PROJECT WHIRLWIND
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FOREWORD

Project Whirlwind makes use of the facilities of the Digital Computer Laboratory. The principal objective of the Project is the application of an electronic digital computer of large capacity and very high speed (Whirlwind I) to problems in mathematics, science, engineering, simulation and control.

Whirlwind I is of the high-speed electronic digital type, in which quantities are represented as discrete numbers, and complex problems are solved by the repeated use of fundamental arithmetic and logical (i.e., control or selection) operations. Computations are executed by fractional-microsecond pulses in electronic circuits, of which the principal ones are (1) the flip-flop, a circuit containing two vacuum tubes so connected that one tube or the other is conducting, but not both; (2) the gate or coincidence circuit; (3) the magnetic-core memory, in which binary digits are stored as one of two directions of magnetic flux within ferro-magnetic cores.

Whirlwind I uses numbers of 16 binary digits (equivalent to about 5 decimal digits). This length was selected to limit the machine to a practical size, but it permits the computation of many simulation problems. Calculations requiring greater number length are handled by the use of multiple-length numbers. Rapid-access magnetic-core memory has a capacity of 32,768 binary digits. Present speed of the computer is 40,000 single-address operations per second, equivalent to about 20,000 multiplications per second.

1. REVIEW AND PROBLEM INDEX

This report covers the specific period of September 17, 1956 to December 23, 1956. During this time, 68 problems made use of 399.46 hours of the 494.06 hours of Whirlwind computer time allocated to the Scientific and Engineering Computations (S&EC) Group. Of the 494.06 allocated hours of computer time, 2.8% was down time because of computer malfunctions. 91.92 hours of the allocated time were used for terminal equipment testing and calibration, demonstrations and various inter-run operations not logged to specific problems.

The 68 problems run during this quarter cover some 18 fields of application. The results of 20 of the problems have been or will be included in academic theses. In these 20 problems, there are represented 17 Doctoral theses, 5 Master's and 1 Bachelor's. Twenty-two of the problems have originated from research projects sponsored at MIT by the Office of Naval Research.

Two tables are provided as an index to the problems for which progress reports have been submitted. In the first table, the problems are arranged according to the field of application and the source and amount of time used on WWI are given. In Table 2-II, the problems are listed according to the principal mathematical problem involved in each. In each table, the letter after the problem number indicates whether the problem is for academic credit and whether it is sponsored. The code is explained in section 2.1, Introduction.

PROBLEM INDEX

Field	Description	Problem Number	Minutes of WWI Time	Supervisor or Programmer
Aeronautical Engineering	Horizontal stabilizer analysis	257 C	1237.1	E. Criscione
	Parametric study of coupling and damping	334 C	32.9	K. Wetmore
	Blast response of rotor blades	364 C	409.1	A. Holland
	Evaluation of velocity potential on wings in supersonic flow	387 C	89.3	S. Gravitz
Aerophysics Research Group	Trajectory calculations for a rocket during powered flight	310 C	2121.3	J. Prigge
	Extraction of stability derivatives from flight test data	317 C	392.4	L. Mazzola
Chemical Engineering	An application of Monte Carlo methods to neutron diffusion	231 C	144.9	M. Troost
	Feed plate location	385 B	85.6	I. Rinard
Civil Engineering	Response of a multi-story building under dynamic loading	203 C	973.1	R. G. Gray
	Response of a single story concrete building to dynamic loading	354 D	24.8	R. Landry
	Design of spherical shell segments	372 B	138.5	E. Traum
Electrical Engineering	Switching circuits	380 B	28.3	C. Roth
	Calculation of prime numbers	382 B	215.0	H. Cohen
	Domain wall motion	399 L.B.	8.4	J. Harrington
Energy Conversion Lab	Optimization of alternator control system	264 C	35.5	J. Dennis
Geology	MIT seismic project	106 C	38.0	S. Simpson
	Fourier synthesis for crystal structures	261 C	176.4	M. J. Buerger
	First approximation solution on ore body	329 N	40.1	N. F. Ness
Hydrodynamics Laboratory	Stokes partical velocities	383 C	26.5	T. Marlow
Lincoln Laboratory	Eigenvalue problem for propagation of electromagnetic waves	193 L	1043.6	H. B. Dwight
	Tropospheric propagation	300 L	1223.5	W. Mason
	Error analysis	312 L	167.5	L. Peterson
	Prediction analysis	327 L	1134.2	L. Peterson
	Coverage analysis	377 L	178.5	L. Peterson
	Magnetic relaxation in thin films	391 L	78.7	D. O. Smith
	Phase error calculation	395 L	10.3	D. MacLellan
Mathematics Department	Nonlinear second order differential equations in the theory of elastic shells	337 N	154.2	H. Weinitschke
Mechanical Engineering	Thermodynamic and dynamic effects of water injection into high temperature high velocity gas streams	120 B.N	123.8	A. Erickson
	Laminar boundary layer of a steady, compressible flow in the entrance region of a tube	199 N	91.0	T. Y. Toong
Meteorology	Investigation of the vorticity field in the general circulation of the atmosphere	226 D	699.6	D. Cooley
	Spectral analysis of atmospheric data	306 D.	51.4	B. Saltzman
	Statistical and dynamic methods in forecasting	341 C	729.7	E. Kelley
	Weather prediction	343 C	253.3	J. Austin
	Long range forecasting	350 D.	190.5	D. Gilman
Naval Supersonic Laboratory	Free convection past a vertical plate	386 C.	72.9	M. Finston
Nuclear Science Laboratory	Response functions of air shower detector arrays	397 N.	28.8	G. W. Clark
	The inverse bremstrahlung spectrum	393 N.	47.3	H. Paul
Office of Statistical Services	Temperature distribution in a beam	369	478.8	F. M. Verzuh
Physics	Determination of phase shifts from experimental cross-sections	162 N.	80.5	F. Epling
	Neutron-deutron scattering	225 B,N.	182.3	L. Sartori
	Theory of neutron reactions	245 N.	85.5	H. Feshbach
	APW as applied to face and body-centered iron	253 N.	318.0	J. Wood
	Evaluation of two-center molecular integrals	262 N.	85.4	H. Aghajanian
	Analysis of air shower data	273 N.	2123.3	F. Scherb
	Multiple scattering of waves from a spatial array of spherical scatterers	274 N.	166.6	P. M. Morse
	Energy levels of diatomic hydrides LiH	278 N.	2546.7	A. Karo
	Atomic wave functions	288 N.	1126.7	R. Watson
	Polarizability effects in atoms and molecules	290 N.	86.5	L. C. Allen
	Pure and impure potassium chloride crystal	309 B,N.	24.8	L. P. Howland
	Diagonalization of matrices	298 A.	232.7	A. J. Morency
Servomechanisms Laboratory	Data reduction	126 C.	1145.6	D. T. Ross
	Temperature distribution in aircraft generators	388 D.	134.4	R. Moroney
Spectroscopy Laboratory	Complex spectrum analysis	346 B.	153.6	J. Lindner
Miscellaneous	Comprehensive system of service routines	100	592.1	S&EC Group
	S & EC subroutine study	141	89.0	M. Callaghan
	Comparison of simplex and relaxation methods in linear programming	219	165.3	D. Arden
	Subroutine study	396	11.3	J. Roseman

Table I Current Problems Arranged According to Field of Application

PROBLEM INDEX

Mathematical Problem	Procedure	Problem Number
1. Matrix algebra and equations		
Eigenvalues	Diagonalization	278 N.
Eigenvalues	Diagonalization	288 N.
Orthonormalization, eigenvalues	Schmidt process, diagonalization	290 N.
Inversion	Crout	312 L.
Linear equations	Crout	317 C.
Diagonalization, inversion	Successive approximation, Crout	341 C.
Inversion	Crout	343 C.
Arithmetic operations	Direct	346 B.
Inversion, eigenvalues	Crout, iteration	364 C.
Arithmetic operations	Direct	377 L.
Boolean algebra, Boolean matrices	McCluskey-Quine reduction	380 B.
Arithmetic operations	Direct	383 C.
Sets of finite difference equations	Direct evaluation	385 B.
Inversion of complex matrix	Crout	387 C.
Simultaneous linear equations	Partitioning	396
Real symmetric matrices	Diagonalization	398 A.
2. Ordinary differential equations		
Seven non linear first order	Fourth order Kutta-Gill	120 B.N.
System	Gill	199 N.
Wave equation	Milne predictor-corrector formula	245 N.
Different sets of differential equations	Step by step	257 C
Second order equations	Gill's method	310 C.
Second order equations	Difference equations	329 A.
Simultaneous differential equations	Finite difference	334 C.
2 simultaneous non-linear, second order	Power series	337 A.N.
Pair of simultaneous non-linear	Kutta-Gill	386 C.
Non-linear differential equations	Finite difference	388 D.
Pair of simultaneous differential equations	Gill	391 L.
Second order non-linear differential equations	Kutta-Gill	399 L.B.
3. Partial differential equations		
First order system	Finite differences	226 D.
First order equations	Difference equations	369
4. Integration		
Stationary point of a variational	Simpson's rule	225 B,N.
Overlap integrals	Evaluations of analytic forms	262 N.
Integrations	Simpson's rule	278 N.
Fresnel integral	Conversion power series with a complex argument	300 L.
Integrations	Barnet and Coulson expansion	309 B.
Integrations	Gauss quadrature	312 L.
Integration	Finite difference	354 D.
Integration	Simpson's rule	368 B,N.
Integrations	Series substitution	372 B.
Integration	Simpson's rule	395 L.
Integration	Trapezoidal rule	397 N.
5. Transcendental equations		
Curve fitting	Least squares	162 N.
System of 47 equations	Step-by-step	231 B,N.
Non-linear equations	Steepest descent	264 C.
Curve fitting	Least squares	273 N
6. Complex algebra		
Complex roots and function evaluation	Iteration	193 L.
7. Data reduction		
Data reduction	Polynomial fitting, etc.	126 C.
8. Fourier series		
Fourier synthesis	Direct evaluation	261 C.
Fourier synthesis	Direct evaluation	306 D.
9. Statistics		
Multiple time series	Prediction by linear operators	106 C.
Evaluation of covariance and variance	Direct	350 D.
10. Integral equations		
Volterra type	Trapezoidal	361 B,N.
11. Linear programming		
Linear programming	Simplex method and relaxation method	219

Table II Current Problems Arranged According to the Mathematics Involved

2. WHIRLWIND CODING AND APPLICATIONS

2.1 Introduction

Progress reports as submitted by the various programmers are presented in numerical order in Section 2.2. Letters have been added to the problem numbers to indicate whether the problem is for academic credit and whether it is sponsored. The letters have the following significance.

