PROJECT WHIRLWIND
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FOREWORD

Project Whirlwind makes use of the facilities of the Digital Computer Laboratory. The principal objective of the Project is the application of an electronic digital computer of large capacity and very high speed (Whirlwind I) to problems in mathematics, science, engineering, simulation and control.

Whirlwind I is of the high-speed electronic digital type, in which quantities are represented as discrete numbers and complex problems are solved by the repeated use of fundamental arithmetic and logical (i.e., control or selection) operations. Computations are executed by fractional-microsecond pulses in electronic circuits, of which the principal ones are (1) the flip-flop, a circuit containing two vacuum tubes so connected that one tube or the other is conducting, but not both; (2) the gate or coincidence circuit; (3) the magnetic-core memory, in which binary digits are stored as one of two directions of magnetic flux within ferro-magnetic cores.

Whirlwind I uses numbers of 16 binary digits (equivalent to about 5 decimal digits). This length was selected to limit the machine to a practical size, but it permits the computation of many simulation problems. Calculations requiring greater number length are handled by the use of multiple-length numbers. Rapid-access magnetic-core memory has a capacity of 32,768 binary digits. Present speed of the computer is 40,000 single-address operations per second, equivalent to about 20,000 multiplications per second.

1. REVIEW AND PROBLEM INDEX

This report covers the specific period of December 24, 1956 to March 17, 1957. During this time, 71 problems made use of 289.51 hours of the 342.66 hours of Whirlwind computer time allocated to the Scientific and Engineering Computations (S&EC) Group. Of the 342.66 allocated hours of computer time, 2.57% was down time because of computer malfunctions. 44.35 hours of the allocated time were used for terminal equipment testing and calibration, demonstrations and various inter-run operations not logged to specific problems.

The 71 problems run during this quarter cover some 15 fields of application. The results of 21 of the problems have been or will be included in academic theses. In these 21 problems, there are represented 16 Doctoral theses, 3 Master's, 2 Naval Engineer's and 1 Bachelor's. Twenty-five of the problems have originated from research projects sponsored at MIT by the Office of Naval Research.

Two tables are provided as an index to the problems for which progress reports have been submitted. In the first table, the problems are arranged according to the field of application and the source and amount of time used on WWI are given. In Table II, the problems are listed according to the principal mathematical problem involved in each. In each table, the letter after the problem number indicates whether the problem is for academic credit and whether it is sponsored. The code is explained in section 2.1, Introduction.

Field	Description	Problem Number	Minutes of WWI Time	Supervisor or Programmer
Aeronautical Engineering	Transient Response of Aircraft Structures to Aerodynamic Heating	236 C.	11.3	L. A. Schmit
	Horizontal Stabilizer Analysis	257 C.	421.3	E. S. Criscione
	Blast Response of Rotor Blades	364 C.	159.4	A. Holland
	Determination of Velocity Potential	387 C.	139.3	R. Stapleford S. Gravitz
	Temperature and Stress Response	400 C.	186.6	J. C. Loria
Aerophysics Research Group	Trajectory Calculations for a Rocket During Powered Flight	310 C	228.2	L. J. Berman
	Extraction of Stability Derivatives from Flight Test Data	317 C.	486.8	M. N. Springer
Civil Engineering	Response of a Multi-Story Frame Building Under Dynamic Loading	203 C	1038.9	R. G. Gray
	Response of a Single Story Concrete Building	354 D	14.6	S. Namyet
	Design of Spherical Shell Segments	372 B	12.9	E. Traum
	Mitchell's Wave-Making Integral	390 B	207.1	W. Tudor
Electrical Engineering	Transportation Problem	219	47.4	J. Dennis
	Production for Transportation Problem	326 C.	545.9	J. Dennis
	Switching Circuits	380 B.	142.7	C. H. Roth, Jr
Geology Department	MIT Seismic Project	106 C.	130.7	S. Simpson
	Fourier Synthesis for Crystal Structures	261 C.	115.2	M. J. Buerger
Lincoln Laboratory	Eigenvalue Problem for Propagation of E. M. Waves	193 L	938.0	R. M. King
	WWI-1103 Translation Program	256 C.	26.6	J. M. Frankovich
	Tropospheric Propagation	300 L.	372.8	W. Mason
	Error Analysis	312 L.	147.4	L. Peterson
	Prediction Analysis	327 L.	838.0	E. Hutcheson
	Atmospheric Propagation of Radio Waves	371 L.	77.5	W. Mason
	Coverage Analysis	377 L.	177.9	E. Hutcheson
	Magnetic Relaxation in Thin Films	391 L.	43.0	D. O. Smith
	Domain Wall Motion	399 L,B.	216.9	J. V. Harrington
Mathematics Department	Nonlinear Second Order Differential Equations in the Theory of Elastic Shells	337 N.	187.7	H. Weinitschke
Mechanical Engineering	Laminar Boundary Layer of a Steady, Compressible Flow in the Entrance Region of a Tube	199 N.	101.8	T. Y. Toong
	Growth of Fatigue Cracks	361 B,N.	31.6	J. B. Walsh
	Supersonic Flow of Air in a Tube	389 D.	87.4	J. Radbill
Meteorology	Statistical and Dynamic Methods in Forecasting	341 C.	753.2	E. Kelley
	Weather Prediction	343 C.	119.6	E. Kelley
	Computation of Variances and Covariances	350 D.	293.3	D. Gilman
Naval Supersonic Laboratory	Free Convection	386 C.	129.9	J. R. Baron
Nuclear Engineering	Core Optimization	404 B.	15.2	B. A. Lee
Nuclear Science Laboratory	Determination of Phase Shifts from Experimental Cross-Sections	162 N.	107.1	E. Mack
	Theory of Neutron Reactions	245 N.	748.2	E. Campbell
	Analysis of Air Shower Data	273 N.	1678.1	F. Scherb
Operations Research	Non-Stationary Queueing Problems	401 N.	102.7	H. Galliher
	Monte Carlo Inventory Control Study	402 N.	1.0	H. Galliher
Physics	Neutron-Deuteron Scattering	225 B,N.	125.4	H. Paul
	APW as Applied to Face- and Body-Centered Iron	253 N.	382.1	J. H. Wood
	Multiple Scattering of Waves from a Spatial Array of Spherical Scatterers	274 N.	202.1	M. Karakashian
	Energy Levels of Diatomic Hydrides LiH	278 N.	767.2	A. M. Karo
	Polarizability Effects in Atoms and Molecules	290 N.	32.9	L. C. Allen
	Pure and Impure Potassium Chloride Crystal	309 B,N.	66.9	L. Howland
	Response Functions of Air Shower Detectors	397 N.	170.9	G. W. Clark
Servomechanisms Laboratory	Data Reduction	126 C.	797.9	D. T. Ross
	Temperature Distribution in Aircraft Generators	388 D.	181.7	R. Moroney
	Automatic Programming for Numerically Controlled Machine Tools	394 C.	210.6	D. T. Ross
Miscellaneous	Comprehensive System of Service Routines	100	366.7	F. Helwig
	S&EC Subroutine Study	141	90.2	M. Callaghan

