MACHINE METHODS OF COMPUTATION
and
NUMERICAL ANALYSIS

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and

PROJECT WHIRLWIND

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
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FOREWORD

This is a combined report for the two projects at the Massachusetts Institute of Technology which are sponsored by the Office of Naval Research under Contract N5ori60.

Project on Machine Methods of Computation and Numerical Analysis

This Project is an outgrowth of the activities of the Institute Committee on Machine Methods of Computation, established in November 1950. The purpose of the Project is (1) to integrate the efforts of all the departments and groups at M.I.T. who are working with modern computing machines and their applications, and (2) to train men in the use of these machines for computation and numerical analysis.

People from several departments of the Institute are taking part in the project. In the Appendix will be found a list of the personnel active in this program.

Project Whirlwind

This Project makes use of the facilities of the Digital Computer Laboratory. The principal objective of the Project is the application of an electronic digital computer of large capacity and very high speed (Whirlwind I) to problems in mathematics, science, engineering, simulation, and control.

The Whirlwind I Computer

Whirlwind I is of the high-speed electronic digital type, in which quantities are represented as discrete numbers, and complex problems are solved by the repeated use of fundamental arithmetic and logical (i.e., control or selection) operations. Computations are executed by fractional-microsecond pulses in electronic circuits, of which the principal ones are (1) the flip-flop, a circuit containing two vacuum tubes so connected that one tube or the other is conducting, but not both; (2) the gate or coincidence circuit; (3) the magnetic-core memory, in which binary digits are stored as one of two directions of magnetic flux within ferro-magnetic cores.

Whirlwind I uses numbers of 16 binary digits (equivalent to about 5 decimal digits). This length was selected to limit the machine to a practical size, but it permits the computation of many simulation problems. Calculations requiring greater number length are handled by the use of multiple-length numbers. Rapid-access magnetic-core memory has a capacity of 32,768 binary digits. Present speed of the computer is 40,000 single-address operations per second, equivalent to about 20,000 multiplications per second.

PART I

COMMITTE ON MACHINE METHODS OF COMPUTATION AND NUMERICAL ANALYSIS

1. General Comments

This report represents the beginning of a transition period in the machine computing activities at MIT. Sometime this spring an IBM 704 machine will be installed in the new MIT Computation Laboratory. At that time a new administration unit, the MIT Computation Center, will be inaugurated. During the spring many members of the staff of the present project will transfer to the staff of the Center and by next summer, Whirlwind will be withdrawn from general use in research and will be turned over to Project Lincoln for its full time use.

At that time the present, ONR-sponsored, project will terminate and these Progress Reports will stop. A new set of Progress Reports, from the Computation Center, will take up the task of reporting on research and training in the use of digital computers at MIT. All those receiving this report, who wish to have copies of the new, Computation Center Progress Reports, should write to the undersigned, who is to be Director of the Computation Center.

As part of the transfer of activities, no part time ONR research assistants were appointed this academic year; IBM research assistants were appointed instead. Consequently, the first half of this Report, which usually contained notes on the research of the ONR Assistants, contains final reports from last year's men, who finished last summer. The next report of this series, and also the remaining two, will only contain what is the second half of this report, a report on the research activities of Whirlwind. The activities of the IBM research assistants and of the staff of the new Computation Center, will be reported in the first issue of the new series of reports, which will be issued early in 1957.

Philip M. Morse
Director, MIT Computation Center

2. GRADUATE SCHOOL RESEARCH

2.1 Index to Reports

2.2 Terminating Reports

PHYSICS DEPARTMENT

A review of work undertaken by the research assistants in physics under contract N5ori60 reveals a remarkably well-defined pattern of growth. Initial efforts were primarily concerned with orientation toward machine computation, a process which required a period of about one year. By the end of this period there was sufficient understanding of machine processes so that fairly complex problems could be attempted, and, indeed, many were carried through to successful conclusions. Not a few of the early problems led to the creation of subroutines which, toward the end of the project, made computations possible which would have been out of the question during the earlier days.

Initial work was centered around the two I.B.M. computers, the 602 and the 604. By the end of the first year, the Whirlwind machine had become available and it rapidly became the center of interest. However, not all research led to machine computation; mathematical analysis and hand computation were sometimes sufficient for the completion of certain projects.

During the first year (beginning in July 1951) the attention of the physics group was largely concerned with such mundane problems as matrix inversion, the evaluation of relatively simple functions, and the integration of Schroedinger's equation to obtain bound states for a coulomb field. Even at this early stage of the game, however, F. J. Corbató was able to contribute a technique for mechanizing certain Fourier series calculations which occur in X-ray diffraction work (see Quarterly Progress Reports 1, 2, 3[1]). This work was all begun on the I.B.M. machines but was transferred to Whirlwind I when that machine became available late in 1951.

Early experience with the computing machines indicated that they could be very useful in treating two, three, and n-body problems in quantum mechanics. Accordingly, attention was focused in this direction. Theoretical work leading to the formulation of a variational principal for nuclear scattering problems was undertaken by S. I. Rubinow (QPR 2-9; 13) and led to the publication of two papers.[2] L. Sartori was able to utilize Rubinow's work as a partial basis for his calculations of nuclear three-body scattering[3] (QPR 14, 15, 16). At the same time M. C. Newstein[4] (QPR 5-13) and J. L. Uretsky[5] (QPR 5-9) carried out variational calculations to describe the scattering of slow electrons by atomic Hydrogen and, in the latter case, Helium. Subsequently, A. Temkin[6] (QPR 16) performed such a calculation for scattering from Oxygen atoms. A somewhat supplementary investigation by L. Mower[7] (QPR 7, 8) was concerned with establishing the accuracy of certain variational techniques when applied to electron-atom scattering problems. Mower's calculations were done by hand, all others were carried out on Whirlwind I.

Bound state problems were not neglected. One of the earliest calculations was that of D. Combelic[8] (QPR 5, 8) who fit a Morse potential to the deuteron using Whirlwind to integrate the Schroedinger equation and iterate for the eigenvalue. H. Yilmaz[9] (QPR 9-13) improved the techniques of Morse, Young and Hurwitz[10] in order to extend their calculations of single-particle eigenfunctions for many-electron atoms. A. Tubis[11] (QPR 14-16) mechanized the procedure so that the relevant quantities would be generated by Whirlwind on a demand basis. M. Rotenberg[12] (QPR 15-16) used Whirlwind to carry out a Hartree-Fock calculation for a heavy nucleus.

Two investigations in solid state physics were accomplished under the contract. F. J. Corbató[13] (QPR 12-19) undertook the machine calculation of energy bands in graphite. Professor P. M. Morse (QPR 16) developed a theory for obtaining electronic wave functions in a crystal lattice. Calculations relative to the latter investigation were initiated by H. Fields and Z. Fried (QPR 18). Corbató's work, in particular, has led to the development of many subroutines which have been useful to other programmers (for examples, see the QPR's referred to above).

One physics assistant chose to do his work in the field of operations research. J. D. C. Little[14] (QPR 7, 8) studied the problem of optimizing the water use in a large hydro-electric system. Again, Whirlwind was used to carry out the computations.

Two long term table-making projects were initiated by the physics group. Sartori, Temkin, Tubis and Uretsky were to develop a Whirlwind Program for producing a table of coulomb wave functions (QPR 2-19). Corbató and Little were to do the same for spheroidal wave functions (QPR 3-11). The first project is still in progress, and the second one has recently achieved publication[15].

The development of a program to generate the expansion coefficients and eigenvalues[15] for the spheroidal wave functions made it possible to carry out a subsequent research problem in quantum physics. J. L. Uretsky used the basic routine of Corbató and Little in order to develop a technique for computing the bound states and zero energy scattering cross-sections for a finite square well potential of spheroidal symmetry[16] (QPR 12-17).

The survey just given is not all inclusive. As in any research program there were many avenues of investigation which were abandoned for one reason or another. Some of these were:

Programming of the "Knight's Tour" on Whirlwind (QPR 2)
Scattering of Electrons from a Diatomic Molecule (QPR 5, 6)
Neutron-Proton Interactions (QPR 5, 6)
Whirlwind Study of Group Dynamics (QPR 5)
Photoproduction of Mesons from Deuterons (QPR 8, 11)

It seems appropriate to remark that many members of the faculty of the M.I.T. Physics Department played indispensable roles in initiating and guiding the research carried out under this contract. In particular, Professors P. M. Morse and H. Feshbach are, in one way or another, associated with nearly all of the investigations discussed above.

Footnotes

1. Quarterly Progress Reports of the M.I.T. Committee on Machine Computation and Numerical Analysis will be referred to by the initials "QPR" followed by the relevant report numbers. Report number 1 was dated September 15, 1951.

2. H. Feshbach and S. I. Rubinow, Physical Review 88, 484 (1952); 96, 218 (1954); 98, 183 (1955).

3. L. Sartori, Ph. D. Thesis - Physics - (title not available).

4. M. C. Newstein, A Variational Treatment of Atomic Scattering, M.I.T. Physics Department, Ph.D. Thesis (1955). M.I.T. Committee on Machine Computation, Technical Report No. 4, February 1955.

5. J.L. Uretsky, The Elastic Scattering of Electrons from Hydrogen and Helium, M.I.T. Physics Department, Master's Thesis (1952).

6. A. Temkin, Ph.D. Thesis - Physics - (title not available).

7. L. Mower, A Variational Calculation of Electron Scattering from a Static Potential, M.I.T. Physics Department, Ph.D. Thesis (1953). Physical Review 89, 947 (1953); 99, 1065 (1955).

8. D. Combelic, A Phenominological Triplet Neutron-Proton Potential, M.I.T. Physics Department, Master's Thesis (1953).

9. H. Yilmaz, Wave Functions and Transition Probabilities of Light Atoms, M.I.T. Physics Department, Ph.D. Thesis (1954). Physical Review 100, 1148 (1955).

10. P. M. Morse, L. A. Young, and E. S. Haurwitz, Physical Review 48, 948 (1935).

11. A. Tubis, Physical Review 102, 1049 (1956).

12. M. Rotenberg, A Self-Consistent Determination of the Nuclear Radius, M.I.T. Physics Department, Ph.D. Thesis (1955). Physical Review 100, 439 (1955). M.I.T. Committee on Machine Computation Technical Report No. 6, September 1955.

13. F. J. Corbató, J. Chem. Physics 24, 452 (1956).

14. J. D. C. Little, Use of Storage Water in a Hydro-electric System, M.I.T. Physics Department, Ph. D. Thesis (1955). J. Operations Res. Soc. of America 3, No. 2 (1955).

15. J. A.Stratton, P. M. Morse, L. J. Chu, J. D. C. Little, and F. J. Corbató, Spheroidal Wave Functions, Wiley (1956).

16. J. L. Uretsky, Some Properties of the Spheroidal Square-Well Potential, M.I.T. Physics Department,Ph.D. Thesis (1956). Physical Review, to be published.

Jack L. Uretsky

MATHEMATICS DEPARTMENT

Introduction

This is a terminating report for work done under DIC project number 6915 by the mathematics members during the period July 1, 1951 to June 15, 1956. The research assistants and associates selected from the Mathematics Department and constituting all the mathematics members - except faculty - that have worked under sponsorship of the Committee at any time during the above period have been the following:

Aronson, Donald G.
Benington, Herbert
Burrows, James W.
Caldwell, William V.
Fox, Phyllis A.
Glantz, Herbert

Handy, Benjamin F.
Hershenov, Joseph
Hicks, Noel J.
Hsu, Pao Tau
Klein, Paul
Lister, Mary
McIlroy, M. Douglas
Phipps, Philip L.
Ralston, Anthony
Rankin, Bayard
Schlesinger, James
Segel, Lee
Weinitschke, Hubertus J.

The main body of this report is made up of abstracts of selected projects that were inaugurated and carried out by ten of the above members. The selection of projects to be abstracted was made largely on the basis of how completely the work was reported and how well the project was completed or led the researchers to clearly stated results. This selection has left out any mention of a fair amount of work that in the course of the projects warranted progress reports but did not seem to lead to any conclusive results. Some of the work done under the committee is too new to report in conclusive form and has not been mentioned for that reason. The projects that have been abstracted encompass the subjects of aerodynamic theory, fluid dynamics, elasticity, differential equations, group theory, cascade shower theory, stochastic processes, and the logic and programming of digital computers.

