found to be in good agreement with the "exact" numerical solution. In addition, the WWI solution has shown satisfactory correlation with some experimental results. It is contemplated that this study will be extended to include the postfailure response of structures that are characterized by plate-like deformation patterns.

R. D'Amato
Aeroelastic and Structures Research Lab.

333 A. BOUNDARY-LAYER CHARACTERISTICS OF A STEADY LAMINAR FLOW OF A COMBUSTIBLE MIXTURE OVER A HOT SURFACE

Solutions were obtained for two specific values of the surface temperature and for a second-order reaction of the Arrhenius type.

The effect of activation energy is to be studied in the future.

Programmers working on this problem are Professor T. Y. Toong and A. Shashaty.

T. Y. Toong
Mechanical Engineering

334 C. PARAMETRIC STUDY OF MODAL COUPLING AND DAMPING

Previously a program had been formulated which would yield the simultaneous solution of three coupled vibratory equations. This program gave the modal response in each of the three modes for one set of initial conditions. During this quarter, the program was altered to yield solutions for five sets of initial conditions in a single run.

Two production runs, each for a specific airplane altitude, have been run successfully on a given airplane for five airplane velocities. Future plans are to run similar parametric studies on several other airplanes.

K. Wetmore
Aeroelastic and Structures Research Lab.

338 C. OPTIMIZATION OF RAM-AIR COOLING SYSTEM

During this quarter, work on problem 338 was completed. The results of the project, which will be published in a Wright Air Development Center Technical Report entitled: "Cooling and Materials Investigations for Aircraft Generator Equipment", are summarized as follows: 1) Optimum ram-air cooling systems have been found for a specific set of flight conditions in the range between Mach 1 and Mach 4, and Sea Level and 100,000 feet; 2) Subroutines for finding the weights of air-to-air heat exchanges and air compressors have been developed; 3) The variation of loss of an aircraft generator, as the machine weight is changed, was computed.

R. Moroney
Energy Conversion Group
Electrical Engineering

339 A. VIBRATING BEAM

A recurrence formula for integrating

$$\frac{\partial^4 \psi}{\partial x^4} + \frac{\partial^2 \psi}{\partial t^2} = 0$$

was experimentally found to be unstable. A program for finding the eigenvalues of the recurrence formula was written and run. The results of this explained the instability and indicated how the recurrence formula should be improved. It is hoped to further





improve the recurrence formula by using higher order approximations to the boundary conditions.

S. H. Crandall
Mechanical Engineering

341 C. STATISTICAL AND DYNAMIC FORECASTING METHODS

As explained in the previous Summary Report, the major effort is being devoted toward the systematization of a method for multiple linear prediction. The programs for computing the covariances and performing the diagonalization were completed. These programs were applied successfully to sea level pressures over a network of 64 stations. In the future it is expected that these programs will be applied to other sets of data, i.e., temperature, pressure change, etc.

This method was applied to the functions (Z's) developed by the Synoptic Climatology project. The functions chosen were those computed for the 700 mb 5-day mean height anomalies. This approach to 5-day prediction will be expanded by the incorporation of temperature parameters. A final expression for the prediction equation will then be formed.

Processing of climatological data for multiple linear prediction by these techniques has demanded continued use of Whirlwind I.

Numerical experiments are being continued to find a satisfactory simplified meteorological forecasting model suitable for generating a sizeable climatic record through extended integration with respect to time. This climatic record may be particularly useful in investigating properties of least squares prediction as applied to a non-linear system similar to the atmosphere.

The work being done is under the supervision of Professor Edward N. Lorenz, Meteorology Department.

Programmers working on this problem are E. A. Kelley, B. Shorr and K.Bryan.

E. A. Kelley
Meteorology

342 B. TRANSIENT HEATING OF SOLIDS

When a slab having a constant initial temperature is suddenly brought into contact with another medium having an invariant temperature (either more or less than that of the slab) a flow of heat will occur within the slab with accompanying temperature changes. The problem of what the temperature is at any specific point demands knowledge of the period of heat flow, physical properties of the material, the initial slab temperature and surrounding medium's temperature which are embodied in Fourier's General Law of Heat Conduction, or

$$\frac{\partial^2 t}{\partial x^2} = \frac{\partial t}{\partial \theta}$$

While the boundary conditions specify the resistance to heat flow,

$$k \frac{\partial t}{\partial x} \bigg|_{x=\pm L} = \pm h(t_a - t_s)$$

This problem is solved using the simplified assumptions associated with semi-infinite and finite isotropic, homogeneous solids with uni-dimensional heat flow, i.e., physical properties are constant irrespective of temperature.

Defined mathematically, a semi-infinite solid is bounded by planes at $x = 0$ and $x = \infty$ while a finite solid has boundaries at $x = -L$ and $x = +L$.

Without a surface resistance to heat flow ($h = \infty$, $m = 0$), a solid immediately upon contact with the surrounding medium assumes the external temperature upon its faces so that $t_s = t_a$.

During the past quarter values were obtained for the following equations and also for the first root of $m\beta = \cot\beta$.

$$Y_m = \frac{4}{\pi} \sum_{i=0}^{\infty} \frac{(-1)^i e^{-(2i+1)^2 \frac{\pi^2}{4} x}}{(2i+1)} \qquad 1.$$

$$Y_{avg} = \frac{8}{\pi^2} \sum_{i=0}^{\infty} \frac{e^{-(2i+1)^2 \frac{\pi^2}{4} x}}{(2i+1)^2} \qquad 2.$$

During the next quarter it is hoped that certain errors in the iteration routine for $m\beta = \cot\beta$ will be ironed out so that the following equations can be readily solved in a manner similar to that used for equations 1. and 2.

$$x = \pm L \quad Y_s = 2 \sum_{i=1}^{\infty} \frac{m}{(m^2\beta_i^2 + m + 1)} e^{-\beta_i^2 x} \qquad 3.$$

$$x = 0 \quad Y_m = 2 \sum_{i=1}^{\infty} \frac{m}{(m^2\beta_i^2 + m + 1)\cos\beta_i} e^{-\beta_i^2 x} \qquad 4.$$

$$Y_{avg} = 2 \sum_{i=1}^{\infty} \frac{e^{-\beta_i^2 x}}{\beta_i^2 \left[m^2\beta_i^2 + m + 1\right]} \qquad 5.$$

R. Gelman
Mechanical Engineering





## 343 C. WEATHER PREDICTION

During the current quarter the research program has been concerned with two phases of Weather Prediction.

Data for December and January 1948-52, inclusive, were utilized to classify weather maps into three classes, according to the vertical structure of the weather systems. Linear prediction equations were developed for each class for the determination of pressure change. An interesting and unexpected result was obtained. It was found that the accuracy of the prediction differed markedly between classes. Attention is now being directed toward an explanation of this result since it should lead to more accurate weather prediction by statistical methods.

The research on the development of new predictors has been completed. With the new predictors it is now possible to work directly with data from reporting stations. It is no longer necessary to draw weather maps and read off grid-point values. Further, these new predictors make it possible to represent the maximum amount of variance of a field with a minimum number of predictors.

In the future it is planned to pursue the work on linear prediction for various classes of weather systems. Data is being processed at the National Weather Records Center in Asheville, N.C. for inclusion in this program. The results to date indicate that this line of approach is a fruitful one.

Work on this problem has been under supervision of Professor James M. Austin, Meteorology Department.

Programmers working on this problem are E. A. Kelley, B. Shorr, H. Brun and R. Huschke.

E. A. Kelley
Meteorology

## 345 B. MATRIX MULTIPLICATION

The portion of the problem which was used for a doctoral thesis is now completed. The thesis was submitted by R. R. Archer to the MIT Mechanical Engineering Department.

In the future, computations will be completed to prepare this problem for publication.

Programmers working on this problem are D. Grine and R. Archer.

R. R. Archer
Mechanical Engineering

## 346 B. ANALYSIS OF COMPLEX SPECTRA

The object of the analysis of the spectrum of an atom is to determine the spacing and quantum numbers of the energy levels of the atom. In principle, this can be done by measuring the wave length of all the spectral lines emitted by the atom, converting these wave lengths to vacuum wave numbers, and then performing all possible subtractions of these wave numbers and looking for repeated differences.

A complex spectrum such as is emitted by the rare earth elements contains several thousand spectral lines. Since none of the wave lengths can be measured exactly, many repeated differences will be obtained when performing the subtractions which do not correspond to true energy level separations in the atom. For this reason, additional data must be obtained. These data are usually obtained by means of Zeeman Spectra in which the sample is excited in a magnetic field. The field will cause the spectral lines to split into many components. The pattern thus obtained enables one to measure two quantum numbers of the energy levels involved in the transition. The consistency of these quantum numbers in the various transitions will separate the true energy level spacings from the false ones.