A implies the problem is NOT for academic credit, is UNsponsored.

B implies the problem IS for academic credit, is UNsponsored.

C implies the problem is NOT for academic credit, IS sponsored.

D implies the problem IS for academic credit, IS sponsored.

N implies the problem is sponsored by the Office of Naval Research.

L implies the problem is sponsored by Lincoln Laboratory.

The absence of a letter indicates that the problem originated within the S&EC Group.

2.2 Progress Reports

100 COMPREHENSIVE SYSTEM OF SERVICE ROUTINES

During the past quarter, the Comprehensive System has been running at its usual routine pace.

Digital Computer Laboratory Staff

106 C. MIT SEISMIC PROJECT

The relatively little computational work we have done this last quarter consisted of cross correlation on a seismic record to locate an echo position; testing the stationarity hypothesis on several seismic traces and seismic well logs; and beginning the construction of theoretical random elastic media with desired scattering properties. Work will be continued and expanded in the next quarter.

Programmers working on this problem are S. Treitel and D. Grine.

S. Simpson
Geology and Geophysics

120 B,N. THERMODYNAMIC AND DYNAMIC EFFECTS OF WATER INJECTION INTO HIGH-TEMPERATURE, HIGH-VELOCITY GAS STREAMS

This problem is connected with the development of a potential gas turbine component, called an "aerothermopressor", in which a net rise in stagnation pressure of a hot gas stream is brought about by cooling of the stream. The cooling of the hot gas is accomplished by evaporation of liquid water which is injected into a high-velocity region of the flow. The concepts underlying the operation of the aerothermopressor are an outgrowth of comparatively recent work in the field of gas dynamics, and its proposed function in the gas turbine cycle is analogous to that of the condenser in a steam power plant.

The device consists of a converging nozzle which accelerates the exhaust gases from the turbine into a circular duct of varying diameter terminated by a conventional conical diffuser, which recovers the kinetic energy of the flow before discharging it to the atmosphere. At the entrance of the duct, special injectors deliver minute jets of water which are in turn atomized by the rapidly moving gas stream.

The changes in state within the aerothermopressor are brought about by the simultaneous thermodynamic and dynamic effects of (a) evaporation of the liquid water, (b) momentum and energy interactions between the phases, (c) friction, and (d) variations in cross-sectional area of the duct. Under proper circumstances, these effects bring about a net rise in stagnation pressure across the device. Further descriptions of this device may be found in earlier reports, beginning with Summary Report No. 32, Fourth Quarter, 1952.

Whirlwind I has played a very important role in the development of the aerothermopressor. The theoretical one-dimensional analysis of the aerothermopressor process involves the simultaneous solution of seven non-linear first-order differential equations for a wide variation of initial conditions. Computation of results in a reasonable time by any method other than a fast digital machine such as Whirlwind would have been impractical.

The agreement obtained between experimental and theoretical results is good. Detailed comparisons are given in the following reports:

1) Gavril, B. D., "A Theoretical Investigation of the Thermodynamic and Dynamic Effects of Water Injection into High-Velocity, High-Temperature Gas Streams."

2) Fowle, A. A., "An Experimental Investigation of an Aerothermopressor Having a Gas Flow of 25 Pounds Per Second."

3) Erickson, A. J., "A Theoretical and Experimental Investigation of the Aerothermopressor Process."

The experimental and theoretical work on the Aerothermopressor Project was carried out at M.I.T. under the sponsorship of the Office of Naval Research, and was directed by Professor A. H. Shapiro of the Department of Mechanical Engineering. The theoretical aspects of the problem treated by Whirlwind I were programmed by B. D. Gavril and carried out by Gavril and Erickson.

A. J. Erickson
Mechanical Engineering

126 C. DATA REDUCTION

Problem 126 is a very large data-reduction program for use in the Servomechanisms Laboratory. The over-all problem is composed of many component sections which have been developed separately and are now being combined into complete prototype programs. Descriptions of the various component sections have appeared in past quarterly reports. After the development and testing of the prototype Whirlwind programs is completed, the programs will be recoded for other, commercially available, large scale computers (probably the ERA 1103, IBM 701 and IBM 704 computers), for use by interested agencies for actual data reduction at other locations. The programs are currently being developed by Douglas T. Ross, David F. McAvinn, Benson H. Scheff, and Dorothy A. Hamilton, Servomechanisms Laboratory staff members, with the assistance of John F. Walsh. This work is sponsored by the Air Force Weapons Guidance Laboratory through Division of Sponsored Research Project 7138.

The nature of the problem requires extreme automaticity and efficiency in the actual running of the program, but also requires the presence of human operators in the computation loop for the purpose of decision making and program modification. For this reason, extensive use is made of output oscilloscopes so that the computer can communicate with the human, and manual intervention registers so that the human can communicate with the computer in terms of broad ideas, while the computer is running, and have the computer program translate these ideas into the detailed steps necessary for program modification to conform to the human operator's decision. The program which does this translation and modification is called the Manual Intervention Program (MIV). The most recent version of the prototype data-reduction program is called the Basic Evaluation Program.

Although the primary task of Problem 126 has been the development of high speed automatic data reduction techniques incorporating human decision making and modifications, additions to the MIV System have consistently been made more elaborate than required by the data reduction problem itself. The results of this seeming

extravagance are that a set of elaborate general purpose utility routines are now available which constitute the central core of a human-computer problem-solving system of wide applicability. The development of this system, called "SLURP" (Servo. Lab. Utility Routine Program), is now receiving a major portion of the programming effort.

SLURP includes all of the facilities of the Comprehensive System developed for the Whirlwind Computer by the Digital Computer Laboratory staff, plus a large number of general purpose routines for extracting and analyzing information about arbitrary computer programs. The governing philosophy of the system is to allow the programmer to work on one small part of a very large and complicated problem, virtually independent of the computer which is being used and those aspects of the over-all problem which are not of immediate concern. The major features of the system which make this possible are as follows:

1) A group control program which allows the automatic incorporation of new sections of programming into the system.

2) A manual intervention (MIV) system of programs which allow the programmer to interrogate and instruct the computer with respect to the over-all problem in terms of a specially designed and easy-to-use language represented by switches, buttons, lights, and visual displays.

3) A logging program which records all the MIV actions taken by the programmer.

4) An editing program which edits the logged information into an easy-to-read record of the manual actions taken, and generates a log playback tape.

5) A log playback program which automatically simulates manual actions in response to the instructions on the log playback tape.

6) An elaborate set of plotting and tabulating programs for data presentation and record keeping.

7) A mistake diagnosis routine (MDR) which may be used to abstract arbitrary intermediate computed quantities from any program and present these quantities in any of the above forms.

8) The SLURP program proper is a simulated, generalized, special-purpose computer which allows the incorporation of all of these routines into a smoothly functioning system, along with the capacity for continued expansion of these facilities.

All of the above facilities are under both manual intervention control and programmed control, through the use of appropriately designed special purpose languages. It is intended that the system be used in the following manner: If a procedure can be thought out in advance, then that procedure should be transcribed in the appropriate languages so that high speed and reliable execution can be insured. The automatic execution, however, may be overridden at any time by manual intervention so that new information or new ideas may be tried out immediately. Since manual actions may be logged, it is possible for a sequence of successful manual actions to be automatically incorporated into the programmed procedure. By using the variety of data presentation routines it is possible to pursue a line of thought with respect to the problem and obtain detailed information for further study.

Work is still in progress on the programming of a part of the above-described system, but the major portion which has been completed has demonstrated its worth many times. The recent increase in memory capacity of the Whirlwind Computer has stimulated a reworking of portions of the system, and the new approaches being used will allow accelerated expansion of the system, so that its completion in 1957 may be predicted.

D. T. Ross
Servomechanisms Laboratory

141 S & EC SUBROUTINE STUDY

During the past quarter three programs have been added to the subroutine library.

1. LSR Fu6a Arc-Sine

This routine computes the arc-sine of an angle in the range $-1 \leq x \leq 1$ using the following approximation:

$$\sin^{-1} x = \frac{\pi}{2} - \sqrt{1-x} \sum_{i=0}^{6} a_i x^i .$$

The routine does not require the use of buffer registers and cycle counters as LSR Fu6 does. The program has been tested for accuracy and the error has been found to be as great as 0.6×10^{-6} for $\sin^{-1} x$ very near 90^o.