The abstracts themselves have tried to outline the particular problem at hand, the mathematical structure of the problem, the techniques used to solve the problem, and the form of the results. Mention has been made of the type of digital computer used, if any. After the title of each abstract some hyphenated numbers identify the references within the progress reports where the work was reported. For example 1-11 refers to progress report no. 1, page 11. Further references in the form of final reports, technical reports or published papers are numbered in square brackets at the end of each abstract and accumulated at the end of this terminating report.

Abstracts

P. T. Hsu, Aerodynamic Equations. 1-11, 2-9, 3-7, 4-9.

The general problem of this project was to obtain approximate solutions of the two-dimensional linear integral equations of lifting surface theory for steady incompressible flow. The problem was confined to the study of a rectangular surface and had interpretation for 2 full span aileron. Specifically, the pairs of quantities desired were the lift and moment, or the lift and hinge-moment. Prof. E. Reissner suggested the method of solution. The numerical technique required was largely numerical integration. Calculations were carried out at first by desk calculator and later by digital computer. Numerical results were obtained and compared with those of Weissinger, another worker in the field.

Mathematically speaking, the basic equation was the following which was to be solved for p when w and k were given:

$$w(x,y) = \frac{1}{\pi}\oint_{-1}^{1} \frac{p(\xi,y)}{x-\xi}\,d\xi + \frac{1}{2\pi}\oint_{-s}^{s} \frac{\partial p}{\partial \eta}\left\{\frac{1}{y-\eta} + K(x-\xi,\,y-\eta)\right\}d\xi\,d\eta$$

This equation was modified in two ways so that the unknown was not the two-dimensional variable p but two one-dimensional variables, lift and moment, or lift and hinge-moment. The modifications, through weighting factors, transformed the equation in each case into two one-dimensional integral equations and it was these pairs of equations that were ultimately solved.

P. Fox, Spherical Wave Propagation. 2-12, 3-6, 4-9.

As an important case of numerical integration of hyperbolic differential equations, the integration of equations governing spherical wave propagation for an adiabatic expansion was studied. The integration was carried out on Whirlwind I by expressing the equations in finite difference form along their characteristic directions. Results were compared with those of J. J. Unwin.

The equations governing the adiabatic propagation of spherical waves have the mathematical form:

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} = -\rho\left(\frac{\partial u}{\partial r} + \frac{2u}{r}\right)$$
$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} = -\frac{c^2}{\rho}\,\frac{\partial \rho}{\partial r}$$

to be solved for u, the radial velocity, and ρ, the density. Along the characteristic direction defined by $dr = (u + c)dt$, the equations take the form

$$\frac{d\rho}{\rho} + \frac{du}{c} = -\frac{2u}{r}\,dt$$

Along $dr = (u - c)dt$ they take the form

$$\frac{d\rho}{\rho} - \frac{du}{c} = -\frac{2u}{r}\,dt$$

It is these latter equations expressed in difference form that were solved. Important computational difficulties arose in regions where terms behaved badly or where convergence was poor. Such regions were at the center of the sphere and where shock waves developed.

This work was later carried on by A. Ralston. Under the authorship of P. Fox it led to part of the first technical report of the project [1] and to a Ph.D. Thesis in mathematics.

P. Fox, Thin Airfoil Theory: Perturbation Solutions. 6-16, 7-17, 8-20.

A perturbation method was used to solve some problems of fluid flow past a thin airfoil. The general method that effects a linearization of the problem was improved with a technique described by M. J. Lighthill. Solutions were obtained mainly for the simple two-dimensional, irrational, incompressible flow, but some results for the more interesting compressible case were obtained. The shape of the airfoil was chosen as an ellipse though some consideration was given to the possible application of the Lighthill technique to hyperbolic types.

The equations involved are, for the velocity components u and v,

$$u_x + v_y = 0$$
$$u_y - v_x = 0$$

The Lighthill technique involves a coordinate transformation based on expanding the independent as well as the dependent variables in terms of a small parameter. The consecutive functional coefficients of the powers of the parameter are adjusted so as to suppress singularities arising in the solution of the problem. This involves a transformation of the coordinate system. The chief difficulties were encountered in the convergence of the modified perturbation series near the leading and trailing edges of the airfoil.

This work led to related techniques in the study of plane wave propagation [2] and to part of the technical report [1] and thesis already mentioned.

N. J. Hicks, Non-Linear Boundary Value Problems. 4-8, 6-9, 7-14, 8-16.

Some series solutions for boundary value problems of ordinary non-linear differential equations were carried out on Whirlwind I. The equations of interest appear in the theory of finite defermations of thin elastic circular plates. These equations were solved by expressing the unknowns in Taylor series, obtaining recurrence relations for the coefficients in these series, and then solving these recurrence relations. In order for this method of solution to go through, it was necessary for some boundary conditions to be replaced by additional initial values obtained through non-linear interpolation. The use of an I.B.M. card machine or a differential analyser for this problem was also considered.

The specific equations of interest to be solved for f(x) and g(x) were

$$f''(x) + \frac{1}{x} f'(x) - \frac{1}{x^2} f(x) = u \frac{g(x)f(x)}{x}$$

$$g''(x) + \frac{1}{x} g'(x) - \frac{1}{x^2} g(x) = \frac{u}{2} \frac{f^2(x)}{x}$$

where u is constant. The form of Taylor series used for substitution in these equations were

$$f(x) = \sum_{n=0}^{\infty} a_n (x-1)^n . \qquad g(x) = \sum_{n=0}^{\infty} b_n (x-1)^n .$$

A. Ralston, Spherical Wave Propagation. 6-11, 7-17, 8-18, 10-10, 11-14, 12-37.

The work discussed above under the same title and initiated by P. Fox was completed. That is, the numerical solution of the two partial differential equations governing the propagation of spherically symmetrical waves was carried to the point where the state of the system was very close to atmospheric equilibrium and was quickly approaching it. The initial density distribution for which complete results were obtained was $\rho = 1 + 2e^{-4r^2}$. The results showed that the shock, which it was thought might form, does in fact not form. The motion was similar qualitatively to that found by J. J. Unwin in investigating the same problem. However, the compression at the center due to the recoil wave was a factor of three times less than Unwin's.

In order to obtain complete results with the initial density distribution specified above, it was necessary to introduce near the t-axis an interpolation - extrapolation procedure to modify the method of two-characteristics previously used by Fox. This modification was then incorporated in a comprehensive program. As a result it was possible with one program to compute as many points as desired in the interior of the plane and on the t-axis for any r-axis spacing. Two integrals relating to mass and density were evaluated numerically in order to provide a physically meaningful check on the accuracy of the results. A mathematical check on the

sensitivity of the solution to the initial r-axis spacing was also made.

Theoretically, the same comprehensive program should have given results for any initial density distribution, for example $\rho = 1+5e^{-4r^2}$. Results for this latter distribution was not forthcoming, however, because errors became too large. A careful investigation into the source of these errors was made. Before calculations were stopped it appeared that a shock was forming in this case, though it had not formed with the previous density distribution.

This work was reported completely in the second technical report of the project [3].

A. Ralston, The Problem of Buckling of a Hyperbolic Paraboloidal Shell Loaded by its Own Weight. 15-12, 16-31.

The problem solved was that of finding the smallest uniformly distributed load which causes the buckling of a shallow hyperbolic paraboloidal shell which has moment free support and whose edge stiffening members are rigid in the direction of their axes and have negligible bending resistance in planes tangent to the shell. The approximate finite problem in a certain special case was solved on Whirlwind I.

The differential equations of the classical linear stability theory for this problem were derived by E. Reissner. They are of the form:

$$\nabla^2 \nabla^2 \phi = Eh \frac{2c}{ab} \frac{\partial^2 w}{\partial x \partial y}$$

$$D\nabla^2 \nabla^2 w = -\frac{2c}{ab} \frac{\partial^2 \phi}{\partial x \partial y} + p_o \frac{ab}{c} \frac{\partial^2 w}{\partial x \partial y}$$

where h is the thickness of the shell, E is Young's modulus, w is the deflection of the shell, ϕ is a stress function, D is a constant, and p_o is the weight of the shell per unit surface area. The boundary conditions were supplemented by convenient conditions for simplifying purposes. The original problem was then replaced by an eigenvalue problem in which the smallest eigenvalue corresponded to the critical shearing stress at which the plate buckled.

This work was reported in full in a published article [4].

M. Lister, Spherical Wave Propagation. 11-15, 12-39.

The differential equations of this project are the same as those investigated by P. Fox and A. Ralston in projects reported above. Here a different method of integration first discussed by D. R. Hartree was used. This numerical process used the properties of characteristics to relate the flow at the beginning and at the end of a given time interval, but did not involve the use of a grid of characteristics as used by Fox and Ralston. It had the advantage of providing the results in a form most likely to be required. The process was adapted in order that a solution could be obtained using the IBM Card Programmed Calculator.

H. Glantz, Aerodynamic Theory. 6-13, 7-14, 8-18, 10-13.

Solutions to the difference equation corresponding to the Laplace equation

$$\nabla^2 U(x,y) = 0$$

were obtained in the upper half plane when the following boundary conditions were specified:

$$\lim_{y \to \infty} U(x,y) = 0$$

$$U(x,0) = 0 , \qquad |x| \geq 1$$

$$\frac{\partial U(x,0)}{\partial y} = g(x) , \qquad |x| \leq 1.$$

Solutions were obtained after reducing the problem to a system of linear algebraic equations. A method was also developed for obtaining approximate solutions to the more general problem in which the boundary condition

$$\frac{\partial U(x,0)}{\partial x} = 0 , \qquad |x| \to 1$$

was substituted for the second one above. The results are significant for two-dimensional thin-wing aerodynamic theory for incompressible fluid flow. Corresponding three dimensional problems were also investigated.

A complete write-up of this work is available in the project's third technical report [5].

L. Segel, Application of Information Theory to Machine Computations. 11-5, 12-12.

It was shown in this project that if a measure of gain of information is associated with the addition of significant digits in a number, then it may pay, in the sense of information carried to the sum or difference of two numbers, to retain digits in the summands otherwise discarded. In general, if a sum or difference is known to lie within an interval β rather than α, $\beta \leq \alpha$, due to the addition of significant digits in the summands, then the information gained by using the extra digits is defined to be

$$\log_{10} \frac{\alpha}{\beta}.$$

This definition is suggested by information theory. This criterion leads one to a more meaningful use of the term "significant digit" than is ordinarily available.

Similar considerations can be carried to products.

L. Segel, Quadratic Theory of Bending for a Circular Plate. 12-6.

A power series solution was obtained for the problem of a circular plate under the action of a uniform compression and a uniform bending moment. The non-linear equations and their boundary conditions were non-dimensionalized and then the equations were substituted by recurrence relations after expressing the two unknown functions in power series. The two unknowns to be solved for were the angle between the horizontal and the middle surface and the product of the radius and the horizontal stress resultant. This series approach rather than an asymptotic approach was

feasible because of the availability of high speed computing machines. A flow chart for the problem was prepared. In addition, series expressions for the ratio of the vertical deflection at the edge to the thickness was obtained.

The non-linear equations to be solved were of the form:

$$L(\varphi) = \frac{1}{Dr} \varphi x$$

$$L(x) = -\frac{Eh}{2r} \varphi^2$$

where L is the equidimensional operator

$$L = \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{r^2}.$$

φ, x are the unknown functions of r. D, E and h are constants. r is the distance from the center of the plate.

A final report of this work was prepared [6].

P. Phipps, Machine Solution of the Diffusion Equation. 12-34, 14-30, 15-13, 16-25.

The solution of the heat conduction equation

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left[K(x)\frac{\partial u}{\partial x}\right]$$

for various choices of K(x) was investigated on the CPC calculator. This equation and the boundary conditions

$$-K(x)\left.\frac{\partial u}{\partial x}\right|_{x=0} = Q , \quad 0 < t$$

$$\left.\frac{\partial u}{\partial x}\right|_{x=\ell} = 0 , \quad 0 < t$$

$$u(x,0) = 0 , \quad 0 < x < \ell$$

were substituted by the corresponding difference formulae. Use of the analytic solution was made to separate the steady state solution from the transient and to determine when the transient became significant. Criteria for variable spacing in the difference approximation were worked out so that computer time could be optimized and convergence was still insured near singularities. Stability conditions were investigated to determine the behavior of error propagation and numerical examples were worked out. Various checks were made on the accuracy of the solution and the effects of a singularity as well as the ability of various spacing to counteract these effects.