The Whirlwind I computer will be used in this problem to convert the measured wave lengths to vacuum wave numbers by solving a three term polynomial equation, perform all the subtractions, search for repeated differences, and check the consistancy of the Zeeman data. This will be done for Erbium which has approximately 2500 spectral lines in the visible and near-ultraviolet. Once the program has been set up, it can be used for the analysis of any other complex spectrum.

During the past quarter, the analysis of the spectra of two elements, Er and Pr, was started. The computer was used to search the Zeeman data of these spectra for repeated values. Then the difference in the wave numbers for these lines was computed. The energy differences which are needed to complete the analysis are found by searching for repeated wave number differences. None of these were obtained in the initial analysis. This is believed to be due to the fact that the data available at this time is not complete enough to give the desired results. The computer was then used to search the complete list of wave numbers for these elements for selected repeated differences. Several repeated differences were found which appear to be significant.

The next step in the analysis consists in obtaining additional Zeeman data and searching for any additional energy level differences. It now appears as if the desired results will be obtained in the near future.

J. W. Lindner
Spectroscopy Laboratory
Physics

350 D. LONG-RANGE FORECASTING

The 30-day forecasting problem is to be attached by means of a method for specifying (or reconstructing) monthly mean pressure anomaly maps in a completely objective mathematical way. The method involves the use of "empirical orthogonal functions", determined from a long series of past data, which will perform analogously to (but more efficiently than) analytical orthogonal functions such as spherical harmonics. The coefficients of the empirical functions will be put into least-squares linear regression forecasting equations. Aside from forecasting possibilities, this approach will give us much valuable information about the characteristics of long-period mean maps.

The original description of the problem stated that WWI will compute as a first step a network of covariances; later it will be used to diagonalize the matrix of these covariances--a matrix which may be as large as 100 x 100. This matrix has been converted to a 90 x 90 matrix of correlations. Because of its size, this matrix must be partitioned and diagonalized in sections, only the major eigenvectors and eigenvalues being retained.

The three sets of variances and covariances required in the original statement of the problem have been computed. These have been combined and converted by hand into a 90 x 90 matrix of correlations. The diagonalization of partitioned sections of the matrix has been completed. The recombination of the matrix sections and final diagonalization will occupy the next few weeks. The set of eigenvectors so obtained will





constitute the "empirical orthogonal functions" mentioned in the original problem statement. The determination by matrix multiplication of the amplitude coefficients of these functions is the next major task for machine computation.

Programmers working on this problem are E. Kelley, B. Shorr, and D. L. Gilman.

D. L. Gilman
Meteorology

351 B. NON-UNIFORM FUEL DISTRIBUTION

A general computational program based on the finite differences approximation of simultaneous differential equations for a 2-group, multi-region reactor (Age-Diffusion equations) has been successfully set up. Fast and thermal distributions of modified fluxes ( r x Flux) for various critical spherical assemblies have been computed and checked with analytical solutions.

In order to obtain a uniform volume power density in the core, the concentration of the fuel is deliberately non-uniformized in various zones of the core. The calculations are performed on a prototype high power reactor. Due to very high flux level, fission poisoning will be taken into account. Criticality will be adjusted by varying the total critical mass and thickness or composition of reflectors.

A. Sutter
Nuclear Engineering

354 D. RESPONSE OF A SINGLE STORY CONCRETE BUILDING TO DYNAMIC LOADING

The object is to study the dynamic response of the roof of a single story reinforced concrete building for various loading functions.

The roof is a system of infinite degree of freedom consisting of a one way slab supported on relatively stiff beams. The roof system was resolved into a two degree of freedom system by assuming deflection shapes for the one way slab and its supporting beams and using Lagrange's equation to obtain the two dynamic equations

$$\ddot{y}_b = \frac{C_1 P + C_2 y_s - C_3 y_b}{M_1}$$

$$\ddot{y}_s = \frac{P - C_4 y_s - C_5 \ddot{y}_b}{M_2}$$

where $\ddot{y}_b$ and $\ddot{y}_s$ are the accelerations of the center of the beam and slab respectively

$y_b$ is the deflection of the center of the beam

$y_s$ is the deflection of the center of the slab relative to the center of the beam

P is the total load on the slab area

$M_1$ and $M_2$ are masses, and

$C_1$, $C_2$, $C_3$, $C_4$, $C_5$ are constants depending on the assumed deflected shapes and resistance characteristics of the system.

The numerical integration portion of the program calculates the deflections of the beam and slab for successive time intervals by utilizing the backward difference equation

$$y_{(t+1)} = 2\,y_{(t)} - y_{(t-1)} + \ddot{y}(\Delta t)^2$$

From these delfections new resistances are computed and the process repeated. This numerical procedure is repeated approximately 100 times per trial loading.

The loading portion of the program is an empirical curve which is a function of the shock wave characteristics (i.e., square wave, triangular wave, etc. and shock strength).

The control portion of the program consists of choosing a new shock strength for the next trial loading on the basis of the maximum deflections of the slab previously obtained.

During the past quarter, the program has been entirely written and debugged and is now running. A great many more runs will be made this summer with smaller programs which are segments of the program which is now running.

B. Landry
Civil Engineering

355 B. QUANTIZATION ERROR

This problem was proposed by Mr. Bernard Widrow in his thesis, "A Study of Rough Amplitude Quantization by Means of Nyquist Sampling Theory." He developed therein a theoretical proof that quantization error is statistically random and independent of quantizer input when Nyquest's restriction, applied to amplitude sampling (quantization) is followed, i.e., sampling rate should be at least twice as high as the highest frequency component of the probability distribution.

Experimental check on this theory can be obtained by determining the effect of roundoff on the autocorrelation function of a set of data. This can easily be accomplished by the WWI computer, using the autocorrelation program devised by Mr. Douglas T. Ross of the MIT Servomechanisms Laboratory, and a supplementary program causing repeated determination of this function (about five times) after binary roundoff, one bit at a time. The critical sampling rate thus determined can then be compared with that predicted by the Nyquist criterion, as a measure of the theory above.

This problem was concluded after determination of the autocorrelation functions of error due to roundoff for three sets of data. These error functions, deduced from computed functions of the data before and after roundoff, agreed with Mr. Widrow's theory that this error is uncorrelated and of flattop probability distribution. The results of this problem were submitted as a Bachelor's thesis to the MIT Department of Electrical Engineering.

H. Pople
Electrical Engineering

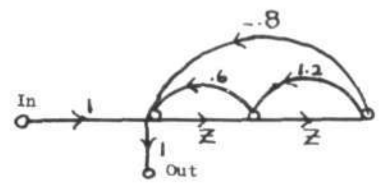




357 B. PROPAGATION OF ROUNDOFF ERROR

The flow graph for the second order difference equation to be studied is shown in Fig. 1. At each multiplication (i.e., ...6, 1.2, -.8), artificial quantization is introduced by means of the shift orders. The input supplies random numbers, in order to obtain statistics for the output. An unquantized solution is obtained for each step, in addition to the quantized solution. The difference between the unquantized and the quantized solution is formed. The mean and second-moment for this difference is obtained for many different runs at each of the 50 steps of the solution. Also, a histogram of the 50th step error is kept.

The problem will be solved, using different levels of quantization. The experimental results will be compared with the theoretical predictions of B. Widrow's Doctoral Thesis Proposal, "A Study of Rough Amplitude Quantization by Means of Nyquist Sampling Theory" (Memorandum 6M-3862, Lincoln Laboratory, MIT).




$z = \epsilon^{-sT}$
T is period between samples

Fig. 1 Second Order Difference Equation

The problem was completed during this quarter. The results compared favorably with theoretical predictions based on B. Widrow's doctoral thesis. The results of the problem, together with the analysis, appeared in my bachelor's thesis ("Propagation of Quantization Error", S.B. Thesis, MIT, Department of Electrical Engineering, June, 1956). Since the desired results were obtained, there are no future plans for the problem.

A. I. Green
Electrical Engineering

358 B. VERTICAL TAIL LOADS DUE TO ROLLING PULL-UP

The purpose of this problem is to find the loads in the vertical tail of my 16.72 airplane design. The critical maneuver to be investigated may be described as a rapid roll in the presence of a high normal load factor. The maximum tail load obtained for this rolling pull-up will then be compared with that value found by neglecting non-linear and quasi-linear terms of the system. From this comparison we should be able to determine how large an effect the non-linearities of the system have on the vertical tail load.