2. LSR Fu9 Arc-Sine-Cosine

This program uses the same approximation as Fu6a with the added relation

$$\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x .$$

The error can be as great as 0.7×10^{-6} for $\sin^{-1} x$ and $\cos^{-1} x$ near 90^o

3. LSR MA9 CM Matrix Transpose

This routine will transpose a square matrix which has been stored in core memory. The transposed matrix is left in the same registers occupied by the original matrix.

M. E. Callaghan
Digital Computer Laboratory

162 N. DETERMINATION OF PHASE SHIFTS FROM EXPERIMENTAL CROSS-SECTIONS

Difficulties were experienced in attempting to minimize the fitting errors in the resonance region. Present and future work will involve attempting to minimize the errors of the function in a different area using the same combination of phase shifts previously treated.

The programmers working on this problem are E. Campbell and E. Mack.

E. Mack
Nuclear Science Laboratory

193 L. EIGENVALUE PROBLEM FOR PROPAGATION OF ELECTROMAGNETIC WAVES

The eigenvalues and eigenfunctions computed previously for the bilinear model at 50 Mc with antenna heights 500' and 30' were used to calculate mode sums at 17 additional distances.

Mode sum calculations at 410 Mc and 3000 Mc using the bilinear model have been extended to higher mode numbers. At 410 Mc convergence has been obtained at five distances. These calculations will be continued until convergence at intermediate distances is obtained.

Eigenvalues, eigenfunctions and mode sums for an inverse-square model at 50 Mc have also been computed. Work on these will continue until sufficiently accurate eigenfunctions are obtained so that convergent mode sums may be calculated.

The programmers working on this problem are H. B. Dwight and R. M. Ring.

R. M. Ring
Lincoln Laboratory

199 N. LAMINAR BOUNDARY LAYER OF A STEADY, COMPRESSIBLE FLOW IN THE ENTRANCE REGION OF A TUBE

Solutions of the third set of the differential equations were obtained for the case where temperature dependence of the viscosity, and thermal conductivity of the compressible fluid is taken into consideration.

More solutions are to be obtained for different entrance Mach numbers and thermal conditions at the tube wall.

T. Y. Toong
Mechanical Engineering

219 COMPARISON OF SIMPLEX AND RELAXATION METHODS IN LINEAR PROGRAMMING

In the past quarter several different methods of selecting the zero X_{ij}, which should be made positive at each step in the solution of the transportation problem (see Summary Report No. 45), have been tested.

Table I gives a comparison of the speed of three different methods when used on Whirlwind I to solve a problem with 30 supplies and 260 demands. Methods of selection require processing of the cost data. The first scheme is the one presented by Dantzig in his exposition of the "stepping stone" method. It involves searching the entire cost matrix to find the zero X_{ij} which will give the best rate of decrease of the total cost as it is made positive. The second method, which is the one originally coded into the Whirlwind program, selects the first zero X_{ij} which will give a rate of decrease of total cost as it is made positive. The last technique is a compromise between the first two in which the best X_{ij} for a given plant is selected, and in the following step the best for the next plant is selected, and so forth.

Table I Comparison of Methods - 30 x 260 Transportation Problem

Method	Number of Iterations	No. of Examinations of Entire Cost Matrix	Total Time, Min.
1. Best X_{ij}	508	508	19.4
2. First X_{ij}	2200	27	25.0
3. Best X_{ij} for a plant	674	44	9.6

It is evident that the compromise method is better by a factor of 2 than either of the others and, hence, will be used for future runs. No further development of the routine is contemplated.

J. B. Dennis
Electrical Engineering

225 B,N. NEUTRON-DEUTERON SCATTERING

The calculation of n-d scattering lengths with an expanded 4-parameter trial function has been performed for the special case μ=0 (no polarization). The results differ very little from those of the earlier two-parameter trial function. The next step will be to use non-zero values of the polarization parameter.

The results of the calculation to date are being presented at the New York meeting of the American Physical Society.* The programmer is H. Paul.

* Bull. Am. Phys. Soc. II 2, 71, 1957.

L. Sartori
Physics Department

231 B,N. AN APPLICATION OF MONTE CARLO METHODS TO NEUTRON DIFFUSION

Progress was made in trouble shooting the assembly of subroutines, but some errors still remain.

M. Troost
Chemical Engineering

244 C. DATA REDUCTION FOR X-1 FIRE CONTROL

Through the use of new data correction achieved by modification of a program tape previously prepared by Dr. J. M. Stark of the M.I.T. Instrumentation Laboratory, data reduction on this problem came to completion during this quarter. The data has now been processed and is in final form.

No more computing time is needed on this problem; however, the program tapes are being saved for possible future use.

S. J. Madden
Instrumentation Laboratory

245 N. THEORY OF NEUTRON REACTIONS

More test runs were made in the vicinity of the P and D waves in order to locate a ξ and c that will give a more satisfactory fit to the experimental values than ξ = .08, c = 1.65. When such values are found a complete case will be done.

E. Campbell
Physics Department

253 N. AN AUGMENTED PLANE WAVE METHOD FOR IRON

In the augmented plane wave method of handling the one-electron problem in crystals, it is necessary to integrate the radial wave equation in the chosen spherical potential. The accuracy of the numerical method (Noumerov method) itself and the accuracy of the Whirlwind routine for carrying out this method have been under investigation. Use has been made in these checks of hydrogen-like potentials where the solutions are analytically known in the form of Coulomb wave functions.

At present it appears that the accuracy of the extend integration routine is quite sufficient for our purposes.

The other routines for the APW method (developed by M. M. Saffren) have been tested and it appears that one should be able to carry through a one-electron energy band calculation via this method in the very near future with an accuracy limited primarily by the approximations of the one-electron method itself.

J. H. Wood
Solid State and Molecular
Theory Group

257 C. HORIZONTAL STABILIZER ANALYSIS

Some production runs have been made with the master program. There are a few minor uncertainties in the problem under investigation which may delay the continuation of the production runs. Once these uncertainties have been eliminated, the production runs shall be carried to completion.

The programmers working on this problem are N. P. Hobbs and E. S. Criscione.

E. S. Criscione
Aeroelastic and Structures
Research Laboratory

261 C. FOURIER SYNTHESIS FOR CRYSTAL STRUCTURES

In the past quarter, the two-dimensional Fourier synthesis programs have been used to develop a theory of partial Fourier synthesis. In this kind of synthesis only part of the total number of Fourier coefficients are used. The resulting synthesis is useful when dealing with crystals which can be said to have a substructure. For these crystals, the Fourier coefficients can be split into two sets, not necessarily equal in number, one of which contributes to the electron-density pattern of the substructure, the other to the electron-density pattern of the complement structure.

Since the complement structure is ordinarily swamped out by the substructure, especially in Patterson syntheses, this method has proved valuable in investigating crystals which can be referred in any way to substructure theory.

M. J. Buerger
Geology Department

262 N. EVALUATION OF TWO-CENTER MOLECULAR INTEGRALS

Some Coulomb integrals were evaluated by Dr. F. J. Corbató's program. The results were compared with those computed previously by an alternative method. The results differed, in some cases agreeing to six decimal places and in others two.

H. A. Aghajanian
Solid State and Molecular
Theory Group

264 C. OPTIMIZATION OF ALTERNATOR REGULATING SYSTEM

The method for minimizing a nonlinear function in the presence of nonlinear equality constraints by the method of steepest descent has been extended to the in-equality constraint problem. This technique may be used to solve the problem of de-signing an optimum system, when said problem is phrased as a nonlinear programming problem. Trial computer runs have been made using the simplified aircraft alternator representation described in the last quarterly report, and have shown that certain un-desirable characteristics of the method are emphasized when inequality constraints are considered.

In order to correct these faults in the program, the analytical study of the techniques used therein has been intensified. This study should be sufficiently advanced by the next quarter so that coding for an appropriately modified program may be initiated.

Programmers working on this problem are J. B. Dennis and R. R. Brown.

R. R. Brown
Energy Conversion Laboratory

273 N. ANALYSIS OF AIR SHOWER DATA

There have been minor changes in the program for the purpose of finding systematic errors in the air shower data. The error analysis is aided considerably by the fact that the parameters of interest are highly overdetermined by the set of measured quantities obtained from the air shower experiment. Thus, inconsistencies in the data are readily noticed.

During the past quarter most of the computer time has been used for data reduction. About 2000 showers have been processed and this work is expected to con-tinue during the next quarter. Programmers working on this problem are F. Scherb and G. W. Clark.

F. Scherb
Physics Department

274 N. MULTIPLE SCATTERING OF WAVES FROM A SPATIAL ARRAY OF SPHERICAL SCATTERERS

During this quarter, the first set of sums have been fitted and some progress has been made into the remaining six sets. The present objective is to find the mini-mum number of fitting formulae for the values of the sums first obtained.

M. Karakashian
Nuclear Science Laboratory

278 N. ENERGY LEVELS OF DIATOMIC HYDRIDES LiH

We are continuing the calculation of the physical properties of the lithium hydride molecule using the electronic wave functions which we have obtained. During the past quarter we have calculated the dipole moment as a function of the inter-nuclear distances for which we have wave functions. The result shows a dipole moment which decreases to zero at infinite separation and at zero separation and which reaches a maximum at an internuclear separation about one atomic unit greater than the equili-brium distance. Although the calculated value is somewhat greater than the experi-mental value, the shape of the curve (dipole moment vs internuclear distance) and the position of the maximum are possibly quite accurate. This awaits experimental verification.