A final report of this work was prepared [7].

M. D. McIlroy, Calculation of Numbers of Structures of Relations on Finite Sets. 15-10, 16-9, 17-14.

Some formulae of R. L. Davis were used to compute how many non-isomorphic structures of relationships may be defined among members of a finite set. The Whirlwind I computer was used and of special interest was the multi-register arithmetic programs that were needed to manipulate numbers of artitrary length.

More precisely, if there are n objects a dyadic relation among these objects specified by an nxn matrix of 1's and 0's, a 1 in the ij place indicating that element i bears the relationship to j. Two relations are non-isomorphic if one cannot be obtained from the other by renumbering the objects.

The project used much of the basic logic of the Whirlwind computer. All operations were conducted in integers with no round-off and during division all remainders were kept. As a result, tables were prepared for n ranging up to 16 giving exact numbers as high as 10^{60}. These tables displayed the number of reflexive, symmetric, irreflexive symmetric, and asymmetric relations. The results were compared with asymptotic formulae.

A final report of the work was prepared [8] .

B. Rankin, The Number Distribution of Electrons and Photons in Cascade. 13-48, 14-11, 18-11.

Results that had been obtained by the author in abstract stochastic process theory were carried into the formulation of computing the number of electrons and photons that emerge from a given thickness of matter when a single high energy electron or photon enters the matter. The theory used suggested certain simplifying physical assumptions and these were found to permit an explicit algebraic expression for the quantity desired. This latter expression, though in closed form, contained many terms which were to be added. The latter operations were programmed for computation on the Whirlwind I computer with the aid of the programming algebra mentioned in the following report.

In the course of the programming certain approximating formulae were obtained for the convolution of many exponential terms and a Monte Carlo technique was developed for selecting distinct, connected paths from an array of points. Complete numerical results are yet to be obtained.

B. Rankin, The Basic Problem of Numerical Analysis Expressed in the Language of Computing Machines. 15-7, 18-7, 19-7.

The three reports cited as references for this project consituted a final and comprehensive report on the subject. In this project the mathematical ingredients of a computer were defined in general and a systematic way to select an optimum program in the sense of time, for a given problem was investigated. In order to avoid impracticality in the use of too general terms, the general problem, once it was formulated, was replaced by the form that it might take for a specific computer. The computer, though imaginary, could be simulated by any modern one. An algebra for programming on this ideal computer was constructed and it was shown how a program once written in this algebra could be optimized in various ways

The algebra used as illustration in this general problem was found to be of rather general aid in programming specific problems irregardless of optimization in time. The translation of programs from this algebra to the codes of any modern computer were discussed. Certain points of basic interest in the logic of computer design were also touched upon.

The results have been published in abstract form [9] .

B. Rankin, Multiple Prediction. 16-7.

The work of N. Wierer on the prediction of single-time series and his work on matrix factorization were combined and extended to bring explicit formulae for the prediction of any of a set of n time series based on the behavior of all the series throughout the past. The series were assumed completely stationary and the prediction operator was assumed linear. Formulae for both the prediction operator and the error of prediction were obtained.

The form of the solution was a series involving the Fourier coefficients of the auto-and cross-correlations of the time series involved. Significant departures from the form of solution already known for the one series case were found. Though all computational difficulties were not investigated, the formulae appeared to submit to calculation with reasonable ease for three series.

The only report given on this subject was considered final [10] .

B. Rankin, Deterministic Processes and Stochastic Processes. 20-7

It was shown how the simple measurable, metrically transitive, and mixing transformation defined by E. Hopf could be used to construct random sequences, simple values of sequences of random variables, and, more generally, sample curves of stochastic processes. The transformation, used as an example in ergodic theory, carries the unit square into itself in such a way that the successive images of a point transformed repeatedly are found within a specified measurable subset of the square with a relative frequency that is proportional to the area of the subset. This is the metrically transitive property and associates relative frequency or time averages with probability. The same transformation deforms a specified subset asymptotically in such a way that its overlap with any other specified subset is proportional to the product of the areas. This is the mixing property and is intimately related to statistical independence.

It was shown how a transformation with the above properties, essentially a deterministic mechanism, can be used in the basic foundations of probability theory, thus leading to the conclusion,the probability theory is adequately interpreted as an asymptotic theory of deterministic processes.

Because of the fact that the Hopf transformation can be simply defined in terms of shift operations on the binary representation of numbers, an implication is immediately gained for the construction of random numbers within a digital computer. This latter point was only suggested and is not yet fully developed.

The single report given was final [11] .

References to Final Reports, Technical Reports and Published Papers.

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[2] Phyllis Ann Fox, "Perturbation Theory of Wave Propagation Based on the Method of Characteristics, Jour. Math. and Phys., 34, (1955) pp. 133-151.

[3] Anthony Ralston, "On Spherical Wave Propagation, Technical Report No. 2, Project for Machine Methods of Computation and Numerical Analysis, M.I.T., June (1954).

[4] Anthony Ralston, "On the Problem of Buckling of a Hyperbolic Paraboloidal Shell Loaded by Its Own Weight," Jour. Math. and Phys., 35 (1956) pp. 53-59.

[5] Herbert Glantz and Eric Reissner, "On the Finite Sum Equations for Boundary Value Problems of Partial Difference Equations," Technical Report No. 3, Project for Machine Methods of Computation and Numerical Analysis, M.I.T., October (1954).

[6] Lee Segel, "Quadratic Theory of Bending for a Circular Plate," Quarterly Progress Report No. 12, Machine Methods of Computation and Numerical Analysis, M.I.T., June (1954) pp. 6-11.

[7] Philip L. Phipps, "Machine Solution of the Diffusion Equation," Quarterly Progress Report No. 16, Machine Methods of Computation and Numerical Analysis, M.I.T., June (1955), pp. 25-29.

[8] M. Douglas McIlroy, "Calculation of Numbers of Structures of Relations on Finite Sets," Quarterly Progress Report No. 17, Machine Methods of Computation and Numerical Analysis, M.I.T., September (1955), pp. 14-24.

[9] Bayard Rankin, "An Algebra for Minimizing Machine Performance Time Within a Class of Algebraically Equivalent Programs," Bull. Amer. Math. Soc., 62, July (1956).

[10] Bayard Rankin, "Multiple Prediction," Quarterly Progress Report No. 16, Machine Methods of Computation and Numerical Analysis, M.I.T., June (1955) pp. 7-9.

[11] Bayard Rankin, "Deterministic Processes and Stochastic Processes," Quarterly Progress Report No. 20, Machine Methods of Computation and Numerical Analysis, M.I.T., June (1956) pp. 7-8.

Bayard Rankin

Part II

Project Whirlwind

1. REVIEW AND PROBLEM INDEX

This report covers the specific period of June 11, 1956 to September 16, 1956. During this time, 80 problems made use of 528.86 hours of the 618.07 hours of Whirlwind computer time allocated to the Scientific and Engineering Computation (S&EC) Group. Of the 618.07 allocated hours of computer time, 2.3% was down time due to computer malfunctions. The remaining 75.15 hours of the allocated time were used for terminal equipment testing and calibration, demonstrations, and various inter-run operations not logged to specific problems.

The 80 problems run during this quarter cover some 16 fields of applications. The results of 29 of the problems have been or will be included in academic theses. In these 29 problems, there are represented 21 Doctoral theses, 8 Master's, and 2 Bachelor's. Twenty-five of the problems have originated from research projects sponsored at MIT by the Office of Naval Research.

Two tables are provided as an index to the problems for which progress reports have been submitted. In the first table, the problems are arranged according to the field of application, and the source and amount of time used on WWI are given. In Table 2-II, the problems are listed according to the principal mathematical problem involved in each. In each table, the letter after the problem number indicates whether the problem is for academic credit and whether it is sponsored. The code is explained in section 2.1, Introduction.

Field	Description	Problem Number	Minutes of WWI Time	Supervisor or Programmer
Aeronautical Engineering	Transient response of aircraft structures to aerodynamic heating	236 C.	20.2	L. Schmit
	Horizontal stabilizer analysis	257 C.	1706.5	E. Criscione
	Parametric study of coupling and damping	334 C.	179.8	K. Wetmore
	Blast response of rotor blades	364 C.	153.5	K. Foss
Aerophysics Research Group	Trajectory calculations for a rocket during powered flight	310 C.	1158.9	J. Prigge
	Extraction of stability derivatives from flight test data	317 C.	790.2	L. L. Mazzola
Chemical Engineering	Critical mass calculations for cylindrical geometry	270 B.	631.1	J. R. Powell
Civil Engineering	Response of a multi-story frame building under dynamic loading	203 C.	601.3	R. G. Gray
	Response of a single story concrete building to dynamic loading	354 D.	79.1	B. Landry
	Dynamic response of shear walls	360 C.	105.5	Shui Ho
	Design of spherical shell segments	372 B.	80.6	E. Traum
	Response of a reinforced concrete roof system to dynamic loading	378 B.	32.5	B. Landry
Electrical Engineering	Production for transportation problem	326 C.	535.6	J. Dennis
	Switching circuits	380 B.	39.2	C. Roth
	Calculation of prime numbers	382 B.	25.4	H. Cohen
Energy Conversion Lab.	Optimization of alternator control system	264 C.	72.6	J. Dennis
Geology	Fourier synthesis for crystal structures	261 C.	464.9	M. J. Buerger
	Fourier synthesis for crystal structure	362 B.	58.1	N. Niizeki
Hydrodynamics Laboratory	Stokes particle velocities	383 C.	18.7	T. Marlow
Lincoln Laboratory	Eigenvalue problem for propagation of E.M. waves	193 L.	1446.0	H. B. Dwight
	Tropospheric propagation	300 L.	1731.4	W. Mason
	Error analysis	312 L.	284.3	L. Peterson
	Prediction analysis	327 L.	713.1	L. Peterson
	Atmospheric propagation of radio waves	371 L.	120.5	W. Mason
	Coverage analysis	377 L.	287.4	L. Peterson
Mechanical Engineering	Thermodynamic and dynamic effects of water injection into high-temperature high-velocity gas streams	120 B,N.	58.3	A. Erickson
	Laminar boundary layer of a steady, compressible flow in the entrance region of a tube	199 N.	60.3	T. Y. Toong
	Rolling bearings	293 C.	108.0	A. Shashaty
	Growth of fatigue cracks	361 B,N.	1.7	J. B. Walsh
	Asymptotic integration of equations concerning Torroidal shells	363 A.	174.0	S. H. Crandall
Meteorology	Spectral analysis of atmospheric data	306 C.	620.9	B. Saltzman
	Statistical and dynamic methods in forecasting	341 C.	917.1	E. Kelley
	Weather prediction	343 C.	124.4	J. Austin
	Computation of variances and covariances	350 D.	131.6	D. Gilman
Nuclear Engineering	Calculations for the MIT reactor	266 A.	498.6	T. Cantwell
	Determination of critical mass	367 B.	275.4	J. Barnett
	Flux leveling in homogeneous reactor - part I	373 B.	101.6	R. Kennedy
	Flux leveling in homogeneous reactor - part II	374 B.	77.5	L. Hoover
Office of Statistical Services	Finding Eigenvalues of an asymmetric matrix	365	209.7	J. Roseman
	Temperature distribution in a beam	369	842.5	F. M. Verzuh
Physics	Coulomb wave functions	122 N.	68.4	A. Tubis
	Determination of phase shifts from experimental cross-sections	162 N.	39.7	F. Eppling
	Energy bands in graphite	172 B,N.	19.7	F. J. Corbato
	Exchange integrals between real slater orbitals	204 N.	61.9	P. Merryman
	Neutron-deuteron scattering	225 B,N.	18.4	L. Sartori
	Theory of neutron reactions	245 N.	2100.5	H. Feshbach
	APW as applied to face- and body-centered iron	253 N.	150.1	J. Wood
	Energy levels of diatomic hydrides	260 N.	130.9	G. Koster
	Evaluation of two-center molecular integrals	262 N.	216.2	H. Aghajanian
	Analysis of air shower data	273 N.	419.2	F. Scherb
	Multiple scattering of waves from a spatial array of spherical scatterers	274 N.	234.0	P. M. Morse
	Energy levels of diatomic hydrides LiH	278 N.	2432.9	A. Karo
	Atomic wave functions	288 N.	2909.1	R. Nesbet
	Polarizability effects in atoms and molecules	290 N.	518.6	L. C. Allen
	Pure and impure potassium chloride crystal	309 B,N.	66.7	L. P. Howland
Servomechanisms Laboratory	Data reduction	126 C.	568.1	D. T. Ross
	Flight simulation	376 N.	549.0	M. Connelly
Spectroscopy Laboratory	Complex spectrum analysis	346 B.	164.1	J. Lindner
Miscellaneous	Ultrasonic delay lines	216 C.	47.1	D. Arenberg
	WWI-1103 translation program	256 C.	101.4	J. Frankovich

