SNR and Disk Drive Performance Symposium

April 27, 1998
8:30 am to 4:30 pm

Presented by
IIST
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Santa Clara, CA 95053

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SNR: The Presentations

SNR Overview:

Roy Gustafson __________ Quantum

Media Noise: Comparisons of Measurements

Dave Wachenswantz _____ Komag

SNR: Measurements, Requirements for High Density Recording

Hans Jurgen-Ricter _____ Seagate Magnetics

Interactions: Noise, Non-Linear Distortions in PRML Channel

Alex Taratorin __________ IBM

Media SNR: Measurents and Projections

Tom Arnoldussen ________ IBM

SNR: Budgets in Drive Design

Giora Tarnopulski ______ Seagate

SNR: Optical Recording

Robert Lynch ___________ Quinta

Panel Discussion:

Joe Rickert _____________ PhaseMetrics

SNR: Overview

Definitions

NSIC
RMS
Carrier/Noise

Noise Sources

Electronics
Media
"System"

Measurements

Techniques
Non-Linearities

System Performance

Error Rate

SNR: Definitions ?? (Cont.)

RMS/RMS

- Vp-p @ FCI
  - Vary FCI; Media Noise, Percolation
- RMS Signal
- RMS Noise
  - DC Erase
  - Head off Disk

Impacted by:

- Data Code
  - Rate
  - Distribution of "Ones"
  - "Resolution"
- Shape/PW50
- Bandwidth

SNR: Definitions (Cont.)

Isolated Pulse Energy / RMS Noise

Amplitude, Width Effects Rolled Up, Not Separated
Independent of Code, ……

Matched Filter Bound

RMS
"Ideal"

Example

NSIC	RMS/RMS	Pulse Energy	M.F.B.
28.5	22.5	31	26.2

Carrier/Noise

Optical

SNR: Measurements

Frequency Domain

- Spectrum Analyzer
- Guzik,...
- Full Channel Bandwidth
- Narrow Band
  - Media

Hybrid

- Amplitude (Viso; 0-p) Time Domain
- Noise (RMS in Specified B.W.)
- Guzik,....

Time Domain

- LeCroy
  - Pseudo-Random Sequence

SNR: Measurements (Cont.)

Complications

Read Non-linearity

10% Yields ~40% Local Gain Compression (At Peak)

Impacts Media Noise

Non-linear Amplitude Loss, Percolation

Shows up as Media Noise

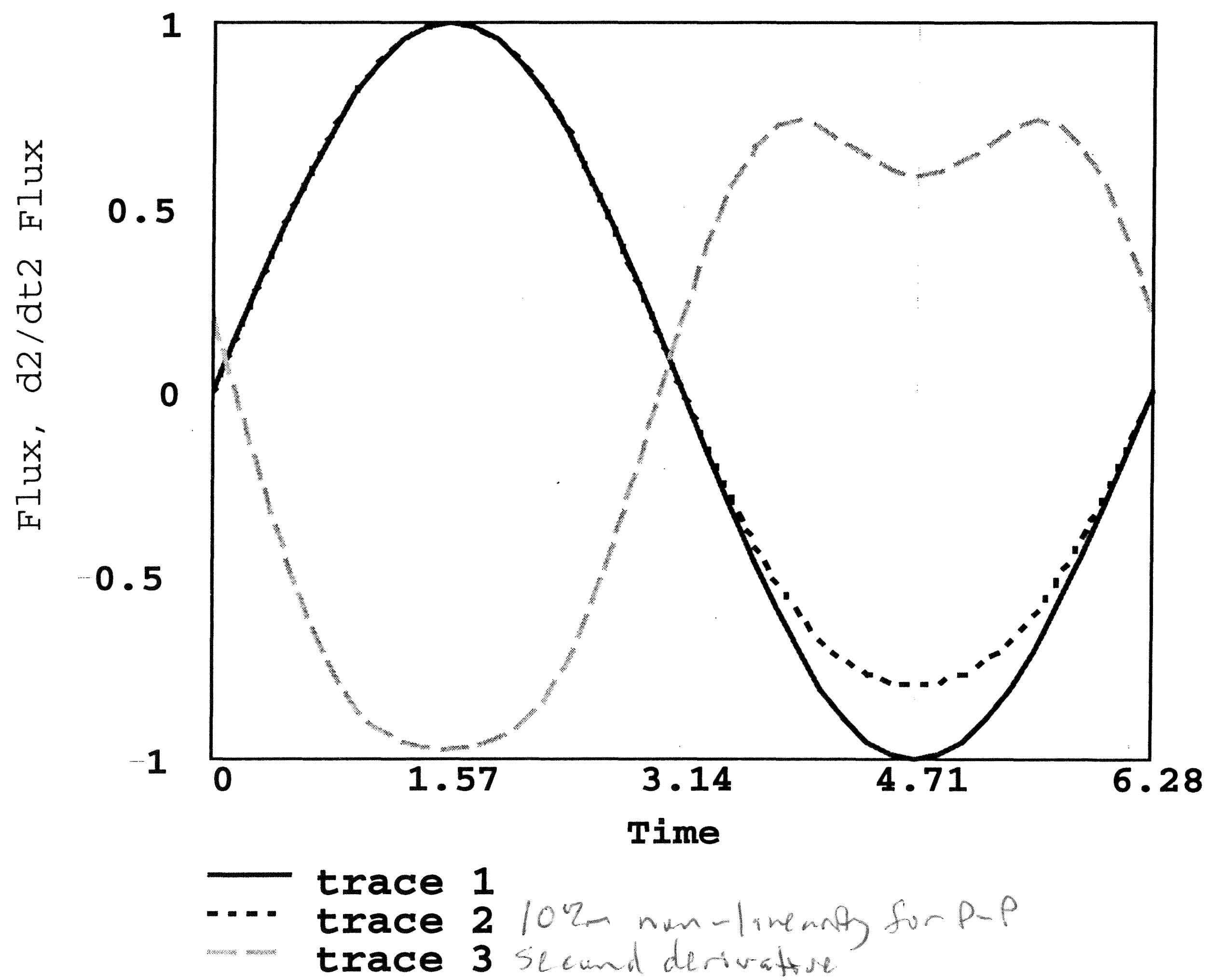
NLTS

Can Impact Media Noise Measurement

Noise Statistics

Not Necessarily Gaussian

Read Non-Linearity for 10% Cos^2 Nonlinearity



SNR: Measurements (Cont.)

Where ??

Preamp Output
Input Filter Output
Equalizer Output
Detector Input

Preamp Output

Preamp Bandwidth; Variable
Tester/Interconnect Parasitics

Input Filter Output

Bandwidth Spec'd by Filter
Rate of Roll-off

Equalizer Output

Convolution of All Blocks

Detector Input

"Bottom Line"
DFE Terms
Determines Pe; …. Statistics?

SNR: Noise Sources

Electronics (AWGN, Real Bandwidth Dependent)

- Preamp Voltage Noise
  - r_b‘, Ie_shot,….
- Resistors
- Shot Noise
  - Sense Currents
  - Amplifier Input Currents

Media Noise (Spatial Bandwidth Dependent)

- Not Stationary; Signal Dependent
- Data Code Dependent
  - d, k
  - Distribution of Transitions
- Distribution a Function of Transition Shape
  - Micro-track Model
  - Shape is Distribution Function
    - Arc-tangent
    - Hyperbolic-tangent

Arnoldussen
– energy of x-sition constant but width/amplitude vary.

is source of noise

μ-track models
– size of μ-tracks is correlation length
– relates to physical dimension in media

SNR: Contributors

SNR =*f* { h, PTR, Ovct, AltRatio, Θ_{yaw}, Vel,... — **[Physical]**

(width σ's)

tau ,H0, g_w, W_w, σWr, Write Pole-trim... — **[Write Process]**

(shield/shield spacing)

Hc ,Mr, δ, S*, s, grain *,... — **[Media]**

W_r, σWr, J, σ_J, Δρ/ρ, Gss, MrHt, ρ_{mr_eff} t_{mr}, t_{spc}, t_{sal}, Hk, Ms,... — **[Read Head]**

R_{leads} , e_{pre}, Channel, Pe, d_{code}, C/a, Brt_{mx},…} — **[System, Preamp]**

(≤3 get percolation)

(how long will transition last)

*** grain =*f* { tmx, Ms, δ , Hk_m, BPI, Ww,…}** — **[Thermal Stability]**

s ~ grain

(cross-track correlation length)

(may be minimum switching unit.)

$(Ku_{eff}\ V) / (K_b\ T) > 40$

for stability ~10 yrs

SNR: Media

SNRmedia = f ("a", BPI, s, W_{read}…)

"a" = f (Hc, dHx/dx, Mr, δ, S*, s, …)

"a" = Transition Width Parameter

δ = Media Thickness

s = Crosstrack Correlation Length

~ grain

$[Wr/s]^{0.5}$ (or $[Wr/grain]^{0.5}$) is key

For Stability with Time:

min. grain = f { tmx, Ms, δ , Hk_m, BPI, Ww }

tmx;	time for magnetization to decay
Ms:	Saturation Magnetization
δ:	Media Thickness
Hk:	Anisotropy Field
BPI:	Max Bit Density
Ww:	Magnetic Track Width

push to lower
bit aspect ratios
3-4:1 for 100Gb/in^2

SNR: Media, Velocity Effects

dHx/dx = *f* ($d_{magnetic}$, gw, H0, Velocity, tau, Pole Geomentry, ….)

write field gradient

d = Magnetic Spacing
gw = Write Gap
H0 = Deep Gap Field
tau = Flux Rise Time

Higher dHx/dx is better
Lower "a"

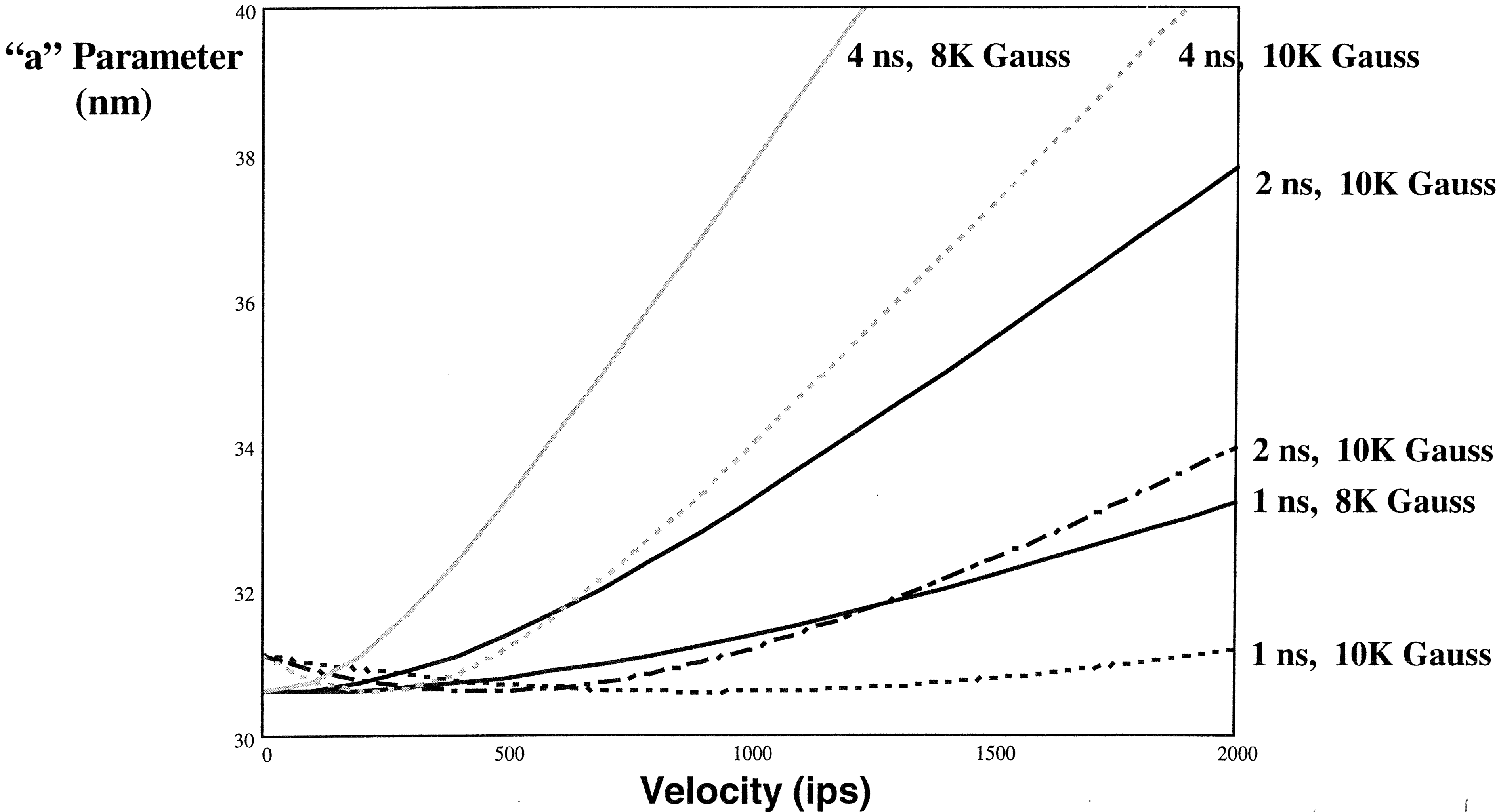
"a" Increases with Velocity

Media SNR Decreases with Velocity

SNR: Media Noise Velocity Effects

"a" Parameter vs Velocity

H0 = 8000, 10000 Gauss d = 3 µin
τ = 1,2,4 nS Mrt = 0.5 memu/cm^2



- a relates to medium noise

SNR: Other Contributors

Track Edge

- **"a" Parameter growth**
- **"Erase" signals**

Next Track

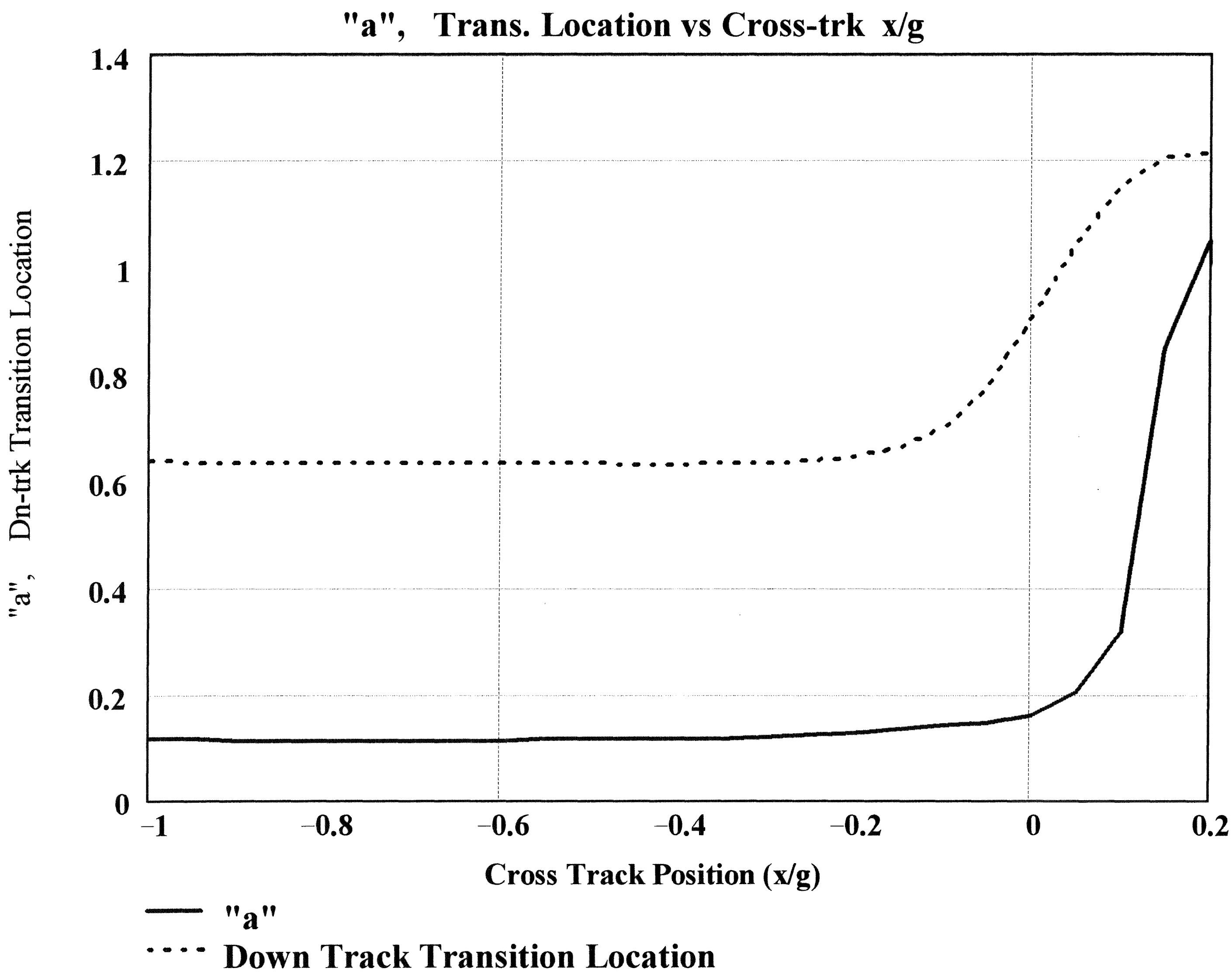
- **Fringing**
  - **Write**
  - **Read**

SNR_system = *f* (Signal, CMRR, PSRR, Interconnect,…)

- **External Cables**
- **Neighbors**

SNR: Edge Noise

"a" Parameter @ Track Edge



SNR: Where Does it Go ?

Equalization Loss = *f* (PW50 /B, target, ...)