The Lateral equations of motion of the airplane are:

$$.00263\,\ddot{\varphi} + .0322\,\dot{\varphi} + .00101\,\ddot{\psi} - .0255\,\dot{\psi} + .026\beta = .00229\,\delta_a(t)$$

$$.00101\,\ddot{\varphi} - .00991\,\dot{\varphi} + .0115\,\ddot{\psi} + .0116\,\dot{\psi} - .063\beta = -.00063\,\delta_a(t)$$

$$-.272\,\sin\varphi + 2.03\,\dot{\psi} + .4\beta + 2.03\dot{\beta} = 0$$

Applying the trapezoidal rule we have in finite difference form:

$$\left[.00263 + .0322\,\frac{h}{2}\right]\ddot{\varphi}_n + \left[.00101 - .0255\,\frac{h}{2}\right]\ddot{\psi}_n + .026\,\frac{h^2}{4}\,\ddot{\beta}_n = .00229\,\delta_a(t)$$

$$-.0322\left[\dot{\varphi}_{n-1} + \frac{h}{2}\ddot{\varphi}_{n-1}\right] + .0255\left[\dot{\psi}_{n-1} + \frac{h}{2}\ddot{\psi}_{n-1}\right] - .026\left[\beta_{n-1} + h\dot{\beta}_{n-1} + \frac{h^2}{4}\ddot{\beta}_{n-1}\right]$$

$$\left[.00101 - .00991\,\frac{h}{2}\right]\ddot{\varphi}_n + \left[.0115 + .116\,\frac{h}{2}\right]\ddot{\psi}_n - .063\,\frac{h^2}{4}\,\ddot{\beta}_n = -.00063\,\delta_a(t)$$

$$+.00991\left[\dot{\varphi}_{n-1} + \frac{h}{2}\ddot{\varphi}_{n-1}\right] - .0116\left[\dot{\psi}_{n-1} + \frac{h}{2}\ddot{\psi}_{n-1}\right] + .063\left[\beta_{n-1} + h\dot{\beta}_{n-1} + \frac{h^2}{4}\ddot{\beta}_{n-1}\right]$$

$$2.03\,\frac{h}{2}\,\ddot{\psi}_n + \left[.40\,\frac{h^2}{4} + 2.03\,\frac{h}{2}\right]\ddot{\beta}_n = .272\,\sin\varphi_{n-1} - 2.03\left[\dot{\psi}_{n-1} + \frac{h}{2}\ddot{\psi}_{n-1}\right]$$

$$-.4\left[\beta_{n-1} + h\dot{\beta}_{n-1} + \frac{h^2}{4}\ddot{\beta}_{n-1}\right] - 2.03\left[\dot{\beta}_{n-1} + \frac{h}{2}\ddot{\beta}_{n-1}\right]$$

Then the tail load:

$$L_v = a\dot{\varphi} - b\,\beta$$

The aileron angle will increase linearly from zero to maximum deflection in two-tenths of a second; from then on it will remain constant. The program will be stopped after the peak tail load is reached. For the first run an increment of .01 second will be used at early time and a coarser increment of .1 second at later time. There will be approximately 200 cycles.

The work on this problem has been successfully completed during the past quarter. A complete description of this problem has been presented in a S.B. thesis submitted to the Department of Aeronautical Engineering.

Nomenclature

| | | |
|---|---|---|
| $\varphi$ | = | angle of bank |
| $\psi$ | = | angle of yaw |
| $\beta$ | = | sideslip angle |
| $\delta_a$ | = | aileron angle |
| $t$ | = | time |
| $h$ | = | increment of time |
| $L_v$ | = | vertical tail load |
| a, b | = | constants |

H. Parechanian
Aeronautical Engineering





359 B. COMPUTATION OF STRESSES IN A TRANSVERSE WEB FRAME

A transverse web frame with non-symmetrical loading can be approximated by a closed ring of variable cross-section. A closed ring is a structure of three degrees of indeterminancies. A web frame has in addition two vertical pin ended stanchion supports which complicate the problem by introducing 2 additional degrees of indeterminancy.

The ring was divided into 49 divisions of varying length. Loads (water pressure loads, weight loads and shear loads) were converted into horizontal and vertical components at each station. Moments at each station were computed and Bruhn's adaptation of Castigliano's method of least work was used to solve for the 5 redundants. Stresses due to bending and direct loads were computed and the process cycled to produce allowable values of these stresses at each station.

$$\Delta x = (x_n - x_{n-1}) \qquad \Delta y = (y_n - y_{n-1}) \qquad \Delta s = \sqrt{\overline{\Delta x}^2 + \overline{\Delta y}^2}$$

$$\sin\theta = \Delta y/\Delta s \qquad \cos\theta = \Delta x/\Delta s$$

$$(\sin\theta)_{av} = \frac{(\sin\theta)_{n-1} + (\sin\theta)_n}{2} \qquad (\cos\theta)_{av} = \frac{(\cos\theta)_{n-1} + (\cos\theta)_n}{2}$$

$F_w = \gamma \times h \times \Delta s$ (Force due water pressure)

$$V_w = F_w(\cos\theta)_{av} \qquad H_w = F_w(\sin\theta)_{av}$$

$$d_{N,A} = \frac{M(\text{about shell})}{A(\text{area of section})} \qquad I_{N,A} = I\,(\text{about shell}) - A\,d_{N,A}^2$$

$M_o$, $P_o$, $Q_o$, R and S are values of redundants.

$$M_{(\text{at each station})} = M_o + P_{oy} - Q_{ox} + R_{xt} + S_x + \Sigma M_1$$

$$\Sigma M_1 = M_w + M_L - M_H - M_v \pm M_{s(\text{shear})} = \sum_1^n (H)_{n-1}\,ds_{n-1} + \Sigma (V)_{n-1}\,ds_{n-1}$$

Redundant unknowns were solved using Bruhn's eqations for conditions of slope and deflection.

Bruhn's Equations

$$\int_o^o \frac{M}{I}\,ds = 0 \quad [\text{Relative change in slope} = 0]$$

$$\int_o^o y\,\frac{M}{I}\,ds = 0 \quad [\text{Deflection in x-direction} = 0]$$

$$\int_o^o x \frac{M}{I} ds = 0 \quad \left[\text{Deflection in y-direction} = 0\right]$$

$$\int_{B_1}^{B_2} (x - x_a) \frac{M}{I} ds = 0 \quad \left[\text{Vertical deflection between } B_1 + B_2 = 0\right]$$

$$\int_{B_3}^{B_4} (x - x_b) \frac{M}{I} ds = 0 \quad \left[\text{Vertical deflection between } B_3 + B_4 = 0\right]$$

Method of Integration

$$\text{Trapezoidal Rule} = \sum_{n=1}^{49} \left[(\text{Ord.})_n + (\text{Ord.})_{n-1}\right] \frac{ds_n}{2}$$

The above integrations formed a matrix of 6 columns and 5 rows. The matrix was solved by Crout's method. After solution the redundants were incorporated into the existing horizontal and vertical loads.

$$\text{Direct loads} = P = \Sigma H \cos\theta + \Sigma V \sin\theta$$

$$\text{Shear loads} = Q = \Sigma H \sin\theta + \Sigma V \cos\theta$$

$$\text{where } \Sigma H = P_o + \Sigma H_w \qquad \Sigma V = -Q_o + R + S + \Sigma V_w + w_t$$

Direct stress = P/A    Bending stress = My/I

Shear stress = Q/A

$\sigma_s = P/A + My/I$

A rational engineering stress allowance was used to compare with computed stresses and the section altered to conform to allowable stress.

During this past quarter, the problem was completed and satisfactory results were obtained. The results of the problem will be found in a Naval Engineering thesis submitted by Brandt, Snyder and Thompson to the Department of Naval Architecture. The title of the thesis is "Applications of Digital Computers to Naval Architecture Problems".

C. R. Brandt
J. C. Snyder
C. R. Thompson
Naval Architecture

363 A. EFFECT OF PRESSURE ON THE BENDING OF CURVED TUBES

The bending of a thin walled toroidal shell under internal pressure is governed by the equations

$$T'' - qT + xS = 0$$

$$S'' \quad -xT = 1$$





and the boundary conditions

$$T(0) = 0 \qquad S'(0) = 0 \qquad \left.\begin{matrix} xT \to -1 \\ x^2 S \to -q \end{matrix}\right\} \text{ as } x \to \infty$$

Numerical solutions have been obtained by Clark and Reissner (Advances in Applied Mechanics II, Academic Press, N.Y., 1951, pp.93-122.) for the case q = 0 which corresponds to zero internal pressure. We plan to obtain numerical solutions for several values of the pressure parameter, q. An iterative method based on a finite difference approximation will be employed. Starting from initial trial values of T'(0) and S(0) the solution will be marched out and its asymptotic behavior compared with the desired behavior. Errors here will be used to determine new initial trial values for T'(0) and S(0). Preliminary estimates indicate that adequate accuracy will be provided by an $0(h^2)$ process using 100 subdivisions or less.

A program has been written and corrected and has been used to calculate T and S for the case q = 0. The results of this calculation check other numerical solutions (for q = 0) available in the literature.