Table 2-I Current Problems Arranged According to Field of Application

PROBLEM INDEX

Mathematical Problem	Procedure	Problem Number
1. Matrix algebra and equations		
Root of determinantal equation	Iteration	266 A.
Linear equations	Crout	270 B.
Eigenvalues	Diagonalization	278 N.
Eigenvalues	Diagonalization	288 N.
Orthonormalization, eigenvalues	Schmidt process, diagonalization	290 N.
Inversion	Crout	312 L.
Linear equations	Crout	317 C.
Diagonalization, inversion	Successive approximations, Crout	341 C.
Inversion	Crout	343 C.
Arithmetic operations	Direct	346 B.
Inversion, eigenvalues	Crout, iteration	364 C.
Eigenvalues, factoring polynomials	Frame, Hitchcock	365
Solution of determinants	Cramer's rule	367 B.
Arithmetic operations	Direct	377 L.
Boolean algebra, Boolean matrices	McCluskey-Quine reduction	380 B.
Arithmetic operations	Direct	383 C.
2. Ordinary differential equations		
Seven non-linear first order System	Fourth order Kutta Gill	120 B,N.
5 simulatneous non-linear differential equations, second order	Gill	199 N.
Wave euqation	Difference equations	203 C.
Different sets of differential equations	Milne predictor-corrector formula	245 N.
Second order equations	Step by step	257 C.
Simultaneous differential equations	Gill's method	310 C.
Sets of simultaneous second order differential equations	Finite difference	334 C.
Pair of simultaneous second order	Backward difference equations	360 C.
Set of differential equations	Finite difference	363 A.
Second order	Kutta-Gill	371 L.
Set of differential equations	Finite difference	374 B.
	Kutta-Gill	376 N.
3. Partial differential equations		
Second order parabolic	Finite difference	236 C.
Second order, variable coefficients	Finite difference, trapezoidal integration	373 B.
4. Integration		
Stationary point of a variational	Simpson's rule	225 B,N.
Overlap integrals	Evaluation of analytic forms	260 N.
Overlap integrals	Evaluation of analytic forms	262 N.
Integrations	Simpson's rule	278 N.
Fresmel integral	Conversion power series with a complex argument	300 L.
Integrations	Barnet and Coulson expansion	309 B.
Integrations	Gauss quadrature	312 L.
Integration	Finite difference	354 D.
Integration	Trapezoidal rule	359 B.
Integrations	Series substitution	372 B.
Integration	Finite difference	378 B.
5. Transcendental equations		
Curve fitting	Least squares	162 N.
Non-linear equations	Steepest descent	264 C.
Non-linear equations	Iteration	272 N.
Curve fitting	Least squares	273 N.
Non-linear equations	Steepest descent	293 C.
6. Complex algebra		
Complex roots and function evaluation	Iteration	193 L.
7. Data reduction		
Data reduction	Polynomial fitting, etc.	126 C.
8. Fourier series		
Fourier synthesis	Direct evaluation	261 C.
Summing series	Direct evaluation	274 N.
Fourier synthesis	Direct evaluation	306 D.
Fourier summation	Direct evaluation	362 B.
9. Statistics		
Evaluation of covariance and variance	Direct	350 D.
10. Integral equations		
Volterra type	Trapezoidal	361 B,N.

Table 2-II Current Problems Arranged According to the Mathematics Involved

2.1 Introduction

Progress reports as submitted by the various programmers are presented in numerical order in Section 2.2. Since this summary report presents the combined efforts of DIC Projects 6345 and 6915, reports on problems undertaken by members of the Machine Methods of Computation (MMC) Group have been omitted from Section 2.2 of Part II to avoid duplication of Part I. For reference purposes, a list of the MMC Group problems appears below.

Letters have been added to the problem numbers to indicate whether the problem is for academic credit and whether it is sponsored. The letters have the following significance.

A implies the problem is NOT for academic credit, is UNsponsored.

B implies the problem IS for academic credit, is UNsponsored.

C implies the problem is NOT for academic credit, IS sponsored.

D implies the problem IS for academic credit, IS sponsored.

N implies the problem is sponsored by the Office of Naval Research.

L implies the problem is sponsored by Lincoln Laboratory.

The absence of a letter indicates that the problem originated within the S&EC Group.

2.2 Progress Reports

100. COMPREHENSIVE SYSTEM OF SERVICE ROUTINES

The Comprehensive System of Service Routines refers to a collection of utility programs which have been in routine use at the Digital Computer Laboratory for the past several years. Included among these are a translator-compiler, a post-mortem program and a utility control program which supervises the operation of the Whirlwind computer.

During the past year no significant changes have been made in the vocabulary of the compiler. Some development has taken place, however, in the utility control program and the post-mortem program.

(1) Director Tapes:

Normal operation of the WWI computer is initiated by submitting a performance request to the operator. The performance request lists the tapes required for the run and their operating instructions. A standard set of abbreviations has been adopted for use here which conveys all of the required information to the operator. The purpose of the director tape is to replace the computer operator in the system. A director tape is prepared by typing the operating instructions from the performance request on a flexowriter. The run can then be executed automatically by reading into the computer the director tape and the tapes forming the run (suitably spliced).

Director tapes have not been as widely used as was expected. There are several reasons for this:

(a) Learning to make use of director tapes places an additional burden on the programmer. Since most runs are fairly simple, most programmers have not felt any need to make use of the system.

(b) The system as presently constructed attempts to duplicate exactly the manual operation of the computer and requires that paper tapes forming a run be spliced together. This has tended to make the system rather unpopular with the tape room.

Director tapes are being used, however, by a small number of programmers whose runs are complicated or out of the ordinary since they guarantee that the set of operating instructions required for their run will be performed correctly

(2) The Post-Mortem Program:

A new generalized post-mortem program is being added but is not yet in routine use.

Request for print-outs will be made using a request tape as before. The programmer may request that specified ranges of storage be printed-out in specified forms (instructions, floating-point numbers, integers, etc.). Both range specification and printed output may be made in terms of floating addresses (previously all information was numerical).

Post-mortem can be obtained as before by reading in a request tape after a run has been executed; however, in the new program the following option is also available: the programmer may specify up to 7 different intermediate break points in his program and associate post-mortem requests with each of these. If any of the break points are reached during the operation of the program the required post-mortem will be given and operation will continue. A final break point (with associated requests) can also be specified. If a program does not terminate at the final break point an "in case of trouble" post-mortem can be specified. Requests using break points must be read in before a program is operated.

F. Helwig
Digital Computer Laboratory

The CS II conversion routine has been modified to accept mnemonic cf as the binary code 00111 in order to make it easier for programmers to use this new instruction and the 4192 more registers of core memory. A further modification prevents an anomalous stop in conversion when there are exactly 256 flads in a tape to be converted. Previously, more than 256 flads were necessary to have the conversion post-mortem print "too many flads". This post-mortem will now be given for 256 flads. The maximum of 255 flads in an acceptable program has not been changed. These modifications have been made in the paper tape of the program and will be transferred to magnetic tape unit 0, at the next opportunity.

A routine for printing out utility programs in parallel columns of octal fractions and Whirlwind instructions, with core memory addresses and drum addresses has been developed. It is used to provide a format suited to annotation and then duplication, so that up-to-date annotated copies of the basic utility programs, e.g., the Utility Control Program, the Generalized Post-Mortem Program, may be available. The output of the program consists of punched paper tapes which can be run through a Flexowriter for print-out.

M. Weinstein
Digital Computer Laboratory

120 B,N. THE AEROTHERMOPRESSOR

During the past quarter, theoretical computations were made to check experimental results. It is expected that the problem will be completed by the end of 1956.

A. J. Erickson
Mechanical Engineering

122 N. COULOMB WAVE FUNCTIONS

The computation program for Coulomb Functions has been terminated temporarily since the present writer has left the machine computation group. The program described in the June 15, 1956 Quarterly Progress Report of the computation group has been used by the author this summer while at the Los Alamos, New Mexico, Scientific Laboratory. Tables of Fo, Go and their derivatives for η = 4.0, 4.2, 4.4, ... 10.0 and ρ = 0, 0.2, ..., 40 were computed on the I.B.M. 704 with an accuracy between 5 to 7 significant digits. The computations could easily be extended to include the range $0 \leq \eta \leq 4$ and $10 \leq \eta \leq 15$.

A. Tubis
Physics Department

126 C. DATA REDUCTION

Problem 126 is a very large data-reduction program for use in the

Servomechanisms Laboratory. The over-all problem is composed of many component sections which have been developed separately and are now being combined into complete prototype programs. Descriptions of the various component sections have appeared in past quarterly reports. After the development and testing of the prototype Whirlwind programs is completed, the programs will be re-coded for other, commercially available, large scale computers, (probably the ERA 1103, IBM 701, and IBM 704 computers), for use by interested agencies for actual data reduction at other locations. The programs are currently being developed by Douglas T. Ross, David F. McAvinn, Walter E. Weissblum, Benson H. Scheff, and Dorothy A. Hamilton, Servomechanisms Laboratory staff members with the assistance of John F. Walsh. This work is sponsored by the Air Force Weapons Guidance Laboratory through Division of Sponsored Research Project 7138.

Because of its nature, the problem requires extreme automation and efficiency in the actual running of the program, plus the presence of human operators in the computation loop for the purpose of decision-making and program modification. Thus, while the computer is running, extensive use is made of output oscilloscopes, so that the computer can communicate with the human and manual intervention registers and so that the human can communicate with the computer in terms of broad ideas. The computer program then translates these ideas into the detailed steps necessary for program modification in confirmation of the human operator's decision. The program which does this translation and modification is called the Manual Intervention Program (MIV). The most recent version of the prototype data-reduction program is called the Basic Evaluation Program.

During the past quarter, work has continued on the Evaluation Program and the MIV System. The evaluation program is being greatly expanded to handle considerably more complicated data reduction. Several more test runs have been made and compared against previously computed values. A special purpose post-mortem program written some time ago for this problem has been expanded and incorporated into the manual intervention system using intervention switches as well as the direct input typewriter keyboard. An elaborate program for controlling the labelling and format of both the scope plot and the post-mortem programs is now being revised. This program will later be expanded to include facilities for giving automatically edited logs of manual intervention operations as well as automatic simulation of manual actions for automatic playback.

As mentioned in previous quarterly reports, programs which are being developed on Whirlwind Computer are prototype programs which are to serve as models for production Univac Scientific 1103 computer programs. The 1103 Input Translation Program of Problem 256 has been jointly sponsored by Problem 126 for the purpose of aiding the transcription of 1103 computer programs. During the past quarter, extensive use has been made of this facility and many programs have been translated and run on the 1103 computer at the Air Force Armament Center, Eglin Air Force Base, Florida. The details of the procedure were worked out in a visit to the Armament Center and at present the debugging of programs is being done entirely by mail and telephone. It is expected that the majority of this work will be completed during the next quarter after which Problem 126 will return to its basic function of research programming on the application of digital computer techniques to selected problems of the Air Force Weapons Guidance Laboratory.

D. T. Ross
Servomechanisms Laboratory

162 N. NUCLEAR SCATTERING PHASE SHIFTS

During this quarter, solutions and phase shift combinations have been found at the finer mesh points necessary for prediction purposes. A theory of behavior has been formulated so that future work will consist of finding phase shift combinations at higher energies (and a few at lower) that are consistent with this behavior.