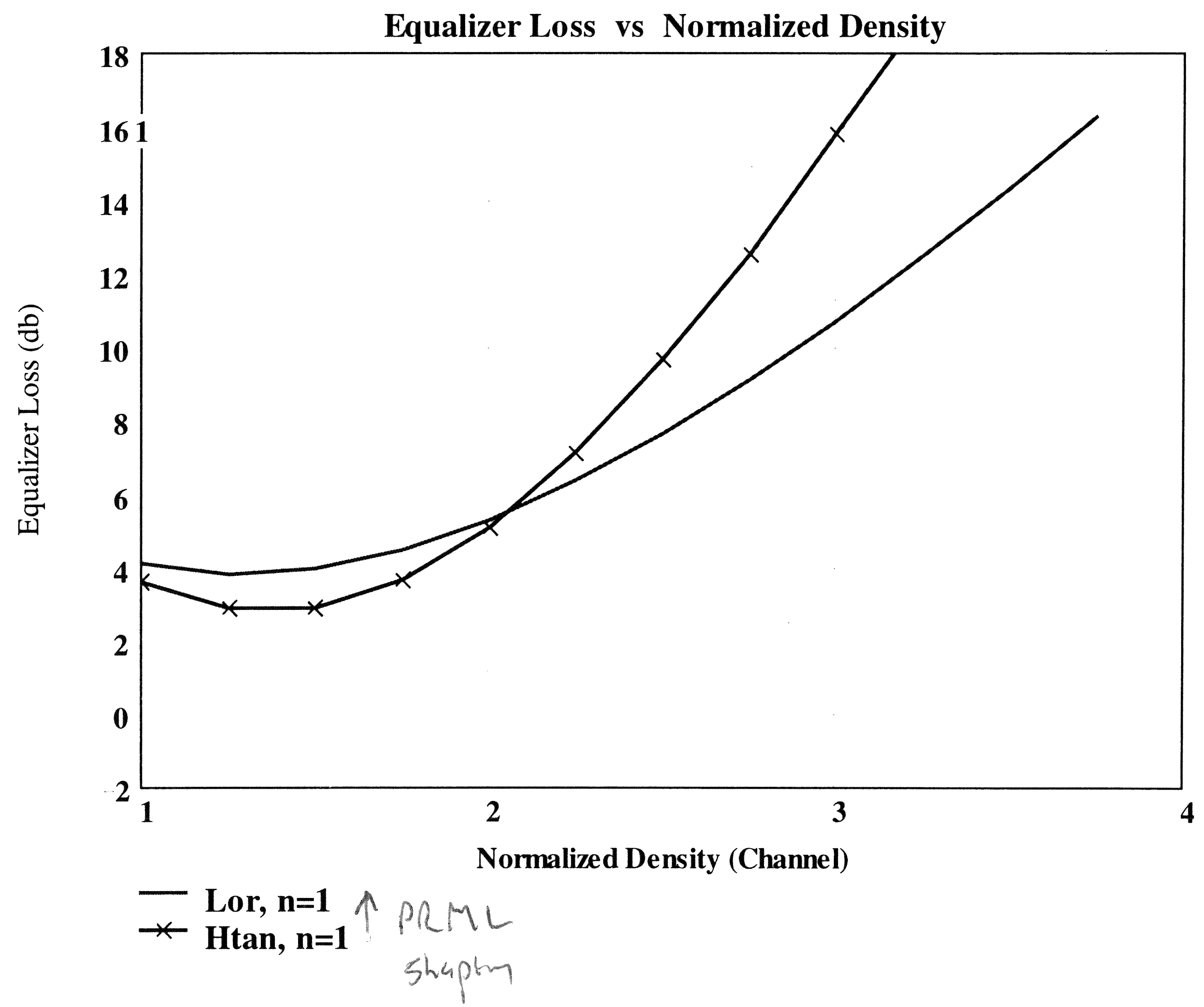
B = Bit Spacing
PW50 = *f* ("a", Mrt, Gss, ...)

Equalization Loss Increases with Velocity, BPI

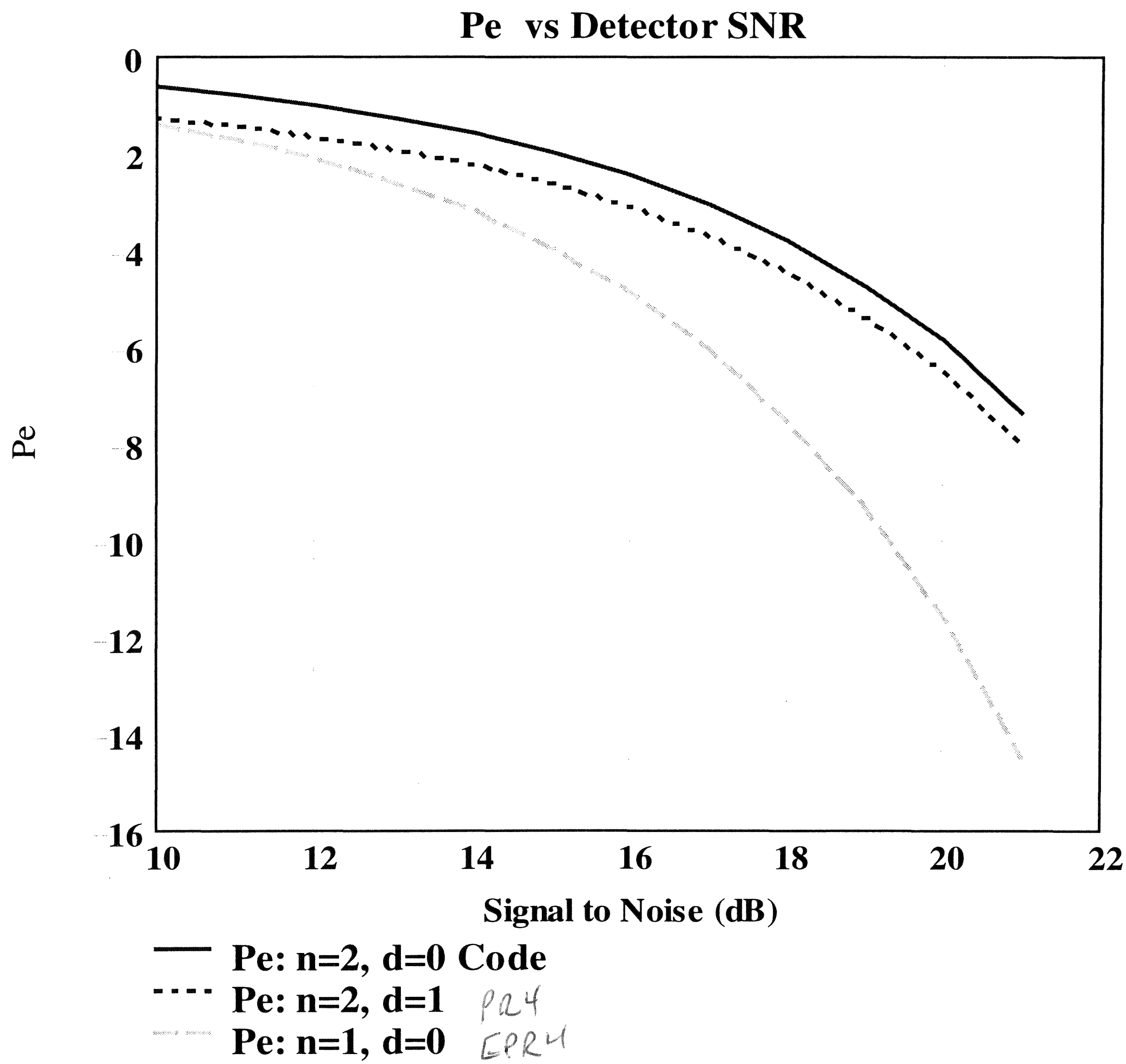
Detector

Pe = K1 erf (-SNR / K2)
Gaussian Noise

SNR: Where Does it Go ?



SNR: What's Required ?



Sources of Media Noise and Comparison Of Methods For the Characterization Of Media Noise

David Wachenschwanz
Komag, Inc.

Noise in Thin Film Media

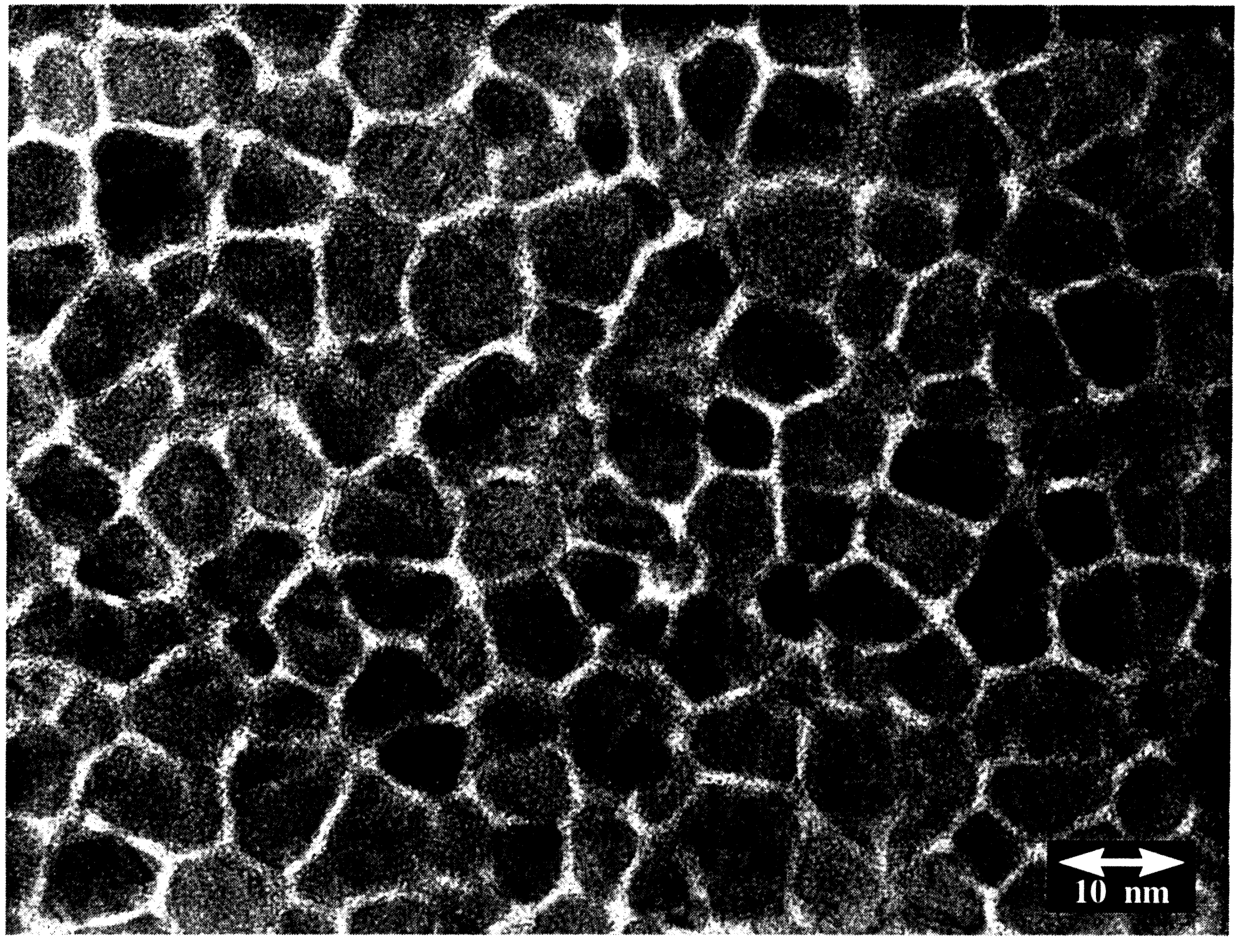
- Media made up of polycrystalline grains; grain sizes may typically vary from 50 to 500 Å

- Noise occurs primarily at written transitions in the media

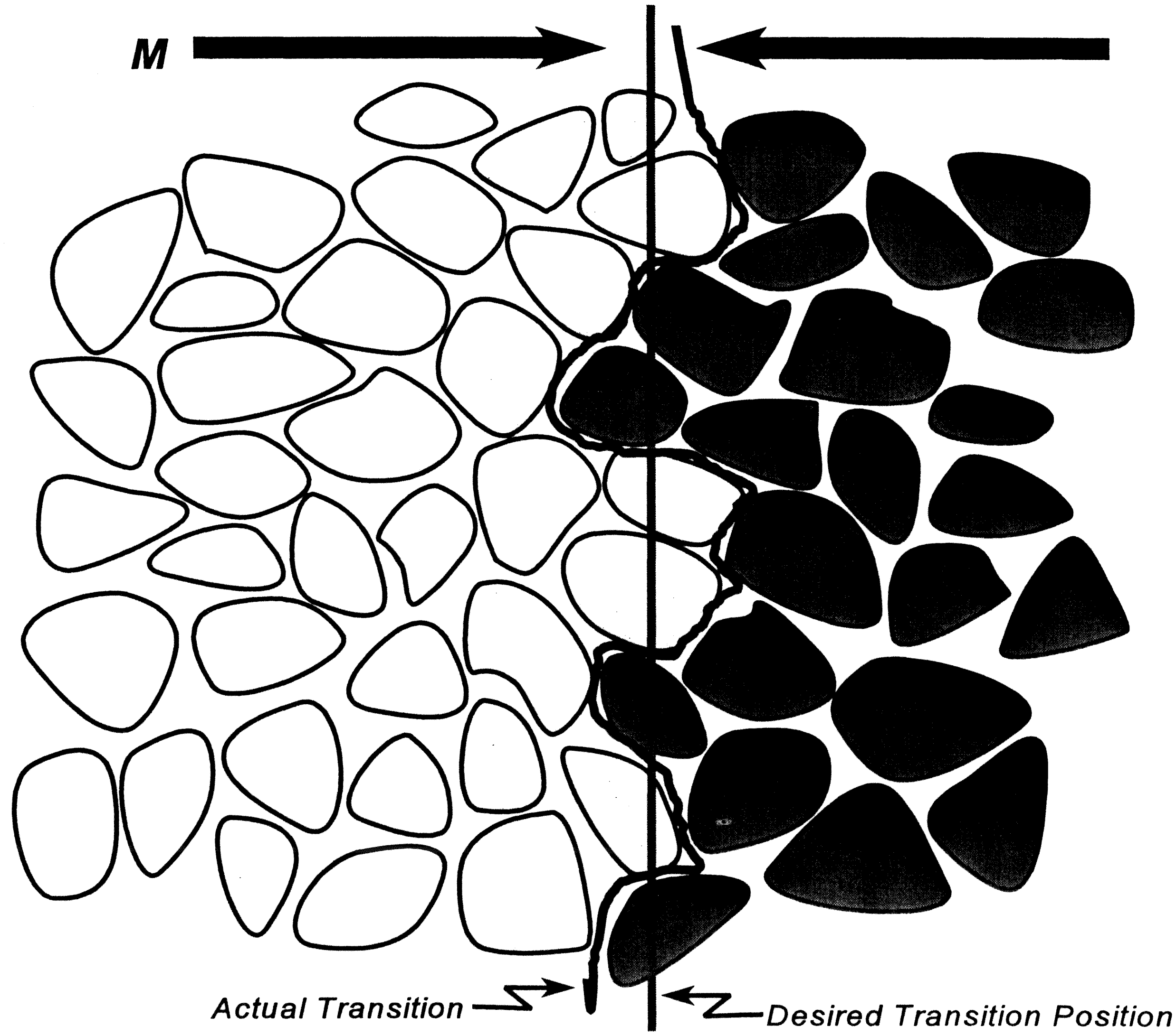
- Two dominant noise modes in the transition region:
    (1) transition position jitter
    (2) transition width fluctuations

- Transition position jitter is usually more significant than transition width fluctuations

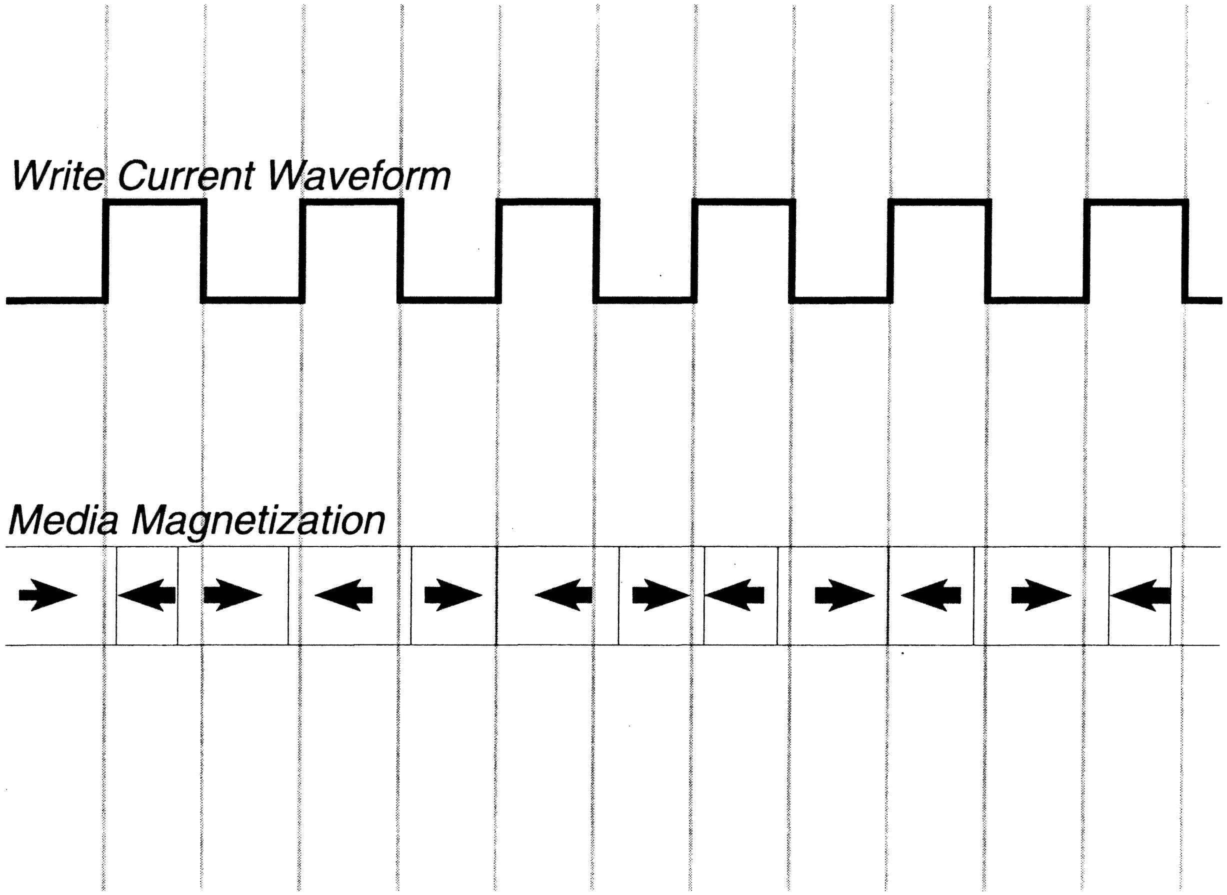
TEM Plan View of A Thin Film Media



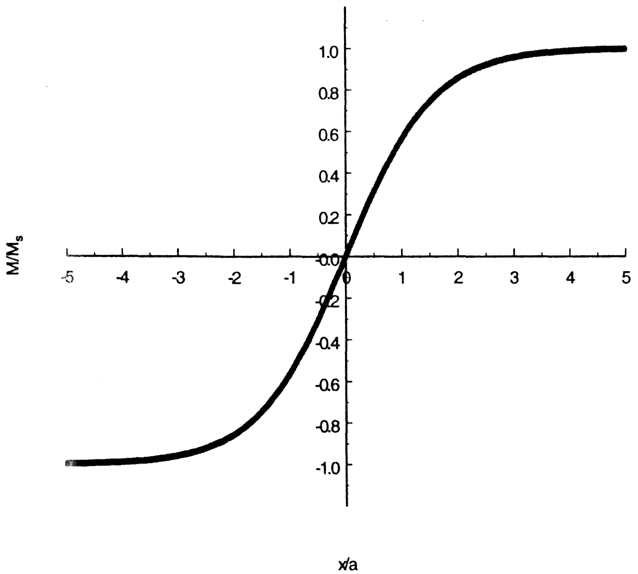
Formation of a Recorded Transition



Position Jitter



Transition Width Variation



At the transition in a thin film media, magnetization flips directions. In real transitions, the flip in direction of magnetization has some finite width over which the direction of magnetization changes.

$$M(x) = M_s \tanh\left(\frac{2}{\pi}\frac{x}{a}\right)$$

where M_s = saturation magnetization and a = transition width parameter.