Table I Current Problems Arranged According to Field of Application

Mathematical Problem	Procedure	Problem Number
1. Matrix algebra and equations		
Eigenvalues	Diagonalization	278 N.
Orthonormalization, eigenvalues	Schmidt process, diagonalization	290 N.
Inversion	Crout	312 L.
Linear equations	Crout	317 C.
Diagonalization, inversion	Successive approximation, Crout	341 C.
Inversion	Crout	343 C.
Inversion, eigenvalues	Crout, iteration	364 C.
Arithmetic operations	Direct	377 L.
Boolean algebra, Boolean matrices	McCluskey-Quine reduction	380 B.
Inversion of complex matrix	Crout	387 C.
Inversion	Crout	404 B.
2. Ordinary differential equations		
System	Gill	199 N.
Wave equation	Milne predictor-corrector formula	245 N.
Different sets of differential equations	Step by step	257 C.
Second order equations	Gill's method	310 C.
2 simultaneous non-linear, second order equations	Power series	337 A,N.
Set of differential equations	Kutta-Gill	371 L.
Pair of simultaneous non-linear equations	Kutta-Gill	386 C.
Non-linear differential equations	Finite difference	388 D.
Pair of simultaneous differential equations	Gill	391 L.
Second order non-linear differential equations	Kutta-Gill	399 L,B.
3. Partial differential equations		
Second order parabolic equations	Finite difference	236 C.
Second order equations	Finite difference	400 C.
4. Integration		
Integration	Simpson's rule	225 B,N.
Integrations	Simpson's rule	278 N.
Fresnel integral	Conversion power series with a complex argument	300 L.
Integrations	Barnet and Coulson expansion	309 B.
Integrations	Gauss quadrature	312 L.
Integration	Finite difference	354 D.
Integrations	Series substitution	372 B.
Integration	Trapezoidal rule	397 N.
Integration	Finite difference	400 C.
5. Transcendental equations		
Curve fitting	Least squares	162 N.
Curve fitting	Least squares	273 N.
6. Complex algebra		
Complex roots and function evaluation	Iteration	193 L.
7. Data Reduction		
Data reduction	Polynomial fitting, etc.	126 C.
8. Fourier series		
Fourier synthesis	Direct evaluation	261 C.
9. Statistics		
Multiple time series	Prediction by linear operators	106 C.
Evaluation of covariance and variance	Direct	350 D.
10. Integral equations		
Volterra type	Trapezoidal	361 B,N.
11. Linear programming		
Linear programming	Simplex method and relaxation method	219

Table II Current Problems Arranged According to the Mathematics Involved

2. WHIRLWIND CODING AND APPLICATIONS

2.1 Introduction

Progress reports as submitted by the various programmers are presented in numerical order in Section 2.2. Letters have been added to the problem numbers to indicate whether the problem is for academic credit and whether it is sponsored. The letters have the following significance.

A implies the problem is NOT for academic credit, is UNsponsored.

B implies the problem IS for academic credit, is UNsponsored.

C implies the problem is NOT for academic credit, IS sponsored.

D implies the problem IS for academic credit, IS sponsored.

N implies the problem is sponsored by the Office of Naval Research.

L implies the problem is sponsored by Lincoln Laboratory.

The absence of a letter indicates that the problem originated within the S&EC Group.

2.2 Progress Reports

100 COMPREHENSIVE SYSTEM OF SERVICE ROUTINES

Staff time used on problem 100 during the past quarter has been mainly consumed by consultation with users of the system and with preparation of annotated copies of the programs involved.

Frank Helwig
Digital Computer Laboratory

106 C. MIT SEISMIC PROJECT

During the past quarter, a program for testing the hypothesis of stationarity of a data sequence has been completed and used on several sets of data. Examples of random elastic media with variable scattering power have also been computed Finally, a spherical Bessel transform program, in WWI code, has nearly been completed.

Plans for the future include the use of the hypothesis testing program on a greater variety of Geophysical data and the use of the transform program to test the possibility that one dimensional data come from isotropically random three dimensional data.

Programmers working on this problem are S. Simpson, S. Treitel and D. Grine.

S. Simpson
Geology and Geophysics

126 C. DATA REDUCTION

Problem 126 is a very large data-reduction program for use in the Servomechanisms Laboratory. The overall problem is composed of many component sections which have been developed separately and are now being combined into complete prototype programs. Descriptions of the various component sections have appeared in past quarterly reports. After the development and testing of the prototype Whirlwind programs is completed, the programs will be re-coded for other, commercially available, large scale computers, (probably the ERA 1103, IBM 701 and IBM 704 computers), for use by interested agencies for actual data reduction at other locations. The programs are currently being developed by Douglas T. Ross, David F. McAvinn, Benson H. Scheff and Dorothy A. Hamilton, Servomechanisms Laboratory staff members with the assistance of John F. Walsh. This work is sponsored by the Air Force Weapons Guidance Laboratory through DSR Project 7668.

The nature of the problem requires extreme automaticity and efficiency in the actual running of the program, but also requires the presence of human operators in the computation loop for the purpose of decision making and program modification. For this reason extensive use is made of output oscilloscopes so that the computer can communicate with the human, and manual intervention registers so that the human can communicate with the computer in terms of broad ideas, while the computer is running, and have the computer program translate these ideas into the detailed steps necessary for program modification to conform to the human operator's decision. The program which does this translation and modification is called the Manual Intervention Program (MIV). The most recent version of the prototype data-reduction program is called the Basic Evaluation Program.

During the past quarter, work has continued in the development of SLURP (Servo Lab Utility Routine Program). A new generalized Group Control program has been designed to take advantage of the increased size of the magnetic core memory of the Whirlwind Computer. The function of the Group Control Program is to allow programs that are much larger than the high speed memory capacity to be operated with ease as though the entire core and drum memory of the computer were individually addressable. The improved Group Control Program includes a modified group structure which is different from the old group structure which has been in use for some time. A Group consists of a number of Sections, each Section consisting of a collection of coding and data which performs some specific function. The new Group Control Program maintains a record of which Sections are in core memory and when a Group is called for, any Sections which are not already in core memory are brought in automatically for execution. Sections and Groups may be of arbitrary lengths and occupy arbitrary locations in the improved Group Control Program. The new program will result in greatly increased efficiency of computer application and increased flexibility as well.

The Slurp program proper has been modified to include several new facilities; the Slurp program operates like a specially designed simulated computer with very unusual control features. Among these are several modes of operations so that programs which differ only slightly may be intermingled to occupy the same storage space and yet perform different functions depending upon the setting of the control element of the simulated computer. The Slurp computer also operates internally in terms of floating addresses of a special type, so that extreme flexibility in program modifications are possible. A basic feature of this Slurp program is that its operation code includes sequences of arbitrary Flexo characters. When such sequences are encountered they are automatically displayed on any type of output equipment which has been previously selected and set up by other Slurp instructions. The control element of the simulated computer is arranged so that any word or sequence of words may be stored only once and yet arbitrary combinations of words and sequences can be assembled for display by making use of the jump-type instructions which are built into the computer. In this way an extremely wide variety of labelling information can be produced with a minimum amount of computer storage. During the past quarter this feature has been used to provide labelling of results from special Slurp post mortem routines and scope plot routines entirely under the control of the Slurp computer.