We are also continuing self-consistent molecular orbital treatment incorpo-rating lithium π orbitals. It will be of interest to see how much improvement in the binding energy and other molecular properties will result from some inclusion of angular correlation.

Programmers working on this problem are A. M. Karo and A. R. Olson.

A. M. Karo
Solid State and Molecular
Theory Group

288 N. ATOMIC WAVE FUNCTIONS

The He configuration interaction work described in earlier reports has been completed. Extensive work has been done with $O^{=}$ functions in various environments (see January 15, 1957 Quarterly Progress Report for the Solid State and Molecular Theory Group). This work has been almost completed. A small amount of time has been spent on Cl^{-} functions using the same number of parameters as in the $O^{=}$ but without a stabilizing environment. We intend to resume calculations with the three Fe configura-tions: $4S^2\ 3d^6$, $4S\ 3d^7$ and $3d^8$ with the object to be to provide total energies and pertinent one-electron energies for the three configurations. The Fe calculations will not be as accurate as the others due to the much larger number of integrals needed for an equivalent calculation and the limited amount of storage in the auxiliary drum groups.

Programmers working on this problem are R. K. Nesbet and R. E. Watson.

R. E. Watson
Solid State and Molecular
Theory Group

290 N. POLARIZABILITY EFFECTS IN ATOMS AND MOLECULES

The mathematical and computational aspects are the same as previously given. A summary of our results to date and an analysis of the physics of the problem as it relates to electronic wave functions for the hydrogen fluoride molecule is given in the January 15, 1957 Quarterly Progress Report of the Solid State and Molecular Theory Group.

L. C. Allen
Solid State and Molecular
Theory Group

300 L. TROPOSPHERIC PROPAGATION

The present phase of the Fresnel integral computations have been completed. The autocorrelation function and power spectrum of a sample of 2290 Mc. field strength data have been determined. A subroutine for generating incomplete elliptic integrals of the first and second kinds has been developed.

An extensive investigation of statistical methods suitable for analyzing data obtained in the Aurora research program will be carried out.

Programmers working on this problem are W. C. Mason and P. A. Duffy.

W. C. Mason
Lincoln Laboratory

306 D. SPECTRAL ANALYSIS OF ATMOSPHERIC DATA

During the quarter most of the computations were completed. It is anticipated that the problem will be terminated during the next quarter.

B. Saltzman
Meteorology Department

309 B,N. PURE AND IMPURE POTASSIUM CHLORIDE CRYSTAL

During the past quarter, the results of the problem were prepared for publication. The results obtained suggested several additional details to be investigated. During the coming quarter, calculations concerning the charge density of the perfect crystal and an investigation of the conduction band are planned.

L. P. Howland
Solid State and Molecular
Theory Group

310 C. TRAJECTORY CALCULATIONS FOR A ROCKET DURING POWERED FLIGHT

In the last quarter, the program was used extensively in carrying out a parametric study of powered-flight trajectories of long-range rockets. The results of this study are presented in the following report: Prigge, J. S., "A Digital Computer

Study of the Powered-Flight Trajectories of Long-Range Ballistic Missiles", Report No. ARG R-1, Pts. I and II, Aerophysics Research Group, Massachusetts Institute of Technology, Cambridge, Massachusetts, December 1956. (CONFIDENTIAL)

This work is sponsored by the U. S. Air Force under Contract AF 33(616)-2392. Programmers working on this problem are J. S. Prigge, Jr. and L. E. Berman.

J. S. Prigge, Jr.
Aerophysics Research Group

312 L. ERROR ANALYSIS

All programming has been completed. Production runs are made from time to time. Programmers working on this problem are E. Hutcheson and L. Peterson.

E. Hutcheson
Lincoln Laboratory

317 C. EXTRACTION OF STABILITY DERIVATIVES FROM FLIGHT TEST DATA

In the past quarter Shinbrot's method using cosine method functions, the derivative method, and a newly developed statistical method have all been applied to a longitudinal response. In all instances excellent results have been obtained for the nonlinear lift-force equation. It has been found that all of these methods and the Shinbrot method with sine method functions only give good results for the nonlinear pitching-moment equations with $C_{m_{\dot{\alpha}}}$ assumed to be zero, but that for the response used (and in fact for all purely longitudinal responses) it is impossible to extract all derivatives from the exact pitching-moment equation.

It is planned to refine the statistical method and to try to extract lateral derivatives from suitable responses by all methods being investigated.

Programmers working on this problem are T. M. Carney, M. E. Hoult, L. L. Mazzola, M. N. Springer and L. E. Wilkie.

M. N. Springer
Aerophysics Research Group

327 L. PREDICTION ANALYSIS

The Maximum Likelihood Criterion Method, first described in Summary Report No. 46, has been programmed. It is hoped that checking out will be completed in the next few weeks.

Programmers working on this problem are E. Hutcheson, L. Peterson and P. Willmann.

E. Hutcheson
Lincoln Laboratory

329 N. FIRST APPROXIMATION SOLUTION ON ORE BODY

The problem of determining the optimum size and shape of the fundamental block to be used in the inverse interpretation of field resistivity data has been solved and production runs of the various geometric configurations have been made. For blocks at or very near the surface of the earth the solution is poor but because of the fact that geologically a layer of overburden of resistivity comparable to the background value always exists, it is unnecessary to use such blocks in the inverse problem.

The cross-section of a general resistivity profile for pole-sender and dipole-receivers has been established and attempts have been made to invert the resulting matrix of conductivity coefficients and geometric factors. Thus far, there has been little success primarily because of the fact that in the use of the Crout Method a great loss of accuracy occurs if the diagonal elements are not greater than the elements to their right. Usually it is possible to rectify this situation by renumbering the equations, but this has not resulted in sufficiently increased accuracy in this problem. It will probably be necessary to determine the corrections to be made from the matrix formed from the residuals, and it is hoped that convergence of this iterative procedure will result. The correct inversion of this matrix will enable a rather detailed inspection of the distribution of the conductivities of the earth useful in mining evaluation.

N. F. Ness
Geology and Geophysics

334 C. PARAMETRIC STUDY OF COUPLING AND DAMPING

During this quarter, a few production runs were made to investigate the possibilities of using this program to investigate airplane flutter.

K. R. Wetmore
Aeroelastic and Structures
Research Laboratory

337 N. NONLINEAR 2ND ORDER DIFFERENTIAL EQUATIONS IN THE THEORY OF ELASTIC SHELLS

During this quarter the main body of the program has been rewritten. It was necessary to split the program into parts, using auxiliary drum groups, in order to obtain solutions of a more complicated type than those obtained during last quarter. Although the new program is considerably slower in time, it allows the extension of the domain of the two parameters which occur in the differential equations in question and for which solutions are obtainable by the present method. A second program has been written and debugged to evaluate quantities of physical interest resulting from the solutions of the differential equations under various boundary conditions.

It is planned to write a similar program solving nonlinear boundary value problems of a slightly more general type arising in the theory of elastic shells. This program will be written for the IBM 704 computer. In view of this, a few preliminary computations are desirable, which are expected to give a hint as to how to make the 704 program to be written more effectively. These calculations will be carried out in the near future. Computations on the Whirlwind I computer are expected to be completed by the end of February.

H. Weinitschke
Department of Mathematics

341 C. STATISTICAL AND DYNAMIC FORECASTING METHODS

Programs for the method of multiple linear prediction described in previous reports were successfully applied to data obtained at the 500 mb level in the atmosphere. The results of the matrix diagonalization procedure were used as a first step in a statistical-dynamic forecast of contour patterns at that level.

Development of a method for predicting the sea-level pressure map based on the coefficients of the empirical orthogonal functions continued. Prediction was based on (1) observations of sea-level pressure on two successive days, and (2) contemporary observations of sea-level pressure and pressure tendency on a single day. With sixteen coefficients as predictors the first method explained 57 per cent and the second 63 per cent of the variance of sea-level pressure the following day. Plans call for further application to statistical forecasting using the empirical orthogonal functions developed for sea-level pressure, pressure tendency, and temperature.

Prediction coefficients for this technique for 5-day data have been obtained. Data are being prepared for the application of these coefficients to an independent sample.

All of the work done on the project has been supervised by Professor Edward N. Lorenz, Department of Meteorology. Programmers are E. Kelley and B. Shorr.

E. Kelley
Meteorology Department

343 C. WEATHER PREDICTION

A complete discussion of the research carried on by the Atmospheric Pressure Change Project under the sponsorship of the Office of Naval Research, contract no. N5ori-07084, may be found in the, as yet unpublished, final report.

Programs for this problem were written under the supervision of James M. Austin, Department of Meteorology.

E. Kelley
Meteorology Department

346 B. COMPLEX SPECTRUM ANALYSIS

The Whirlwind computer was used in an extensive search for energy levels of the erbium atom. The Zeeman data from over 400 erbium lines were used as the starting point. The computer carried out an extensive search for repeated intervals between lines with consistent data, but no intervals were found. Therefore the search was widened to include all of the spectral lines of erbium listed in the M.I.T. wavelength tables. The computer checked this long list for all of the suspected intervals of erbium, but none could definitely be established as significant.

A similar search was carried out on the spectra of Pr and Dy. No intervals have been found for Rr at the present time, but work on this analysis is still being carried out. Several intervals were found for the spectrum of DyII. The work on this analysis is also being continued.

This marks the first time that work of this sort has been carried out on digital computers. The computer has proved to be an extremely valuable tool in the analysis of spectra, and it will probably be used extensively for this work in the future.