Due to uncertainty in the shape of the transition, each individual transition's width parameter has some fluctuation δa. This produces a variation in the width and the amplitude of the read-back transition.

If we assume a tanh transition shape, then

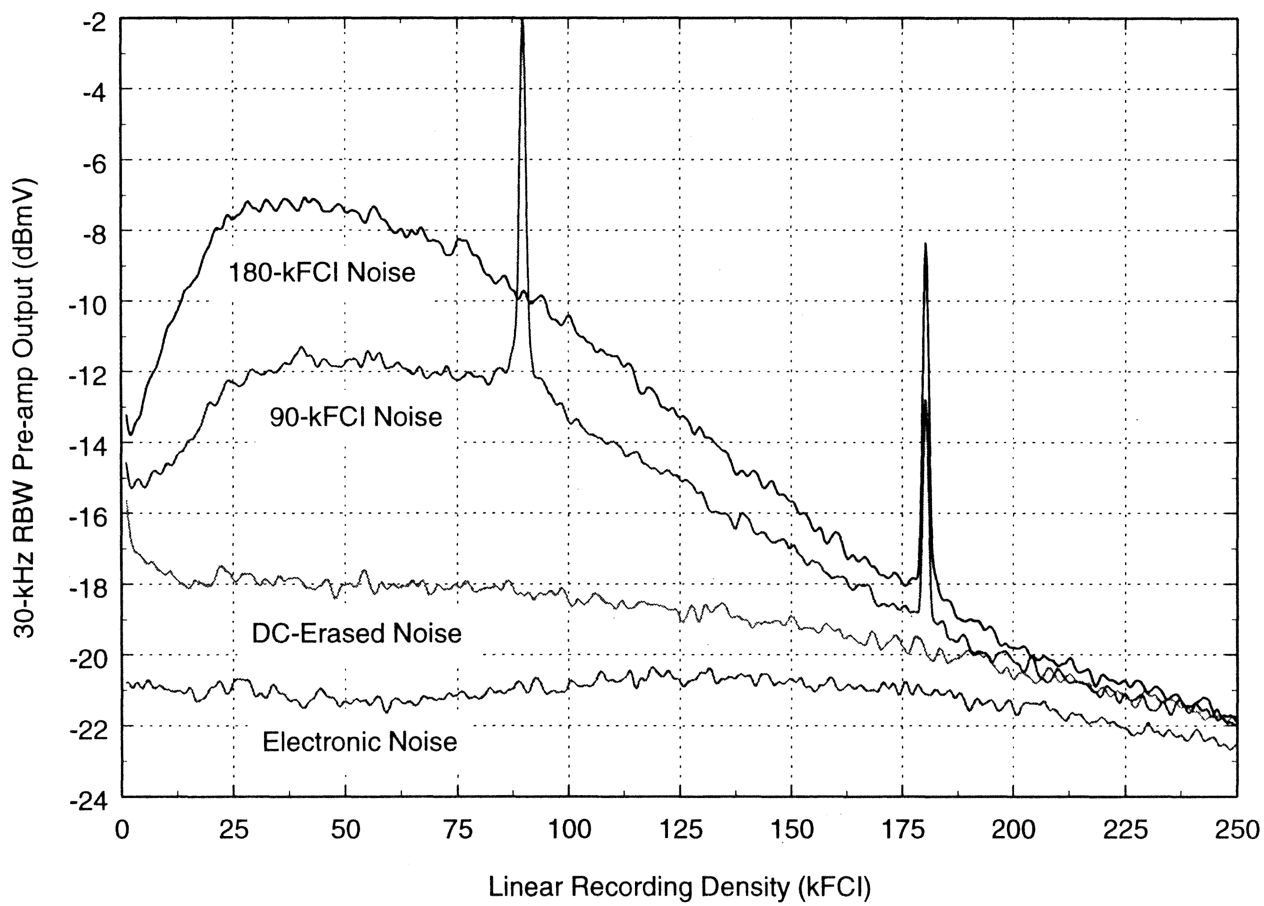
$$\sigma_x^2 = \frac{\pi^4}{48}\frac{sa^2}{W}, \qquad \sigma_a^2 = \frac{\pi^4}{60}a^2\sigma_x^2 = \frac{\pi^8}{2880}\frac{sa^4}{W}$$

where

σ_x = *transition position jitter*, σ_a = *transition width fluctuations*

s = *cross-track correlation length*, a = *transition length* W = *trackwidth*

Noise and Signal Spectra As Measured By A Spectrum Analyzer



- higher 180kFCI noise may just be due to # transitions/ unit time
- normalize noise to transitions @ each density
- he suggests subtracting DC erase noise point by point from trace w/ X-ther noise.

David Wachenschwanz

KOMAG

Thin Film Media Transition Noise Power Spectral Density is

$$PSD_{trans}(k) = \frac{\sigma_x^2}{B} k^2 \, |V_{sp}(k)|^2 + \frac{\sigma_a^2}{B} k^4 \, |V_{sp}(k)|^2$$

where

σ_x = *transition position jitter*, σ_a = *transition width fluctuation*

$V_{sp}(k)$ = *Fourier transform of the isolatedpulse*, B = *flux change length*, $k = \frac{2\pi}{\lambda}$

Measurement of Integrated Media Noise

(1) A constant frequency square wave is recorded on a track.

(2) A spectrum analyzer is used to measure the power spectral density (PSD) of the total noise (media noise, electronic noise DC-erased noise) vs. frequency (*Note: Removal of the signal peaks from the spectrum trace and correction of the spectrum analyzer readings in order to measure noise proper is required*).

(3)

$$Total\ Integrated\ Noise\ Power = \int PSD_{Total\ Noise}(k)\ dk$$

(4) The track is then DC-erased and the power spectral density vs. frequency is measured.

(5)

$$DC\text{-}Erased\ Integrated\ Noise\ Power = \int PSD_{DC\text{-}Erased\ Noise}(k)\ dk$$

(6)

$$Media\ Transition\ Integrated\ Noise\ Power = Total\ Integrated\ Noise\ Power - DC\text{-}Erased\ Integrated\ Noise\ Power$$

(7) Signal-to-Integrated Media Noise Ratio

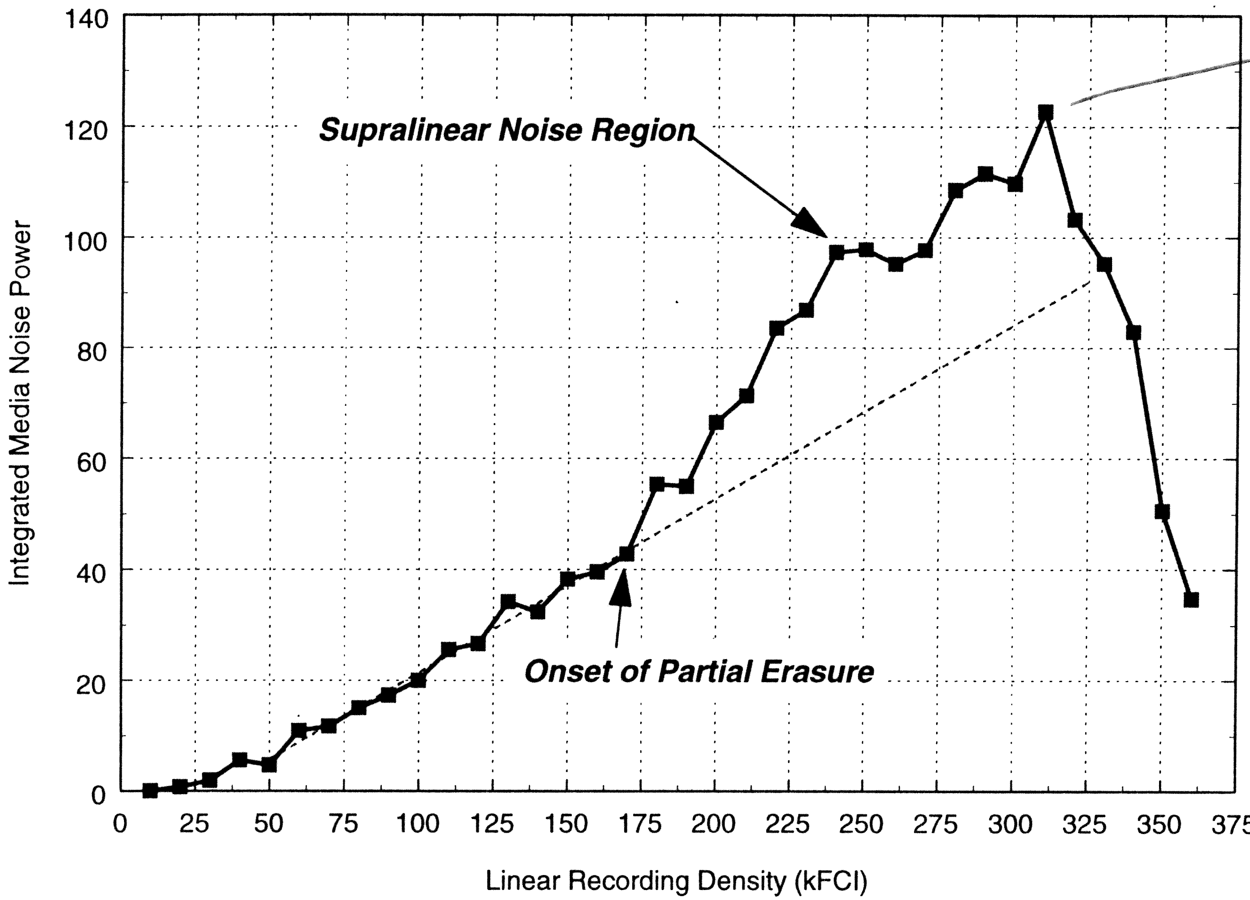
-***Which signal to use?***

(a) RMS signal power of fundamental harmonic of signal in step (1)?

(b) 0-peak or peak-to-peak signal power of signal in step (1)?

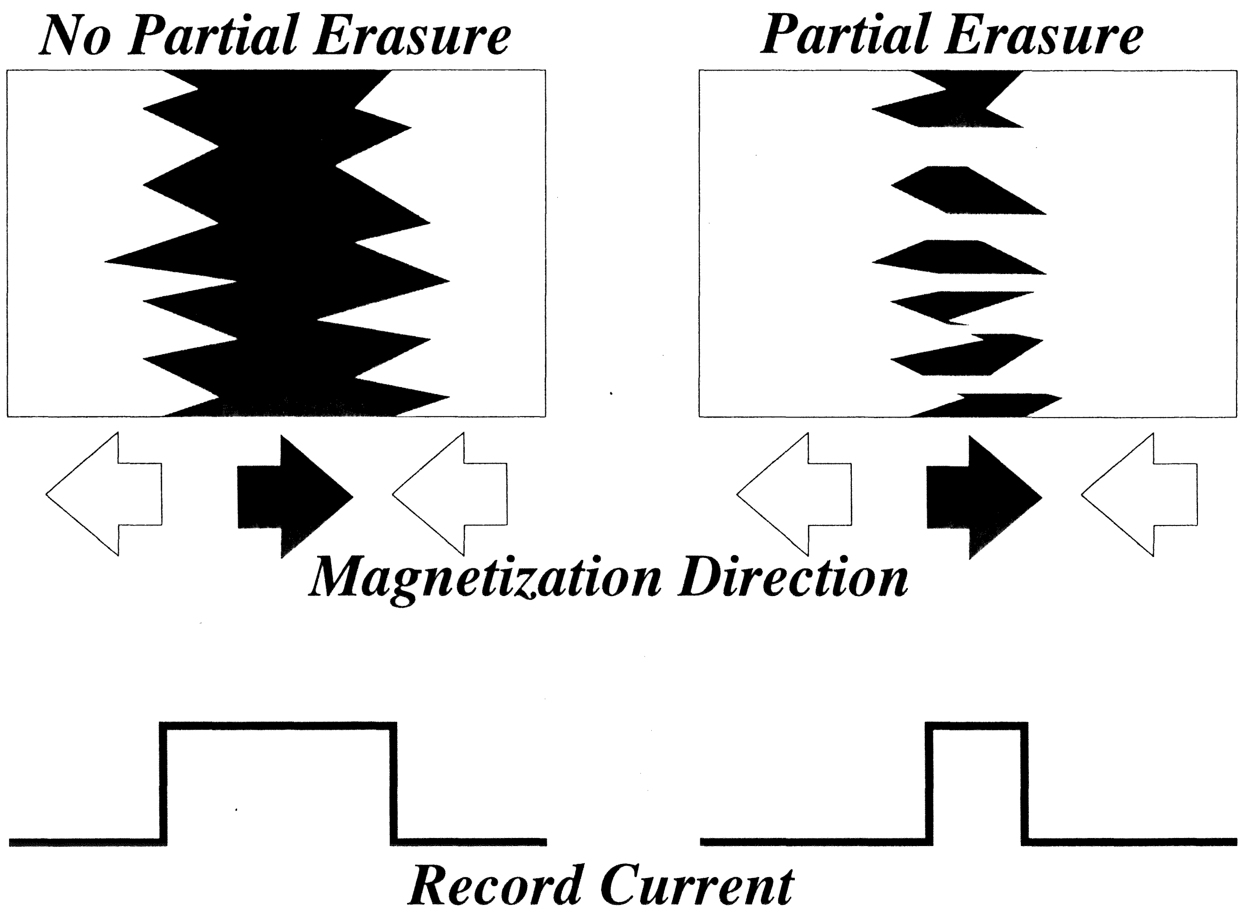
(c) RMS, 0-peak, or peak-to-peak signal power of some other signal?

Integrated Media Noise Power vs. Density



point where most of track became erased

Straight line is due to # of X-sitions under analysis.


No Partial Erasure
Partial Erasure
Magnetization Direction
Record Current

Measurement of Media and Electronic Jitter on Thin Film Media

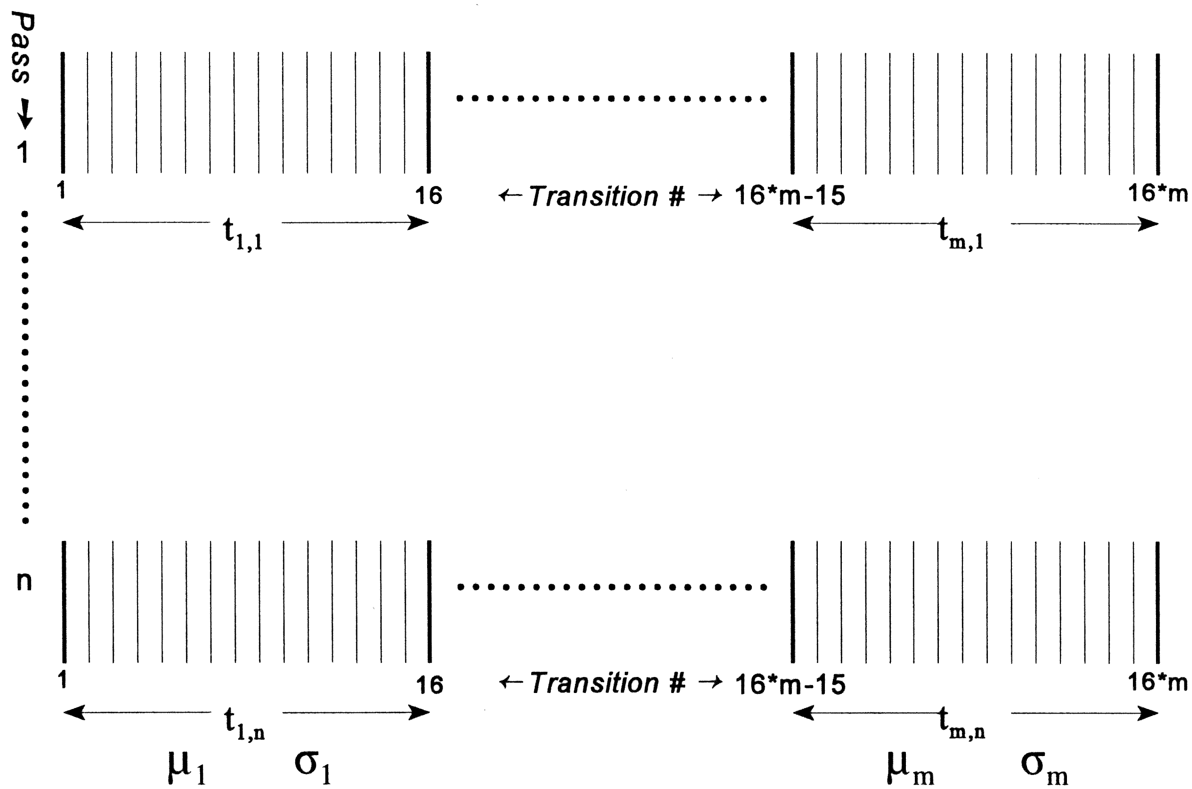
(1) A constant frequency square wave is recorded on a track.
(2) The signal is read back and the time interval between M sets of transitions is measured. This time interval information is then stored. The measurement of the time between transitions can be done using a time interval analyzer or by digitizing the signal and determining the time between the positions of transitions.
(3) Step (2) is repeated N number of passes making sure that the time intervals are measured on the same sets of transitions for each pass.
(4) See the next figure for calculation of media jitter σ_x and electronic jitter σ_e.