Plans for the future are to use the established program to calculate T and S for cases where $q \neq 0$.

Programmers working on this problem are Professor S. H. Crandall and Professor N. C. Dahl.

N. C. Dahl
Mechanical Engineering

364 C. BLAST RESPONSE OF ROTOR BLADES

This problem involves the determination of the stresses and the flapping of the blades of a helicopter rotor which while in flight is enveloped by a blast wave. The c.g. of the helicopter is permitted to move in a direction parallel to the rotor shaft; non-uniform mass and stiffness distributions of the flexible rotor blades are taken into account. In the interest of simplicity, quasi-steady rather than unsteady airforces due to the blast are used in the analysis. Later the influence on unsteady airforces on the response of the rotor will be studied.

The transient rotor response is obtained by solving, by numerical integration, nm + n + 1 simultaneous differential equations having variable coefficients, where m is the number of blades on the rotor and n is the number of normal modes of each rotating blade. The unknowns are the nm modal amplitudes, n β-flapping motions, and the vertical translation of the c.g. By taking advantage of previous studies on blast response of fixed-wing aircraft, a somewhat simpler formulation of the above problem can be accomplished. This is done by neglecting mass coupling and airforce coupling terms between the rigid-body response and the flapping and/or the modal vibrations, as previous studies have shown to be permissible. In this way the vertical translation equation becomes a single-degree-of-freedom equation with the blast forces acting as the forcing function - it does not involve the flapping equations or the modal-vibrations equations. The flapping equations involve the flapping response and the previously-computed rigid-body response; they are then solved. To compute the modal vibration, the previously-computed rigid-body and flapping responses are integrated.

Whirlwind I is used to solve for the normal modes, shapes and frequencies of the hinged, rotating, non-uniform (mass, stiffness, chord) blade. Whirlwind is also used to numerically integrate the equations of motion; integrations with respect to blade span and with respect to azimuthwise position of the rotating blade are performed.

During the past quarter, programs have been written to compute the difference equations and matrices required to find the eigenvalues and eigenvectors which will be used to calculate the normal modes and frequencies desired.

The first phase of this problem will be completed sometime in the next quarter.

This problem was initiated by J. Dugundi and K. A. Foss of the Aeronautical Engineering Department, MIT.

M. E. Callaghan
Digital Computer Laboratory

365. FINDING EIGENVALUES OF AN ASYMMETRIC MATRIX

This problem is concerned with finding eigenvalues of an asymmetric matrix. We will be using LSR MA 1 which finds the largest and smallest eigenvalues. A routine will be written to find the characteristic equation of a matrix using Frame's method. The characteristic equation will be solved by Hitchcock's method if possible. In particular, we have four 9 x 9 matrices to solve.

During the past quarter, Frame's method has been written and debugged, and does find the characteristic equations of the 9 x 9 matrices. However, we are having trouble solving the characteristic equations using Hitchcock's method.

During the next quarter, we will try to find a better method of finding eigenvalues of asymmetric matrices in factoring polynomials.

This problem is being done in close connection with the MIT Office of Statistical Services, whose Director, Dr. F. M. Verzuh, initiated this problem on Whirlwind I. Richard Steinberg and Eleanor Donovan are doing the programming on the OSS IBM 650.

J. Roseman
Digital Computer Laboratory

367 B. DETERMINATION OF CRITICAL MASS IN A CYLINDRICAL REACTOR

The problem is to apply two-group diffusion theory to the prediction of critical masses for finite cylindrical reactors which are completely reflected.

The procedure is to write the diffusion equation

$$\nabla^2 \phi_{ij} + \mu_{ij}^2 \phi = 0$$

for the group of thermal neutrons and the group of fast neutrons. The resulting two equations containing average values of the constants for each group are solved, and appropriate boundary conditions applied.

For the completely reflected cylinder being considered, exact solutions are not possible because of the inconsistency of the boundary conditions at the junction of side and top or bottom reflectors. Instead, a method of superposition is used to give





two solutions which are then converged. First, a cylindrical reactor with only a radial reflector is assumed and the exact solutions are solved. Second, a cylinder with only end reflectors is assumed, and a second set of exact solutions are obtained. The two sets of solutions are then made to converge.

This problem differs from number 270-43 because of the low enrichment in the uranium used as fuel. Allowance is made for the presence of $U^{238}$ by putting into the program fictitious values of the constants for $U^{235}$ which are really effective values for the $U^{235}$ - $U^{238}$ mixture.

The following equations will be used:

These four boundary conditions result in 4 equations in four unknowns.

By Cramer's rule, for non-trivial solutions, the determinant of the coefficients must equal zero.

$$\Delta_1 = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & - & - & - \\ a_{31} & - & - & - \\ a_{41} & - & - & - \end{vmatrix} = 0$$

This determinant will be solved by Whirlwind I.

<u>End Reflectors Only</u>

Core Flux:

Fast: $\phi_{1c} = \left[A'\theta + BW + C'U + EV\right] J_o (\mu_r r)$

Slow: $\phi_{2c} = \left[S_1 (A'\theta + BW) + S_2 (C'U + EV)\right] J_o (\mu_r r)$

Reflector Flux: <u>Top</u>

Fast: $\phi_{1r} = \left[P_1 F'\right] J_o (\mu_r r)$

Slow: $\phi_{2r} = \left[S_3 P_1 F' + G'P_2\right] J_o (\mu_r r)$

<u>Bottom</u>

Fast: $\phi_{1r} = \left[MQ_1\right] J_o (\mu_r r)$

Slow: $\phi_{2r} = \left[S_3 MQ_1 + NQ_2\right] J_o (\mu_r r)$

where A', B', C', E, F', G', M, N are unknowns.

and $\theta = \text{Sin} (\mu_z z)$ $\qquad P_1 = \text{Sinh} \, \xi_{1z} \left[\frac{H_z}{2} + T_z - z\right]$

$$W = \cos(\mu_z z) \qquad P_2 = \sinh \xi_{2z} \left[\frac{H_z}{2} + T_z - z\right]$$

$$U = \sinh(\nu_z z) \qquad Q_1 = \sinh \xi_{2z} \left[\frac{H_z}{2} + T'_z - z\right]$$

$$V = \cosh(\nu_z z) \qquad Q_2 = \sinh \xi_{2z} \left[\frac{H_z}{2} + T'_z - z\right]$$

Radial Reflector Only

Core Flux:

Fast: $\phi_{1c} = \left[AX + CY\right] \cos(\mu_z z)$

Slow: $\phi_{2c} = \left[S_1 AX + S_2 AY\right] \cos(\mu_z z)$

Reflector Flux:

Fast: $\phi_{1r} = Fz_1 \cos(\mu_z z)$

Slow: $\phi_{2r} = \left[S_3 Fz_1 + Gz_2\right] \cos(\mu_z z)$

where

$$X = J_o(\mu_r r)$$

$$Y = I_o(\nu_r r)$$

$$Z_1 = I_o(\xi_{1r} r) - \frac{I_o\left[\xi_{1r}(R + T)\right]}{K_o\left[\xi_{1r}(R + T)\right]} K_o(\xi_{1r} r)$$

$$Z_2 = I_o(\xi_{2r} r) - \frac{I_o\left[\xi_{2r}(R + T)\right]}{K_o\left[\xi_{2r}(R + T)\right]} K_o(\xi_{2r} r)$$

$$\mu_r^2 + \mu_z^2 = \mu^2$$

$$v_r^2 - \mu_z^2 = \nu^2 = \nu_r^2 + \nu_z^2$$

$$\xi_{1r}^2 = \kappa_{1r}^2 + \mu_z^2$$

$$\xi_{2r}^2 = \kappa_{2r}^2 + \mu_z^2$$

Boundary Conditions:

At $r = R_c$: $\phi_{1c} = \phi_{1r}$ and $\phi_{2c} = \phi_{2r}$ Flux Continuous





$$D_{1c} \frac{\partial \phi_{1c}}{\partial r} = D_{1r} \frac{\partial \phi_{1r}}{\partial r}$$

$$D_{2c} \frac{\partial \phi_{2c}}{\partial r} = D_{2r} \frac{\partial \phi_{2r}}{\partial r}$$

Neutron Current Continuous

Applying the same boundary conditions as before but in the z direction;

at $z = \pm H/2$: $\phi_{1c} = \phi_{1r}$ and $\phi_{2c} = \phi_{2r}$

$$D_{1c} \frac{\partial \phi_{1c}}{\partial z} = D_{1r} \frac{\partial \phi_{1r}}{\partial z}$$

$$D_{2c} \frac{\partial \phi_{2c}}{\partial z} = D_{2r} \frac{\partial \phi_{2z}}{\partial z}$$

plus the additional condition that the flux equal zero at the outer surface of all reflectors, results in eight equations in eight unknowns. Again Cramer's rule is applied to secure a non-trivial solution.