J. W. Lindner
Spectroscopy Laboratory

350 D. LONG RANGE FORECASTING

The computation of eigenvalues mentioned in the last report has been completed. Lag correlations for temperature and pressure forecasting have been obtained from them, as have contemporary correlations for temperature specification and pressure map reconstruction. To complete the problem, pressure forecasts still have to be computed.

Programmers working on the problem are B. Shorr and D. L. Gilman.

D. L. Gilman
Meteorology Department

354 D. RESPONSE OF A SINGLE STORY CONCRETE BUILDING

The effect of various parameters is being investigated. It is expected that the problem will be completed early in 1957.

S. Namyet
Civil and Sanitary Engineering

361 B,N. GROWTH OF FATIGUE CRACKS

The integral equation described in the last progress report has been changed in order to determine the rate of crack growth from notches of finite radius. This new equation has been programmed and is currently being debugged. Upon completion of the debugging, about 5-10 runs of about 3 minutes each will be necessary to cover all parameter variation.

The finite difference solution for another plasticity problem (the torsion of an axially symmetric notched bar) is now under way, using Whirlwind. A solution for one set of parameter values is being done manually, using relaxation techniques. It is planned that the Whirlwind work will be completed by May.

J. B. Walsh
Mechanical Engineering

364 C. BLAST RESPONSE OF ROTOR BLADES

Routines for calculating the approximate normal modes of rotor blades from a finite set of assumed modes have been completed.

In the future, these "normal modes" will be used to integrate the equations

of motion of the rotor blade under various blast loadings.

A. Holland
Aeroelastic and Structures
Research Laboratory

369 TEMPERATURE DISTRIBUTION IN A BEAM

The original set of thirty-four partial differential equations describing the transient heat transfer problem was programmed for solution on both the IBM 650 Magnetic Drum Calculator and the Whirlwind Computer as described in the preceeding Quarterly Report. During the present quarter, a number of solutions were obtained for the entire system using several different physical configurations and parameter values.

Two of these key studies were solved on the 650 Magnetic Drum Calculator and these, in turn, served to check the accuracy of the results obtained on succeeding case studies on the Whirlwind Computer. The results on the two computers agree to 7 and in some cases 8 decimal digits.

As part of this study, it became necessary to write a special compiler for Whirlwind to achieve a shorter machine solution time. The following section describes the characteristics of this compiler.

Special Programmed-Arithmetic Compiler for Whirlwind

A 30, 15 programmed-arithmetic compiler has been written for use in the Heat Transfer program with the primary objective of increasing the speed of solution to a maximum.

The basic compiler is composed of the following routines:

(a) Multiplication - Multiplies the contents of the three register accumulator by the contents of the three register AR leaving the AR undisturbed and placing the result in the accumulator. This routine is accurate to 28 binary digits (minimum).

(b) Addition - Adds the contents of the accumulator to the contents of the AR without disturbing the AR--leaving the result in the accumulator. This routine is used for subtraction by placing the negative of the number in the AR. The addition routine is good to 29 binary digits (maximum). Addition of two numbers whose binary exponents differ by more than 28 is not performed.

(c) Division - This routine divides the contents of the accumulator by the contents of the AR (without affecting the AR) and places the result in the accumulator. This routine is good to 28 binary digits.

A number of additional routines have been included in the basic compiler. The following 30, 15 routines have been written:

(a) Exponential - This routine is entered with x in the accumulator, and exits with e^x in the accumulator. This routine used the complete basic compiler.

(b) Logarithm - This routine is entered with x in the accumulator, and exits with $\log_e x$ in the accumulator. This routine uses the complete basic compiler.

(c) Printout - This routine prints (without disturbing the content of the accumulator) on Tape Unit No. 3 or 4 using 8 decimal digit mantissa and 2 decimal digit exponent. The print routine used the multiplication routine

The Scope coordinate routine plots the divided coordinates on the scope. It does not use the compiler. Point plotting routine unpackages the number in the accumulator and plots it in correct position on the scope. The print routine does not disturb the accumulator or use the compiler.

All the routines operate on the accumulator and the AR therefore the desired numbers should be placed there before transfer of control is effected. All routines operate on normalized (scale factored) numbers. The insertion of an unnormalized number into the compiler will cause a divide alarm.

In order to save time, there are no built-in checks in the routines, and all possible alarms are regular Whirlwind alarms. The following alarms are possible:

(a) Arithmetic overflow - if the binary exponent of a number gets larger than 2^{15} or smaller than 2^{-15}.

(b) Divide alarm in case an attempt is made to divide a non-zero number by zero. (A zero divided by zero will result in a zero).

The routines are written to provide the fastest possible speed and no attempt was made to minimize the number of instructions.

The comparative speed and length of the basic routines are given below:

	CS II MS	New MS	Length of New Routine
Multiplication	1.2	0.645 - 0.500	38
Addition	2.0	1.000 - 0.200	129
Subtraction	2.0	1.000 - 0.200	
Division	2.2	1.217 - 1.150	66
Clear and Add	0.7	0.111	6
Clear and Subtract	0.7	0.111	6
Store	0.9	0.111	6
Jump	0.5	0.015	1
Conditional Jump	0.4	0.030	2

In order to increase the speed of operation, the major and minor parts of the mantissa of both the accumulator and the AR have been placed in the flip flops. The availability of the above compiler reduced the machine solution time by a factor of two. In certain cases, as shown above, operations were performed 8 times as fast as previously.

Programmers working on this problem were A. Zabludowsky of the Digital Computer Laboratory, and Mary Hermann and Eleanor Donovan of the Office of Statistical Services. A detailed description of the results of this study are contained in Statistical Services Report S-32.

F. M. Verzuh
Office of Statistical Services

372 B. DESIGN OF SPHERICAL SHELL SEGMENTS

The objective of the present problem was essentially achieved and all stresses in a spherical shell segment due to its own weight have been determined. In the future one program only must be slightly modified and its solution will complete the numerical results for the present problem.

E. Traum
Civil Engineering

377 L. COVERAGE ANALYSIS

The data handling section of the problem has been programmed and checked out. Runs are now being made. This program is only tentative, however, as the analysis may be extended or modified.

E. Hutcheson
Lincoln Laboratory

380 B. SWITCHING CIRCUITS

A program which uses the McCluskey-Quine reduction procedure for simplifying switching functions of 15 or fewer variables has been completed. The function to be simplified is specified in standard sum form (with a maximum of 512 terms included in the standard sum). First the prime implicants, and then one or more minimum sums are determined and printed out in binary form. The part of the program which determines the prime implicants from the standard sum has been completely checked out, and the remainder of the program should be working within a short time.

Possibilities of writing a program for synthesis of non-series-parallel switching networks by means of Boolean matrices have been further investigated, and additional work will be done on this when the McCluskey-Quine reduction program has been completely checked out.

C. Roth, Jr.
Electrical Engineering

382 B. CALCULATION OF PRIME NUMBERS

Except for running off a few sample copies of the prime calculation program, I have now completed my work on the Whirlwind computer. In about three weeks I intend to submit a report telling exactly what each of my programs does. For anyone who intends to do further work on my programs, I suggest reference to my thesis, and I will be available during the semester, February - June, 1957, for personal advice.

At present, there are two programs that will calculate prime numbers. The more efficient one is almost a complete program lacking only a check routine. However, there are several modifications which could be made so as to increase the efficiency of the program and to provide for a less bulky output. Also an initial prime calculation subroutine which is already prepared could be incorporated into the program to make future use of the program easier. These modifications will be more carefully explained in my thesis.

H. I. Cohen
Electrical Engineering

383 C. STOKES PARTICLE VELOCITIES

Experimentally obtained parameters were fed to WWI and all data has been reduced and plotted. Further work will consist of running only a few specialized parameters through the computer to aid in the writing of a final report.

Programmers working on this problem are T. A. Marlow and J. G. Housley.

T. A. Marlow
Hydrodynamics Laboratory

385 B. FEED PLATE LOCATION

The range of optimum feed plate locations was determined for each of twelve ternary distillation systems at five different values of the reflux ratio for each system. The data obtained from these computer calculations were sufficient to permit an analysis of the factors influencing the optimum feed plate location in multicomponent distillation systems.

Pending final acceptance of the thesis written using the data mentioned above, the problem is completed.

I. Rinard
Chemical Engineering

386 C. FREE CONVECTION PAST A VERTICAL PLATE

Problem Description

The total differential equations (representing momentum and energy conservation) to be solved simultaneously are the following:

Note that ()' refers to differentiation with respect to the independent variable, η .

$$\begin{cases} f''' + (1 + q)\, ff'' - (1 + 2q)\, f'^2 = -Bh \\ (Pr^{-1})\, h'' + (1 + q)\, fh' - (1 + 4q)\, f'h = 0 \end{cases}$$

subject to the boundary conditions:

$\eta = 0$: $\quad f = f' = 0, \quad h = 1$

$\eta = \infty$: $\quad f' = h = 0$

It is proposed to employ the Gill-Kutta routine for integration purposes. Estimates will be made of $f''(0)$ and $h'(0)$ in lieu of $f'(\infty)$ and $h(\infty)$ and the integration will be repeated until satisfactory and point conditions are met. An influence coefficient routine will be employed to enable the computer to determine a successive sequence of $f''(0)$ and $h'(0)$ values which will converge upon the proper values on the basis of successive $f'(\infty)$ and $h(\infty)$ results. This technique was used previously by the present programmer for two-point boundary condition problems.