Important caveat:

- Want to determine noise statistics for individual transitions
- Step (2) measures the time interval between two transitions. Thus, this data is the combination of noise at two transitions.
- If the noise statistics at both of the transitions are the same and also uncorrelated, the the variance calculated for the time interval between transitions can simply be divided by 2 to obtain the variance of the position uncertainty at an individual transition.
- In a magnetic recording channel, "tails" from adjacent transitions interact and thus cause the noise between adjacent transitions to be correlated.
- To avoid this correlation, the time intervals measured in step (2) should be "distant" transitions.
- Measuring the time between every 16th transition is normally adequate

* problems w/ same equipment algorithm

@ each density



μ_i = mean value for repeated measurements of interval t_i

σ_i = standard deviation for repeated measurements of interval t_i

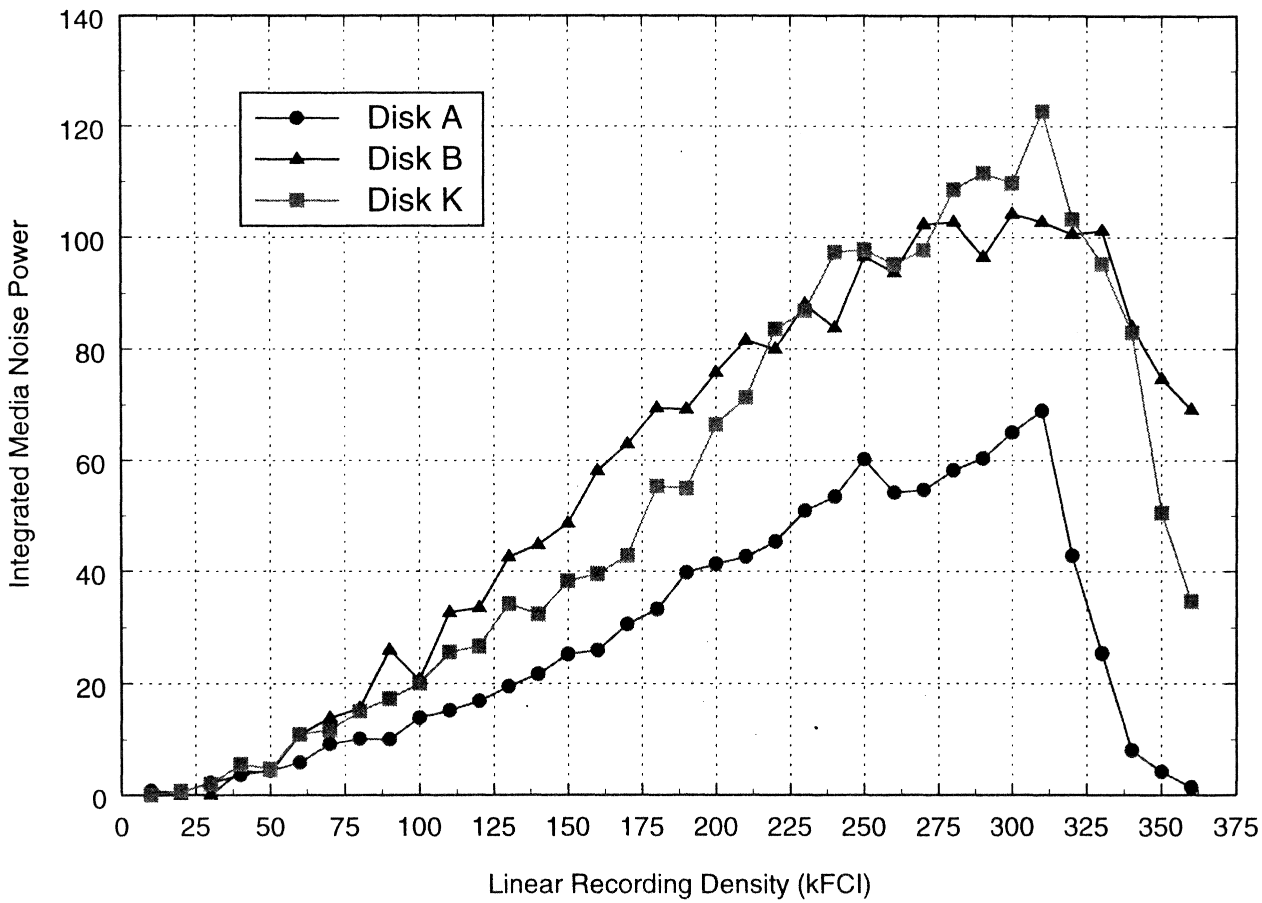
$$\sigma_x = \sqrt{\frac{\sum_{i=1}^{m} (\mu_i - \bar{\mu})^2}{2(m-1)}} \qquad \sigma_e = \sqrt{\frac{\sum_{i=1}^{m} \sigma_i^2}{2m}}$$

Recording Performance For Several Disks

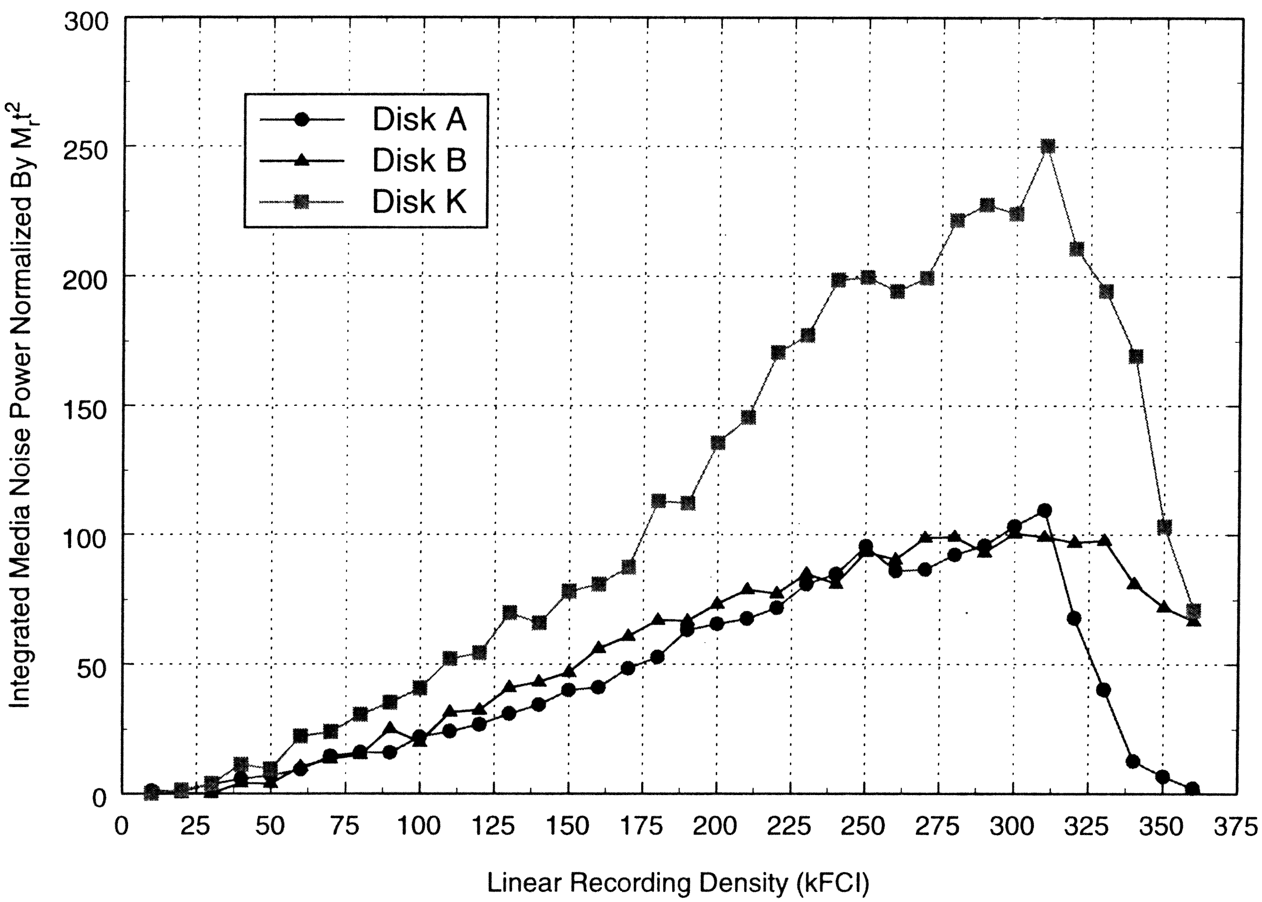
Disk	H_c Oe	M_rt memu/cm²	S*	HF mVpp	MF mVpp	LF mVpp	HF/LF Resolution %	PW50 ns	OW dB	Media Jitter nm	SNR dB
Disk A	2843	0.79	0.840	0.1641	0.4478	0.5452	30.12	17.51	34.46	3.13	27.21
Disk B	2310	1.02	0.771	0.1519	0.4261	0.5715	26.60	19.90	40.69	4.75	22.84
Disk C	2465	1.03	0.796	0.1657	0.4826	0.6071	27.23	19.36	39.35	4.52	23.22
Disk D	2377	0.99	0.791	0.1697	0.4577	0.5686	29.89	19.00	40.05	4.75	23.42
Disk E	2453	0.92	0.832	0.1759	0.4662	0.5780	30.44	18.43	37.59	4.32	23.78
Disk F	2544	0.89	0.860	0.1548	0.4630	0.5846	26.49	18.78	36.20	3.15	26.58
Disk G	2613	0.93	0.843	0.1891	0.5104	0.6235	30.40	18.17	39.21	4.03	23.92
Disk H	2526	0.88	0.842	0.1930	0.5045	0.5998	32.23	17.74	39.12	3.73	24.51
Disk I	2322	0.78	0.787	0.1544	0.3985	0.4984	30.98	18.11	41.39	3.82	25.30
Disk J	2719	0.73	0.839	0.1761	0.4393	0.5211	33.82	17.30	41.28	4.26	23.60
Disk K	2400	0.70	0.857	0.2087	0.4786	0.5409	38.60	16.23	41.81	3.47	26.11

- Media Jitter Measured At 110 kFCI
- SNR is 110-kFCI RMS Signal-to-Integrated Media Noise Ratio
- HF Density = 220 kFCI, MF Density = 110 kFCI, LF Density = 37 kFCI

Integrated Media Noise Power vs. Linear Recording Density For Three Different Media



Integrated Media Noise Power Normalized By $(M_rt)^2$ For Three Different Media



David Wachenschwanz

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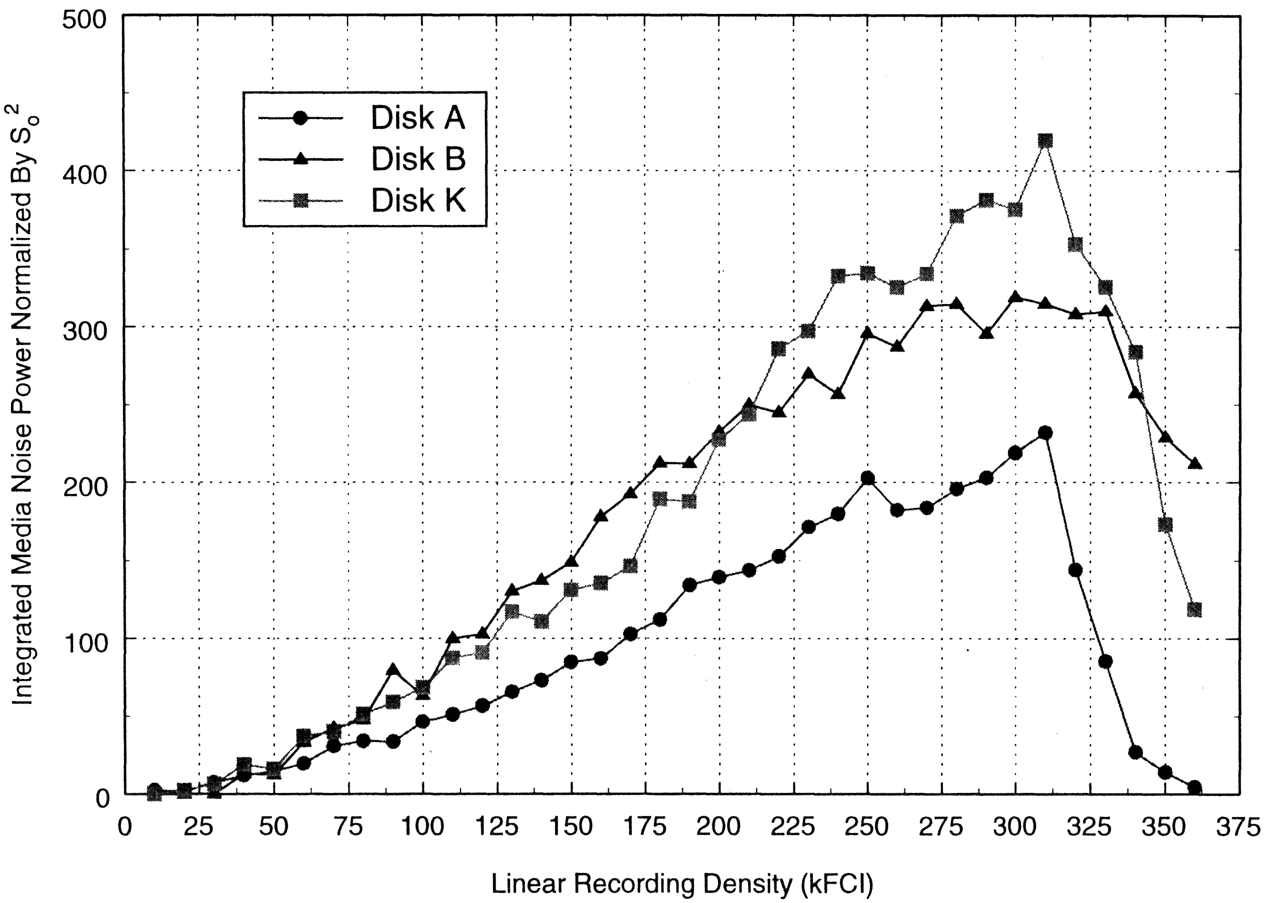
Integrated Media Noise Power Normalized By S_o^2 For Three Different Media

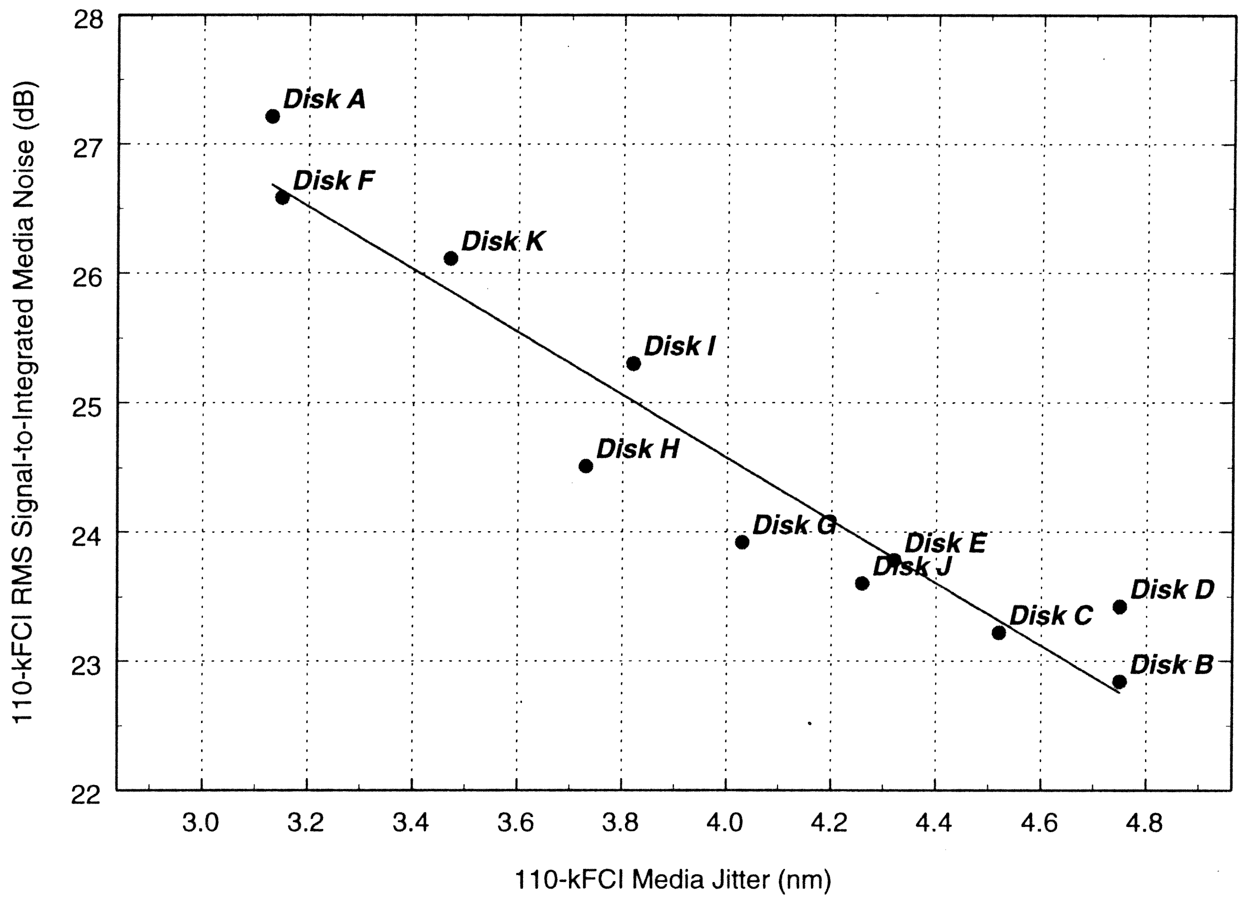
— since noise power



- S_o is the isolated pulse 0-peak signal amplitude

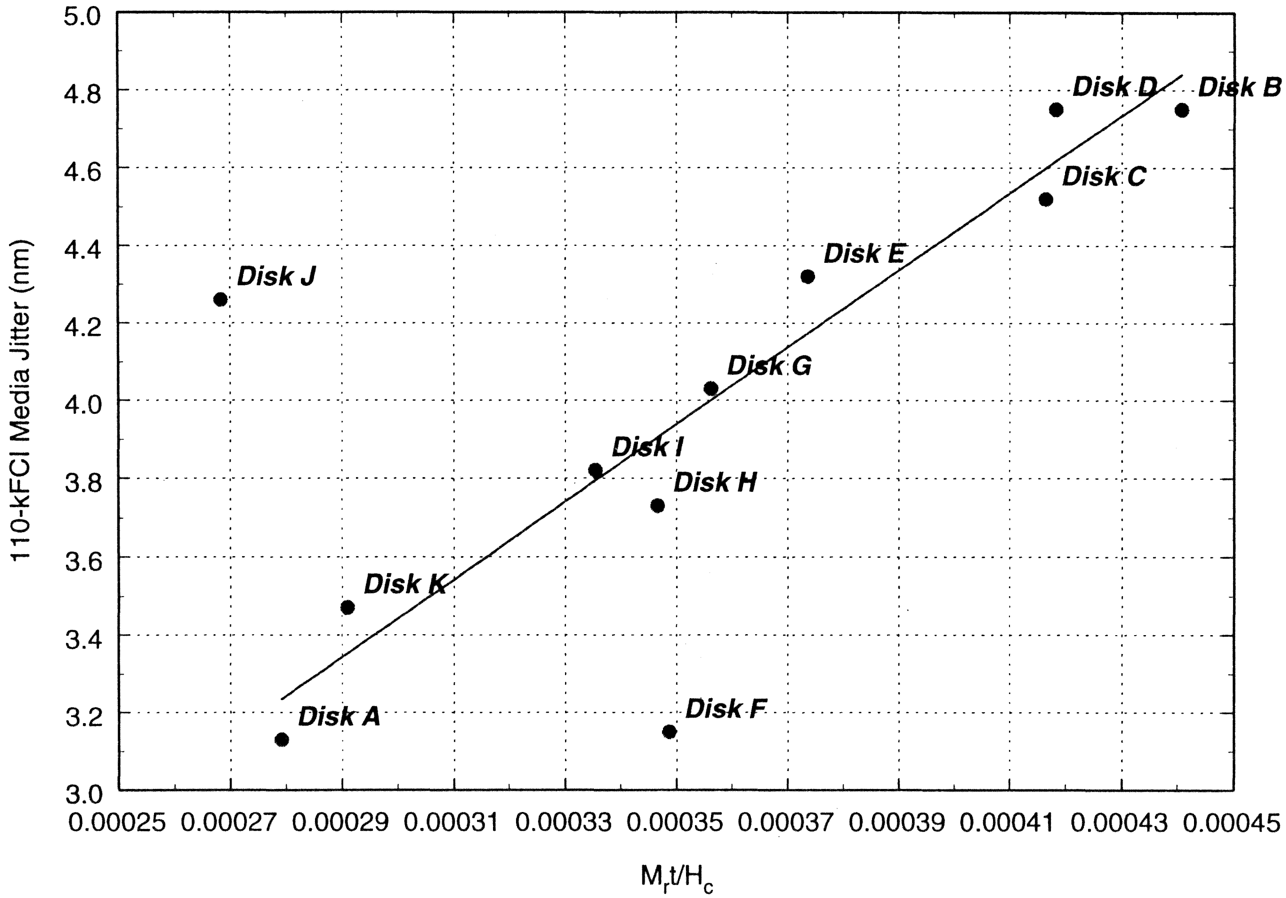
RMS Signal-to-Integrated Media Noise Ratio vs. Media Jitter