For superposition to be valid both determinants must be satisfied simultaneously for the reactor to be critical.

The solution procedure is to fix all the properties of the materials and the reactor dimensions. A critical mass is assumed, and the values of total buckling ($\mu^2$ and $\nu^2$) are computed. Iteration continues until the 4th order determinant is solved giving values of $\mu_r^2$ and $\nu_r^2$. With these values $\mu_z^2$ and $\nu_z^2$ can be obtained. Finally, all these values are substituted into the 8th order determinant. If it does not equal zero, a new critical mass is assumed and the calculations repeated until a predetermined limit of convergence is reached.

The following symbols not elsewhere identified are:

$\phi_{ij}$ = neutron flux $i^{th}$ energy group, $j^{th}$ region

$D_{ij}$ = diffusion coefficient

$r$ = distance in radial direction

$z$ = distance in longitudinal direction

$\mu_z^2$ = longitudinal buckling

$\mu_r^2$ = radial buckling

$\nu_z^2 = f(\mu_z^2)$ Imaginary bucklings

$\nu_z^2 = f(\mu_r^2)$

$S_1, S_2, S_3$ = constants which are coupling coefficients between fast and slow flux

$$\kappa^2_{1r} = \frac{1}{\tau_2} = \frac{1}{\text{Age}}$$

$$\kappa^2_{2r} = \frac{1}{L^2_{2r}} = \frac{1}{(\text{thermal diffusion length})^2}$$

During the past quarter, a series of runs using three different programs have been made using data from an operational reactor to determine the success of each type of calculation in predicting the critical mass. As a result of these comparisons one program has been discarded. One of the remaining two is undergoing corrections and modifications. The third program tried has given very accurate results to compare with the measured critical mass.

Plans are to adopt the most satisfactory of the two remaining programs and to determine through its use the critical masses of several types of reactor cores over a range of uranium enrichment up to a maximum of 20% $U^{235}$. These results should permit selection of the optimum design.

Programmers working on this problem are J. W. Barnett, M. Troost and J. R. Powell.

J. W. Barnett
Nuclear Engineering

369. TEMPERATURE DISTRIBUTION IN A BEAM

This problem is concerned with the solution of a set of 30 non-linear equations which arose in connection with a study of heat transfer in a cylindrical ring. First order finite difference approximations were used to solve partial differential equations which were present in this problem.

As part of this study the same problem was solved on the IBM Type 650 Magnetic Drum Calculator where the drum calculator was used essentially as a desk calculator to obtain check solutions for the Whirlwind program. That is to say, only a limited portion of the overall problem was solved -- time levels 0, 1, 40 and 41 respectively. Time levels 40 and 41 were selected to overcome a common pitfall encountered preparing test solutions on a large-scale calculator, namely, that of using as a check-point values which are non-zero. Specifically, usual techniques for initiation of check solutions at the beginning of a problem are dangerous because a number of the values are zero or non-zero. Obviously, a zero result obtained by any correct or incorrect method gives no measure of the correctness of the program. Selecting step 41 eliminated the non-zero limitation.

Results obtained on Whirlwind I and the 650 agree to 6 decimal digits which is quite good considering the differences in the two interpretive systems used on Whirlwind and the 650 respectively.

Programmers working on this problem are A. Zabludowsky of the Digital Computer Laboratory, and M. Hermann of the Office of Statistical Services.

F. M. Verzuh
Office of Statistical Services





371 L. ATMOSPHERIC PROPAGATION OF RADIO WAVES

The problem consists of the investigation of the influence of the dielectric inhomogeneities of the lower atmosphere and ionosphere upon the propagation of electromagnetic waves to great distances and elevations. In the first phase of this work the stability of rays at low angles to the horizon will be investigated from solutions of the non-linear differential equation:

$$\frac{dh}{dR} - \frac{1}{n(h)} \left\{ 1 - \left[ \frac{r_o n_o \cos \epsilon_o}{n(h)\ (r_o + h)} \right]^2 \right\}^{1/2} = 0 \qquad (1)$$

where R = ct, c is the velocity of radiation in a vacuum, t is the time required for a disturbance to reach a point (R,h) on a ray trajectory whose angle with the horizon at (0,0) is $\epsilon_o$. h is the height of a point (R,h) on the trajectory and n(h) is the refractive index of the atmosphere given as a function of height. $r_o$ is the radius of the earth. Equation (1) is to be solved for a large number of functions n(h) corresponding to measurements of refractive index in the lower atmosphere and ionosphere at various wavelengths. These solutions are expected to provide important information about radio optics in the atmosphere. Seasonal daily and hourly fluctuations in the trajectories about mean trajectories will be obtained.

In the second phase of this problem, the astronomical refraction R of radio stars at various wavelengths will be computed from the integral

$$R(\epsilon_o) = -\int_o^{\infty} \frac{1}{n} \frac{dn}{dh} \left\{ \left[ (1 + \frac{h}{r_o}) \frac{n}{n_o} \sec \epsilon_o \right]^{-2} - 1 \right\}^{-1/2} dh$$

Fluctuations of $R(\epsilon_o)$ will be obtained from the use of measured functions n(h). Additional phases of this work are to be specified.

Two methods for solving the refraction equation have been tested, -- the Kutta-Gill method and the Tchebyshev method of integration. The latter method was found to be more rapid than the former and appears to provide a satisfactory degree of accuracy. The radio range versus height for elevation angles from $0^o$ to $5^o$ have been determined by this method up to heights of 300,000 feet for different seasons of the year.

In the future we plan to: 1) extend the problem using the above method from heights of 300,000 feet on up through the ionosphere at a number of frequencies; 2) investigate, using the above method, the hourly variations in atmospheric refraction.

W. Mason
Lincoln Laboratory

### 372 B. DESIGN OF SPHERICAL SHELL SEGMENTS

The problem proposed for solution on Whirlwind I is part of a doctoral thesis, carried out under the supervision of Professor C. H. Norris, on the subject: "The design of precast domes". The general idea is to construct a reinforced concrete spherical shell out of individual pie-shaped segments, which are assembled by means of prestressing to form the complete dome. Each such precast segment will be hoisted into its final position and be temporarily supported at its crown and bottom ring.

The present problem, to be solved on Whirlwind, deals with the determination of the membrane stresses in this precast spherical triangle during construction.

In shell thoery it is normally found, that an explicit solution in closed form for the stresses in the shell can be obtained only for the simplest forms of shape and loading. However, for the given shape of a spherical triangle, where there is no rotational symmetry, one must use series substitutions and numerical methods to evaluate the stresses.

The proposed method is based on the following theory: our object is to determine the membrane stresses in a shell segment under its own load, when supported at its crown and bottom ring. This state of stresses will be the same as in a complete dome in which the vertical planes bordering the individual segment form planes of antisymmetry. For such a case of antisymmetric load, one can express the load and stresses in terms of a Fourier series and can thus integrate the equilibrium equations. As a result of these expressions the shear stress, for example, can be expressed in the following form:

$$N_{\theta\phi n} = \Bigg\{ K_{1n} \frac{tg^n (\frac{\phi}{2})}{\sin^2\phi} + K_{2n} \frac{cotg^n (\frac{\phi}{2})}{\sin^2\phi} + \frac{2n}{k} \frac{tg^n (\frac{\phi}{2})}{\sin^2\phi} \int_0^\phi \Big[ n \cos\phi - \cos^2\phi - \sin\phi \Big] \sin\phi \, cotg^n (\tfrac{\phi}{2}) \, d\phi + \frac{2n}{k} \frac{cotg^n (\frac{\phi}{2})}{\sin^2\phi} \int_0^\phi \Big[ n\cos\phi + \cos^2\phi + \sin\phi \Big] \sin\phi \, tg^n (\tfrac{\phi}{2}) d\phi \Bigg\} \sin n\theta$$

This consists for each value of $n$ of three terms, two of them proportional to a constant of integration and one a numerical value. Expressions of this form must be evaluated for the following values of $n$ :

Number of Segments

| | |
|---|---|
| k = 2 | n = 1 |
| k = 4 | n = 2 |
| k = 12 | n = 6, 18, 30, 42 |
| k = 24 | n = 12, 36, 60, 84 |
| k = 36 | n = 18, 54, 90, 126 |





The range of $\phi$, the angle from the vertical axis of rotation should be $0 \leq \phi \leq \frac{\pi}{2}$ and values should be determined in intervals close enough to give an accurate plot of the function.

It is obvious , that in view of the very high values of $n$ numerical integration must be used.