The scope of the computing problem is defined by the following:

(i) $0 \leq \eta \lesssim 5.0$, i.e., $\eta = 5.0 \Rightarrow \infty$

(ii) $\Delta\eta = 0.1$ for integration increments

(iii) q = -.10, -.05, or -.01
(iv) Pr = 0.01, 1.0, or 100
(v) B = 0.01, 1.0, or 100

} present interest is in 11 combinations of the constants q, Pr, and B.

(vi) Results to be printout (presumably on magnetic tape) of f, f', f'', h, h' as functions of η for each case.

The required estimates of $f''(0)$ and $h'(0)$ should be fairly accurate since exact solutions are available for q = -.25 and -.20 from which the approximate values needed here may be extrapolated for the q = -.10 case, etc.

Progress

Several similarity solutions for the dynamic and thermal boundary layer profiles have been obtained during the past quarter. The solutions correspond to a family of surface temperature distributions of the form $1 + Bx^{\omega}$ where B and ω are constants and x represents distance along the plate. Several additional values of B are contemplated for future computations. All results are presently based upon a Prandtl number of unity.

J. R. Baron
Naval Supersonic Laboratory

387 C. DETERMINATION OF VELOCITY POTENTIAL

Problem Description

In order to evaluate the unsteady lifts and moments acting on wings under various loading conditions at supersonic speeds, the wing planform is divided into a grid of elementary areas, or boxes. The velocity potential influence coefficients (VPIC) evaluate the velocity potential at each given box due to unit downwash at another box. The VPIC's are then properly weighted and multiplied by the actual downwashes to obtain the velocity potential distribution on the planform. The velocity potential is suitably integrated to determine the generalized forces acting on the planform for each Mach number and reduced frequency.

The portion of the problem that will utilize WWI centers around the following matrix operations, where most of the numbers involved are complex.

1. Weighting of the coefficient matrices, term by term, to account for the effect of partial boxes on the planform

$$\left[\begin{array}{c|c} \bar{C}_{DD} & \bar{C}_{DP} \\ \hline \bar{C}_{PD} & \bar{C}_{PP} \end{array}\right] \times \left[\begin{array}{c|c} R_{DD} & R_{DP} \\ \hline R_{PD} & R_{PP} \end{array}\right] = \left[\begin{array}{c|c} C_{DD} & C_{DP} \\ \hline C_{PD} & C_{PP} \end{array}\right]$$

2. Solving sequentially for the downwashes in the mixed regions near the edges of the planform:

$$\left\{ W_D \right\} = - \left[C_{DD} \right]^{-1} \left[C_{DP} \right] \left\{ W_P \right\}$$

where $\left[C_{DD} \right]$ is a diagonal matrix.

3. Solving for the velocity potential distribution on the planform

$$\left\{ \varphi_P \right\} = \left[C_{PD} \,\vdots\, C_{PP} \right] \left\{ \begin{array}{c} W_D \\ \hline W_P \end{array} \right\}$$

where: D = mixed or diaphragm region
P = planform region
$\bar{C}$ = unweighted velocity potential coefficients
C = weighted
R = weighting factors to account for the effect of partial boxes
W = downwash
φ = velocity potential

A total of about 18 cases will be investigated. The largest matrix will be 46 x 73 complex.

Progress During the Past Quarter

A generalized routine has been formulated for complex matrix multiplication capable of handling matrices up to order about (18 x 18) for the premultiplier and (18 x 1) for the postmultiplier (or equivalent) where all the numbers are complex.

A generalized routine has been formulated for a sequential solution of complex equations, capable of solving up to about 18 equations. The matrix of coefficients is triangular and all the numbers are complex.

For determination of supersonic velocity potentials on wing planforms a generalized routine has been devised which consists of 2 long tapes:

a) Input tape which stores the basic aerodynamic influence coefficients, their appropriate weighting factors, and a given mode shape on the drum;

b) Computation tape which takes the input data and computes the resultant velocity potentials corresponding to the given mode shape.

The above described routines have been devised and debugged and have been used to yield satisfactory velocity potentials corresponding to two mode shapes of a delta planform at a given Mach Number and reduced frequency of oscillation.

Future Plans

Additional computations will be performed utilizing the existing routines for determination of velocity potentials for the delta wing, corresponding to many more combinations of mode shape, Mach Number and reduced frequency.

The existing routines will be modified (the modifications will be relatively minor) to handle the velocity potentials for a straight wing in supersonic flow and corresponding calculations will be performed.

The program will be modified (these modifications are also expected to be relatively simple) to handle swept wings and corresponding calculations will be performed.

S. Gravitz
R. Stapleford
Aeroelastic and Structures
Research Laboratory

388 D. TEMPERATURE DISTRIBUTION IN AIRCRAFT GENERATORS

Problem Description

Present aircraft generators are cooled by directing a blast of air through internal ducting. No analytical investigation of internal temperature distribution has yet been made, with the result that placement of ducts is somewhat arbitrary, and prediction of machine temperature capabilities is impossible.

The present problem represents an attempt to utilize relaxation methods for determining generator temperature distributions, along with numerical integration of fluid flow equations to calculate the heat-transfer coefficient in each part of a machine. For solution of the entire problem, it is planned to use the superior speed and storage capacity of the 704, but Whirlwind will be used for preliminary "order-of-magnitude" fluid flow calculations, intended to determine which flow obstructions are most significant.

Theory may be idealized as a calculation of the flow in a tube, with heat addition and friction. The appropriate equations are integrated by taking small enough steps so that an average fluid temperature may be used.

Progress

During the past quarter there has been substantially no progress because of difficulties with the underlying theory.

R. Moroney
Servomechanisms Laboratory

391 L. MAGNETIC RELAXATION IN THIN FILMS

Problem Description

Thin films (~ 1000 Å) of vacuum evaporated permalloy may be useful as bistable memory elements for digital computers. Elementary theory indicates that the relaxation time with coincident current is of the order of 10^{-9} seconds, which is substantially faster than present memory elements. Experiment has substantiated the expectation of fast relaxation and has revealed a situation more complex than the simple theory can handle. A more adequate theory based on the Landau-Lifshitz equation of motion of the magnetization can only be treated by numerical methods on a machine such as Whirlwind I.

The specific problem is to solve the pair of coupled differential equations

$$\dot{\theta} = mg\Psi + r(\theta)$$

$$\dot{\Psi} = -m\Psi + g\, r(\theta),$$

with $r(\theta) = h_s \sin\theta + h_\perp \cos\theta - \sin\theta \sin\theta$, and m and g known constants of the magnetic material.

Solutions must be obtained for a set of points in "h-space", $0 < h_s < 2$ and $0 < h_\perp < 1$. It is proposed to use the program of Laning and Zierler which translate equations directly into Whirlwind I language.

Progress During the Past Quarter

Laning's program was used and was found to be too slow. The problem has been reprogrammed using the Comprehensive System.

Programmers working on this problem are J. Frankovich and D. O. Smith.

D. O. Smith
Lincoln Laboratory

393 N. THE INVERSE BREMSTRAHLUNG SPECTRUM

The yield $Y(E_m)$ of a photonuclear reaction produced by a bremstrahlung spectrum $S(E_m,k)$ is measured as a function of the maximum energy E_m of the spectrum. It is usually desired to know the cross-section c(k) for the reaction as a function of photon energy k.

$$Y(E_m) = \int_0^{E_m} S(E_m,k)\, c(k)\, dk \quad \text{, or approximately}$$

$$Y([m+1/2]t) = t \sum_{h=1}^{E_m k - 1/2} S([m+1/2]t, nt)\, c(nt) \quad \text{and}$$

$$c(nt) = t^{-1} \sum_{m=1}^{\frac{E_m}{t} - 1/2} S^{-1}(nt, [m+1/2]\, t)\; Y([m+1/2]\, t).$$

S^{-1} is the inverse of the triangular matrix S.

Using the Schiff bremstrahlung spectrum for S, Whirlwind will be used to compute the elements of S^{-1} for ten twenty-by-twenty S matrices. Three of these will cover the range 5-25 Mev with t = 0.50 Mev and the other seven will cover the range 8-18 Mev with t = 0.125 Mev.

H. Paul
Nuclear Science Laboratory

394 C. AUTOMATIC PROGRAMMING FOR NUMERICALLY CONTROLLED MACHINE TOOLS

The completion of the MIT Numerically Controlled Milling Machine in 1952 introduced a new era in the automatic translation of design requirements into finished parts. The primary problem associated with this type of machine tool, the preparation of detailed instructions for making a part, has been studied using the Whirlwind computer. Under problem 132 a library of subroutines was developed to assist in data preparation for the MIT Numerically Controlled Milling Machine (see Whirlwind Summary Reports 33 through 43). Under problem 250 the feasibility of applying modern automatic programming techniques to machine tool programming was successfully demonstrated for plane parts bounded by straight line and circular segments (see Whirlwind Summary Reports 41 and 42). Problem 394 is concerned with the extension and elaboration of these automatic programming techniques to arbitrary surfaces which can be machined by 3 and 5 axis numerically controlled milling machines.

The problem may be viewed as the design of an APT System (Automatically Programmed Tool System) combining the human operator, the machine tool itself, and an APT computer, which is a combination of a general purpose computer and existing machine tool directors. The objective is a system of Whirlwind computer programs which will simulate an APT computer with a language which is convenient for human expression of machined part designs. A design expressed in this language will be automatically translated by the APT computer into machine tool instructions.