(his argument is this is not too definitive

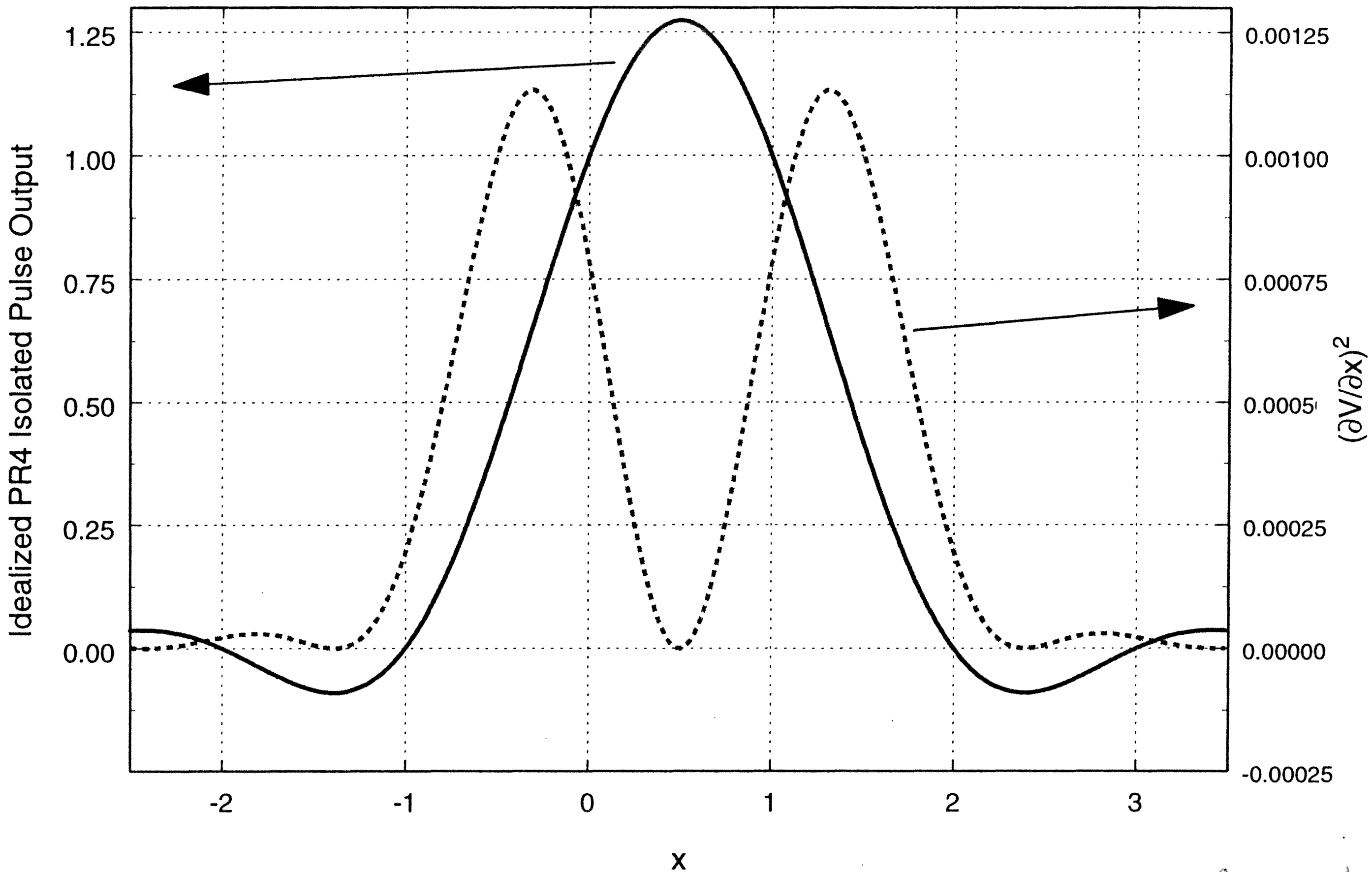


J & F may have different magnetic grain size (different process than rest of disks)

Media Jitter vs. M_rt/H_c



The Effect of Media Position Jitter on a PR4 Isolated Pulse



$$\sigma_v^2(x) = \sigma_x^2 \left(\frac{\delta V}{\delta x} \right)^2$$

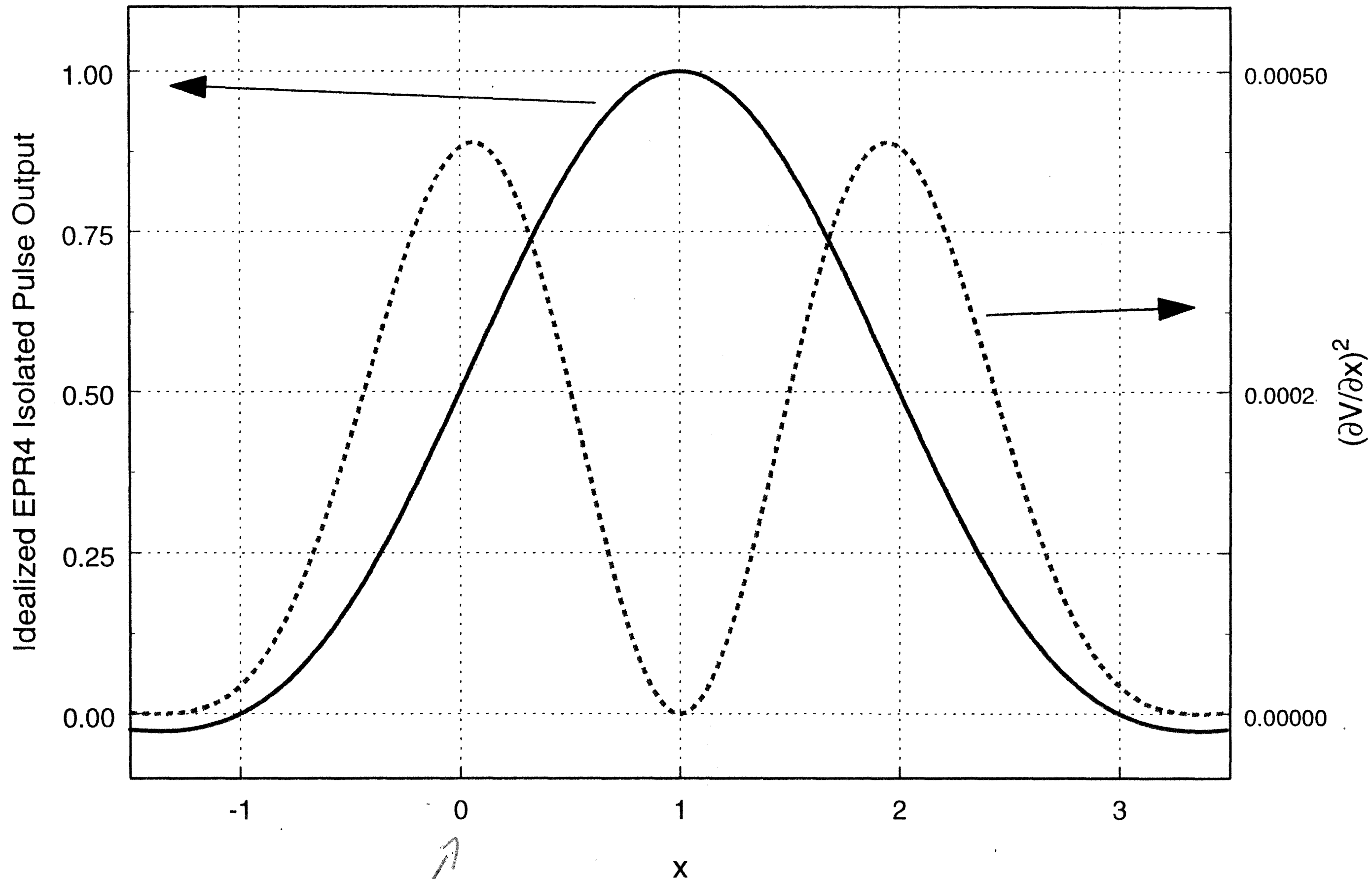
voltage variance

jitter variance (time)

– if position uncertainty is predominate noise source

David Wachenschwanz

The Effect of Media Position Jitter on an EPR4 Isolated Pulse



sample point @ max jitter variance

$$\sigma_v^2(x) = \sigma_x^2 \left(\frac{\delta V}{\delta x} \right)^2$$

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References

"Time Domain Characterization of Magnetic Disk Drives", HP Application Note 358-3.

Xinzhi Xing and H. Neal Bertram, "Analysis of Transition Noise in Thin Film Media", IEEE Trans. Mag., Vol. 33, No. 5, Sept. 1997, pp. 2959-2961.

T.C. Arnoldussen and L.L. Nunnelley, Editors, *Noise In Digital Magnetic Recording*, World Scientific Publishing, 1992. (Chapter 8 is especial good for understanding issues related to integrated noise measurements)

Joe Caroselli and Jack Keil Wolf, "A new model for media noise in thin film magnetic recording media", SPIE, Vol. 2605, pp. 29-37. microtrack model

Jian-Gang Zhu et al., "Nonlinear partial erasure and its correlation with transition noise in longitudinal thin-film media", J. Appl. Phys, 79 (8), 15 April 1996, pp. 4906-4908.

SNR Measurements and SNR Requirements for High Density Magnetic Recording

Hans Jürgen Richter,
Seagate Recording Media, Media Technology

IIST Symposium on SNR and Disk Drive Performance

Outline

a) medium noise: types and physical background

b) noise measuring techniques
(noise integration, Jitter)

c) extraction of media parameters out of noise measurements

d) SNR requirements
(correlation between BER, Offtrack performance and SNR)

e) Challenges in making media which satisfy future requirements

(a) Noise Sources in Media

media are granular => statistics: “number of particles per bit”

“particle noise” }
“transition noise” } ⟷ granularity

“texture (modulation) noise ”

Hans Jürgen Richter
Seagate Recording Media
IIST SNR Symposium, 27th Apr. 98

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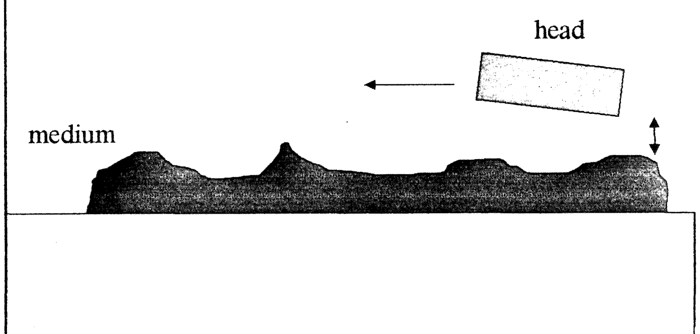
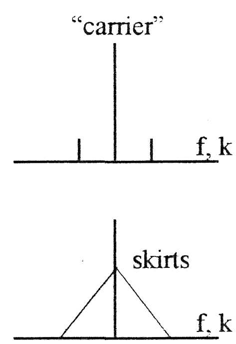
Information, the way you want it.

(a) Texture (Modulation) Noise



head medium spacing modulation



the effect of texture (modulation) noise on performance of today's media is hard to detect at all
=> no further consideration

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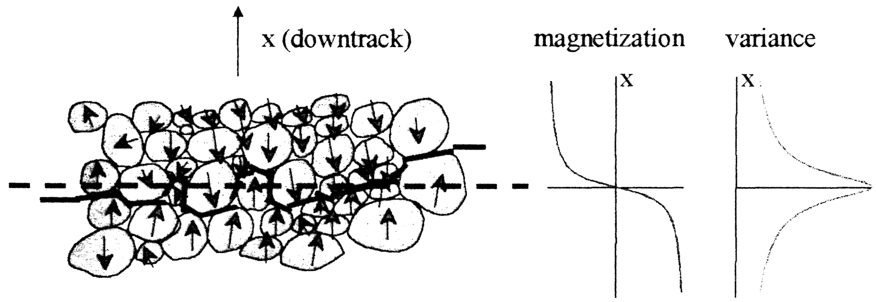
Information, the way you want it.

(a) Effects of Medium Granularity

(no magnetic interaction assumed)



if all magnetic grains are magnetized, say, up, the variance is minimum, $\sigma^2 = 1 - p$
(there is no statistical freedom left)

$\sigma^2(x) = 1 - p\, m(x)^2$

p = (volumetric) packing density

statistical uncertainty is maximum at the transition center

Hans Jürgen Richter
Seagate Recording Media
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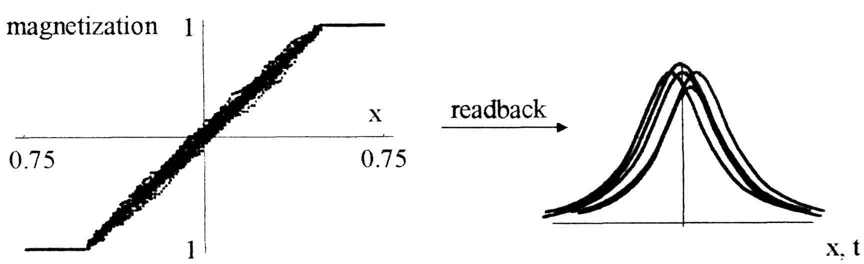
(a) Jitter

"noisy transition"



uncertainty in
- pulse position (Jitter)
- pulse shape (height)

Hans Jürgen Richter
Seagate Recording Media
IIST SNR Symposium, 27th Apr. 98

(b) Medium Noise Measurements

- SNR definition, measurements with a spectrum analyzer
- direct Jitter measurement
- other methods (noise correlation) not discussed here

(b) SNR Definition

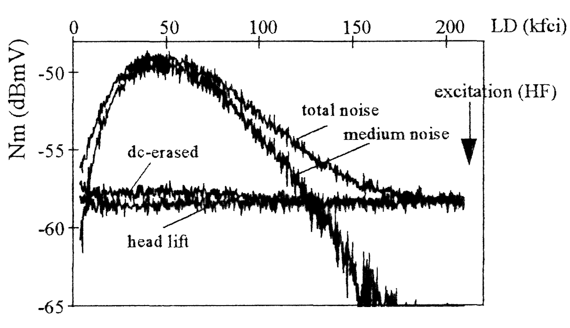
B = shortest transition spacing

Define:

LD = 1/(2B) [HF]

Signal: rms value at LD/2 = 1/(4B) [MF = HF/2]

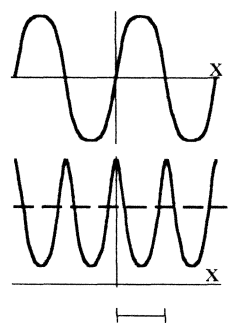
Noise: rms value of noise (integrated) [HF]



"mid-frequency" scheme

- weights resolution not too strong
- signal power in PR4 at 1/4B

(b) Noise in Recording



magnetization

variance

- magnetization fluctuation is not stationary
- magnetization fluctuation is pattern dependent
- spectrum analyzer measures average

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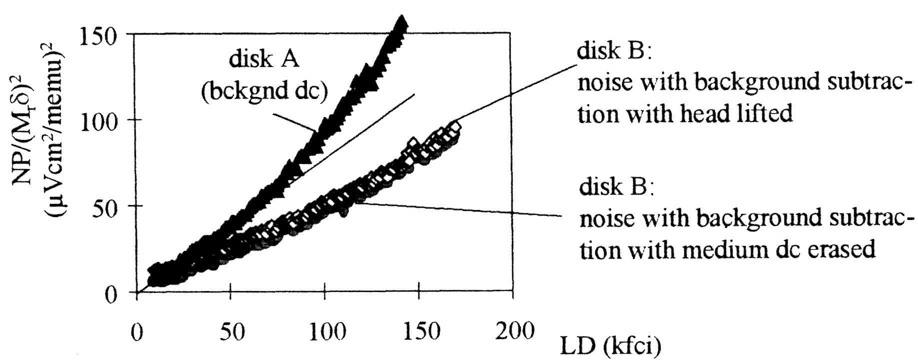
(b) Noise as Function of Linear Density



- dc erased state has a very low noise
- noise increases with LD (non-stationarity!)
- supralinear noise increase

head: RW = 2.3 μm

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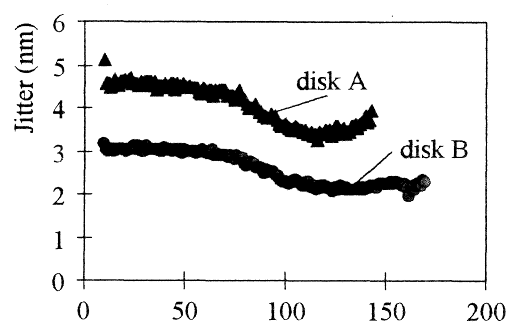
(b) Direct Jitter Measurement

time interval analyzer measurement



Jitter is defined to be the standard deviation of the scatter of the pulse position

Jitter drops due to intersymbol interference
(see G. Hughes, IEEE Trans. Magn. 33, p 4475, 1997)

head: RW = 2.3 µm
threshold 45%
40 MHz filter, LF

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(c) Extraction of Medium Parameters

- theoretical conversion of Jitter into SNR
- experimental comparison
- crosstrack correlation length

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(c) SNR versus Jitter, Theoretical (1)

After Giora Tarnopolsky, one can convert Jitter into SNR.

$$SNR = \frac{\sqrt{\pi}}{2} U^{1.5} \boxed{\frac{B}{\sigma}} \exp\left(-\frac{U\pi}{4}\right)$$

jitter SNR

$U = PW_{50}/B$
(channel density)

The derivation of the equation assumes

- small perturbations of pulse position and shape
  (after read-out process)
- Lorentzian, non-interacting pulses (no supralinear noise)
- Formally, σ absorbs both Jitter and pulse shape fluctuations
  (Jitter dominates)

(c) SNR versus Jitter, Theoretical (2)

Strictly speaking, position Jitter (which is the dominant noise source) is a modulation and not noise (*).