As described in the last quarterly report, an important feature of the Slurp system is a Logging Program which creates a logical record of all manual intervention actions and displays during a computer run on magnetic tape. During the past quarter an Editor Generator Program, originally coded by F. C. Helwig of the S and EC Group, has been incorporated with Slurp. The Editor Generator Program represents another level of simulated computer, and its function is to write or generate editing programs which make use of the facilities of Slurp itself to provide easily read edited versions of the information placed on magnetic tape by the Logging Program. The Editor Generator Program has an operation code which consists of individual Flexo characters so that it is attached to Slurp as though it were another output device. The editing programs themselves after they have been generated in turn make use of Slurp to operate actual output equipment as above.

D. T. Ross
Servomechanisms Laboratory

141 S & EC SUBROUTINE STUDY

During the past quarter, a double register fixed point sign agreement routine has been tested and added to the subroutine library. This routine will test for the sign agreement of the major and minor parts of a number in 96 μsec. If the signs disagree the sign of the minor will be made to agree with that of the major. If the major is equal to zero its sign will be set equal to the sign of the minor. The routine takes a maximum of 512 μseconds.

M. E. Callaghan
Digital Computer Laboratory

162 N. DETERMINATION OF PHASE SHIFTS FROM EXPERIMENTAL CROSS-SECTIONS

In the last quarter we have been attempting to fit the phase shift combinations in an area of our function containing an anomaly. We have found a few significant facts about this region leading to a decision to change our approach between the energies of 2.6 MEV and 2.72 MEV. Future plans include using our present approach to determine fits beyond the energy of 2.72 MEV.

Programmers working on this problem are B. Campbell and E. Mack.

E. Mack
Nuclear Science Laboratory

193 L. EIGENVALUE PROBLEM FOR PROPAGATION OF E M. WAVES

Calculations of eigenvalues, eigenfunctions and mode sums have been continued for the bilinear model at 410 Mc and 3000 Mc. Convergent sums have been obtained at four additional distances at 410 Mc, and at the two largest distances used for 3000 Mc.

More accurate programs for obtaining eigenfunctions and eigenvalues were written for the inverse-square model. These calculations are being made at 50 Mc.

R. M. Ring
Lincoln Laboratory

199 N. LAMINAR BOUNDARY LAYER OF A STEADY COMPRESSIBLE FLOW IN THE ENTRANCE REGION OF A TUBE

Solutions were obtained for the case where temperature dependence of the viscosity and thermal conductivity of the compressible fluid is taken into consideration.

More solutions are to be obtained for different entrance Mach numbers and thermal conditions at the tube wall.

T. Y. Toong
Mechanical Engineering

203 C. RESPONSE OF A MULTI-STORY FRAME BUILDING UNDER DYNAMIC LOADING

Time during the past quarter has been used in examining the effect of variation of certain parameters. A generalizing program to handle other structures has also been worked on.

R. G. Gray
Civil Engineering

219 TRANSPORTATION PROBLEM

See report for problem 326.

J. Dennis
Electrical Engineering

225 B,N. NEUTRON-DEUTERON SCATTERING

During the past quarter, calculations for a non-zero value of the polarization parameter, $\mu = .02$, with the expanded four parameter trial function, were made. As in the case of no polarization, these results differ little from the two-parameter case. The next step will be to try a higher value of μ.

H. Paul
Nuclear Science Laboratory

236 C. TRANSIENT RESPONSE OF AIRCRAFT STRUCTURES TO AERODYNAMIC HEATING

For the past three months Whirlwind routines for solving secular equations have been used to aid the solution of certain heat flow problems.

L. A. Schmit
Aeroelastic and Structures
Research Laboratory

245 N. THEORY OF NEUTRON REACTIONS

During this quarter, more tests were made in critical areas to determine better values of c and ζ . When these are determined a complete case will be done for the cross-sections and angular distributions.

E. Campbell
Nuclear Science

253 N. APW AS APPLIED TO FACE- AND BODY-CENTERED IRON

An investigation of the accuracy of the Noumerov method of integrating the radial wave equation has been completed. This check was carried out by investigating the solutions of the Coulomb wave equation, for which the analytical solutions are known.

Currently an empty lattice test for the face-centered structure is being carried out. Programmers working on this problem are M. M. Saffren and J. H. Wood.

J. H. Wood
Solid State and Molecular
Theory Group

256 C. WWI-1103 TRANSLATION PROGRAM

During the past quarter the input language vocabulary for the translation program has been extended by several modifications to the program. The form of octal and decimal numbers has been generalized to permit factors of powers of two and ten. The work "FEED" permits programmers to obtain enough blank tape where desired in the translated tape to meet the dimensional problems of the 1103 input paper tape reader.

Programmers working on this problem are F. C. Helwig and J. M. Frankovich.

J. M. Frankovich
Lincoln Laboratory

257 C. HORIZONTAL STABILIZER ANALYSIS

Some production runs were made during the last quarter; others will be made during the next quarter. The program is also being used to investigate

several parametric variations.

Programmers working on this problem are N. P. Hobbs and E. Criscione.

E. Criscione
Aeroelastic and Structures
Research Laboratory

261 C. FOURIER SYNTHESIS FOR CRYSTAL STRUCTURES

During the past quarter, the original problem was in the process of being finished.

M. J. Buerger
Geology and Geophysics

273 N. ANALYSIS OF AIR SHOWER DATA

During the past quarter about 1500 air showers have been analyzed by the WWI computer. Very little programming time has been spent on the problem since the program works satisfactorily and has required only occasional minor modification. We plan to continue with the reduction of air shower data using the WWI computer and also to develop a similar program for the new IBM 704.

Programmers working on this problem are G. Clark and F. Scherb.

F. Scherb
Nuclear Science Laboratory

274 N. MULTIPLE SCATTERING OF WAVES FROM A SPATIAL ARRAY OF SPHERICAL SCATTERERS

A satisfactory fit, $F(k^2,K)$ has been obtained for the sum $2\ {}^{2}{}_{kM}{}^{o}_{o}$ for all seven directions of K. $F(k^2,K)$ takes the form:

$$(a_o + b_oK + c_oK^2 + d_oK^3 + e_oK^4) + (a_1 + b_1K + c_1K^2 + d_1K^3 + e_1K^4)\ k^2 + (a_2 + b_2K + c_2K^2 + d_2K^3 + e_2K^4)(k^2)^2 + (a_3 + b_3K + c_3K^2 + d_3K^3 + e_3K^4)(k^2)^3 + (a_4 + b_4K + c_4K^2 + d^4K^3 + e_4K^4)(k^2)^4.$$

Work is now in progress to obtain fits for the remaining six sums.

M. Karakashian
Physics Department

278 N. ENERGY LEVELS OF DIATOMIC HYDRIDES LiH

In order to obtain a better approximation to the true molecular wave function for the lithium hydride molecule, $2p_\pi$ orbitals are being included for both the lithium and hydrogen atoms. While radial correlation has been reasonably well considered in the treatment so far, no angular correlation has been included since π orbitals were omitted.

The necessary integrals are being evaluated at the present time by means of the programs, written by Dr. Corbató, which have previously been used. Several internuclear distances are being considered for which we have results of earlier calculations. The self-consistent field calculation will be carried out using Dr. Nesbet's programs. If there is time, a configuration interaction treatment will be included.