However, this expression for the meridional stress given above still is a function of two constants of integration. To determine them the equilibrium conditions for the complete shell segment are used. In writing these out, one comes across expressions of the following form:

$$\int_0^\phi N_{\theta\phi n} \sin\phi \, d\phi \qquad \int_0^\phi N_{\theta\phi n} \sin^2\phi \, d\phi$$

$$\int_0^\phi N_{\theta\phi n} \cos\phi \, d\phi \qquad \int_0^\phi N_{\theta\phi n} \cos\phi \, (1 - \cos\phi) \, d\phi$$

which again must be evaluated by numerical procedures for each value of $n$ as detailed above.

As a result of the integrations outlined above a series of curves will be plotted which will finally give all the necessary data for the determination of the membrane stresses in any segment of a spherical shell for any opening angle $\phi$.

During the past quarter, part of the program was developed and checked for a particular case. The result obtained verified the numerical solution.

In the future, the program will be set up for the determination of stresses in a more general form and with the results from the computer, graphs will be plotted to determine the membrane stresses at any segment of a spherical shell.

Programmers working on the problem are E. Traum and Assistant Professor S. Namyet.

S. Namyet
Civil Engineering

### 373 B. FLUX LEVELING IN HOMOGENEOUS REACTOR - PART I

Spatial distribution of neutrons in a nuclear reactor usually prevents full exploitation of the outer regions of the reactor, since heat generation, which follows the spatial distribution of neutrons, is limited by heat transfer and metallurgical problems at the reactor center. Neutron distribution may be made more nearly uniform by decreasing moderation in the core and allowing moderation in the reflector to increase neutron concentration in the outer core.

The diffusion equation, which describes neutron distribution, is solved for a spherical (one-dimensional) reflected reactor for the time independent case. Coefficients of this equation, which vary with neutron energy, are replaced with averaged constant values by considering n energy groups of neutrons, each of which then has its own diffusion equation.

These n diffusion equations are non-dimensionalized, converted to a standard form (to permit a standard iteration method) and integrated. WWI solves n equations of the following form by trapezoidal integration:

$$Y(x) = \exp\left(\int_0^x \frac{f\,dx}{ax^2}\right)\left[\int_1^x \exp\left(-2\int_0^x \frac{f\,dx}{ax^2}\right)\left\{\int_0^x \exp\left(\int_0^x \frac{f\,dx}{ax^2}\right)\frac{Z\,dx}{ax^2}\right\}dx\right]$$

where
- x is dimensionless space parameter r/R
- a is a material property of the core
- Z is the source term, representing fission neutrons, or neutrons moderated to a lower energy group
- f is given by a finite difference recursion formula, and includes leakage and absorption of neutrons. WWI also solves for f.

Conversion from an assumed distribution to the correct one is rapid, since the error in an integral is normally less than the error in an integrand. Three iterations have proved sufficient in hand calculations. Twenty hours are required for hand computations (not including time lost in errors) while WWI is estimated to require six seconds, both figures based on a single iteration.

During the past quarter, three runs (13 minutes) were required to debug the program. On the fourth run, WWI gave a correct solution to the problem which had previously been hand-calculated.

At present data are being collected to feed WWI for final results.

R. W. Kennedy
Nuclear Engineering

377 L. COVERAGE ANALYSIS

This problem is concerned with the study of radar coverage, and the properties of the observations for assumed radar configurations and tactical situations. In the first part of the problem, trigonometric-algebraic equations are solved for each combination of parameters. The program is designed to handle a maximum of 4096 combinations. Each combination results in six associated functional values that will be stored on the drum for accuracy requirements in order to make maximum use of available storage. (Some will be reduced from double length words to single length and others to half register lengths.)

The second part will be mainly a data-handling problem using the computed functional values to analyze the coverage and characteristics of the distribution of the observations.

The problem is in the process of being programmed. Part of the first half has been completed.

Programmers working on this problem are E. Hutcheson and L. Peterson.

L. Peterson
Lincoln Laboratory



APPENDIX

1. SYSTEMS ENGINEERING

Since 1954, the MIT Servomechanisms Laboratory has been using the WWI manual intervention and display equipment in the development of high-speed data reduction techniques. In order for them to expand their research into computer applications, it was essential that more versatile manual inputs be made available on the WWI computer. Besides requiring additional on-off switches, many of the new programs will be so complex and will require so many parameters that the only reliable way to instruct them will be to use specially designed mnemonic languages and translation programs. In order to have this general language structure available on a manual intervention basis, it is necessary to have a keyboard such as a Flexowriter for direct input to the computer.

The MIT Scientific and Engineering Computations Group have contemplated the following applications for the new facility:

1. Demonstration programs would be a great deal more effective if this form of input were available for control purposes.

2. Typewriter input for Comprehensive System Flexowriter and post-mortem request tapes. Short program modifications and post-mortem requests can presently be inserted in the insertion registers. However, errors are easily made because the required vocabulary is awkward. A typewriter input facility would make available a normal mnemonic vocabulary for such purposes.

3. Experimental use of a typewriter facility for direct operator control of the computer. Here we would consider using the typewriter to replace the button-pushings required of the operator during normal operations. Vocabulary similar to that of director tapes and performance requests would be devised for these purposes. This could easily prove to be an extremely convenient and efficient method of computer operation.

The new input installation will be available for use by 4 July 1956. Much of the information to be inserted via the keyboard will be the same as is now introduced via a free running photoelectric tape reader using punched paper tapes. The keyboard input will also be treated as a free running device, i.e., selection of the facility by the computer may be followed by an arbitrary number of read instructions, each of which reads the next character which has been struck on the keyboard. The total equipment requirements amount to 15 relays and 20 tubes.

WWI RELIABILITY

The following is the WWI Computer reliability for the past quarter:

| | |
|---|---|
| Total Computer Operating Time in Hours | 1915 |
| Total Time Lost in Hours | 24.3 |
| Percentage Operating Time Usable | 98.7 |
| Average Uninterrupted Operating Time Between Failure Incidents in Hours | 28.6 |
| Total Number of Failure Incidents | 66 |
| Failure Incidents per 24-hour Day | 1.2 |
| Average Lost Time Per Incident in Minutes | 22 |
| Average Preventive Maintenance Time Per Day In Hours | 1.8 |

## 2. VISITORS

Tours of the Whirlwind I installation include a showing of the film, "Making Electrons Count", a computer demonstration, and an informal discussion of the major computer components. During the past quarter, 10 groups totalling 190 people visited the computer installation, representing the following:

| | |
|---|---|
| April 23 | University of New Hampshire, Student Branch of the AIEE |
| April 27 | United Nations |
| May 3 | MIT Course entitled "Electronic Computational Laboratory" |
| May 3 | Class from Northeastern University |
| May 8 | U. S. Representatives of S.A.C.M., France |
| May 16 | Foxboro Corp |
| May 17 | MIT Class in Aeronautical Engineering |
| May 17 | MIT Class in "Machine-Aided Analysis" |
| May 29 | Gillette Co. |
| May 31 | Phillips Eindhoven, The Netherlands |
| June 8 | British Joint Services Mission |

The procedure of holding Open House at the Laboratory on the first and third Tuesday of each month has continued. A total of 155 people attended the four Open House tours during the quarter, representing members and friends of the MIT Community, Weston High School, Hood Rubber, Northeastern University, Purdue, Tufts, the Massachusetts Department of Commerce, Dunlop of France, Godfrey L. Cabot Co., Rand, Inc., Microfine Instrument Co., Massachusetts General Hospital, Edward Devotion School, Raytheon Mfg. Co., Westinghouse Electric Co., Remington Rand, Production Systems, Inc., Elliot Bros. London office, General Electric, the U.S.Weather Bureau, Forsyth Dental School, Boston Museum of Fine Arts, Harvard University and Boston University.

During the past quarter, there were also 53 individuals who made brief tours of the computer installation at different times. Represented by these individuals were the Office of Naval Research, Emmanual College, Dartmouth, General Radio Corp., Middlebury College, Seoul University, Dean Witter & Co., W. R. Grace Co., Institute of Statistical Mathematics in Japan, Sperry Gyroscope Co., Monsanto Chemical Co., British Ministry of Supply, Bell Telephone Mfg. Co., Ramo-Wooldridge Corp., Rutgers University and the U.S. Patent Office.