Programmers working on this problem are E. Mack and E. Compbell.

E. Mack
Nuclear Science Laboratory

172 B,N. ENERGY BANDS IN GRAPHITE

The energy bands of two-dimensional graphite have been calculated. These results, which were reported in essential detail in the previous progress report, formed the basis of a Ph.D. thesis, (Physics Dept., Sept. 1956).

In view of the two-dimensional results, a survey of three-dimensional graphite energy bands has become highly desirable. The major obstacle to this extension of the calculation is the writing of a new matrix element generation program. Current plans are to construct such a program.

F. J. Corbató
Physics Department

193 L. EIGENVALUE PROBLEM FOR PROPAGATION OF E. M. WAVES

Mode sum calculations at 410 Mc and 3000 Mc, using the bilinear model, have been extended to higher mode numbers.

Eigenvalues and eigenfunctions have been obtained for an inverse-square model at 50 Mc, with 500 and 30 foot antenna heights, using both power series and asymptotic series. Mode sums were also computed.

It is planned to continue these calculations until convergence of the mode sums is obtained.

Programmers working on this problem are H.B. Dwight, R.M. Ring and W.C. Mason.

R. M. Ring
Lincoln Laboratory

199 N. LAMINAR BOUNDARY LAYER OF A STEADY, COMPRESSIBLE FLOW IN THE ENTRANCE REGION OF A TUBE

A program is being prepared for the solution of the third set of differential equations for the case where temperature dependence of the viscosity and thermal conductivity of the compressible fluid is taken into consideration.

Solutions were obtained for the case where air flowing at supersonic speed is heated at the tube wall.

T. Y. Toong
Mechanical Engineering

203 C. RESPONSE OF A MULTI-STORY FRAME BUILDING UNDER DYNAMIC LOADING

During the past quarter programming work has been continued. When this work is satisfactorily completed we expect to use it as a basic program which will be modified accordingly, to the special conditions associated with a series of appropriate buildings, to be analyzed for various types of loading.

Programmers working on this problem are R.G. Gray, Shui Ho and E.A. Lawlor.

R. G. Gray
Civil and Sanitary Engineering Dept.

204 N. EXCHANGE INTEGRALS BETWEEN REAL SLATER ORBITALS

A portion of the numerical procedure has been redesigned to give a greater accuracy and to conserve computing space. It is expected that the useful parameter range will be notably enlarged. A complete statement of the alteration will appear in the next progress report with a summary of the results of testing and subsequent operation.

P. Merryman
Laboratory of Molecular
Structure and Spectra
Department of Physics
University of Chicago

216 C. ULTRASONIC DELAY LINES

Programming has been completed and new designs will be worked on from time to time.

Programmers working on this problem are Richard Bishop and John Ackley.

R. Bishop
Arenberg Ultrasonic Laboratory, Inc.

225 B,N. NEUTRON-DEUTERON SCATTERING

During the past quarter, the calculations which formed the basis of the author's Ph.D. thesis have been extended through program testing. The results of these revised calculations should be completed and affirmed during the next quarter.

Programmers working on this problem are L. Sartori and H. Paul.

L. Sartori
Physics Department

236 C. TRANSIENT RESPONSE OF AIRCRAFT STRUCTURES TO AERODYNAMIC HEATING

During the past quarter the WWI has been used only to evaluate characteristic values and characteristic vectors for matrix equations of the form $Ax = \lambda Bx$. These matrix equations arise from temperature distribution problems and dynamic analyses with which we are concerned.

At this time, it appears that future use of the WWI Computer on this problem will be limited to:

a) production runs of existing programs,
b) determination of characteristic values and vectors for matrix equations,
c) inversion and multiplication of matrices.

L. A. Schmit
Aero-elastic and Structures
Research Laboratory

245 N. THEORY OF NEUTRON REACTIONS

The calculations of the cross-sections for c=1.65 and γ =.08 were completed. For these same parameters the angular distributions were done for potential wells of depth 38 Mev. and 42 Mev. These results were presented at the Amsterdam meeting on Nuclear Physics in July by Professor Weisskopf. Since then the angular distributions for 4.1 Mdv. have been calculated. At the moment small test runs are being made with various values of c and γ in hopes of getting a better fit to the experimental data for the P wave maximum around x = 9.0. If this is possible another case will be done for the values of c and γ which give the best fit there and at zero energy.

E. Campbell
Nuclear Science

253 N. APW AS APPLIED TO FACE- AND BODY-CENTERED IRON

Little machine work has been done on the problem during the last quarter. At present the particular electronic states at the k point (0,0,0) are being investigated.

Programmers working on this problem are J. Wood and M. Saffren.

J. H. Wood
Solid State and Molecular Theory Group

256 C. WWI-1103 TRANSLATION PROGRAM

A translation system for the ERA 1103 computer has been developed under a contract with the Servomechanisms Laboratory at M.I.T. With this system a programmer can write programs for the 1103 Computer (in a specified language closely resembling that used in the comprehensive system compiler) and have them translated by WWI into binary tapes which are acceptable to the 1103. The program is working satisfactorily at present.

F. Helwig
Digital Computer Laboratory

257 C. HORIZONTAL STABILIZER ANALYSIS

In this quarter the complete program which analyzes the response of a present day fighter to a gust type disturbance has been tested. The analysis employs numerical integration techniques to solve twelve non-linear differential equations of motion. At the present time a study is under way to assess the importance of some of the parameters associated with the problem. Within the next quarter it is anticipated that many production runs will be made in an effort to complete the problem.

Programmers working on this problem are N. Hobbs and E. S. Criscione

E. S. Criscione
Aero-elastic Laboratory

260 N. ENERGY LEVELS OF DIATOMIC HYDRIDES

The dipole moment of OH was calculated for three values of the internuclear distance. An electron distribution analysis was carried out using the results of the SCF-MO calculation described earlier.

The problem has been terminated. The results are being written up for publication.

A. J. Freeman
Solid State and Molecular
Theory Group

261 C. FOURIER SYNTHESIS FOR CRYSTAL STRUCTURES

The crystal structure investigations of pectolite, diglycine hydrochloride, and diglycine hydrobromide, mentioned in earlier reports, are complete and have been submitted for publication.

The refinement of the structure of wollastonite has been successfully continued and the structure of rhodonite is under investigation.

Programmers working on this problem are M. J. Buerger and T. Hahn.

M. J. Buerger
Geology Department

262 N. EVALUATION OF TWO-CENTER MOLECULAR INTEGRALS

The values of hybrid integrals evaluated by Dr. Corbató's routine were checked against the values of the same integrals obtained elsewhere and they were found to agree to four decimal places. This accuracy is not enough for the specific problem which is being calculated and therefore, attempts are being made to evaluate these integrals by P. Merryman's routine.

H. Aghajanian
Solid State and Molecular
Theory Group

264 C. OPTIMIZATION OF ALTERNATOR CONTROL SYSTEM

A program has been written to perform simple optimization with respect to the weight of an aircraft alternator. The optimization problem was simply phrased for the first computer runs in order to provide information regarding the operation of the program when the functions involved are highly nonlinear. The problem was phrased as follows: minimize the weight of the alternator holding the full load, zero power factor losses constant, by varying the magnetic flux density in the air gap and the armature stack length. This simple, 2-variable, one-equality constraint problem was run successfully for several different terminal conditions. The results of these optimization runs indicate that the program developed for minimizing a nonlinear function in the presence of nonlinear equality constraints by the method of steepest descent can be made to operate successfully.

A complete description of the techniques used can be found in WADC Technical Note 56-383, "Design of Systems Using Digital Computers", (to be published). This report also contains a description of the difficulties which arise when this technique is applied to a practical design problem. These difficulties are being investigated in order to minimize them and to extend the class of problems which can be solved by the technique.

It is anticipated that an optimization of the aircraft alternator subject to several constraints will be run in the ensuing quarter. The results of such a run should provide practical information regarding the physical and performance characteristics of the optimum machine.

Programmers working on this problem are J. B. Dennis and R. R. Brown.

R. R. Brown
Energy Conversion Laboratory

266 A. CALCULATION FOR THE MIT REACTOR

Since the design is now firm for the M.I.T. Reactor, the core was calculated using these final properties. A new type of core, with only 20% U-235 in the fuel, was also calculated.

At the moment no future calculations are planned except for those completing the above sets which are 90% complete now.

Programmers working on this problem are F. Bradley, and J. O'Connor.

T. Cantwell
Chemical Engineering

270 B. CRITICAL MASS CALCULATIONS FOR CYLINDRICAL GEOMETRY

During the past quarter, all program work was finished. The results are currently being drafted into a thesis: Nuclear Characteristics of Heavy Water Moderated, Enriched, Homogenous Reactors, which will appear shortly. Readers are referred to this report for full details of the programs and results achieved.

James R. Powell
Chemical Engineering

273 N. ANALYSIS OF AIR SHOWER DATA

During the past quarter we have used 7 hours of computer time to analyze 500 events and to do a Monte Carlo study of the resolution of the experiment. We plan to continue the analysis of data and finish the Monte Carlo calculations. A complete description of the program and method of analysis will be written in the next month.

F. Scherb
Cosmic Ray Laboratory

274 N. MULTIPLE SCATTERING OF WAVES FROM A SPATIAL ARRAY OF SPHERICAL SCATTERERS

All values needed to compute the nine M_ℓ^o expressions for seven directions have been obtained.

A program is now being tested to fit polynomials to the M_ℓ^o's by the method of least squares.

This is being done to facilitate interpolating for the M_ℓ^o's.

M. Karakashian
Nuclear Science Laboratory

278 N. ENERGY LEVELS OF DIATOMIC HYDRIDES LiH

During this quarter we have extended the calculation of the electronic energy of lithium hydride to internuclear distances of 7 and 8 atomic units, using the non-orthogonal atomic orbitals and the Löwdin formalism described in previous progress reports. The potential energy values which were derived from these results for the ground state and the first excited state are consistent with our previous work and the potential energy curves drawn through all the points show no discontinuities.

A self-consistent field calculation and a subsequent configuration interaction treatment using orthogonal molecular orbitals were carried through for the 9 internuclear distances at which we have evaluated the electronic energy. This second procedure shows remarkable agreement with the non-orthogonal case.

Using the molecular wave functions which have now been evaluated, work can be continued in evaluating other molecular quantities such as the dipole moment. A more detailed description of these recent calculations is given in the current Quarterly Progress Report of the Solid State and Molecular Theory Group.

Programmers working on this problem are A. M. Karo and A. R. Olson.

A. M. Karo
Solid State and Molecular Theory Group

288 N. ATOMIC WAVE FUNCTIONS

Calculations for various atomic functions are being carried out, such as:

1) He configuration interaction
2) transition element calculation
3) Wave functions for an MnO band calculation
4) four electron systems

Professor Nesbet's program has been used to do these first three sets of SCF calculations:

1) in an atomic He where 20 configurations are taken in interaction. (Corbato's matrix diagonalization is also used here.)

2) in various configurations of transitions metals in their atomic state, such as: Fe's, $(15)^2$ $(25)^2$ $(2p)^6$ $(35)^2$ $(3p)^6$ $(4s)^2$ and $(15)^2$ $(25)^2$ $(2p)^6$ $(35)^2$ $(3p)^6$ $(3d)^7$ $(4s)^1$ and $(15)^2$ $(29)^2$ $(2p)^6$ $(35)^2$ $(3p)^6$ $(3d)^8$.

3) in calculating $M_n{++}$ and $0^=$ ionic functions, which will be used as the basis of an energy band calculation for M_nO. (This last calculation will be done under a new problem number.)

4) Calculations on four electron systems, LiH, He2, and Be have been completed. Calculation of the magnetic hyperfine coupling constant of Li has been completed. Calculation of the binding energy of CH_4 by the one-center method, with d and f orbitals, is in progress.

During the past quarter, the He work was essentially completed. The 10 ingoing parameters were varied in order to produce the best final energy. Numerical discrepancies were found in some of the results of our transition element work. During the next quarter we hope to complete this work.

The M_nO problem is a major calculation, which has just been started under another problem.