– time axis shift not "noise"

Medium quality is related to:

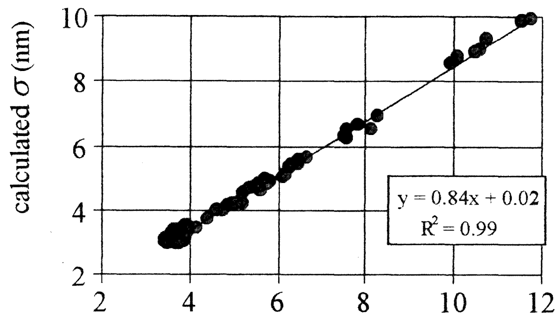
$$Q = \frac{B}{\sigma}$$

σ : effective Jitter

(*) V. B. Minuhin, " Parasitic Transition-Shift Phenomenon from Viewpoint of Modulation Theory", to be published.

(c) Experimental, Jitter versus SNR

Comparison of calculated σ (from SNR) with directly measured Jitter.
The same head, various disks.



Jitter (nm)
direct measurement

Linear density: 50 kfci

$$\sigma = \frac{\sqrt{\pi}}{2} U^{1.5} \frac{B}{SNR} \exp\left(-\frac{U\pi}{4}\right)$$

Note: calculated $\sigma \neq$ directly measued σ.

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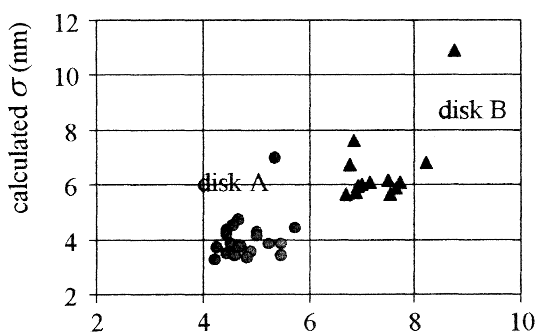
(c) Experimental, Jitter versus SNR

Comparison of calculated σ (from SNR) with directly measured Jitter.
Various heads, two disks.



Jitter (nm)
direct measurement

Linear density: 50 kfci

$$\sigma = \frac{\sqrt{\pi}}{2} U^{1.5} \frac{B}{SNR} \exp\left(-\frac{U\pi}{4}\right)$$

- σ or Jitter is not unique to a disk
- ranking of media works

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(c) Crosstrack Correlation Length

- crosstrack correlation length is the “magnetic grain size”

$$\sigma \propto a\sqrt{\frac{s}{R_W}}$$

a	transition parameter
R_W	reader width
s	crosstrack correlation length

- models differ in proportionality factor
- models differ in transition shape
- this dependence is universal

- typically, s is predicted to be unrealistically ~~high~~ ~~Small~~ small
  (note: all PW_{50} predictions are poor)

should be larger than grain size.

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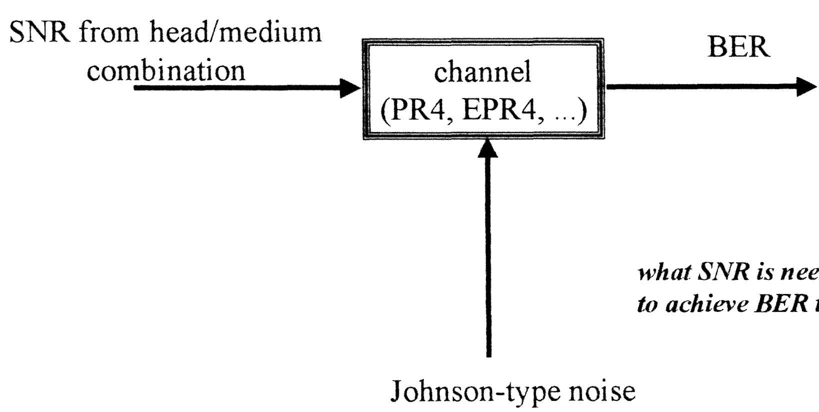
(d) SNR Requirements, Correlation to BER

“black-box approach”



what SNR is needed to achieve BER target?

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(d) Treatment of Noise-Mix

for the next slides: simplified view

Johnson noise and medium noise are equally harmful

$$SNR_{tot} = \frac{S}{N_{tot}}$$

typically 60-90% of the noise power is from the medium

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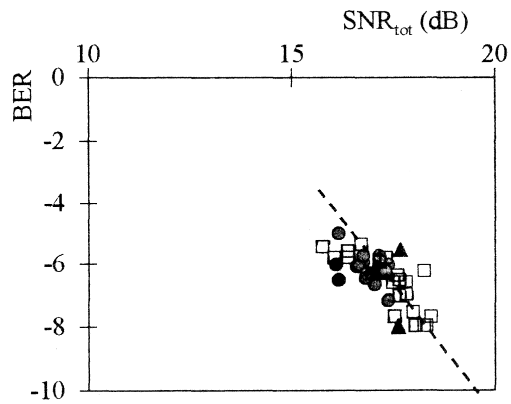
(d) Correlation SNR <=> BER



LD[kbpi]	221 [235]
tpi	10500
DR[Mb/s]	93

channel:
EPR4, tuned to every surface

- required (total) SNR for 10^{-10} error rate is 19.5dB

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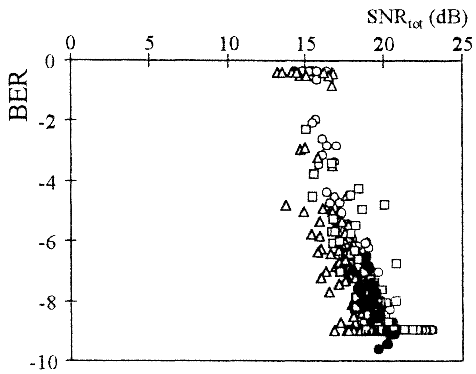
(d) Correlation SNR <=> BER, cont'd



various conditions do not alter the SNR BER relationship.

- different testers
- different channels (all EPR4)
- different components
- various crosstrack positions
- AD 1.5 ... 3.2 Gbit/inch^2

differences between channel types are found to be 1...1.5 dB
(after code correction)

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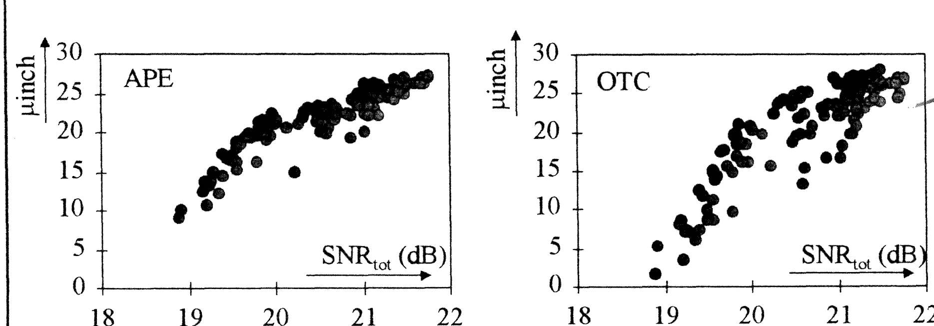
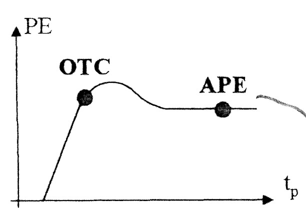
(d) Correlation: SNR -Offtrack Performance



also track edge effects operating (BER cons. and random events introduce scatter)



average position error

SNR_{tot}: on-track

one head - various media

LD 152 kbpi
TD 6700 tpi
DR 67 Mb/s
Channel: PR4

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(d) SNR Requirement

- noise mix: keep the relative amount of medium noise constant
- no matter what theory, the ratio

$$\frac{\sigma}{B} = \text{constant} = \text{requirement } req$$

(needs to be conserved when increasing areal density)

σ scales with reader width as: $\sigma \propto \frac{1}{\sqrt{R_W}}$

- simple scaling leads to:

$$AD \propto \frac{req^{4/3}}{\sigma^{4/3} R_W^{2/3} As^{1/3}}$$

As = bit aspect ratio

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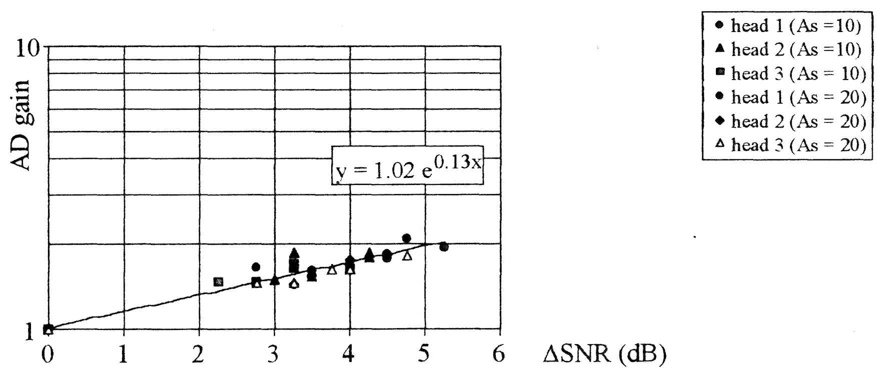
(d) SNR Requirement, cont'd

For media development, the head is fixed and one monitors the gain in SNR.



- doubling the density thus requires + 5.2 dB SNR when measured at "old" conditions.
  (compare N. Bertram et al. Intermag 98, HB01, to be published: 4.5 dB)
- at new conditions, SNR is expected to be const.

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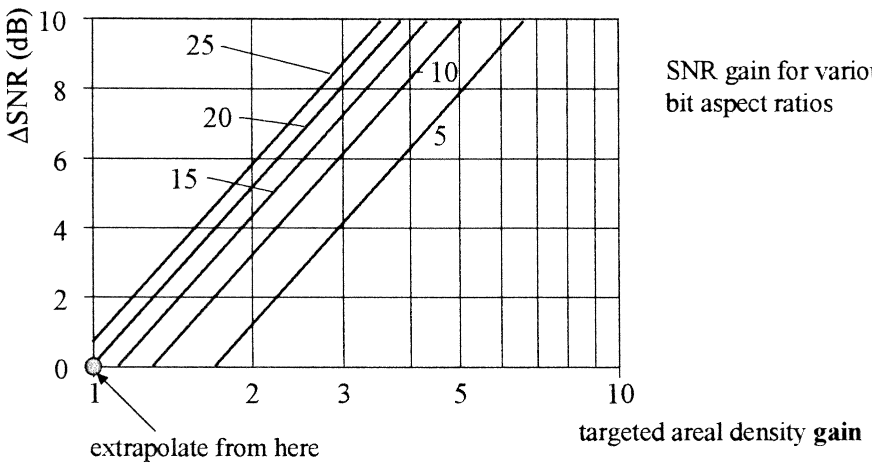
(d) Requirement, Summary Chart



SNR gain for various bit aspect ratios

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(e) Challenges for Future Media

a back-on-the-envelope calculation:

$$AD \propto \frac{1}{\sqrt[3]{As}} \frac{1}{s^2} \left(\frac{H_r}{M_r} \right)^{\frac{2}{3}}$$

item	comment
aspect ratio	mild dependence
H_r, M_r	mild dependence
s	strong dependence

some simplifications involved

s needs to be reduced, which is in the end particle size.

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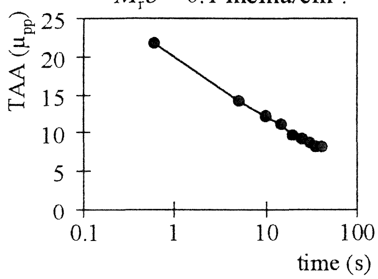
(e) Challenges: Time Effects

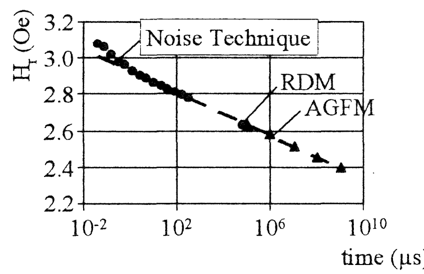
too small magnetic particles become magnetically unstable:

10 kfci recording
$M_r\delta = 0.1$ memu/cm².



time dependence of coercivity increases for small particle size

$M_r\delta = 0.7$ memu/cm².



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Interactions Between Noise and Non-Linear Distortions in PRML Channels

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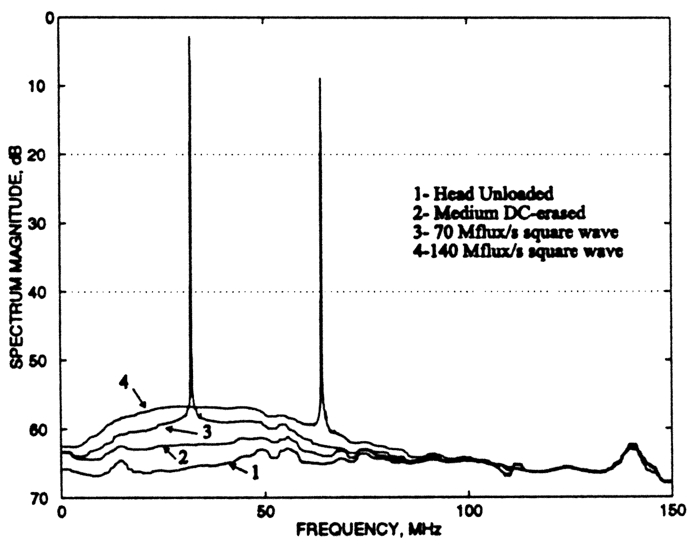
Outline:

- Naïve Look at Transition Noise: Power spectrum derivation
- Pattern-dependent Transition Noise
- Supralinear regime: NLTS, Partial Erasure, Track Edges and Noise
- Noise and PRML Error Rates: AGWN paradigm versus colored noise
- Noise, Non-Linearity, Pattern Dependence and Error Rate of PRML channels
- Conclusions

Naïve Look at Transition Noise



- Noise spectrum is different with transitions and without transitions
- Larger noise when high frequency square wave pattern is written.

Transition Jitter: Power Spectrum (without "small" jitter assumption)

- Series of delta-functions $D(t)=\sum_{k=-\infty}^{\infty} a_k \delta(t-kT-\xi_k)$ ← timing jitter
- Read-Back pattern: $s(t)=\sum_{k=-\infty}^{\infty} a_k p(t-kT-\xi_k)=p(t)*\left[\sum_{k=-\infty}^{\infty} a_k \delta(t-kT-\xi_k)\right]$
- Read-Back Spectrum: $S(\omega)=P(\omega)\left[\Im\left\{\sum_{k=-\infty}^{\infty} a_k \delta(t-kT-\xi_k)\right\}\right]=P(\omega)D(\omega)$
- Power Spectrum:

$$\left\langle |S(\omega)|^2\right\rangle = |P(\omega)|^2 \left\langle |D(\omega)|^2\right\rangle$$

$$\left\langle |D(\omega)|^2\right\rangle = \lim_{N\to\infty}\frac{1}{N}\left\langle \Im\left\{\sum_{k=-N/2T}^{N/2T} a_k \delta(t-kT-\xi_k)\right\}\Im^*\left\{\sum_{m=-N/2T}^{N/2T} a_m \delta(t-mT-\xi_m)\right\}\right\rangle$$

Power Spectrum of Jittering delta-functions:

- Calculate Fourier Transform
- Find cross-product, treat the cases k=n and k != n

$$D(\omega)=\Im\left\{\sum_{k=-N/2T}^{N/2T} a_k \delta(t-kT-\xi_k)\right\}=\sum_{k=-N/2T}^{N/2T} a_k e^{-i\omega kT} e^{-i\omega \xi_k}$$

$$D(\omega)D^*(\omega)=\sum_{n=-N/2T}^{N/2T}\sum_{k=-N/2T}^{N/2T} a_k a_m e^{-i\omega(k-n)T} e^{-i\omega(\xi_k-\xi_n)}=$$

$$=\sum_{k=m}\sum a_k a_m e^{-i\omega(k-n)T} e^{-i\omega(\xi_k-\xi_n)}+\sum_{n=-N/2T}^{N/2T}\sum_{\substack{k=-N/2T\\ k\neq N}}^{N/2T} a_k a_m e^{-i\omega(k-n)T} e^{-i\omega(\xi_k-\xi_n)}=$$

$$=\sum_{k=-N/2T}^{N/2T} a_k^2+\sum_{n=-N/2T}^{N/2T}\sum_{\substack{k=-N/2T\\ k\neq N}}^{N/2T} a_k a_m e^{-i\omega(k-n)T} e^{-i\omega(\xi_k-\xi_n)}$$

Power Spectrum Calculation:

- K/T - Total Power of modulating pattern per modulating period.
- Assume that jitter is Gaussian, use characteristic function property:

$$\langle \exp(-i\omega\xi) \rangle = \exp(-\frac{\omega^2\sigma^2}{2})$$

- Expression for power spectrum:

$$\langle | D(\omega) |^2 \rangle = \lim_{N\to\infty} \frac{1}{N} \left[\frac{NK}{T} + \left\langle \sum_{m=-N/2T}^{N/2T} \sum_{\substack{k=-N/2T \\ k\neq m}}^{N/2T} a_k a_m e^{-i\omega(k-n)T} e^{-i\omega(\xi_k-\xi_n)} \right\rangle \right] =$$

$$= \frac{K}{T} + \left\langle \sum_{m=-\infty}^{\infty} \sum_{\substack{k=-\infty \\ k\neq m}}^{\infty} a_k a_m e^{-i\omega(k-n)T} e^{-i\omega(\xi_k-\xi_n)} \right\rangle = \frac{K}{T} + e^{-\omega^2\sigma^2} \lim_{N\to\infty} \frac{1}{N} \left[\sum_{m=-N/2T}^{N/2T} \sum_{\substack{k=-N/2T \\ k\neq m}}^{N/2T} a_k a_m e^{-i\omega(k-n)T} \right] =$$

$$= \frac{K}{T}(1-e^{-\omega^2\sigma^2}) + \frac{e^{-\omega^2\sigma^2}}{T} \left[\sum_{m=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} a_k a_m e^{-i\omega(k-n)T} \right]$$