A. M. Karo
Solid State and Molecular Theory Group

290 N. POLARIZABILITY EFFECTS IN ATOMS AND MOLECULES

A large part of this quarter was used in analysing and organizing the results of previous computer works and in writing up the results as part of a thesis submitted to the M.I.T. Physics Department. Most of the physical results and interpretation have been or will be reported in the Quarterly Progress Reports of the Solid State and Molecular Theory Group.

We have recently started to use the computer again and an intensive program is planned for the remainder of S&EC Whirlwind operations. There are several general points which can be made in regard to the use of the computer on this problem and it seems appropriate to give them here. Over the course of several years a large number of routines and sub-routines have been built up that are especially applicable to the solution of problems in the electronic structure of atoms, molecules, and solids. Most of these have been written by Corbató, Meckler and Nesbet (with additions and modifications by others) and have been adequately described in Whirlwind I Quarterly Progress Reports. The capability of these programs have now been thoroughly explored and their use on a more or less production basis has resulted in significant advances in solid state and molecular theory. In particular, in our works we have been concerned with ab initio electronic wave functions for atoms and simple molecules and these programs have enabled us to carry various aspects of the problem considerably further than has been possible before.

In looking forward to the I.B.M. 704, initially we will be at a considerable disadvantage since competent programmers are not now available in sufficient numbers to transform these routines in a short time. It may be expected that there will be a gap of one to two years in re-establishing certain of these programs. Because of this and because we have these routines in a form well tailored to our specific type of molecule problem, production runs in the next few months will be extremely valuable. It is reasonable to hope that this will allow us to obtain adequate theoretical predictions for experimentally inaccessible quantities of direct aid to spectroscopy and upper atmosphere physics.

L. C Allen
Solid State and Molecular Theory Group

300 L. TROPOSPHERIC PROPAGATION

A subroutine for generating incomplete elliptic integrals of the first and second kinds has been programmed and tested. There has been no other activity this quarter.

W. Mason
Lincoln Laboratory

309 B,N. PURE AND IMPURE POTASSIUM CHLORIDE CRYSTAL

Certain additional details of the structure of the valence band of KCI were obtained by solving secular equations at new points in reciprocal space. Some calculations of the charge density in KCI have also been made. In the near future, it is planned to attempt a density of states calculation for the valence band of KCI and to obtain the charge density to a somewhat higher accuracy.

L. P. Howland
Solid State and Molecular Theory Group

310 C. TRAJECTORY CALCULATIONS FOR A ROCKET DURING POWERED FLIGHT

By eliminating intermediate printout, total computation time per run has been considerably reduced. Parametric variations are planned for the immediate future.

L. J. Berman
Aerophysics Research Group

312 L. ERROR ANALYSIS

A modification of 100-word length has been written and tested. Production runs are being made. There may be other modifications of the problem in the attempt to use the main program for satellite computations. Programmers working on this problem are E. Hutcheson and L. Peterson.

L Peterson
Lincoln Laboratory

317 C. EXTRACTION OF STABILITY DERIVATIVES FROM FLIGHT TEST DATA

In the past quarter, this project has been concerned primarily with the problem of extracting lateral stability derivatives. A number of simulated responses to aileron and rudder inputs have been obtained. Generally these responses lie in the range within which aerodynamic non-linearities are unimportant, but for many of them, nonlinear inertial terms ("cross-coupling" terms) are significant. From these responses extractions of the derivatives have been made by both Shinbrot's method and a new statistical method developed by this project. The effects of varying the type of response, the assumed precision of measurement, the number of frequencies used and other factors influencing the accuracy of results have been investigated. It is proposed to continue the present investigations in the next quarter and to obtain responses from which other quantities in the equations of motion of an aircraft which have not thus far been considered can be extracted. Among these additional quantities are I_z, I_{xz}, $C_{m_{\dot{a}}}$, and parameters determining nonlinear lateral aerodynamic terms.

Programmers working on this problem are M. E. Hoult, L. L. Mazzola, M. N. Springer, L. E. Wilkie and J. F. White.

M. N. Springer
Aerophysics Research Group

326 C. PRODUCTION FOR TRANSPORTATION PROBLEM

During the past quarter, additional data was obtained on the computation time used in solving transportation problems, when the best method of selecting the next step is used (see Summary Report No. 48). This data is presented in Fig. 1.

No further computer use is contemplated for this problem.

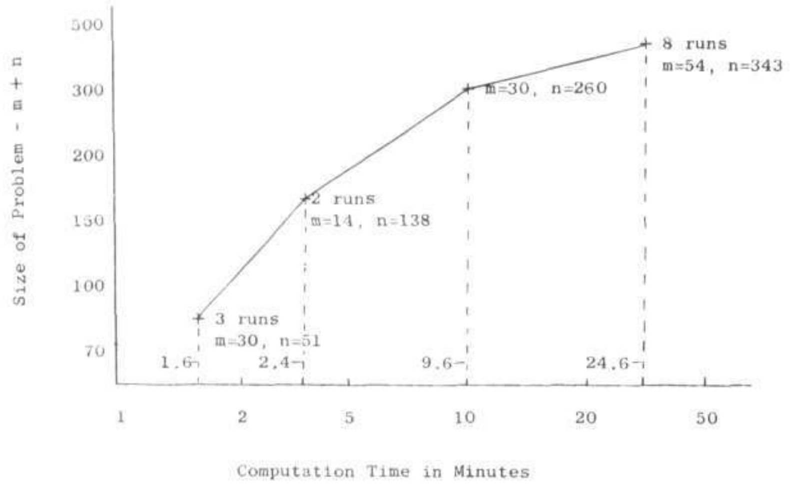

Fig. 1 - Computation Time Data - Whirlwind Transportation Routine.

m = No. of Supplies
n = No. of Demands

J. Dennis
Electrical Engineering

327 L. PREDICTION ANALYSIS

The program has been completed and production runs are being made. Modifications may be made from time to time. Programmers working on this problem are L. Peterson, E. Hutcheson and P. Willmann.

E. Hutcheson
Lincoln Laboratory

337 N. NONLINEAR SECOND ORDER DIFFERENTIAL EQUATIONS IN THE THEORY OF ELASTIC SHELLS

The program evaluating physical quantities of interest in the present

problem, which was mentioned in the last progress report, has been successfully used to obtain stresses and displacements for a wide range of parameters of the differential equations in question. These results are included in a report written up for publication. Some experiments have also been made during the last quarter with a modified program in order to obtain "asymptotic solutions" of the differential equations subject to suitable boundary conditions. However, no reliable results have been obtained as yet. We feel that further modifications of the present program will be necessary to get the desired results, a point under investigation at the present time.

H. Weinitschke
Mathematics Department

341 C. STATISTICAL AND DYNAMIC METHODS IN FORECASTING

Studies of sea-level pressure by empirical orthogonal functions continued. Certain functions of pressure and pressure tendency, selected by a statistical screening procedure, were used as predictors in a linear forecast method. Programs were developed for Whirlwind to test this new method on a set of independent data.

A method for predicting the surface 5-day-mean temperature anomaly based on the coefficients of the empirical orthogonal functions was tested on an independent sample. Verifications of the resulting forecasts are now in progress.

Work continues on a program for a simplified numerical model of the general circulation of the earth's atmosphere developed in terms of spherical harmonics.

Another phase of the Statistical Forecasting Project has been devoted to the development of a statistical-dynamic method of numerical weather prediction. Work on Whirlwind has been concerned mainly with the determination of certain constants appearing in the prognostic equations.