Programmers working on this problem are R. K. Nesbet and R. Watson.

R. Watson
R. Nesbet
Physics Department

290 N. POLARIZABILITY EFFECTS IN ATOMS AND MOLECULES

The Whirlwind techniques now being used are the same as those described in previous reports. However, during the last quarter a great number of results of physical interest have been obtained. These are described in the Solid State and Molecular Theory Quarterly Progress Report for October 15, 1956.

L. C. Allen
Solid State and Molecular
Theory Group

293 C. ROLLING BEARINGS

A new iterative gradient method for solving non-linear simultaneous equations was programmed and tested. Although successful when used on a sample set of equations, it gave negative results when used on the bearings equations, because of the ill-conditioned nature of the latter. No further computer work has been undertaken.

A. J. Shashaty
Mechanical Engineering

300 L. TROPOSPHERIC PROPAGATION

It was found desirable to introduce a third type expansion involving exponentials which would be valid for intermediate values of the argument. As the program now stands a power series is used for small arguments, a sum of exponentials for intermediate values and an asymtotic expansion for large arguments.

The scheme which was developed for locating the maxima and minima automatically has been working successfully. The results have been checked for the C=0 or un situation which reduces to the ordinary Fresnel integral and agrees to six figures with calculations made by J. A. Greenwood of Harvard.

It is expected that the problem will be completed in the next quarter.

W. C. Mason
Lincoln Laboratory

306 D. SPECTRAL ANALYSIS OF ATMOSPHERIC DATA

During the quarter most of the computations were completed. It is anticipated that the remaining computations will be completed during the coming quarter.

B. Saltzman
Meteorology Department

309 B,N. PURE AND IMPURE POTASSIUM CHLORIDE CRYSTAL

The problem has been completed and presently is being written up in detail. The perfect crystal calculation led to values for the widths of occupied bands in KCI and predicted many details of the band structure. The results seemed more reasonable in some respects than earlier results, but direct confirmation of the results by comparison with experiment is not possible at the present. Calculations of the crystal energies and electronic structure of KCI with a positive ion vacancy and an attached hole gave less clear results because of necessary empirical connections for effects of electronic polarizations. The results are consistent with the present belief that the imperfection is not stable and does not exist in nature. The results of the calculation provided interesting information which should be applicable to other imperfection calculations. It is planned to use Whirlwind in the near future for investigating a few problems which still remain in the imperfect crystal case after the principal calculations discussed above.

L. P. Howland
Solid State and Molecular
Theory Group

310 C. TRAJECTORY CALCULATIONS FOR A ROCKET DURING POWERED FLIGHT

The entire program has been completely checked during this quarter, and many trajectories have been computed. At present a parameter study is being carried on, which will be continued during the next quarter.

This work is sponsored by the U.S. Air Force under Contract AF 33(616)-2392.

J. S. Prigge
Aerophysics Research Group

312 L. ERROR ANALYSIS

There has been a revision in the method of programming the modification described in QPR No. 46. This revision was necessary because of difficulties encountered in the use of the Programmed Arithmetic. Production runs are being made with this revision.

Programmers working on this problem are L. Peterson and E. Hutcheson.

L. Peterson and E. Hutcheson
Lincoln Laboratory

317 C. EXTRACTION OF STABILITY DERIVATIVES FROM FLIGHT TEST DATA

During the past three months investigation of Shinbrot's method for the extraction of stability derivatives has been continued, covering such aspects of the problem as effects of changing the number of frequencies and of varying the assumed precision of the simulated response "measurements". Determinations of lateral as well as longitudinal derivatives have been made. An investigation has been initiated covering a modification of Shinbrot's method using differences of cosines as method functions as well as the sine functions suggested by Shinbrot. Also, a new method for stability derivative extraction based on applying the principle of maximum likelihood to an assumed probability distribution for response measurements and substituting statistical estimates for the quantities in the resulting equations for which explicit analytical expressions do not exist is being considered. In the coming quarter, it is intended to continue the investigations of Shinbrot's method that are presently being carried on and to determine whether either of the new methods offers promise of better results. For a more detailed description of the work done and proposed on the problem, consult the Progress Reports on the Dynamic Stability and Control of Aircraft and Missiles of the Aeorphysics Research Group.

Programmers working on this problem are T. Carney, M. E. Hoult, L. L. Mazzola, M. N. Springer and L. E. Wilkie.

M. N. Springer
Aerophysics Research Group

326 C. PRODUCTION FOR TRANSPORTATION PROBLEM

A number of large transportation problems have been solved during the past quarter using the Whirlwind transportation routine.* These problems concerned optimum shipping from 30 to 60 plants to 260 to 291 customers.

The computer time required to solve these problems ranged from 30 to 50 minutes.

Programmers working on this problem are J. Dennis and W. Jewell.

(*See description under Problem 219 in Summary Reports 45 and 46.)

J. B. Dennis
Electrical Engineering

327 L. PREDICTION ANALYSIS

The general methods previously applied in this problem are now being applied to test the effects of radar measurement errors on a power-series-approximation method of prediction. The effects of the non-spherical earth on the prediction are also being tested. The programming involved in these two studies is largely completed and checked out.

The large system study mentioned in QPR No. 46 is now being programmed. It is a Maximum Likelihood Criterion method as applied to the prediction analysis problem previously programmed. Large sections of programs 312 and 327 are being combined to form a part of this much more elaborate analysis.

Programming is now in progress.

Programmers working on this problem are L. Peterson, E. Hutcheson, and P. Willman.

L. Peterson

Lincoln Laboratory

334 C. PARAMETRIC STUDY OF COUPLING AND DAMPING

During this quarter, production runs have been completed on several airplanes at various altitudes and airspeeds.

Future work on this problem is dependent upon a complete evaluation of the present results.

K. Wetmore
Aero-elastic and Structures
Research Laboratory

341 C. STATISTICAL AND DYNAMIC FORECASTING METHODS

During the past quarter, the development and application of programs for the method of multiple linear prediction, as described in previous reports, was continued.

These programs were applied successfully to both surface temperature and pressure change observations over a network of 64 stations. Development of a method for predicting the sea-level pressure map was undertaken. Those eight linear combinations of the original sea-level pressure observations which contributed most to the variance were computed by Whirlwind. These functions were used both as predictors and predictands. It was found that 50% of the variance was explained with a one day lag. Future plans call for extension of the method to two and three day lags and inclusion of the temperature and pressure change functions as predictors.

The attempt to incorporate temperature parameters into the 5-day prediction equation has been completed. The grid field used to obtain the 700 mb 5-day mean temperature information necessitated the computation of 28 functions (Z's) developed by the Synoptic Climatology Project. These functions were then combined with similar functions previously computed for the 700 mb 5-day mean height anomaly field of the same size. These 56 (Z's) were diagonalized. Results of the diagonalization show that 86% of the original temperature and height variance is explained by 20 of the new functions. It is planned to use the 20 functions as predictors of the 5-day mean surface temperature anomalies at several United States stations. The computation of the equation coefficients and the application of the equation to independent data will require the use of Whirlwind I.

Work was continued on a simplified, non-adiabatic model of the atmosphere. Tests were conducted using different values of heating and friction in order to determine empirically the best values to use for correspondence to actual atmospheric motions. In order to extend the number of iterations in these experiments without the time factor becoming infeasible, a revision of the original program was undertaken.

All of the work done on the project has been supervised by Professor Edward N. Lorenz, Department of Meteorology.

E. A. Kelley
Meteorology Department

343 C. WEATHER PREDICTION

During the last quarter work has continued on the development of routines for work on linear prediction for various classes of weather systems. Because of the large amount of data to be analyzed, a conversion routine for input of numerical information only was written. Use of this routine reduces read-in time a significant amount.

Future plans call for further development of linear prediction equations. Quite recently additional data arrived from the National Weather Records Center, Asheville, N.C. to further the investigation.

Work on this program has been under supervision of Professor James M. Austin, Meteorology Department.

Programmers working on this problem are Herb Brun and E. A. Kelley.

E. A. Kelley
Meteorology Department

346 B. COMPLEX SPECTRUM ANALYSIS

All available Zeeman data was used to search for energy intervals. No definite intervals have been found as yet, but several possible ones have been found recently. These will be checked in the near future, and if necessary more Zeeman data will be obtained. It is believed that some definite results will be obtained within the next quarter.

J. Lindner
Spectroscopy Laboratory

350 D. COMPUTATION OF VARIANCES AND COVARIANCES

The set of eigenvectors mentioned in the last report have been obtained. The determination by matrix multiplication of the amplitude coefficients of these vectors is the next major task for machine computation.

Programmers working on this problem are Bernard Shorr and Donald L. Gilman.

D. L. Gilman
Meteorology Department

354 D. RESPONSE OF A SINGLE STORY CONCRETE BUILDING TO DYNAMIC LOADING

The original program for this problem was designed to solve a specific problem and the required results were obtained. Our recent effort has been to generalize the program so that it would be able to handle any structure subjected to empirical loads, provided that the dynamic model was limited to either one or two degrees of freedom. The program is ready for debugging and will be used to obtain additional results.

B. Landry
Civil and Sanitary Engineering

360 C. DYNAMIC RESPONSE OF SHEAR WALLS

During the past quarter, production runs were completed, using the original program. The results are contained in a Sc.D. Thesis, by A. Finerman, Course I, 1956. In the future a new program will be written to account for base fixity, the previous results being obtained for two-point base support. This work will be carried on by Mr. Shui Ho, who replaces Mr. Finerman.

A. Finerman
Civil and Sanitary Engineering

361 B,N. GROWTH OF FATIGUE CRACKS

In reference 1, the problem of determining the rate of growth of a fatigue crack in a rectangular bar compound of an ideally plastic material was analyzed for the case of torsion in the fully-plastic range. An integral equation expressing the growth rate was obtained in finite difference form; this equation was programmed for Whirlwind, and, during the last quarter, results were obtained for several values of the parameter, Xs.

The analysis mentioned above is valid only for cracks originating from a sharp notch. The analysis was redone for cracks originating from notches of radius λ_o and resulted in an integral equation similar to the one obtained in the sharp-notch analysis. The new equation was programmed for Whirlwind and is now in the process of being corrected.

1. Quarterly Progress Report No. 20, Machine Methods of Computation on Numerical Analysis. June 15, 1956, pp. 14-16.

J. B. Walsh
Mechanical Engineering

362 B. FOURIER SYNTHESIS FOR CRYSTAL STRUCTURE

Crystal structure determination and refinement work of the mineral jamesonite has been completed.

N. Niizeki
Geology Department

363 A. ASYMPTOTIC INTEGRATION OF EQUATIONS CONCERNING TORROIDAL SHELL

Asymptotic integration of a pair of simultaneous second order ordinary differential equations was accomplished for several values of pressure, using finite difference method. The future plan is to obtain a series solution to the problem, taking enough terms in the series to obtain satisfactory agreement with the asymptotic solution. In this manner a solution will be available for all values of the important geometric variable. The results to date were reported at the 9th International Congress of Applied Mechanics in Brussels during September and were also reported before the 11th Annual Meeting of the Association for Computing Machines at Los Angeles, in a paper: Optimum Recurrence Formulas for a Fourth Order Parabolic Partial Differential Equation, (August 23-29, 1956).

Programmers working on this problem are N. C. Dahl and S. H. Crandall.

N. C. Dahl
S. H. Crandall
Mechanical Engineering

364 C. BLAST RESPONSE OF ROTOR BLADES

During the past quarter a program has been written to calculate the final matrix of the first phase of this problem. This program, together with a preliminary one which computes the elements of the matrices by the solution of difference equations, is being used with various parameters to solve for the desired normal modes, shapes and frequencies.

M. Callaghan
Digital Computer Laboratory

365. FINDING EIGENVALUES OF AN ASYMMETRIC MATRIX

This problem has been completed. The 3 largest eigenvalues were obtained using the Frame and Hitchcock methods. More work investigating the Hitchcock method will be done in connection with problem 396.

J. Roseman
Digital Computer Laboratory

367 B. DETERMINATION OF CRITICAL MASS

A total of seven different reactor core designs were investigated to determine at what value of U^{235} enrichment each design would require its minimum critical mass of uranium.

The results of these calculations were incorporated into a thesis entitled: An Engineering and Economic Comparison of Reactor Designs for Package Power Applications.