Power spectrum of Read-Back with Jitter

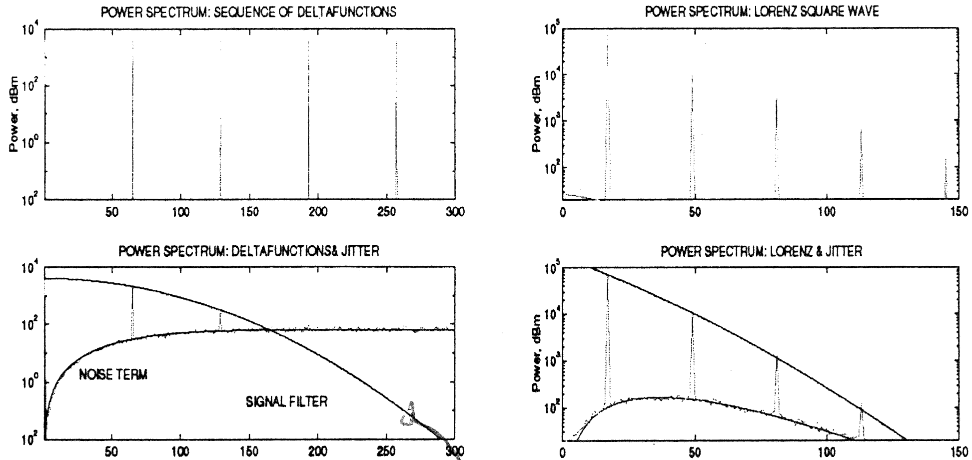
- Second term is the weighted power spectrum of the modulating process.
- Example: Let the sequence of the modulating binary pattern is stationary, i.e. $\langle a_k a_m \rangle = \alpha_{k-m}$ Then, using the Poisson summation formula:

(A(w) is the power spectrum of the modulating process):

$$\langle | D(\omega) |^2 \rangle = \frac{K}{T}(1-e^{-\omega^2\sigma^2}) + \frac{e^{-\omega^2\sigma^2}}{T} \left[\sum_{m=-\infty}^{\infty} \alpha_m e^{-i\omega nT} \right] =$$

$$= \frac{K}{T} P(1-e^{-\omega^2\sigma^2}) + \frac{e^{-\omega^2\sigma^2}}{T^2} \sum_{m=-\infty}^{\infty} A(\omega - m\omega_0)$$

Model Results:



Lorentzian Pulses

Top- sequence of delta-functions power spectrum
Bottom - same, with jitter. Noise term is 1-exp, harmonics are weighted by exponential filter

Top- Lorenzian square wave PW50/T=2, no jitter
Bottom - same, with jitter. Noise term is product of Lorenzian spectrum with 1-exp, harmonics are weighted by exponential filter

fits to harmonics

Read-Back Power Spectrum:

$$\left\langle | S(\omega) |^2 \right\rangle = \frac{K}{T} | P(\omega) |^2 (1 - e^{-\omega^2\sigma^2}) + \frac{e^{-\omega^2\sigma^2}}{T^2} | P(\omega) |^2 \sum_{m=-\infty}^{\infty} A(\omega - m\omega_0)$$

Observations:

- Continuous spectrum term does not interfere with pattern
- Pattern harmonics are weighted by the Low-pass filter:

$$\exp(-\omega^2\sigma^2)$$

Spectrum for Square-Wave pattern

$$\langle |S(\omega)|^2 \rangle = \frac{|P(\omega)|^2}{T}(1-e^{-\omega^2\sigma^2}) + \frac{e^{-\omega^2\sigma^2}}{T^2}|P(\omega)|^2 \sum_{m=-\infty}^{\infty}\delta(\omega - m\omega_0)$$

- For small jitter: $1-\exp(-\omega^2\sigma^2) \approx \omega^2\sigma^2$

- The final equation is almost identical to [Bertram, Tarnopolsky] except for the weighting of the pattern harmonics:

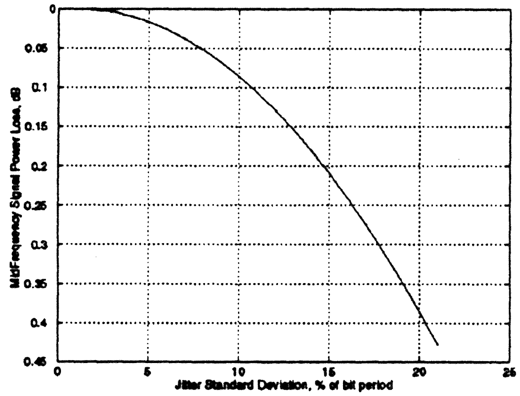
$$\langle |S(\omega)|^2 \rangle = \frac{|P(\omega)|^2\,\omega^2\sigma^2}{T} + \frac{1}{T^2}\sum_{m=-\infty}^{\infty}(1-m^2\omega_0^2\sigma^2)|P(m\omega_0)|^2\,\delta(\omega - m\omega_0)$$

SNR calculation:

- Noise power: $N = \frac{K}{T}\int_0^B |P(\omega)|^2\,(1-\exp(-\omega^2\sigma^2)d\omega$
- Signal Power: $S = \frac{1}{T^2}\int_0^B |P(\omega)|^2\,A(\omega)\exp(-\omega^2\sigma^2)d\omega$
- Simplified SNR for a square wave recording (similar to G. Tarnopolsky et.al)

$$SNR = \frac{(V_{peak})^2}{P_N} \approx \frac{2TPW_{50}\exp(-\pi^2\sigma^2/T^2)}{\pi\sigma^2}\left[\frac{\pi PW_{50}/2T}{\sinh(\pi PW_{50}/2T)}\right]^2$$

Influence of pattern filtering on SNR:



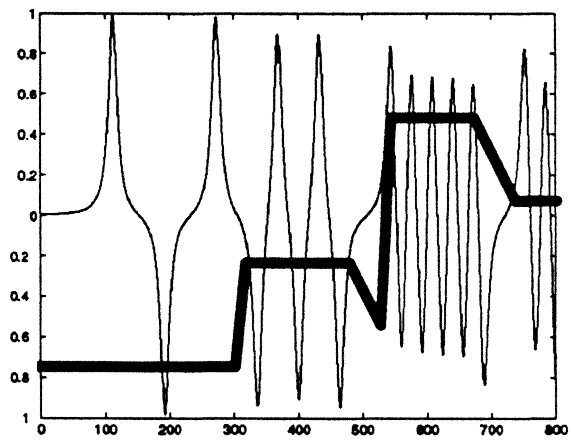
- acts like additive noise

- SNR correction (fundamental or mid-frequency to integrated noise power)
- T=150 nm, sigma = 6 nm, fundamental is scaled as 0.98 of the signal power, <0.1 dB loss;
- Below the Nyquist frequency the losses are negligible when jitter <5% of bit period.
- At 10% jitter maximum signal loss is 0.4 dB at Nyquist frequency

Transition Jitter, Power spectrum, SNR: Naïve Look

- "Small parameter" approximation works very well
- No interactions between noise power spectrum and pattern components (except for small Low - Pass filtering effect). This effect is negligible for jitter <10% below Nyquist frequency and in a range of densities PW50/T >2
- Medium Transition Noise spectra is additive to the pattern
- Is Medium Transition Noise similar to colored Gaussian noise???
- The power spectrum is averaged over a period of a modulating process (binary pattern). Locally, the process is non-stationary and pattern-dependent.

Variable-Density pattern and Medium Noise

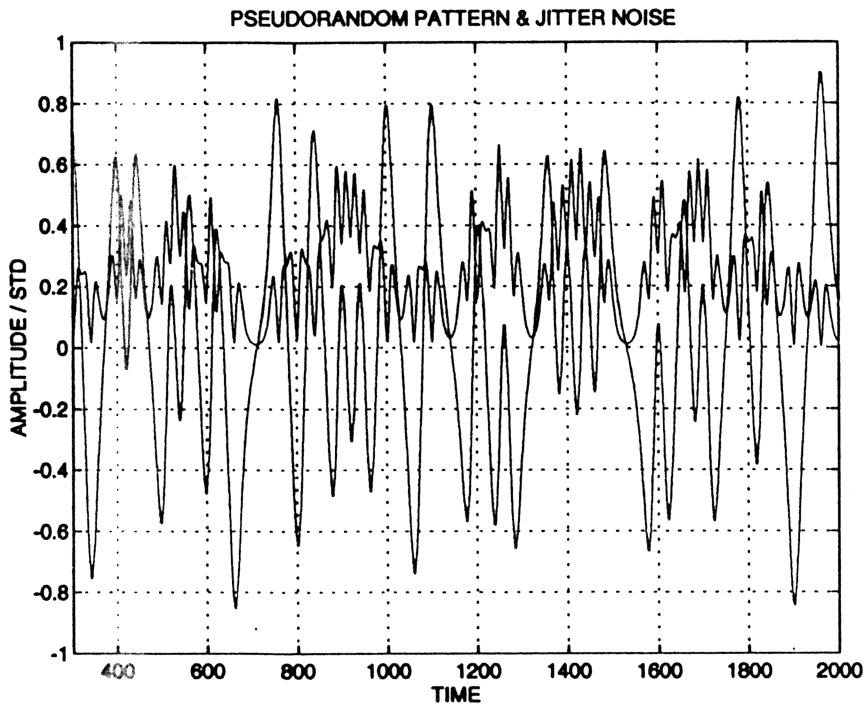


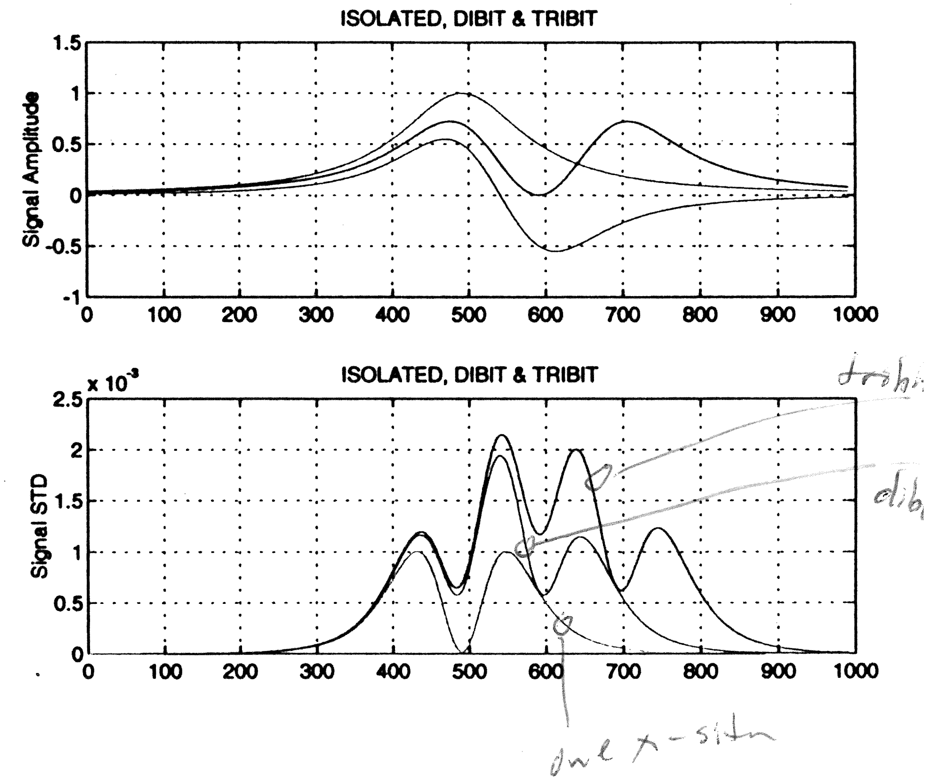
- Noise is created by transitions
- Locally higher transition density creates higher density of noise power

Signal Amplitude (samples) deviation: Isolated, Dibit and Tribit Transitions

- Generate multiple realizations of transition with Gaussian jitter
- Calculate position-dependent standard deviation
- Maximum deviation at maximum derivative (as expected according to the model)
- Higher noise for a dibit transition - superposition of jitter from two adjacent transitions into the integral waveform
- Same - for tribit
- Distribution of position-dependent noise for random pattern
- Transition noise is locally density-dependent

power
— max noise @ max signal position



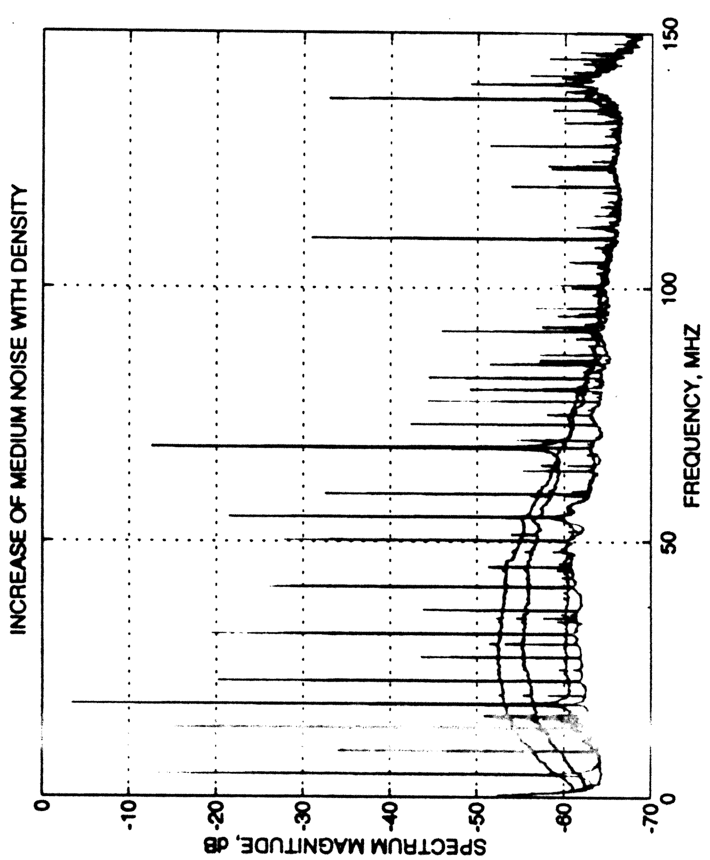


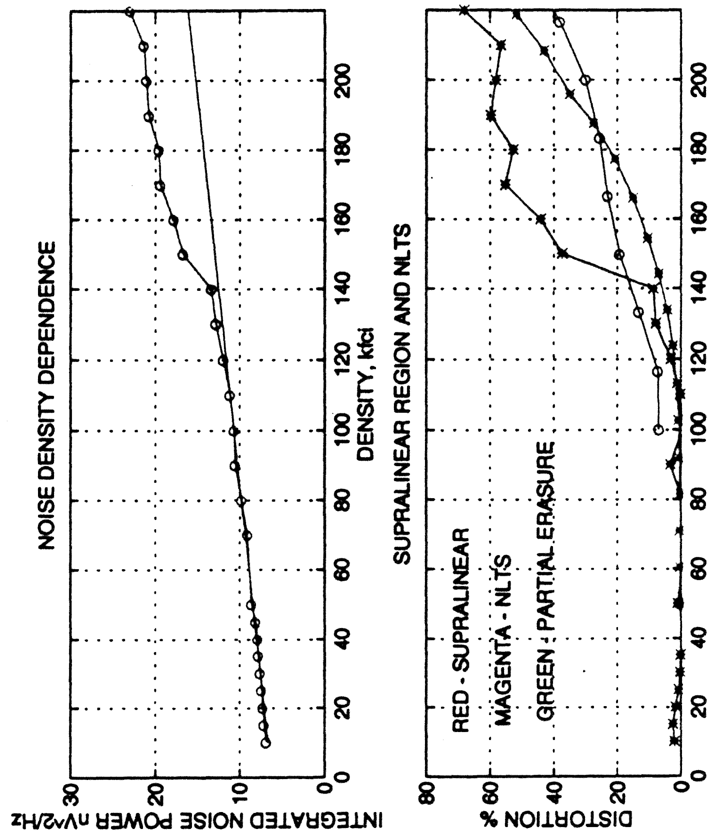
Density dependence of Media Noise: Linear or supralinear?

- Forget about position dependence: use a square wave recording and integrate noise in a bandwidth
- According to theory, the noise term (ignoring signal harmonics):

$$\langle |N(\omega)|^2 \rangle = \frac{|P(\omega)|^2}{T}(1 - e^{-\omega^2\sigma^2})$$

- Noise Power is linearly proportional to density (1/T).
- Deviation from linear dependence (supralinear region) is often observed and correlated with onset of percolation (partial erasure) and Non-Linear Transition Shift.