Whirlwind programs have been written which allow programming of a part by means of arbitrary curves in the surface of that part. At present the system is restricted to planes and spheres but may be arbitrarily expanded. Work is now in progress on an advanced system which will allow programming by means of entire regions of the part. Following completion of this advanced simulated computer, attention will be directed to the determination of the most appropriate English-like coding language for communication with these APT computers.

Associated studies are being made for the determination of appropriate languages for the feedback of information from the APT computer to the human operator. A program has been written by which the Whirlwind computer can draw on the output oscilloscope an axonometric projection (near perspective) of the part being made. The program allows arbitrary orientation of axes and arbitrary scaling so that any view of any portion of the part can be shown. The program has several modes of operation, but at present the picture is displayed as the path of cut vectors swept out by the motion of the cutting tool over the surface of the part. English-like computer to human languages will also be devised.

The programs are currently being developed by Douglas T. Ross, Harry E. Pople, Jerome R. Wenker, George Zames, Samuel M. Matsa, Servomechanisms Laboratory staff members. This work is sponsored by the Air Materiel Command through Project 6873 in the Division of Sponsored Research.

D. T. Ross
Servomechanisms Laboratory

395 L. PHASE ERROR CALCULATION

The problem involved is one of calculating the far-field energy distribution of a radar antenna that has the form of a parabolic torus. In using an antenna of this configuration there is a certain inherent phase error problem whose effect can be determined by constructing a model and measuring energy distrubution patterns. As an enhancement of this method, one may also calculate the energy pattern of the antenna. An expression for doing this is as follows:

$$g(\theta,\emptyset) = \int_{o}^{a}\int_{-b/2}^{b/2} e^{-29f^2}\left[\cos\frac{2\pi}{\lambda}\left\{\Delta + \sin\theta\ (x\cos\emptyset + y\sin\emptyset)\right\}\right] dxdy$$

$$+ j\int_{o}^{a}\int_{-b/2}^{b/2} e^{-29f^2}\left[\sin\frac{2\pi}{\lambda}\left\{\Delta + \sin\theta\ (x\cos\emptyset + y\sin\emptyset)\right\}\right] dxdy$$

where

$$\Delta = -2f\left[1 - \frac{\left\{\left(1-\frac{x^2}{4f}\right)^2 - y^2\right\}^{\frac{1}{2}}}{1-\frac{x^2}{4f}}\right]\left[1 - \frac{(1-f)\left\{\left(1-\frac{x^2}{4f}\right)^2 - y^2\right\}^{\frac{1}{2}}}{\left(1-\frac{x^2}{4f}\right)\left(\frac{x^2}{4f}+f\right)}\right]$$

To integrate this expression, one may use Simpson's Rule. Choosing some angle θ ($0^\circ \leqq \theta \leqq 5^\circ$), one sweeps through the angle $\emptyset$ ($0^\circ \leqq \emptyset \leqq 180^\circ$) in 1° steps. At each (θ, $\emptyset$), sum up the energy contributions from all x and y. This is done by choosing some value of x ($0.00 \leqq x \leqq 0.48$) and moving through all y ($-0.24 \leqq y \leqq 0.24$) in steps of 0.01. Repeat this for all x in steps of 0.01. Having summed the energy contributions from all x and y at each $\emptyset$ for a particular value of θ, change θ in 0.1° steps and repeat the above calculations for each value of θ.

Number of iterations:

y - 48	θ - 50
x - 48	∅ - 180

Total number of iterations:

$$48 \times 48 \times 50 \times 180 \cong 20 \times 10^6 \text{ iterations}$$

During the past quarter, phase errors have been obtained. These will be incorporated in calculations of the radiation pattern of a toroidal reflector type antenna.

D. C. MacLellan
Lincoln Laboratory

396 SUBROUTINE STUDY

This problem is concerned with the investigation of different methods known to invert matrices. The standard methods, such as the elimination (using back substitution), Craig's, Crout's and Frovenius', may not work for ill-conditioned matrices. In particular, we have a 16 x 16 complex, ill-conditioned matrix to invert. What we would like to do is:

1) Find the inverse of the particular matrix;

2) Develop a subroutine incorporating the "best" method available for ill-conditioned matrices.

We would also like to develop a subroutine that will invert matrices whose order is too large to fit into core memory. We may possibly be able to do this by some sort of partitioning.

During the past quarter, the problem was programmed to solve a 16 x 16 complex matrix, first by partitioning and then by Crout's method. By substituting the results into the original equation, the answers were found significant to 5 digits. Different methods will be compared in the near future.

J. Roseman
Digital Computer Laboratory

397 N. RESPONSE FUNCTIONS OF AIR SHOWER DETECTOR ARRAYS

Air showers are to be detected by a specified array of detectors. The problem is to determine the efficiency with which the array detects air showers of size N whose axes fall within a certain area A and whose arrival directions lie within a certain solid angle ω.

We call $R(N\theta)dNd\theta$ the rate of detection of showers with sizes between N and N + dN and zenith angles between θ and $\theta + d\theta$. We call $S(N\theta)dNd\omega$ the rate of showers with sizes between N and N + dN which arrive with a zenith angle θ within the solid angle $d\omega$ and cross unit area perpendicular to the direction of arrival. R is related to S by the equation

$$R(N\theta) = 2\pi \sin\theta \cos\theta \, S(N\theta) \int_0^{\infty} dr \int_0^{2\pi} d\alpha \int_0^{2\pi} \epsilon(N,\theta,r,\alpha,\varphi) \, d\varphi$$

where

$$\epsilon = \prod_{i=1}^{n} P_i \left[a_i(\theta,\varphi), \rho(N,\theta,r,\alpha,\varphi)\right]$$

$$\rho = \frac{N}{2\pi} f(R) ,$$

$$R = \left\{ (x - x_i)^2 + (y - y_i)^2 - \left[\ell(x - x_i) + m(y - y_i)\right]^2 \right\}^{1/2} ,$$

$$\ell = \sin\theta \sin\varphi ,$$

$$m = \sin\theta \cos\varphi ,$$

$P_i(a_i, \rho)$ = the probability that the i^{th} detector with projected area $a_i(\theta,\varphi)$ be successfully triggered by a shower whose density at the i^{th} detector is ρ, and (x_i, y_i) = the coordinates of the i^{th} detector.

We call

$$F(N\theta) = R(N\theta)\left[\pi \sin 2\theta \, S(N\theta)\right]^{-1} = \int_0^{\infty} dr \int_0^{2\pi} d\alpha \int_0^{2\pi} \epsilon \, d\varphi$$

the response function of the array.

The program is designed to evaluate this function for an arbitrary choice of the functions a_i, P_i, and f, and of detector locations (x_i, y_i) also the integrations can be performed between definite limits other than 0, 2π, and ∞. The integrations are performed by the trapezoidal rule. The density of the net of points at which the integrand is evaluated is adjustable. Experience with hand computations indicates that a net containing about 500 points will be required for the desired accuracy in the evaluation of F for one set of the parameters and one choice of N and θ.

The program has been tested with a trial case, and errors are being eliminated.

G. W. Clark
Nuclear Science Laboratory

398 A. DIAGONALIZATION OF MATRICES

In the collective model of the nucleus, the nucleons move independently in a potential which represents the major portion of their mutual interactions. The moments of inertia of ellipsoidally deformed nuclei in their ground states are, in such a potential, the rigid moments of inertia corresponding to the rotation of the mass distribution about an axis perpendicular to the symmetry axis. The observed moments of inertia tend, however, to be less than half this value. The reduction of the moments of inertia has been explained as an effect of that part of the interaction between nucleons not included in the common potential.

To obtain a rough estimate of the effectiveness of these residual interactions in reducing the moments of inertia, these calculations are being made with a greatly simplified model in which the particles move in an anisotropic harmonic oscillator potential and the residual interactions are represented by an attractive, two-body, delta function interaction. Basis wave functions are formed from single particle harmonic oscillator wave functions in Slater determinants and the secular equation is solved approximately (considering only states within the same shell) using the program prepared by F. J. Corbató. The eignefunctions are then used to find the moment of inertia of the ground state in the usual way.

Calculations are being made for two and three particles outside a closed shell and for various deformations and strengths of residual interaction.

It is expected that work on this problem will be completed early in 1957.

A. J. Morency
Physics Department

399 L,B. DOMAIN WALL MOTION

In studying the motion of 180° magnetic domain walls in magnetization reversal processes in ferrite toroids an equation of the form

$$\ddot{x} + 2d\,\dot{x} + f(x) = h$$

occurs where x is the normalized distance the wall moves, 2d is a damping parameter, f(x) is a non-linear restoring function with maximum and minimum amplitudes of ± 1 and is periodic in x with period 2, h is a normalized driving force. The problem is to obtain solutions of this equation for various values of d and h. In particular, a plot of $\dot{x}$ or wall velocity vs. time is desired. It is expected that such plots will have the same general character as the actual voltage response observed in magnetic cores during switch processes under the influence of varying driving currents.

Most of the time during this quarter was spent in straightening out the program. One or two good runs have been obtained for certain values of the damping constant δ in the basic equation. Solutions for other values of δ and h are to be attempted during the coming quarter.

Programmers working on this problem are J. V. Harrington and P. Dineen.