All of the work done on the project has been supervised by Professor Edward N. Lorenz, Department of Meteorology. Programmers working on this problem are E. A. Kelley, B. Shorr and K. Bryan.

E. A. Kelley
Meteorology Department

343 C. WEATHER PREDICTION

During the past quarter, the computations on this problem were completed. The results of this work may be found in the final report Research on Atmospheric Pressure Changes.

E. A. Kelley
Meteorology Department

350 D. COMPUTATION OF VARIANCES AND COVARIANCES

Empirical functions of temperature have been computed and their lag and contemporary correlations with empirical functions of pressure found. No further machine computations are planned for the immediate future.

Programmers working on this problem were B. Shorr and D. Gilman.

D. L. Gilman
Meteorology Department

354 D. RESPONSE OF A SINGLE STORY CONCRETE BUILDING

During the past quarter the problem was completed.

S. Namyet
Civil Engineering

361 B,N. GROWTH OF FATIGUE CRACKS

During the past quarter, the integral equation expressing crack growth was debugged and several runs were made. A program to determine the stress distribution in a round bar with a circumferential crack in it was set up and is now being debugged.

J. B. Walsh
Mechanical Engineering

364 C. BLAST RESPONSE OF ROTOR BLADES

During the past quarter routines were debugged. In the future it is hoped that the equations of motion will be integrated.

A. Holland
Aeroelastics and Structures
Research Laboratory

371 L. ATMOSPHERIC PROPAGATION OF RADIO WAVES

A rapid method of reasonable accuracy for integrating the range-height equation (described in earlier Quarterly Progress Reports) has been programmed. This program will be used with large sets of experimental data and statistical studies will be carried out on the results.

W. Mason
Lincoln Laboratory

372 B. DESIGN OF SPHERICAL SHELL SEGMENTS

During the past quarter final results were obtained and the full purpose of the program was accomplished. No future work on this program is necessary.

E. Traum
Civil and Sanitary Engineering

377 L. COVERAGE ANALYSIS

A series of production runs is being made. Additional modifications may be made in the future. Programmers working on this problem are L. Peterson and E. Hutcheson.

E. Hutcheson
Lincoln Laboratory

380 B. SWITCHING CIRCUITS

A program for simplification of switching functions of 14 or fewer variables by means of the McCluskey-Quine reduction procedure has been completed and checked out and 89 problems have been solved. The switching function to be simplified is specified in standard sum form. The program compares the terms in the standard sum to form the prime implicants and then selects certain of these prime implicants to form the minimum sum.

A program has been written for synthesis of non-series-parallel contact networks by using Boolean matrix methods. The standard sum which represents the transmission of the network to be synthesized is represented by an octal fraction. By means of successive node insertions, a Boolean matrix is obtained which represents a network having the desired transmission. After elimination of redundant contacts, the matrix representation of the contact network is printed out. In approximately half of the 67 synthesis problems which have been solved, minimum networks were obtained. In most of the remaining cases, the networks contained only one or two extra contacts. The program is capable of synthesizing a network to represent any four-variable switching function, and the solution time ranges from a fraction of a second to several minutes.

The computer work on this problem is practically completed. After solution of a few more problems with each program, the results will be written up in a Master's thesis entitled, "Digital-Computer Solution of Switching-Circuit Problems."

C. Roth, Jr.
Electrical Engineering

386 C. FREE CONVECTION

Additional similarity solutions have been completed corresponding to several choices for the constants defining the surface temperature variation ($\sim BX^q$, X being the length parameter in the flow direction). Final computations are expected to be complete within the next quarter.

J. R. Baron
Naval Supersonic Laboratory

387 C. DETERMINATION OF VELOCITY POTENTIAL

The complete program for the delta wing has been debugged and is available and has been used for production runs to yield useful information. In addition, the program has been expanded in scope so that the machine takes the basic aerodynamic influence coefficients and combines them with the proper weighting factors to form the various component matrices used in the determination of the velocity potentials. Previously this was done on desk calculators and was a tedious, time-consuming, and error-prone process. The increase in time per production run for inclusion of this step is a small fraction of the total machine time required. At present, three reduced frequencies at the lowest Mach number and one reduced frequency at the intermediate Mach number have been investigated for the delta. Remaining are runs for two reduced frequencies at the intermediate M and 3 reduced frequencies at the high M.

A first draft of the complete program for the straight wing has been completed and is in the process of being debugged. The swept wing program awaits the successful conclusion of the delta and straight wing programs. The swept and straight programs are essentially merely modifications of the successful delta program. For each of the three planforms, three reduced frequencies must be investigated for each of three Mach numbers.

R. Stapleford
S. Gravitz
Aeroelastic and Structures
Research Laboratory

388 D. TEMPERATURE DISTRIBUTION IN AIRCRAFT GENERATORS

A routine which produces a generator rating chart from design parameters has been written and is being tested. This program, when checked out, will be the final result of the present problem.

R. M. Moroney
Servomechanisms Laboratory

389 D. SUPERSONIC FLOW OF AIR IN A TUBE

Problem Description

Physical Model. A supersonic stream of air enters a round tube with a uniform Mach number across the stream. Gas, which may be different from that in the entering stream, is forced through the wall which has uniform porosity. The flow in the entrance length of the tube is assumed to consist of two concentric regions: a boundary layer and an isentropic core.

Required Solution. Profiles of velocity, temperature and injected gas concentration are required as a function of the distance from the entrance of the tube. Pressure distributions, friction factors and heat-transfer coefficients are also required. The effect of entrance Mach number, foreign gas molecular weight, injection rate and heat transfer rate on the above quantities is required.

Basic Differential Equations. The partial differential equations describing the problem are obtained from the Navier Stokes equations in cylindrical coordinates by an order of magnitude analysis. These equations are as follows:

1. Overall Continuity

$$\frac{\delta}{\delta r}(r\rho u) + \frac{\delta}{\delta z}(r\rho w) = 0$$

2. Momentum or Equation of Motion

$$\rho\left\{\frac{u\,\delta w}{\delta r} + w\,\frac{\delta w}{\delta z}\right\} = \frac{1}{Re}\,\frac{1}{r}\,\frac{\delta}{\delta r}\left\{r\mu\,\frac{\delta w}{\delta r}\right\} - \frac{\delta p}{\delta z}$$

3. Single Species Continuity

$$Sc_o\,\frac{Re_o}{2}\,\rho\left\{u\,\frac{\delta c}{\delta r} + w\,\frac{\delta c}{\delta z}\right\} = \frac{1}{r}\,\frac{\delta}{\delta r}\left\{r\rho\mathcal{D}_{12}\left[\frac{\delta c}{\delta r} + \alpha c(1-c)\,\frac{\delta(\ln T)}{\delta r}\right]\right\}$$