The program tape (367-268-104) has been left on file in the Nuclear Engineering Department with instructions for its use in the event there is need for a further investigation in this field.

Programmers working on this problem are J. Barnett, J. Powell and M. Troost.

J. W. Barnett
Nuclear Engineering

369 TEMPERATURE DISTRIBUTION IN A BEAM

A mathematical statement of this problem has been given in preceding quarterly reports. During the present quarterly period a number of solutions were obtained for different physical configurations and parameter variations.

The I.B.M. Type 650 Magnetic Drum Calculator was used to obtain check solutions for the Whirlwind program as well as to carry out the solution of two limiting case problems. These solutions were quite valuable from the standpoint of providing both a check solution for the runs obtained on the Whirlwind Computer,as well as providing useful information for the limiting cases.

The results obtained on WWI and the 650 agreed to six and occasionally seven decimal digits which is exceedingly good considering that two different interpretive systems were used on Whirlwind and the 650 Calculator respectively.

Programmers working on the problem are A. Zabludowsky of the Digital Computer Laboratory and M. Hermann of the Office of Statistical Services.

F. M. Verzuh
Office of Statistical Services

371 L. ATMOSPHERIC PROPAGATION OF RADIO WAVES

Various methods of placing observed data for atmospheric refraction in the Tchbyshev Integration Method for the refraction equation have been tried. A least square fit proved impractical, and a linear segment approximation is now contemplated.

In the future, continued attempts will be made to use data for hourly variation of atmospheric refraction as mentioned above.

Extension of calculations from 300,000 feet through the ionosphere will also be attempted in the future.

Programmers working on this problem are W. Mason and P. Duffy.

W. Mason
Lincoln Laboratory

372 B. DESIGN OF SPHERICAL SHELL SEGMENTS

During this quarter the membrane stresses of a spherical shell segment were computed The future part will deal with benoling stresses and displacements.

E. Traum
Civil Engineering

373 B. FLUX LEVELLING IN HOMOGENEOUS REACTOR, PART I

The results of this problem have been submitted to the Chemical Engineering Department in a Master's thesis entitled Uniform Power Distribution in a Reflected Homogeneous Reactor, by L. N. Hoover and R. W. Kennedy.

For Problem 373 the theory and programming were supplied by the undersigned.

The problem solution on Whirlwind I was very satisfactory. Results were in agreement with physical considerations. However, comparison with experimental data was not possible since such data are not available.

I would like to express my appreciation for the outstanding assistance of the staff of the Digital Computer Laboratory. Both tape room personnel and computer operators did their utmost to minimize my errors, all of which were directly my own fault.

It has been a privilege and pleasure to work with the computer laboratory.

R. W. Kennedy
Nuclear Engineering

374 B. FLUX LEVELING IN HOMOGENEOUS REACTOR PART II

A uniform fission rate throughout a nuclear reactor core will increase the performance and efficiency of the reactor by permitting each region of the core to operate near its temperature limit, which is determined by heat transfer and metallurgical properties

By employing a suitable reflector with a homogeneous core, it is possible to greatly flatten the thermal flux and low energy flux distribution in a spherical reactor. For an epithermal reactor, this is equivalent to approaching a uniform fission rate in the core.

The results of diffusion theory are applied by means of finite difference approximations to i energy groups and n spatial regions for a reflected spherical reactor with homogeneous properties in the core and reflector respectively.

The finite difference equations solved are:

$$F^i_{n+1} = k^i F^i_n - F^i_{n-1} - I^i_n$$

where k^i and I^i_n are parameters determined by the material properties of the reactor.

The core radius is varied in successive iterations until the critical condition is achieved.

L. N. Hoover
Nuclear Engineering

376 N. FLIGHT SIMULATION

A total of 41 apparently correct runs have been made. Each of these runs represents 10 seconds of flight of the aircraft. The numerical data is being processed, and if it proves to be accurate and conclusive, the problem is completed.

H. M. Bourland
Servomechanisms Laboratory

377 L. COVERAGE ANALYSIS

The first half of the program, as described in QPR No. 46 has been completed; production runs have been made. The data-handling aspects of the problem are now being considered, initial programming having been done by R. Karp.

Programmers working on this problem are L. Peterson, E. Hutcheson and R. Karp.

L. Peterson and E. Hutcheson
Lincoln Laboratory

378 B. RESPONSE OF A REINFORCED CONCRETE ROOF SYSTEM TO DYNAMIC LOADING

The object of this problem was to study the dynamic response of the roof of a single story reinforced concrete building for various loading functions.

The problem was the same as 354 except for the loading portion, and for that reason similar portions were not repeated.

The loading portion of the program consisted of three types of loading curves: triangular, square, and sinusoidal. For each of these types the slab and beam response was found for three different loading time durations. The results of the response of the system for each type of curve were interpolated to find the peak value of the curve of the same shape and loading time duration which would yield a slab deflection in a specified range.

During the past quarter, this problem has been completed.

B. Landry
Civil and Sanitary Engineering

380 B. SWITCHING CIRCUITS

Two programs have been written for analysis of switching networks by means of Boolean matrices. The first of these programs is for analysis of two-terminal contact networks to be analyzed is represented by a symmetric Boolean matrix, and the terms above the main diagonal are fed into the computer in a simple coded form. After translation of the code, the computer reduces the size of the matrix by removal of successive nodes until a two-by-two matrix remains. The standard sum which represents the transmission between the output terminals of the network is obtained from this matrix and printed out in decimal form. An 8-variable, 42-node network can be completely analyzed in less than two minutes, and small networks require only a few seconds. The second of these programs is for analysis of multiple-output contact networks of 4 to 8 variables and 3 to 40 nodes. The terms on and above the main diagonal of the symmetric Boolean matrix which represents the contact network are fed into the computer, and the computer squares the matrix successfully until an output matrix is obtained. The transmissions between any or all node pairs in the network can then be obtained directly from this output matrix.

The McCluskey-Quine reduction procedure is currently being programmed to determine minimum sum forms of switching functions of 15 or fewer variables with a maximum of 512 terms in the standard sum. The part of the program which determines the prime implicants from the standard sum has been completed and checked, and the remainder of the program should be completed in a few weeks. After the performance of this program has been evaluated, a program will be written for synthesis of non-series-parallel switching circuits by means of Boolean matrices.

Charles Roth
Electrical Engineering

382 B. CALCULATION OF PRIME NUMBERS

Problem Description

The feasibility of determining prime numbers on the Whirlwind computer, the minimization of the time involved and the limiting factors are to be determined. The sieve of Eratosthenes lists all the numbers to be tested and, then, strikes out those divisible by 2, those divisible by 3, those divisible by 5,..., those divisible by p_1, where p_1 is prime and less than Y, the greatest number to be tested. This method will be modified in the following ways in order to make the problem applicable to the computer:

1. Only primes equal to or less than the square root of Y need be considered for eliminating composite numbers

2. X, the least number to be tested, may be reduced mod p, for each elimination prime, and a least negative residue can be obtained in each case. Since $X = r_1 \bmod p_1$ implies that $X+1 = [r_1+1] \bmod p_1$, the negative residues can be used for the elimination of composite numbers in lieu of the corresponding primes. If at any time $X+a = 0 \bmod p_1$ we know that X+a is divisible by p_1 and hence composite. This process of adding one to each negative residue and seeing if it becomes zero may be continued until all numbers between X and Y are tested.

Two variations of the above procedure will be considered to determine which will take less time. The first variation checks every residue on each number to be tested. The second goes onto the next number to be tested as soon as one prime divisor has been determined. Although the second method seems shorter, there is more time involved in resetting the residues to congruent negative values.

Limiting factors such as number of memory storage registers, number of bits that can be stored in a single register and economical time will be considered in anticipation of further attempts at determining prime numbers on the Whirlwind computer.

Progress During Quarter

I have divided my problem into four parts, each of which operates independently of the others. The first part computes the smallest (in absolute value) negative remainders of the fraction, X/p_1*, for each prime, p_1. I have already obtained a program that will do this for X's that are small enough to fit in a single register, but when this routine is altered slightly to accommodate two register numbers it fails. At present I am working on a program that will allow me to check the formation of the residues at each step.

Once the residues have been formed, the smallest prime divisor greater than 1, of each number n_1 between X and Y is recorded. I have developed a program that will do this if given the proper negative residues. I also have a second program that can do the same operation, and I intend to determine which of the two is more economical.

The third part of the problem is recording the least prime divisors in a suitable fashion. The outputs that I am using at the present time would not be practical for the computation of, say, a million numbers, although it is quite satisfactory for the few hundred numbers that I am using to troubleshoot the program. I have not yet begun to work on an output that can handle the large amount of numbers that I will get ultimately.

The final part, which is dispensable and which will be completed only if I have time, is the making of a program that will find and store all primes, $p_1 \leq \sqrt{Y}$ This part is not absolutely necessary since the primes could be found in tables and recorded on the actual input program. The recording of these primes is somewhat laborious and this last part will facilitate further attempts to determine prime numbers on the Whirlwind computer.

H. Cohen
Electrical Engineering

383 C. STOKES PARTICLE VELOCITIES

Problem Description

The purpose of this problem is the solution of G. B. Stokes' equations for the motion of water particles in oscillatory waves of finite amplitude. The entire program is being carried out under Contract DIC 7116 sponsored by the Humble Oil and Refining Co. of Houston, Texas, and entitled, "Wave Forces on Offshore Structures."

The problem is being solved in an effort to compare the theoretical velocities with experimentally measured velocities. The comparison will aid in evaluating the rationality of various relations derived from Newton's Laws for determining inertial forces on objects submerged in oscillatory waves.

The output of Whirlwind is to be Flexowriter tape for print out of the horizontal and vertical components of particle velocity versus wave phase angle. The velocities are to be computed for every 15° of phase at three depths for each of 12 waves.

Progress During Quarter

Tapes have been prepared and run to check the solution. The problem is now ready for production runs as soon as experimental parameters are known. It is expected that these parameters will be known in a month and all work will be completed by November 31, 1956.

Programmers working on this problem are T. Marlow and J. Hougley.

T. Marlow
Hydrodynamics Laboratory

APPENDIX

1. SYSTEMS ENGINEERING

WWI RELIABILITY

The following is the WWI Computer Reliability for the past quarter:

Total Computer Operating Time in Hours	2572.2
Total Time Lost in Hours	32.9
Percentage Operating Time Usable	98.8
Average Uninterrupted Operating Time Between Failure Incidents in Hours	22.3
Total Number of Failure Incidents	114.0
Failure Incidents per 24 Hour Day	1.06
Average Lost Time Per Incident in Minutes	17.3
Average Preventative Maintenance Time Per Day in Hours	2.2

HIGH-SPEED MEMORY

The storage facilities available at WWI include magnetic tape units, magnetic drums, and magnetic core memory. The access time to both the tape and drum storage is long in comparison to that required of the core memory. WWI is frequently scheduled to provide solutions to many real time problems. To meet these demands, the high-speed core memory facilities have been expanded by a factor of three. The addition of 4096 registers raises the high-speed core memory complement to 6144 registers (each register is composed of 17 bits). The WWI computer is designed to employ a 16 bit work in which 5 bits form the instruction and 11 bits address the desired register. Therefore, one-third of the registers (2048) can be actively engaged in problem solving, while the remaining two-thirds are passive. However, through use of a new computer instruction, change core memory fields (cf), it is possible to activate or deactivate any of the six core memory fields (a field is composed of 1024 registers) in 20 microseconds. The cf instruction provides several flexible features in having computer pulses interrogate the address bits for the cf instruction to determine if any of the following actions are to be performed:

1. program pass control to the succeeding register or to any other register;
2. replace a single field, both fields, or leave the field assignments unchanged;
3. assign any of the six fields to appear as fixed addressed registers 0 - 1023 or 1024 - 2047.

To further assist WWI users in solving real time problems, the core memory access time has been reduced from 8 to 7 microseconds.

2. VISITORS

Tours of the Whirlwind I installation include a showing of the film, "Making Electrons Count", a computer demonstration, and an informal discussion of the major computer components. During the past quarter, the following 10 groups totalling 494 people visited the computer installation. Those attending the MIT summer courses represent businesses, colleges, and government agencies all over the United States and several other countries.