J. V. Harrington
Lincoln Laboratory

APPENDIX

1. SYSTEMS ENGINEERING

WWI RELIABILITY

The following is the WWI Computer Reliability Record for the past quarter:

Total Computer Operating Time in Hours	1836.5
Total Time Lost in Hours	35.8
Percentage Operating Time Usable	98.1
Mean Free Time Between Failure Incidents (in hours)	23.6
Total Number of Failure Incidents	84
Failure Incidents per 24 Hour Day	1.1
Average Lost Time per Incident in Minutes	25.2
Average Preventive Maintenance Time per Day in Hours	2.3

2. VISITORS

Tours of the Whirlwind I installation include a showing of the film, "Making Electrons Count", a computer demonstration, and an informal discussion of the major computer components. During the past quarter, the following 7 groups totalling 237 people visited the computer installation.

November 1	Northeastern University	27
November 6	Springfield High School	21
November 7	MIT Course 2.215, "Methods of Engineering Analysis"	19
November 13	Naval War College	13
November 15	Northeastern University	28
November 19	American Society of Civil Engineers	113
December 17	MIT Course 6.25, "Machine-Aided Analysis"	16

The procedure of holding Open House at the Laboratory on the first of the month has continued. A total of 56 people attended the 3 Open House tours during the quarter, representing members and friends of the MIT community, Queen's College, Simmons College, and the Stromberg-Carlson Company.

During the past quarter, there were also 42 individuals who made brief tours of the computer installation at different times. Represented by these individuals were MIT, University of Leyden, Netherlands, U.S. Air Force, U.S. Navy, General Electric, Radio Corporation of America, N.H. Ball Bearing, Inc., Jackson & Moreland, Nippon Electric Company, Blue Cross, Curtiss Wright, American Cyanamid, and North American Aviation.

LET'S TAKE A TRIP

The weekly Columbia Broadcasting System television program, "Let's Take a Trip", was broadcast live from the Digital Computer Laboratory on Sunday, October 21, 1956 at 12 noon, on Channel 12.

Young Ginger MacManus and Pud Flanagan were led by Sonny Fox on a tour through the Laboratory. Dr. Francis Bitter, professor of Physics and Associate Dean of MIT's School of Science, explained, with the aid of simple experiments, the scientific background that led to the development of vacuum tubes, magnetic core memory, and other components of the computer.

Dr. Jay W. Forrester, director of the Laboratory from its inception in 1951 until July 1956, showed the WHIRLWIND computer in action, explaining the demonstration programs which were run by chief operator, Michael Solomita. Dr. Forrester also pointed out some of the practical applications of the computer to scientific, engineering and industrial problems.

Dr. Vannevar Bush, a member of the MIT Corporation, closed the program with a few words on the impact that computers have had on science, broadening the whole scope of scientific investigation and opening exciting and unlimited possibilities for the future.

The show was produced by Stephen Fleischman and directed by Roger Englander, both of Columbia Broadcasting System.

3. PUBLICATIONS

Project Whirlwind technical reports and memoranda are routinely distributed to only a restricted group known to have a particular interest in the Project, and to ASTIA (Armed Services Technical Information Agency) Document Service Center, Knott Building, Dayton, Ohio. Requests for copies of individual reports should be made to ASTIA.

The following is a list of memoranda published by the Scientific and Engineering Computations Group during the past quarter.

No.			
DCL-140	A Program to Solve the Classical Transportation Problem	J. Dennis	8-28-56
DCL-146	A Proposal for an Algebraic Compiler to be Used on the MIT 704.	F. Helwig A. Siegel	10-16-56
DCL-148	List of Available SHARE Material	F.M. Verzuh	10-22-56

DCL-150	Simplex Method Subroutine	R. Bishop	10-29-56
DCL-153	An Argument for Selecting a Certain Size Block of Temporary Storage for a Post Mortem Routine is Explained	S. Best	11-21-56
DCL-155	MIT 704 Post Mortem Routine	A. Siegel	11-26-56

4. ACADEMIC

1. Introduction

There were a number of graduate subjects in automatic computation, numerical analysis, and data processing offered at MIT during the Fall Term of 1956. Specifically, the following subjects directly relating to machine computation were offered.

Subject	Description	Units	Year	Instructor	No. of Students
2.215	Methods of Engineering Analysis	3-9	G	Crandall	17
6.25	Machine-Aided Analysis	3-6	4	Arden	132
6.567	Switching Circuits	3-6	G	Caldwell	52
15.542	Management Information Systems	3-6	G	Gregory	30
M371	Operations Research	3-9	G	Wadsworth	33
M39	Methods of Applied Mathematics	3-9	G	Hildebrand	152
M411	Numerical Analysis	3-2-7	G	Hildebrand	37

It is apparent from the above list that the subjects are predominantly Graduate A subjects--which indicates that the subject is primarily for graduate students. However, it is perfectly possible for undergraduate students to take these subjects on an elective basis. Although the total number of students attending these classes is 453, this does not mean that there are that many different people attending these subjects since a number of the students are taking several of the subjects.

2. 650 Subroutine Developments in the Office of Statistical Services

During the Fall Term further developments on interpretive systems and compilers for the 650 Magnetic Drum Calculator were carried out in the Office of Statistical Services:

a) MITSS -- MIT Selective System
b) Solution of Linear Systems Using a Modified Elimination Procedure
c) Evaluation of Roots of Non-Symmetric Matrices Using the 650 Calculator
d) Polynomial Factorization Research Using the Down-Hill Method.

The selective systems have been so arranged to permit a two-pass use of the Symbolic Optimal Assemble Program (SOAP) system on all programs which are written for the 650 Calculator. Details describing the above programs are available in other reports prepared by the Office of Statistical Services.

3. MIT Computation Center

During the Fall Term continued progress has been made in setting up the plans for the MIT Computation Center. Specifically, the machine is to be installed in the new Karl T. Compton Laboratory, and the space is rapidly nearing completion. It is expected that the machine will be in operation early in April, 1957.

During the Fall Term a number of IBM-MIT Research Assistants were appointed as part of this program. They include the following:

Department	Name
Mathematics	Mr. P. W. Abrahams
Mechanical Eng.	Mr. D. M. Baumann
Civil Eng.	Mr. C. R. Calladine
Physics	Dr. F. J. Corbató
Schl. Ind. Mgment.	Mr. M. J. Erdei
Electrical Eng.	Mr. T. J. Glass
Chemical Eng.	Mr. K. Hansen
Mathematics	Mr. J. Hershenov
Physics	Mr. C. S. Naiman
Geology and Geophysics	Mr. N. F. Ness
Nuclear Eng.	Mr. M. Troost
Mathematics	Mr. H. J. Weinitschke

In addition, a number of IBM Research Assistants and Associates have been appointed at the participating colleges; the following list indicates the present representation:

College	Appointee	Title
Boston Coll.	Dr. W. E. Perrault Mr. J. A. Riley	Res. Assoc. Res. Asst.
Bowdoin	Dr. R. L. Chittim Dr. C. E. Huntington Dr. E. O. LaCasce	Res. Assoc. Res. Assoc. Res. Assoc.

College	Appointee	Title
Brown	Dr. W. Freiberger	Res. Assoc.
	Mr. J. E. Panarelli	Res. Asst.
Conn., U. of	Mr. M. J. Casarella	Res. Asst.
Dartmouth	Dr. T. E. Kurtz	Res. Assoc.
	Prof. J. McCarthy	Res. Assoc.
Harvard	Mr. B. Cohen	Res. Asst.
	Mr. G. Kraft	Res. Asst.
	Mr. C. Varsavsky	Res. Asst.
New Hamp., U. of	Mr. F. J. Lorenzen, Jr.	Res. Asst.
Northeastern	Dr. M. J. Carrabes	Res. Assoc.
	Mr. R. W. Taylor, Mr. R. F. Trocchi	Res. Asst.
Tufts	Dr. K. S. W. Champion	Res. Assoc.
	Dr. D. B. Devoe	Res. Assoc.
	Mr. P. S. Swartz	Res. Asst.
Wesleyan	Mr. J. M. Kyle	Res. Asst.
Yale	Mr. P. R. Hoffman	Res. Asst.
	Mr. R. N. Rosett	Res. Asst.
	Prof. R. W. Southworth	Res. Assoc.

A number of programs have been set up for the effective utilization of the 704 Calculator; these include a project with the Rockefeller Foundation and a program with the National Science Foundation.

4. Preparation of Programs for the 704 Computer

During the Fall Term considerable thought was given to the preparation of a suitable post-mortem system for use on the 704 Calculator. This program is currently being tested out on the General Electric 704 machine at Lynn and it is expected that it will be available for use when the 704 arrives in March of 1957.

A more general algebraic compiler is also receiving attention; however it will not be available until later in the academic year.

F. M. Verzuh
Assistant Director
MIT Computation Center

PERSONNEL ON THE PROJECT

COMMITTE ON MACHINE METHODS OF COMPUTATION AND NUMERICAL ANALYSIS

Faculty Supervisors:

Philip M. Morse, Chairman	Physics
Samuel H. Caldwell	Electrical Engineering
Stephen H. Crandall	Mechanical Engineering
David Durand	Industrial Management
Herman Feshbach	Physics
Jay W. Forrester	Industrial Management
Francis B. Hildebrand	Mathematics
Chia-Chiao Lin	Mathematics
J. Francis Reintjes	Electrical Engineering

Ex-Officio

Dean N. Arden	Electrical Engineering
Fernando J. Corbato	Computation Center
Frank M. Verzuh	Computation Center

PROJECT WHIRLWIND

Frank M. Verzuh, Head
Dean N. Arden
Sheldon F. Best
Marion Callaghan
Frank C. Helwig
Jack Roseman
Arnold Siegel
Murray Watkins
Munroe R. Weinstein
Alexander Zabludowsky