4. Energy

$$Re_o\,\rho\,Cp\left\{u\,\frac{\delta T}{\delta r} + w\,\frac{\delta T}{\delta z}\right\} = Re_o M_o^2\,(\gamma_o-1)w\,\frac{\delta p}{\delta z} + \frac{2}{Pr_o}\,\frac{1}{r}\,\frac{\delta}{\delta r}\left\{r\,\frac{\lambda\delta T}{\delta r}\right\}$$

$$+ \frac{2}{Sc_o}\rho\mathcal{D}_{12}\left\{\frac{\delta c}{\delta r} + \alpha c(1-c)\,\frac{1}{T}\,\frac{\delta T}{\delta r}\right\}\frac{\delta}{\delta r}\left\{T\left[Cp_1 - Cp_2 + \frac{\gamma_o-1}{\gamma_o}\,\frac{\alpha M_{21}}{c(M_{21}-1)+1}\right]\right\}$$

$$+ \frac{\gamma_o-1}{\gamma_o}\,\frac{\alpha M_{21} T\,Re_o\rho}{c(M_{21}-1)+1}\left\{u\,\frac{\delta c}{\delta r} + w\,\frac{\delta c}{\delta z}\right\} + M_o^2\,(\gamma_o-1)\,\mu\left(\frac{\delta w}{\delta r}\right)^2$$

5. Equation of State and Properties

$$p = \rho RT$$

$$h = CpT$$

In these equations all lengths are referred to the tube radius, α, all velocities to the initial free stream velocity, w_o, all properties to free stream entrance properties, μ_o, $\mathcal{D}_{12_o}$, λ_o and the pressure to $\rho_o w_o^2$. The subscript $_o$ refers to entrance free stream conditions.

Transformations. The overall continuity equation is satisfied by the introduction of a stream function:

$$\rho ur = -\frac{\delta\Psi}{\delta z}$$

$$\rho wr = -\frac{\delta\Psi}{\delta r}$$

A modified length Reynolds Number ζ and a wall distance parameter η are defined by:

$$\zeta = \left(\frac{2z}{aRe_o}\right)^{1/2}$$

$$\eta = \frac{1}{4\zeta}\left[1 - \left(\frac{r}{a}\right)^2\right]$$

where

$$Re_o = \frac{2wa}{\nu_o}$$

The boundary layer quantities Ψ, T, $\mathring{\alpha}$, C are expanded in series in ζ with coefficient which are functions of η.

$$\Psi = -a^2 \rho_o W_o \sum_{i=1}^{\infty} \zeta^i f_i(\eta)$$

$$T = T_o \sum_{i=1}^{\infty} \zeta^{i-1} \theta_i(\eta)$$

$$C = \sum_{i=1}^{\infty} \zeta^{i-1} g_i(\eta)$$

The properties in the isentropic core are described by series in ζ.

$$W_c = W_o \left(1 + \sum_{i=1}^{\infty} \zeta^i K_i\right)$$

$$p = p_o \left(1 + \sum_{i=1}^{\infty} \zeta^i \beta_i\right)$$

$$T = T \left(1 + \sum_{i=1}^{\infty} \zeta^i t_i\right)$$

$$\rho = \rho \left(1 + \sum_{i=1}^{\infty} \zeta^i \alpha_i\right)$$

The coefficients for ρ, T_c, $\mathring{\alpha}$, ρ_c may be related to the coefficient by use of the isentropic relations for the core.

When the series for T, Ψ, $\mathring{\alpha}$, C are substituted into the partial differential equations, the coefficients of the powers of ζ in each equation may be equated to zero, since these equations must hold anywhere in the region of validity of the analysis. Three infinite sets of ordinary differential equations in f_i, θ_i, g_i, $\mathring{\alpha}$ η are obtained. Each set of three determines the coefficients of one power of ζ in the series. The equations for the first terms are:

$$f_1''' = -\left\{\frac{f_1 + \mu_1'}{\mu_1} + 2\,\frac{\theta_1'}{\theta_1}\right\} f_1'' - \left\{\theta_1'' + \theta_1' \frac{(f_1 + \mu_1')}{\mu_1}\right\} \frac{f_1'}{\theta_1}$$

$$\theta_1'' = -(Pr_o f_1 + \lambda_1') \frac{\theta_1'}{\lambda_1} - \frac{M_o^2}{4} \frac{Pr_o}{\lambda_1} \mu_1 (\gamma_o - 1)(f_1'\theta_1' + f_1''\theta_1)^2$$

where: $\mu_1 = {}_1\omega({}_1T + {}_2T\theta + {}_3T\theta)$, $\lambda_1 = {}_1\sigma({}_1T + {}_2T\theta + {}_3T\theta)$

$\mu' = {}_1\omega({}_2T + {}_3T\theta)\theta_{13}'$ $\qquad \lambda_1' = {}_1\sigma({}_2T + T_3\theta)\theta_1'$

and ${}_1\omega$, ${}_1\sigma$, ${}_1T$, ${}_2T$, $\mathring{\alpha}_3T$ are all constants.

Only the three first order equations are nonlinear. The higher order equations are linear and contain functions calculated from the lower order equations, but their complexity increases rapidly with the order of the equation.

The boundary conditions for the first order equations are:

$$\eta = 0;\ f_1 = 0,\ f_1' = 0,\ \theta_1' = 0,\ g_1' = 0$$

$$\eta \to \eta_c \approx \infty;\ \lim_{\eta \to \eta_c} f_1' = 2,\ \lim_{\eta \to \eta_c} \theta_1 = 1,\ \lim_{\eta \to \eta_c} g_1 = 0.$$

Numerical Solution. The numerical part of the solution to be computed on Whirlwind will use a subroutine to calculate the highest derivatives in each of the equations for arbitrary values of the variables in conjunction with the Kutta-Gill integration method. Unknown values of θ_1, g_1, $\mathring{\alpha}$ f_1'' must be guessed at a value of η equal to zero so that f_1', θ_1, $\mathring{\alpha}$ g_1 will approach the proper limits at large values of η. Two initial calculations are made, then linear interpolation is used to establish the conditions for the third and successive iterations based on the two most recent values. When the boundary conditions at the second point are satisfied within the prescribed tolerance, the quantities which depend on the profiles such as heat-transfer coefficients and friction factors are calculated.

The linear equations for the higher order terms are solved by obtaining particular solutions which satisfy the conditions at the initial point and adding to these solutions which contain parameters which may be adjusted to satisfy the second point conditions.

Progress

A routine has been programmed and checked to calculate the first terms in the series solution. This method is dependent for convergence on the trial values of initial conditions with which it is started. Therefore, exploratory calculations or a modification of the method will be necessary.

The routine to calculate the second terms in the series is now being programmed and a hand calculation being made to check it. Calculation of a third set of terms may be undertaken if required for convergence of the series

J. Radbill
Mechanical Engineering

390 B. MITCHELL'S WAVE-MAKING INTEGRAL

Problem Description

Mitchell's integral expression in the form

$$I = \int_1^{\infty} e^{-t^2 y} \cos(xt) \frac{t^2 dt}{\sqrt{t^2 - 1}}$$

expresses the mutual wave resistance between any two elements on the hull of a ship (progressing at a uniform velocity) when multiplied by the mean areas and the mean slopes of the two elements concerned. In the above expression:

t is the variable of integration
y is a function of the vertical spacing of the areas and of speed
x is a function of the horizontal spacing of the areas and of speed.