## 3. ACADEMIC

Introduction

There are a number of graduate subjects in automatic computation, numerical analysis, and now electronic data processing, offered at M.I.T. The present list of subjects directly related to machine computation includes the following:

| Subject | Description | Units | Year | Instructor |
|---|---|---|---|---|
| 6.25 | Machine-Aided Analysis | 3-6 | 4 | Linvill |
| 6.535 | Digital Computer Coding and Logic | 3-6 | G | Arden |
| 6.538 | Electronic Computational Laboratory | 3-1-5 | G | Verzuh |
| 6.54 | Pulsed-Data Systems | 3-6 | G | Linvill |
| 6.567 | Switching Circuits | 3-1-6 | G | Caldwell |
| 6.568 | Switching Circuits | 3-1-6 | G | Caldwell |
| 2.215 | Methods of Engineering Analysis | 3-9 | G | Crandall |
| 15.542 | Management Information Systems | 3-6 | G | Gregory |
| M39 | Methods of Applied Mathematics | 3-9 | G | Hildebrand |
| M411 | Numerical Analysis | 3-2-7 | G | Hildebrand |
| M412 | Numerical Analysis | 3-2-7 | G | Hildebrand |

It is apparent from the above list that these subjects are predominantly Graduate A subjects -- which indicates that this subject is primarily for graduate students. However, it is perfectly possible for undergraduate students in their senior year to take these subjects as an elective.

Machine-Aided Analysis - 6.25

Subject 6.25 -- Machine-Aided Analysis -- was offered for the first time during the fall term 1953-1954. This subject differs from other computational subjects in the following respects:

1. It is a fourth-year elective subject designed for the undergraduate,
2. It is a survey subject which covers numerical analysis, analog computation, and digital computation,
3. It provides the student with a general knowledge of some of the numerical methods necessary for machine computation.

Admittedly, such a survey subject cannot be all inclusive, and because of the large enrollment, it is no longer possible to provide the student with actual physical contact with any computer.

Since 1953, this subject has turned out to be increasingly popular. As a matter of fact, it is now offered both fall and spring terms. The attendance is quite high. There were 106 students in 6.25 during the Fall Term 1955, and 47 students during the Spring Term 1956. This subject offered at the undergraduate level enables students to decide whether they are interested in taking additional subjects in this area, and if they are, there are 10 graduate subjects in which they may enroll.

Introduction to Digital Computer Coding and Logic - 6.535

This subject was, as usual, again offered during the Spring Term 1956, and there were 97 students enrolled. As a result, it was necessary to divide the subject into two sections taught by D. Arden and F. Helwig, respectively.

Because of the large enrollment in this subject, no attempt was made to make actual use of any computer. As a matter of fact, the usual detailed coding for the Whirlwind Computer was not presented, and students were not allowed to use the computer because of the large size of the group. Since the departure of Professor Adams, the content of the course has been changed somewhat, and during the spring term considerable emphasis has been devoted to a study of the logical design of the internal system aspects of several large computers -- Whirlwind I, NORC, XD-1, etc.

Electronic Computational Laboratory - 6.538

The subject matter offered in 6.538 since 1947 provided the student with two basic types of material. Specifically, some 80% of the laboratory work was devoted to a study of electronic circuit components by actual laboratory work on the following experiments:

I. Time Measurement Equipment and Study of Synchroscopes and Oscilloscopes.

II. Pulse Generating Equipment
An experiment on blocking oscillators, delay-line pulse generators, sharpening and squaring circuits, and ringing circuits.

III. Bistable Circuits and Flip-Flops
An experiment covering various flip-flop circuits including Thyratron, single-tube pentode circuits, Eccle-Jordan circuits, high-speed pentode circuits, and transistorized flip-flops.

IV. Electronic Counters and Counting Circuits
An experiment covering Thyratron counters, binary decade counters, screen-coupled counters, biquinary counters, respectively.

V. Coincidence Circuits and Gate Circuits
An experiment considering various types of coincidence circuits: diodes, triodes, pentodes, and other multiple coincidence devices.

VI. Design of Bistable Circuits
A project-type of experiment in which the student designs (on paper) a flip-flop to meet certain operating conditions, and tests his design by actual construction of the flip-flop, and testing its performance in the laboratory.

VII. Magnetic Recording of Pulsed Information
An experiment on magnetic recording in which the magnetic material, head, recording frequency, pulse amplitude, tape velocity, are varied and the resulting performance evaluated.

VIII. Time Pulse Distributor
An experiment studying vacuum tubes and diodes as control elements by pulse distribution of voltage signals.

IX. Square-Hysteresis-Loop Magnetic Storage Elements
An experiment on the use of magnetic cores as a storage media, number of turns on various windings, current amplitude of recording and sensing turns, are used as parameters.

Detailed information regarding the above nine experiments was given to the students by means of a written set of notes (230 pages) which were written as reference notes and



experiment notes, respectively. In addition to the above nine experiments, two laboratory experiments were devoted to the use of punched card equipment. Specifically, one on the card punch, sorter, and tabulator, and the second on the Card Programmed Calculator, respectively. The class lectures were primarily devoted to a discussion of basic construction of digital computers, and their use in the solution of scientific computations encountered in the solution of simultaneous algebraic equations, inversion of matrices, solution of ordinary differential equations, etc.

Because of the availability of the Type 650 Magnetic Drum Calculator, it was decided that the subject content of 6.538 should be revised considerably. Accordingly, during the Spring Term 1956 all of the above-mentioned electronic experiments were deleted from the subject content and the subject was devoted entirely to the application and use of digital computing equipment. The students were taught the basic principles of digital computers by performing experiments on card punches, sorters, accounting machines, the Card Programmed Calculator, and the Type 650 Magnetic Drum Calculator. The principle emphasis was actually devoted to a study of the use of the 650 machine in the solution of "scientific and business data processing problems".

Students were taught to program the 650 by becoming familiar with the basic 650 language. After the basic language was understood, they were taught the following interpretive systems:

| | | |
|---|---|---|
| 1. | MITSS | MIT Selective System |
| 2. | MITILAC | FLOATING DECIMAL Interpretive-mnemonic coding (1600) |
| 3. | NOPI | Fixed and Floating Interpretive-numeric coding (600) |
| 4. | SOAP | Symbolic Optimal Assembly Program |
| 5. | FLIMSY | Floating Interpretive Matrix System (815) |
| 6. | BTL | Fixed and Floating Interpretive-numeric coding (1000) |

It is obvious that the students were not able to become completely familiar with all of the above systems. All students instructed now understand the 650 language, MITSS, and MITILAC. However, only certain students became familiar with each of the systems depending upon their particular interest.

The following list of homework and experiments were performed during this subject:

I. Problem 1 - Laboratory use of the Type 026 Card Punch and Type 082 Sorter
The student spends two hours in the laboratory using the card punch and sorter to prepare and sort several hundred IBM punched cards, thus insuring complete familiarity with these two machines.

II. Problem 2 - Design of Punched Card System to Handle Business Data Processing Work at the Eastern Joint Computer Conference
This problem teaches the importance of overall flow charts, the design of suitable IBM card layout forms, the preparation of accurate instructions for machine operating personnel, and the estimation of the personnel and machine time required to handle the registration for 2,000 attendees at the conference.

III. Problem 3 - The Solution of a First Order Differential Equation on the Type 650 Magnetic Drum Calculator

The student is given a block diagram of a 650 program used to solve an ordinary differential equation using fourth-order Runge-Kutta formulae. As part of the assignment, the student prepares the actual coding of the 650 program using basic language, prepares a test solution for this program, computes a few check points, debugs his program on the 650, and finally carries out the solution of this problem on the machine. After completing this particular experiment, the student has a very intimate knowledge of the 650 machine.

IV. Problem 4 - Solution of an Ordinary Differential Equation on the 650
The student solves the identical problem stated in the preceding section using the Card Programmed Calculator. This teaches them the difference between the internally-stored program features of the 650 and the card-programmed characteristics of the Card Programmed Calculator.

V. Problem 5 - Evaluation of the Roots of a 98 degree Polynomial
A 650 program using the NOPI system was prepared which evaluates the roots of the polynomial using Hitchcock's method.

VI. Problem 6 - Computation of the Actuarial Life Insurance Tables
Since a high degree of accuracy in this computation was needed, the basic 650 language was modified to perform 20-digit arithmetic in the solution of this problem.

VII. Problem 7 - Inversion of Matrices using Modified Elimination Procedure
A basic 650 program was prepared which automatically includes a "pivotal-condensation" technique. This program correctly handles matrices which have zero elements on the main diagonal.

VIII. Problem 8 - Calculation of Nuclear Transformations and Radioactive Decay
A MITILAC program was prepared which computes the number of atoms present in the nth chain member in the simultaneous transformation by nuclear reaction in radioactive decay of a given target nuclide in a given neutron flux at any time.

IX. Problem 9 - Computation of Interest Calculations for Savings Banks
A basic 650 program was prepared which automatically handles any of the six different methods for performing interest calculations arising in the savings bank.

X. Problem 10 - Computation of Bivariate Frequency Distributions on the 650
A basic 650 program was prepared which computes the bivariate frequency distribution by corresponding class intervals (for various parameters), and also computes certain statistics obtained from the above-mentioned distribution. An effective method of storage allocation is employed which employs a neat trick for obtaining the desired arguments and functions in an internally-stored table of values.