June 26	MIT Summer Course 6.569, "Switching Circuits"	18
June 27	MIT Summer Course 2.219, "Vibration, Shock, and Noise"	94
June 28	MIT Summer Course 6.569, "Switching Circuits"	21
July 3	MIT Summer Course 16.259, "Aerodynamic Heating of Aircraft Structures in High-Speed Flight"	45
July 7	MIT Summer Course 15.579, "Operations Research"	33
July 9	MIT Science Teachers Summer Program	67
August 9	MIT Summer Course 15.548, "Electronic Computers and Business Problems"	40
August 10	MIT Summer Course 1.559, "Structural Design for Dynamic Loads"	52
August 15	MIT Summer Course 15.549, "Business Management and Electronic Data Processing"	58
August 29	MIT Training Course for the IBM 704	66

The procedure of holding Open House at the Laboratory on the first, and sometimes the third, Tuesday of each month has continued. A total of 125 people attended the 4 Open House tours during the quarter, representing members and friends of the MIT community, Harvard, University of Virginia, Yale, New York University, Johns Hopkins University, Sperry Rand, Lybrand Ross Brothers and Montgomery, Shell Development Co., Lahey Clinic, National Shawmut Bank, Provident Institution for Savings, Latrola Steel Co., Ethyl Corp., Army Signal Corps, Pittsburgh Plate Glass Co., Dow Chemical Co., and Hawaiian Airlines.

During the past quarter, there were also 90 individuals who made brief tours of the computer installation at different times. Represented by these individuals were Computing Devices of Canada, Rutgers University, University of Illinois, Stanford University, Barbour, Stockwell & Co., North American Aviation, Union Carbide and Carbon Co., Laval University, Quebec, Philco Co., British Broadcasting System, Institute for Advanced Study, Federal Telecommunications Laboratory, Brandeis University, Detroit Public School System, Stewart & Lloyd, Ltd., University College, London, Allied Control Co., Waseda University, Japan, Datamatic Corporation, Metropolitan Life Insurance Co., U.S. Rubber Co., Monsanto Chemical Co., and the Radio Corporation of America.

3. ACADEMIC

1. Introduction

There were a number of special summer programs offered this year which made use of the computing facilities in either the Office of Statistical Services or the Digital Computer Laboratory, respectively. The following list of subjects involved direct use of the computers:

Subject	Description	Instructor	Computer
2.138	Fundamentals of Strain Gauge Techniques	W. Murray	WW
2.219	Vibration Shock and Noise	S. Crandall	WW
2.769	Control Systems Engineering	W. Seifert	650
6.539	Analog-to-Digital Conversion	A. Susskind	WW
6.569	Switching Circuits	S. Caldwell	WW & 650
8.002	Science Teachers Program	G. Wadsworth	WW
15.548	Electronic Computers in Business Problems	R. Gregory	650
15.549	Business Management and Electronic Data Processing	R. Gregory	650
15.579	Operations Research	P. M. Morse	WW & 650
16.259	Aerodynamic Heating of Aircraft Structures in High-Speed Flight		WW

In certain cases, such as Subject 2.769, 15.548, 15.549 and 15.579, Dr. Verzuh presented a number of lectures to the classes involved. Similarly, Professor Arden presented a number of lectures in 15.579. In other cases, the computers were used either on a demonstration or laboratory basis.

2. Statistical Services Summer Academic Program

There were a number of different academic projects conducted in the Office of Statistical Services during the summer of 1956. Specifically, these included:

a. 6.68 - Special problems in Electrical Engineering,
b. Individual staff member research on the development of interpretive systems,
c. Master's theses research using the 650 Calculator,
d. Foreign Student Summer Project (FSSP) Participation

3. Subject 6.68

Mr. C. W. Turk enrolled in 6.68 and carried out a 15-unit program concerned with the following project: "Evaluation of the M.I.T. Selective System (MITSS) for the Type 650 Magnetic Drum Calculator". During the course of the summer, Turk evaluated the MITSS System and obtained quantitative information regarding the time required to effect the following subroutines:

a. floating addition,
b. floating multiplication,
c. floating division,
d. fixed and floating square root,

e. fixed and floating sin x,
f. fixed and floating cos x,
g. fixed and floating exponential,
h. fixed and floating logarithm,
i. fixed and floating arctangent,

respectively. MITSS was designed to minimize storage requirements--only 405 registers are used.

4. 650 Routines Available in Statistical Services

It should be mentioned that at the present time there are a number of interpretive systems available in the Office of Statistical Services for use with the 650 Magnetic Drum Calculator:

a. MITSS-MIT Selective System (405 registers),
b. MITILAC - floating decimal interpretive mnemonic coding (1600 registers),
c. NOPI - fixed interpretive system numeric coding (600 registers),
d. FLIMSY - floating interpretive matrix system (815 registers),
e. BTL - fixed and floating interpretive numeric coding (1000 registers),
f. SIR - symbolic interpretive routine (768 registers).

SOAP is actually a two-pass operation and does not require internal storage during machine operation. In addition to the above interpretive systems, there are a number of special-purpose programs which have been developed for use on the 650 Calculator. No attempt will be made to describe these here since a separate Statistical Services Report has been written to describe them. For example, REPORT S-29, entitled MIT Selective System for the 650 Magnetic Drum Calculator.

5. MIT Theses Using the 650 Calculator

During the summer there were nine theses completed using the 650 Calculator and facilities of Statistical Services. These are:

Student	Title	Degree	Supervisor
R. E. Lyons	Automatic Coding of a Digital Computer by the Compiler Technique	Engineering (E.E.)	Verzuh
S. Matsa	Evaluation of the Methods for Solving Simultaneous Linear Equations on a Digital Computer.	M.S. (E.E.)	Verzuh
A. Sage, Jr.	An Evaluation of Digital Computer Methods for Solving Ordinary Differential Equations Associated with a Spheroidal Antenna	M.S. (E.E.)	Verzuh
R. Hamlin	An Electronic Data Processing System for Savings Banks	M.S. (Bus.Adm.)	Verzuh
C. H. Roth	Study of Series-Parallel Switching Functions	M.S. (E.E.)	Huffman

C. Woodruff	Evaluation of a Method of Time Domain Synthesis	M.S. (E.E.)	Guillemin
C. Sherbrooke	A Digital Computer Analysis for Production Scheduling	B.S. (Bus.Adm.)	Arden
R. Follett	Solution of a Rectangular Game Using the 650 Calculator	B.S. (Math)	Wadsworth
D. Binner	A Digital Computer Program for the Computation of Commutation Tables	B.S. (Math)	Franklin

The actual number of machine hours required to complete a particular thesis varies considerably depending upon the type of thesis. Generally speaking, a master's thesis requires much more time than a bachelor's thesis; on the other hand, certain theses require more machine usage because of their nature.

6. Foreign Student Summer Project

During the summer, the following individuals spent the summer months in the Office of Statistical Services:

Dr. Heinz Gumin (SC.D.) has received his doctorate from the University of Muenster, Germany in 1954. He has spent the last two years working on the logical design for a transistorized computer for Siemens & Halske, Munich, Germany.

During the summer he obtained first-hand knowledge of the applicational aspects of computer design, using the 650 Magnetic Drum Calculator on several scientific problems. He is now revising the specifications for his transistorized computer in light of the applicational experience he received--emphasizing the importance of index counters, cycling, etc.

Mr. Gunnar Dundblad, of the Dator Corporation, Sweden, spent a number of weeks in the Office of Statistical Services becoming failiar with the available equipment. His particular interest was that of a developmental nature, rather than applicational. Hence, his research was devoted to transistors, printed circuit components, reliability studies, etc.

Lt. Djalma S. Ferreira spent the summer in the Office of Statistical Services studying Evaluation of the Roots of Nonsymmetric Matrices. The entire work in this area was performed using the 650 Calculator. A report describing this work is available.

It is apparent from the above activities that the summer program in Statistical Services was extremely active and very beneficial both from the participants and the operational standpoint. Specifically, these studies revealed that a large amount of concentrated time and effort of qualified personnel is needed to achieve effective solutions to these problems. A number of reports have been prepared describing the work of the above individuals.

7. Special 704 Programming Course

In addition to the previously-described academic programs, the first MIT course of programming instruction on the 704 Computer was offered during the period

August 20-31, 1956. This course was an intensive program consisting of sixty hours of instruction over a two-week period. One hundred and twenty people from twenty-four different New England Universities attended this program on a special-invitation basis. This program was one of the first educational contributions of the new M.I.T Computation Center.

8. M.I.T. Computation Center

The M.I.T. Computation Center represents a cooperative venture between M.I.T., at least 23 New England Colleges and Universities and the I.B.M. Corporation. The following colleges and universities--in addition to M.I.T.--are currently participating in the program:

Boston College	Northeastern University
Bowdoin College	Rhode Island, University of
Brandeis University	Tufts University
Brown University	Vermont, University of
Connecticut, University of	Wellesley College
Dartmouth College	Wesleyan University
Harvard University	Williams College
Mount Holyoke College	Worcester Polytechnic Institute
New Hampshire, University of	Yale University

It is expected that additional New England Colleges and Universities will become active participants in the very near future.

The principal objective of this program is to increase the number of students, engineers and scientists qualified to use computers in their research efforts. The Type 704 Computer will be used solely for non-classified, publishable work originating at M.I.T., the participating New England Colleges and Universities and the I.B.M. Corporation, respectively.

The 704 Computer and associated electronic accounting machine (EAM) equipment will be located in a two-story annex which has been added to the new Karl T. Compton Laboratory (Building 26) which is currently being erected. According to present plans, the building should be completed in February, 1957 and the EDPM equipment will be in operation shortly thereafter. The actual machine complement which will be available for use in the M.I.T. Computation Center is as follows:

Machine Complement in the M.I.T. Computation Center

Quantity	Type	Description
1	704	Analytical Control Unit
1	711	Punched Card Reader
1	716	Alphabetic Printer
1	721	Punched Card Recorder
1	733	Magnetic Drum Unit (8192 words)
1	736	Power Frame #1
2	737	Magnetic Core Storage Units (8192 words)
1	740	CRT Output Recorder
1	741	Power Frame #2
1	746	Power Distribution Unit
1	753	Magnetic Tape Control Unit
10	727	Magnetic Tape Units
1	780	CRT Display Unit

Off-Line Equipment

1	714	Card Reader
1	717	Alphabetic Printer
1	722	Card Punch
2	727	Magnetic Tape Units
1	757	Printer Control Unit
1	758	Punch Control Unit
1	759	Card Reader Control Unit

Auxiliary Machines

1	024	Key Punch
5	026	Key Punches
1	056	Verifier
1	066	Printing Card Unit) Data Transceiver
1	068	Telephone Signal Unit) Data Transceiver
1	077	Collator
1	082	Sorter
1	407	Accounting Machine
1	519	Reproducer
1	552	Interpreter
1	650	Magnetic Drum Calculator

The above complement of machines represents the initial installation. As time goes on and the number and type of applications change, the complement of machines will vary accordingly. Specifically, the two Type 737 Magnetic CORE Storage Units (8192 words) will be replaced by a single Type 738 CORE Storage (32,000 words) some time in 1958. In a similar manner, the amount of transceiver equipment, etc., will vary as need arises.

9. Preparation of Programs for the 704 Computer

Considerable thought has been given to the choice of methods for operating the 704 Computer in the M.I.T. Computation Center. As a result of a six-month study it has been decided that the basic language to be used for programming will be the SHARE language. SHARE, an organization of 704 users, was formally organized in August, 1955. Since that time, SHARE has done an outstanding job of coordinating the efforts, developing new methods and distributing useful programs and materials to the SHARE membership.

A more general algebraic compiler, sophisticated post-mortem routines and and elaborate input-output selection routines are being written for New England users of the M.I.T. Computation Center.

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M.I.T. Computation Center

PERSONNEL ON THE PROJECT

COMMITTEE ON MACHINE METHODS OF COMPUTATION AND NUMERICAL ANALYSIS

Faculty Supervisors:

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Samuel H. Caldwell	Electrical Engineering
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Jay W. Forrester	Electrical Engineering
James B. Reswick	Mechanical Engineering
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PROJECT WHIRLWIND

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