At the lower limit this integral is infinite as are also all previous transformations of this integral. No analytical expression for this integral has yet been found. However, a transformation has been made of this integral to the form:

$$I = \frac{1}{\pi^{1/2} y^{3/2}} \int_0^\infty e^{-t/y} \left\{ J_1(2t^{1/2} + x) + J_1(2t^{1/2} - x) \right\} dt$$

in which J_1 is the Bessel function of the first kind of order one. This transformation is finite at all points (y is never zero for a finite area), thus enabling numerical integration to be performed without the use of questionable formulae near the discontinuities. The convergence of the integrand is reasonably rapid in all cases for such an evaluation to be practical. With a constant x and constant y, t can be varied in small increments and use of Simpson's rule then gives the value of the integral. This is the part of the problem that Whirlwind can perform.

Progress

The final form used for numerical integration with Simpson's three-point rule is

$$I = 2 \int_0^\infty \frac{e^{-y(t^2+1)^2} \cos x(t^2+1)}{\sqrt{(t^2+1)+1}} (t^2+1)^2 \, dt$$

rather than the initially proposed transformation containing Bessel functions.

Values of the above integral expression have been obtained on WWI These values apply to ship forms that are less than 400 feet and advancing at normal speeds. The remaining work is centered around a refined grid in the neighbourhood of the small region where the integral takes on large values and obtaining values of the integral for ships longer than 400 feet.

This data is being incorporated in Table II of my Sc.D. thesis entitled "The Slope-Area Concept for determining the Wave Resistance of a Ship".

W. Tudor
Civil Engineering

391 L. MAGNETIC RELAXATION IN THIN FILMS

The programming has been perfected so that future calculation only involves parameter change (also, three runs have been made bearing on current experiments). The amount and exact form of future calculation will depend on the course of the concurrent experimental study of film relaxation.

Programmers working on this problem are J. Frankovich and D.O. Smith.

D. O. Smith
Lincoln Laboratory

394 C. AUTOMATIC PROGRAMMING FOR NUMERICALLY CONTROLLED MACHINE TOOLS

The completion of the MIT Numerically Controlled Milling Machine in 1952 introduced a new era in the automatic translation of design requirements into finished parts. The primary problem associated with this type of machine tool, the preparation of detailed instructions for making a part, has been studied using the Whirlwind Computer. Under problem 132 (Whirlwind Progress

Reports 33-43) a library of subroutines was developed to assist in data preparation for the MIT NCMM. Under problem 250 (Whirlwind Progress Reports 41, 42) the feasibility of applying modern automatic programming techniques to machine tool programming was successfully demonstrated for plane parts bounded by straight line and circular segments. Problem 394 is concerned with the extension and elaboration of these automatic programming techniques to arbitrary surfaces which can be machined by 3 and 5 axis numerically controlled milling machines. The programs are currently being developed by Douglas T. Ross, Harry E. Pople, Jerome R. Wenker, George Zames, Samuel M. Matsa, Servomechanisms Laboratory Staff members. This work is sponsored by the Air Materiel Command Through DSR Project 6873.

The problem may be viewed as the design of an APT System (Automatically Programmed Tool system) combining the human operator, the machine tool itself, and an APT computer, which is a combination of a general purpose computer and existing machine tool directors. The objective is a system of Whirlwind computer programs which will simulate an APT computer with a language which is convenient for human expression of machined part designs A design expressed in this language will be automatically translated by the APT computer into machine tool instructions.

During the past quarter coding and debugging has continued on Whirlwind programs which allow programming of a part by means of arbitrary curves on the surface of a part, and also in terms of entire regions of the part. A number of additional functions have been put into the system programmed by curves, such as automatic indexing of curves to allow a specialized type of region programming. The oscilloscope output routine which draws near perspective views of the part being made has been incorporated into these routines and a number of test parts have been drawn automatically by the programs. Progress on Problem 394 was demonstrated in March as part of a special course on Programming for Numerical Control presented by the Servomechanisms Laboratory staff under the auspices of the Summer Session Office. Work is now underway on more general programs. The design of special purpose, English-like languages for programming APT Systems will be begun in the next quarter.

Associated studies are being made for the determination of appropriate languages for the feedback of information from the APT computer to the human operator. A program has been written by which the Whirlwind Computer can draw on the output oscilloscope an axonometric projection (near perspective) of the part being made. The program allows arbitrary orientation of axes and arbitrary scaling so that any view of any portion of the part can be shown. The program has several modes of operation, but at present the picture is displayed as the path of cut vectors swept out by the motion of the cutting tool over the surface of the part. English-like computer to human languages will also be devised.

D. T. Ross
Servomechanisms Laboratory

397 N. RESPONSE FUNCTION OF AIR SHOWER DETECTORS

The response of the detector array used on the roof of Building 6 has been completed. An array now in use in India has been analyzed in part. An array for use in the study of altitude variations of air showers will be analyzed.

G. W. Clark
Physics Department

399 L,B. DOMAIN WALL MOTION

Problem Description

In studying the motion of 180° magnetic domain walls in magnetization reversal processes in ferrite toroids an equation of the form:

$$\ddot{x} + 2d\,\dot{x} + f(x) = h$$

occurs where x is the normalized distance the wall moves, 2d is a damping parameter f(x) is a nonlinear restoring function with maximum and minimum amplitudes of ±1 and is periodic in x with period 2, h is a normalized driving force. The problem is to obtain solutions of this equation for various values of d and h. In particular a plot of $\dot{x}$ or wall velocity vs. time is desired. It is expected that such plots will have the same general character as the actual voltage response observed in magnetic cores during switch processes under the influence of varying driving currents.

Progress

During the past quarter, the initial computations were completed. It is now planned to reprogram the problem along somewhat different lines. Programmers working on this problem were J. V. Harrington and G. Dineen.

J. V. Harrington
Lincoln Laboratory

400 C. TEMPERATURE AND STRESS RESPONSE

Problem Description

This problem is concerned with the temperature and stress responses of various helicopter blades exposed to thermal radiation on one surface. The thermal inputs include a step input and time-varying pulses. The mathematical analysis consists of a second order partial differential heat conduction equation which describes the absorption and diffusion of heat energy and the Timoshenko-Goodier stress equations for predicting the thermal stresses involved. These equations, reduced to finite difference form for solution by the Whirlwind computer are as follows:

Temperature Equations:

$$T_{i,j} = \frac{\Delta Q_{ij}(A_{i\,exp})}{A_{i\,cr}\gamma c_p} + E_i T_{i,j-1} + G_i T_{b,j-1} + H_i T_{c,j-1} + F_i T_{d,j-1} + I_i T_{e,j-1}$$

where $E_i + G_i + H_i + F_i + I_i = 1$

and $T_{i,j}$ = temperature in element i at time j.

$\Delta Q_{ij}(A_{i\,exp})$ = heat absorbed by element i in the time interval t_{j-1} to t_j where $A_{i\,exp}$ is the exposed surface area of element.

$A_{i\,cr}$ = cross-sectional area of element i.

γ = weight density of element.

c_p = specific heat.

$T_{b,c,etc}$ = temperature of adjoining elements

Stress Equations:

$$\sigma_i = -E_i C_i \Delta T_i + \frac{E_i \sum_{i=1}^{m} E_i L_i \Delta T_i A_i}{\sum_{i=1}^{m} E_i A_i} + \frac{E_i X_{ie} \sum_{i=1}^{m} E_i L_i \Delta T_i X_{ie} A_i}{\sum_{i=1}^{m} X_{ie}^2 E_i A_i}$$

$$+ \frac{E_i Y_{ie} \sum_{i=1}^{m} E_i L_i \Delta T_i Y_{ie} A_i}{\sum_{i=1}^{m} Y_{ie}^2 E_i A_i}$$

The number of repetitions of the temperature equations depends on the time increment chosen and the time values of interest. The temperatures are stored at the times of interest for use in the stress equations. The stress equations are repeated only for the time points of interest.