XI. Problem 11 - Study of Optimum Programming on the 650 Calculator
A basic 650 program was prepared which studied various methods for effecting semi-optimum data storage on the drum. A compact decimal system for word storage and selection using the ten branch instructions (BRD 1 - 10, respectively).

XII. Problem 12 - Solvent Extraction of Metals
A basic 650 program was prepared for computing the distribution coefficients of zircon and hafnium in the aqueous and organic phases.

XIII. Problem 13 - Solution of the Heat Flow Equation
A basic 650 program was prepared for the solution of a partial differential equation describing the heat flow problem using several different boundary conditions.



XIV. Problem 14 - Iterative Method for Solving Simultaneous Equations on the 650
A basic 650 program was prepared which evaluates the convergence rate, accuracy and solution time using Craig's method, Conjugate Gradient method, and Lanczos's method, respectively.

XV. Problem 15 - 650 Solution of the Vertical Resistivity Problem
A MITSS program was prepared for the solution of the integral equation arising in geophysical prospecting.

XVI. Problem 16 - Preliminary Analysis of an Inventory Control System
A non- machine study of the many problems arising in a systems study of the overall inventory control problem. Detailed description is given of the nature of transactions involved, considerations involving volume of data, rate of information flow, need for rearrangement of computational procedure, study of the effects of errors, breakdown, and interruptions, and finally a consideration of problem areas of interest to management to provide better overall control.

XVII. Problem 17 - Solution of a Payroll Problem on the 650 Calculator
A basic 650 program was prepared for a hypothetical payroll involving 200 employees. The study included a consideration of piece-work as well as the regular payroll in which employees switch back and forth several times each day. The usual items such as, regular pay, overtime pay, bond deduction, hospitalization, income tax, union dues, credit union, Community Chest, and other miscellaneous deductions, were included.

XVIII. Problem 18 - Hitchcock's Method for Evaluating the Roots of a Polynomial
A basic 650 program was prepared for evaluating the roots of a polynomial. Particular emphasis was given to a consideration of polynomials which cannot by handled by this method.

XIX. Problem 19 - Study of Series-Parallel Switching Functions
A basic 650 program was prepared for evaluating the switching functions of six or fewer variables. The program employed is a modification of the McCluskey Quine reduction procedure by means of which the preliminary implicants and minimum sum forms of the switching functions were obtained. The solutions of 86 different problems were performed during the course of this study.

XX. Problem 20 - Solution of Spheroidal Antennae Equation Using the 650
A NOPI program was prepared for the solution of this equation using Milne's predictor-corrector formulae.

XXI. Problem 21 - 650 Solution of the Production Scheduling Problem
A MITSS program was prepared for the solution of this problem which determined production curves which minimize costs for a single arbitrary sales forecast. Graphical results were obtained which related the sensitivity of the solutions for various system parameters using a quadratic production cost function.

XXII. Problem 22 - Solution of the Inventory Control Problem on the 650 Calculator
A basic 650 program was prepared for a simplified inventory control problem. Specifically, the program was designed to handle any number of stock-numbered items up to 34-per-day items. The program was limited to control and reporting of customer products only. No attempt was made to control the component parts for the items. Similarly, no attempt was made to report the raw material requirements.

XXIII. Problem 23 - Study of Brokerage Accounting

A basic 650 program was prepared for calculating the items appearing in the transaction statement of a customer's brokerage account. The following items were included in this study: 1) Commercial postage and insurance, 2) Federal transfer taxes, 3) State transfer taxes, 4) S.E.C. fees.

Naturally a program of this type cannot be all inclusive and a number of simplifying assumptions had to be made to effect the solution of the program in a reasonable amount of time.

A consideration of the above list reveals that a wide variety of problems were considered. It must be emphasized that the latter group of subjects were performed on a "term paper" basis. That is, these studies were performed on an individual basis, and elaborate reports were prepared describing the problem statement, the actual 650 program, the results obtained, and a summary and evaluation of the paper. Although the term paper was expected to require 30-40 hours of home and laboratory work, the actual time put in by the students varied enormously. For example, this time ranged from 30 hours to 360 hours, respectively. In the latter case, students intend to use this topic as a basis for their master's theses during succeeding terms.

It is apparent from the above course description that an attempt was made to cover both scientific computing and electronic data processing in the subject. This can indeed be done, particularly when individual term papers are assigned. However, as time goes on, it may be advisable to separate this subject into two distinct subjects, one dealing solely with scientific computing and the other with business data processing.

This subject was taught by Dr. F. M. Verzuh, Director, Office of Statistical Services. There were 26 students enrolled in Subject 6.538 during the Spring Term 1956 of which there were 12 Electrical Engineering students, 4 Chemical Engineering students, one Geophysics student, 3 Economics students, 2 Business Administration students, and 4 Mathematics students, respectively. This was the first time students were able to use the 650 Calculator as part of their laboratory work. Judging by the large number of this group -- 12 students -- who are now using the machine for the solution of their thesis problems, it appears that the subject was highly successful.

In addition to the solution of scientific computing and business data processing problems, a number of classroom lectures were devoted to the consideration and evaluation of the relative merits of the many commercially-available electronic data processing systems. Specifically, this was achieved by a study of 16 representative intermediate-size drum computers, and seven of the large-scale systems -- BIZMAC, Datamatic 1000, ERA 1103, IBM 704, IBM 705, Raycom, and UNIVAC, respectively. Since many of the students were entering various large business, industrial, and government concerns, it was important that they have some knowledge of some of the relative merits of the individual systems. Finally, several lectures were devoted to the IBM 704 Calculator which will be installed in the M.I.T. Computation Center in the near future.

Summary of Student Enrollment Statistics

A detailed description of the course content of the subjects listed in the introduction is given in the M.I.T. Catalogue.

An indication of the interest in machine computation may be obtained by considering the following enrollment in each of these subjects during the academic year 1955-1956:



| Subject | Name | No. Students Fall | No. Students Spring | Total |
|---|---|---|---|---|
| 2.215 | Methods of Engineering Analysis | 18 | 0 | 18 |
| 6.25 | Machine-Aided Analysis | 106 | 47 | 153 |
| 6.535 | Digital Computer Coding and Logic | 0 | 97 | 97 |
| 6.538 | Electronic Computational Lab. | 0 | 26 | 26 |
| 6.54 | Pulsed-Data Systems | 0 | 60 | 60 |
| 6.567 | Switching Circuits | 48 | 0 | 48 |
| 6.568 | Switching Circuits | 0 | 30 | 30 |
| 15.542 | Management Information Systems | 0 | 22 | 22 |
| M39 | Methods of Applied Mathematics | 130 | 123 | 253 |
| M411 | Numerical Analysis | 45 | 0 | 45 |
| M412 | Numerical Analysis | 0 | 16 | 16 |
| | Total | 347 | 421 | 768 |

It is apparent from the above list that the total attendance in these 11 subjects was 768 students. This does not mean, however, that there were 768 individuals involved since many of the students took a number of the above subjects, thus the actual number of individuals exposed to machine computation is considerably less.

F. M. Verzuh
Office of Statistical Services

## PERSONNEL OF THE PROJECTS

### MACHINE METHODS OF COMPUTATION AND NUMERICAL ANALYSIS

Faculty Supervisors:

| | |
|---|---|
| Philip M. Morse, Chairman | Physics |
| Samuel H. Caldwell | Electrical Engineering |
| Herman Feshbach | Physics |
| Jay W. Forrester | Electrical Engineering |
| James B. Reswick | Mechanical Engineering |
| Chia Chiao Lin | Mathematics |

Research Associate:

| | |
|---|---|
| Bayard Rankin | Mathematics |

Research Assistants:

| | |
|---|---|
| Joseph Hershenov | Mathematics |
| M. Douglas McIlroy | Mathematics |
| James W. Schlesinger | Mathematics |
| Hurbertus J. Weinitschke | Mathematics |
| Fernando J. Corbato | Physics |
| Harvey Fields | Physics |
| Zoltan Fried | Physics |
| Raymond F. Stora | Physics |
| Arnold Tubis | Physics |
| Aaron Temkin | Physics |
| Jack L. Uretsky | Physics |
| Jukka A. Lehtinen | Mechanical Engineering |
| Joseph B. Walsh | Mechanical Engineering |
| Marius Troost | Chemical Engineering |
| Norman F. Ness | Geology |
| Ralph G. Gray | Civil Engineering |

### PROJECT WHIRLWIND

Staff Members of the Scientific and Engineering Computations Group at the Digital Computer Laboratory:

Frank M. Verzuh, Head
Dean N. Arden
Sheldon F. Best (Abs.)
Frank C. Helwig
Leonard Roberts
Jack Roseman
Arnold Siegel
Murray Watkins
Munroe R. Weinstein
Alexander Zabludowsky