Progress

During the past quarter, the program has been modified to include the response to a thermal pulse. Programmers for the problem are J. C. Loria and J. F. Duvivier.

J. C. Loria
Aeroelastics and Structures
Research Laboratory

401 N. NON-STATIONARY QUEUEING PROBLEM

Problem Description

1. Let $P_i = \{P_{i0}, P_{i1} \ldots\}$, i = 1,...1400/h be a set of discrete distribution functions defined recursively by

$$P_{in} = \sum_{j=0}^{r+1} P_{i-1,\,n+1-j}\, S_{j,i} \quad \text{for } i \bmod 1440/h$$

where $S_{j,i} = \frac{(\lambda_i h)^j e^{-\lambda_i h}}{j!}$, where the λ_i and h are specified, the λ_i being periodic with period $i = \frac{1440}{h}$. The problem is to calculate the stationary vectors P_i.

2. The application is : 1) λ_i is the arrival rate in the i^{th} interval of a calendar day of airplanes at an airport; 2) h is the servicing time of an airplane and is either 1 or 2 minutes as specified; 3) P_{in} is the probability that at the end of the i^{th} interval there are n-1 airplanes waiting to land and one landing, unless n=0 in which case none are waiting or landing.

3. The S_j are computed in (24,6), the P_n in (15,0).

Progress

During the past quarter the problem was successfully completed. Eight production runs were made.

H. P. Galliher
Operations Research Project

402 N. MONTE CARLO INVENTORY CONTROL STUDY

Problem Description

The problem includes various studies of the effect of variability of demand for a stored product and variability of replenishment times, on the inventory and ordering frequency. The studies include both single and multiple warehouse problems, the latter being an actual case study in the Ordnance Corps, U.S. Army, the former theoretical studies.

Random numbers are generated by a multiplicative congruential technique. All probability distributions are sums or probability combinations of independent exponential variates. All computation is done in (15,0) except for natural logarithm computations.

Progress

During the past quarter two programs were written. It is planned to start trouble shooting in the immediate future. Programmers working on the problem are H. P. Galliher and M. Simond.

H. P. Galliher
Operations Research Project

404 B. CORE OPTIMIZATION

This problem is an extension of Problem 270 with the following changes:

1. Core materials to be low enrichment uranium and water moderator.
2. Cladding materials stainless steel, zirconium and aluminum.
3. Water reflector.
4. Cylindrical geometry only.
5. Experimental data from Brookhaven and Bettis critical experiments and EBWR.

(See Quarterly Progress Report No. 16 for a description of Problem 270.)

Progress

Time during the past quarter was used in the checking of results against experimental data and adjusting questionable parameters. Programmers working on this problem are R. Gardner and B. A. Lee.

B. A. Lee
Nuclear Engineering

APPENDIX

1. VISITORS

Tours of Whirlwind I installation include a showing of the film, "Making Electrons Count", a computer demonstration, and an informal discussion of the major computer components. During the past quarter, the following 6 groups totalling 206 people visited the Laboratory.

January 9	MIT Course, "Machine-Aided Analysis"	48
January 15	MIT Course, "Introduction to Digital Computers - Coding and Logic"	11
January 28	Lexington High School	14
March 4	Belmont Hill School	13
March 11	Librarians from Harvard University and Radcliffe College	57
March 28	MIT 10-day Course, "Programming for Numerically Controlled Machine Tools"	63

The procedure of holding Open House at the Laboratory on the first Tuesday of each month has continued. A total of 38 people attended the 3 Open House tours during the quarter, representing members and friends of the MIT community, Brown University, Remington Rand, Westinghouse, Philips-Endhoven in Holland, and Boston Latin School.

During the past quarter, there were also 51 individuals who made brief tours of the computer installation at different times. Represented by these individuals were the Mathematical Center in Amsterdam, Georgia Institute of Technology, United States Air Force, Royal Canadian Air Force, Lockheed, National Cash Register Company, Max Planck Institute in Germany, Norden-Ketay, Amherst High School in Buffalo, Stanford Research Institute, Stone and Webster, Radio Corporation of America, Christian Science Monitor, RIAS, Houghton Mifflin Publishing Company, General Radio Company, Yokogawa Electric Works and Japan Overseas Company.

2. ACADEMIC

Introduction

During the spring term 1957 there were a number of graduate subjects in automatic computation, numerical analysis, and electronic data processing offered at MIT. In particular, the following subjects related directly to machine computation were offered during the spring term:

Subject	Description	Units	Year	Instructor
6.535	Digital Computer Coding and Logic	3 - 6	4	Arden
6.538	Electronic Computational Laboratory	3 - 1 - 5	G	Verzuh
6.54	Pulsed-Data Systems	3 - 6	G	Widrow
6.568	Switching Circuits	3 - 1 - 6	G	Caldwell
15.542	Management Information Systems	3 - 6	G	Gregory
M352	Advanced Calculus for Engineers	3 - 6	G	Hildebrand
M39	Methods of Applied Mathematics	3 - 9	G	Hildebrand
M412	Numerical Analysis	3 - 2 - 7	G	Hildebrand
N22	Nuclear Reactor Theory II	3 - 9	G	Clark

It is apparent from the above list that these subjects are predominantly Graduate A subjects - indicating that the subject is primarily for graduate students. However, a large number of undergraduate students in their junior and senior year do take these subjects on an elective basis.

Electronic Computational Laboratory - 6.538

The subject matter offered in 6.538 during the spring term 1957 was altered to provide the student with more time on the 650 Magnetic Drum Calculator. Specifically, each student had at least 8 hours of time on the 650 machine. In this manner a number of very proficient programmers were developed by the end of the term. A complete description of the course content including problems, term papers, and other assignments will be provided in the next quarterly report covering the period April - June.

The material previously offered on the Card Programmed Calculator has been discontinued and instead this portion of the course has been devoted to a study of the physical and functional features of the Type 704 Electronic Data Processing Machine. Specifically, the students were taught a limited amount of programming on the 704 machine. Unfortunately, however, the machine was not in operation in time to allow them to use the machine to carry out actual machine runs prior to the completion of the term. Again, a detailed description of the 704 material, physical plant, etc., will be given in the succeeding quarterly report.

F. M. Verzuh
Assistant Director
MIT Computation Center

PERSONNEL ON THE PROJECT

COMMITTE ON MACHINE METHODS OF COMPUTATION AND NUMERICAL ANALYSIS

Faculty Supervisors:

Philip M. Morse, Chairman	Physics
Samuel H. Caldwell	Electrical Engineering
Stephen H. Crandall	Mechanical Engineering
David Durand	Industrial Management
Herman Feshbach	Physics
Jay W. Forrester	Industrial Management
Francis B. Hildebrand	Mathematics
Chia-Chiao Lin	Mathematics
J. Francis Reintjes	Electrical Engineering

Ex-Officio

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PROJECT WHIRLWIND

Frank M. Verzuh, Head
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Jack Roseman
Arnold Siegel
Murray Watkins
Munroe R. Weinstein