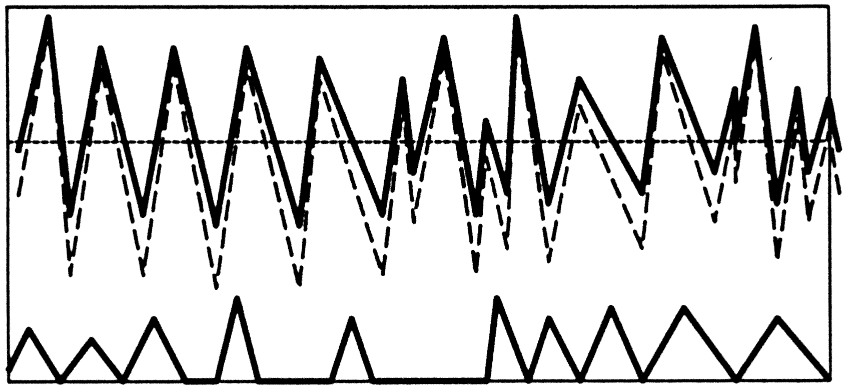




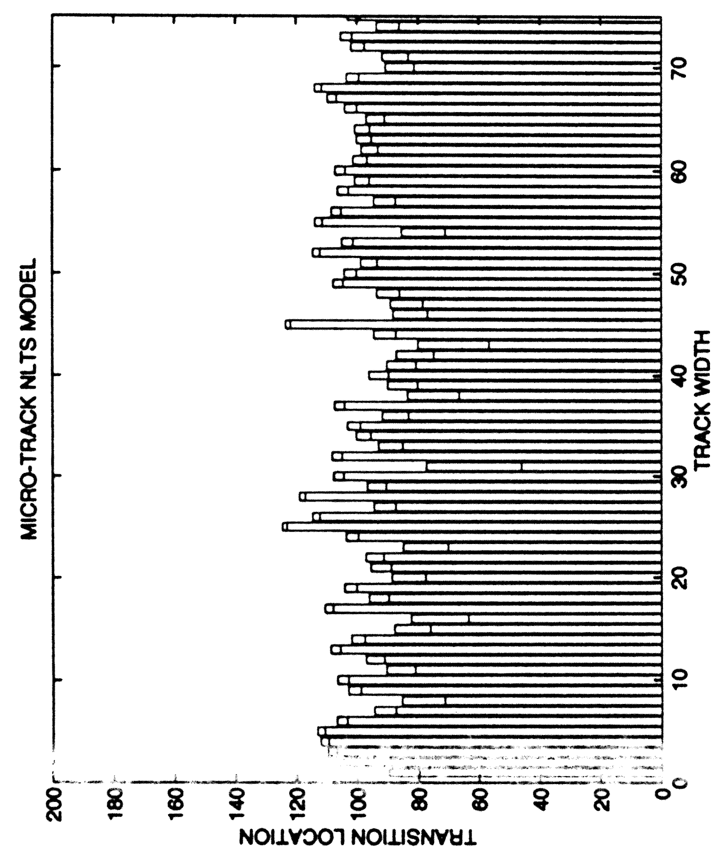
Experimental results:

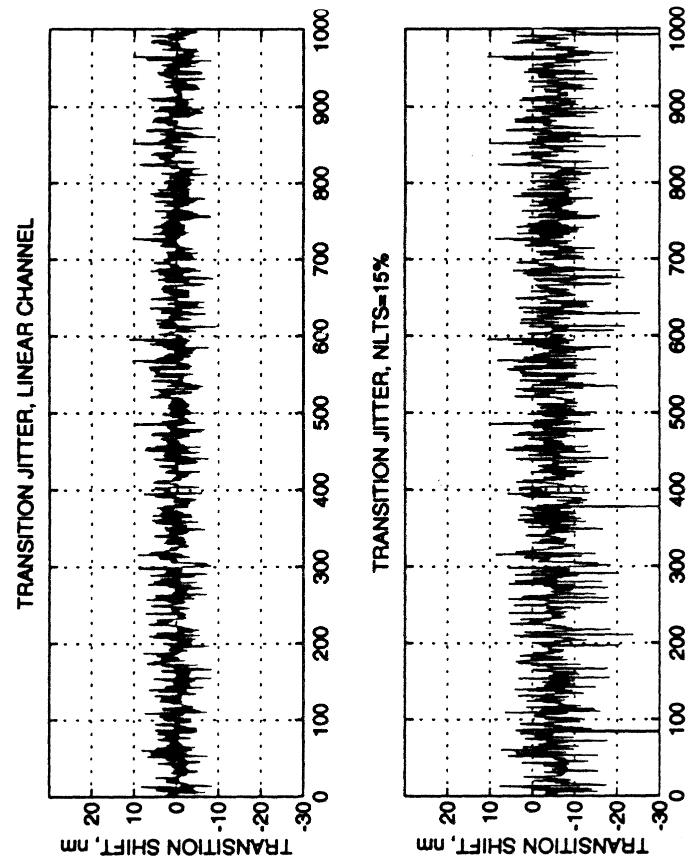
- Deviation from linear dependence correlates with the onset of NLTS and Partial Erasure (J. Zhu et al., J.Appl. Phys, vol.79, p.4904 April 1996, E. Yen, IEEE Trans. Mag., vol. 33, p. 2701, 1997)
- Possible contribution of track edge noise - percolation starts at track edges first (J.Zhu, TMRC -96, INTERMAG-97)
- Very difficult to distinguish contributions of NLTS and Partial Erasure
- Simplified read-back model: non-interacting micro-tracks (70-80 microtracks). Aspect ratio = 15, typical transition jitter 3 nm

Simplified micro-track model: NLTS only

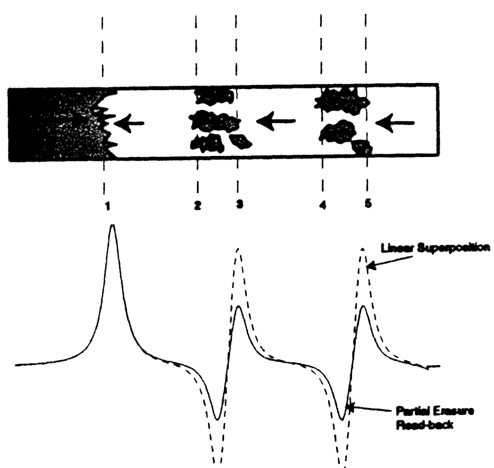


- Each Zig-Zag adjacent to the previous transition has more probability to switch
- NLTS=15% of bit period, N=K/T^3
- Distribution of micro-track positions becomes asymmetrical, standard deviation increases





Partial Erasure: similar mechanism + percolation



- As the second transition is written, zig-zag tips percolate. Effective transition irregularity increases, plus amplitude drops

Media Noise + Non-Linearity: Summary

- Transition jitter noise cannot be analyzed as the signal-independent, additive colored Gaussian process: this noise is position- and density-dependent. Higher noise is observed for high frequency patterns (dibits and transition bursts)
- Onset of non-linearities (NLTS and Partial Erasure) increases transition noise for highest-density segments of the data pattern.
- Medium noise and non-linearities "reinforce" each other, causing extra degradation of error rate performance
- What will happen with PRML Error Rates?

Error Rate of the PRML Channels: Analysis

Received Sequence of Samples:

$$y(k)=s0(k)+n(k)$$

$s0(k)$ - ideal sample values

$n(k)$ - noise samples

Arbitrary allowable sequence

$$b(k)=s0(k)+m(k)A$$

A - Step between PRML levels

m(k) - integer number of levels

Errors of the ML detector

$$\sum_{k=1}^{N}[s(k)-s_0(k)]^2 > \sum_{k=1}^{N}[b(k)-s(k)]^2$$

$$\sum_{k=1}^{N}[n(k)]^2 > \sum_{k=1}^{N}[m(k)A-n(k)]^2$$

- Error is made when: $$\frac{1}{\sum_{k=1}^{N}[m(k)]^2}\sum_{k=1}^{N} m(k)n(k) > \frac{A}{2}$$

m(k) are coefficients of the probable error sequences, for example PR4: m(k)={1,0,-1} or {1,0,0,0,-1} etc.

Random noise versus Non-Linearities: Theoretical Analysis

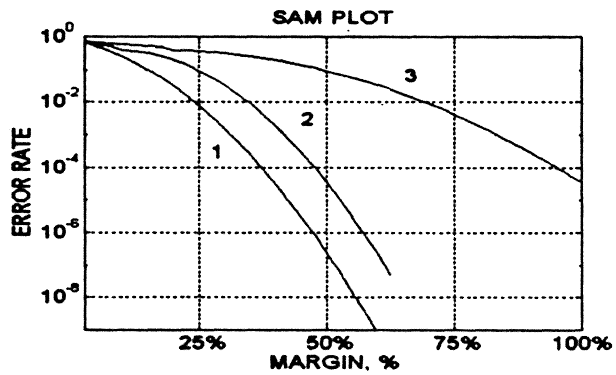
Total Noise:

N(k) = n(k) + d(k)
n(k) - random noise (Medium + electronics)
d(k) - deterministic shape distortions (NLTS, partial erasure etc).
Therefore error is made when:

$$\frac{1}{\sum_{k=1}^{N}[m(k)]^2}\left(\sum_{k=1}^{N} m(k)n(k) + \sum_{k=1}^{N} m(k)d(k)\right) > \frac{A}{2}$$

PRML Channels: Error Rate and Noise

- 1 - On-track
- 2- misequalized
- 3 - off track



- Noise decreases the slope of the SAM plot
- Shape distortions: shift of SAM plot

Noise versus Shape distortions

$$\frac{1}{\sum_{k=1}^{N}[m(k)]^2}\left(\sum_{k=1}^{N} m(k)n(k)+\sum_{k=1}^{N} m(k)d(k)\right)>\frac{A}{2}$$

First term: filtered noise with pdf *p(f)*

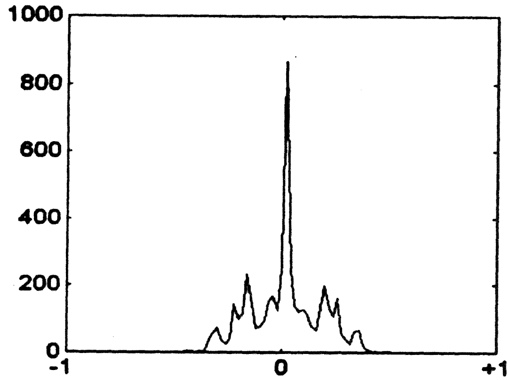
Second term: linear combination of shape distortion terms with distribution *H(f)*

Example: Partial Erasure: s(k)= {0.8,0,-0.8}, PR4: m(k)={1,0,-1}

d(k) ={0.2,0,-0.2}

H(f) has peaks at {-0.4,-0.2,0,+0.2,+0.4}

Distribution of Shape Distortion term for NLTS (signal injection experiment)



- Random pattern, 25% NLTS

Mixing Random and Shape Terms

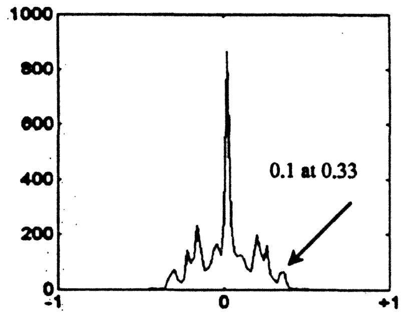
$$P(f) = \int_{-\infty}^{\infty} p(f-x)H(x)dx$$

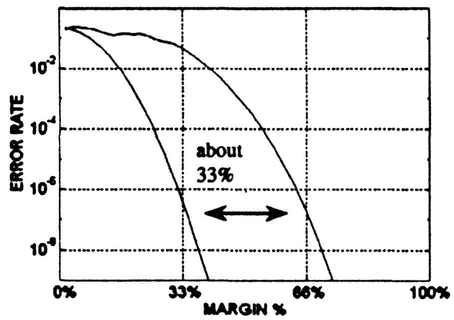
- Assume statistical independence of $p(f)$ and $H(f)$

$$G(f) = \int_{f}^{\infty} P(f)df = \int_{-\infty}^{\infty} Q(f-x)H(x)dx$$

- Error Margin for PRML channel is given by convolution of the error margin of the $Q(f)$ with $H(f)$
- $Q(f)$ is the error margin of linear, ideally equalized PRML channel with only random noises
- Every peak in "shape" term $H(f)$ degrades error rate. The most distant peaks are the most critical

Explanation of Margin Plots



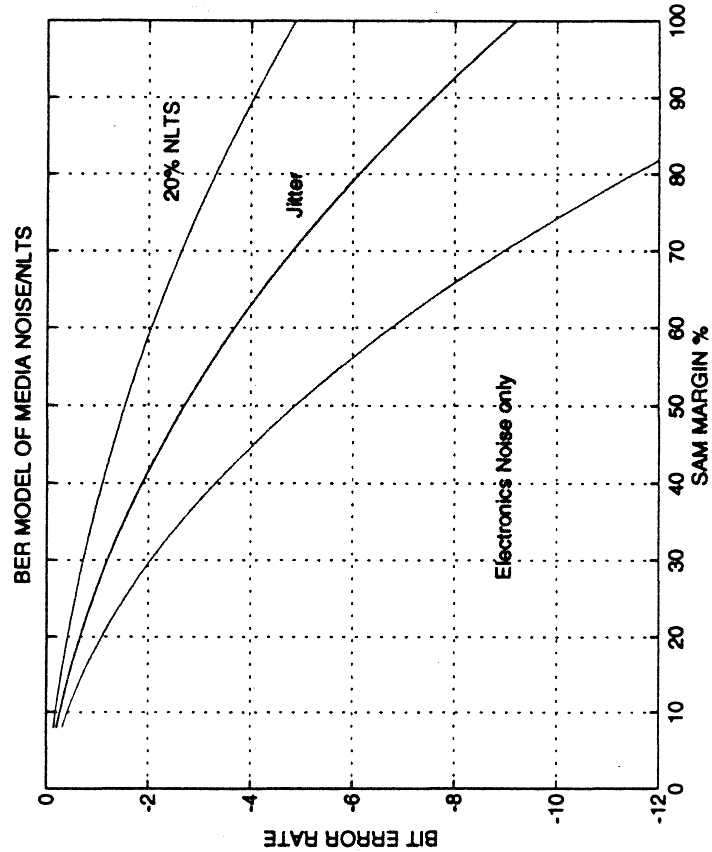


- Function *H(f)* (left). Right:Convolution of a Gaussian error function *Q(f)* (left curve) with function *H(f)*. Result of convolution (right curve) is close to a shifted copy of *Q(f)*.
- 10% peak at 0.33 in *H(f)* causes approximately 33% shift of the margin plot

Media Noise, Non-Linear Distortions and Error Rates of PRML Channels

Accurate analysis is complicated:

- Pattern-dependent noise distribution
- Pattern-dependent non-linearities
- Consider only worst-case error event for PR4: dibit-type errors {-1,0,1},{-1,000,1}, etc.
- Isolated dibit and dibit in the middle of the burst of transitions are different. Worst-case error: dibit in a burst. At the same time this dibit has less NLTS compared to an isolated dibit
- Medium noise without NLTS/Percolation is approximately twice higher than for an isolated transition
- Then add amplitude loss and see what happens
- VERY APPROXIMATE MODEL!!!



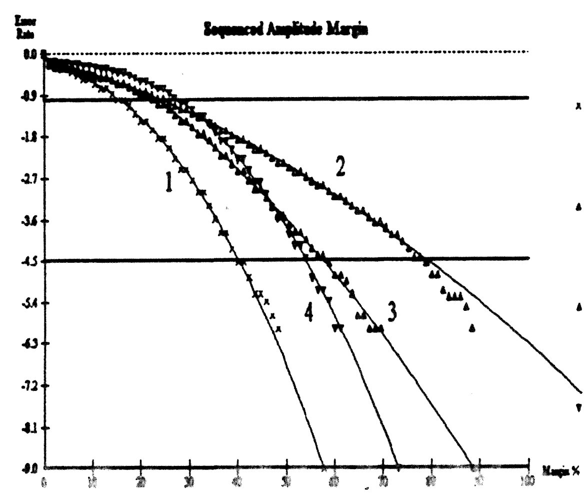
- worst case dibit in middle of pattern

Sequence Amplitude Margin

35

Pattern Dependent Equalization

- 1 - Isolated pulses, optimal equalization
- 2 - equalization is same as (1), random pattern
- 3 - Equalizer is adjusted for random pattern
- 4 - same equalization as (3), isolated pulses



Conclusions:

- Media noise-dominated detection is complicated
- Media noise is pattern-dependent
- Magnitude of media noise is coupled with non-linear distortions
- Model predicts significant error rate degradation for worst-case pattern (dibit-type error).
- Minimization of media noise - media design, thermal stability....
- Operate in linear region if the channel is media-noise dominated

Media SNR Measurements and Projections

Thomas C. Arnoldussen
IBM Corporation

IIST Symposium on SNR and Disk Drive Performance
Santa Clara University

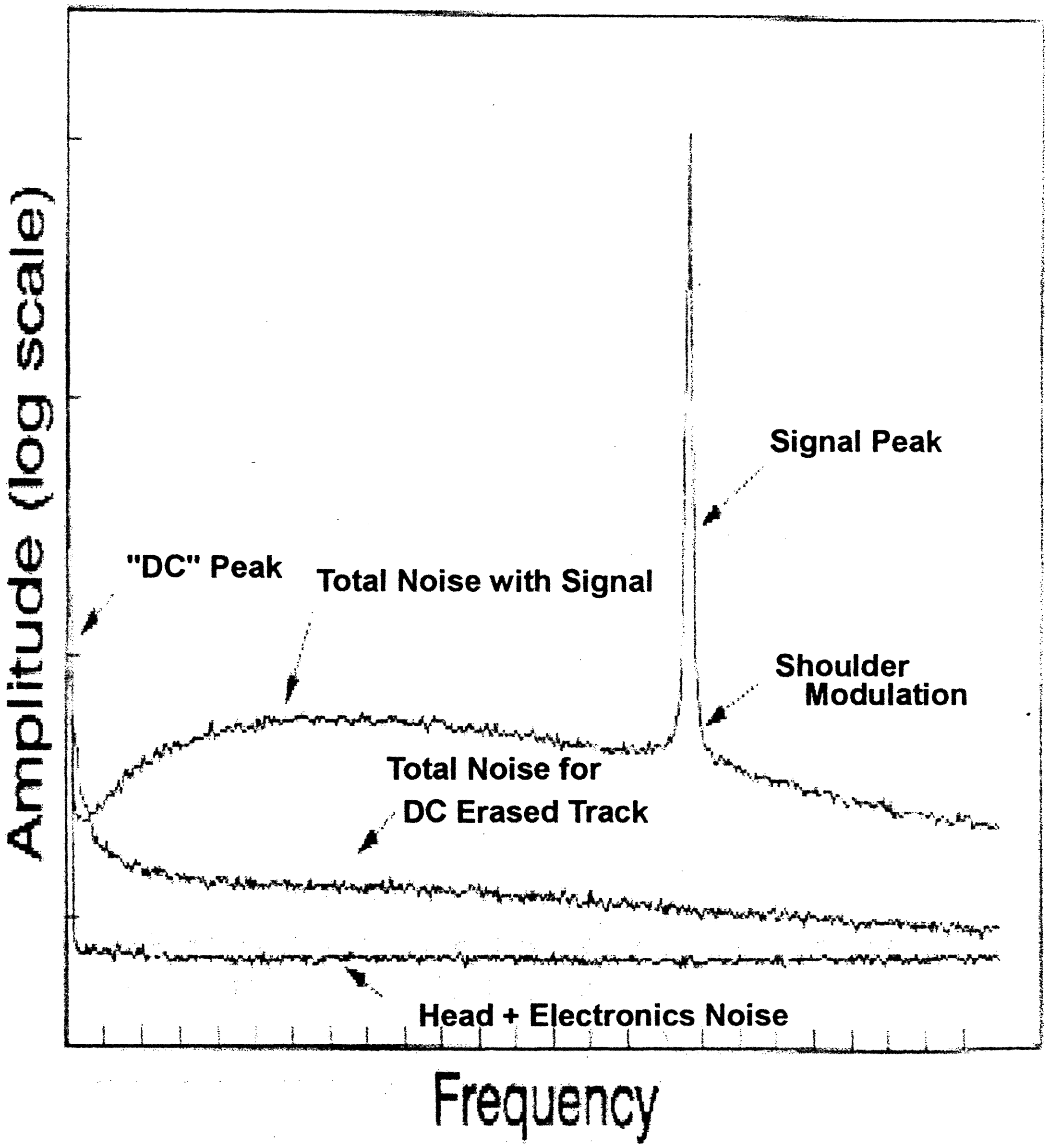
April 27, 1998

THE PROBLEM:

- **What is the best measurement of Media SNR?**

- **Transition Noise is not an Intrinsic Property of the Media, but Depends on Write/Read Head as well as Magnetic Spacing.**

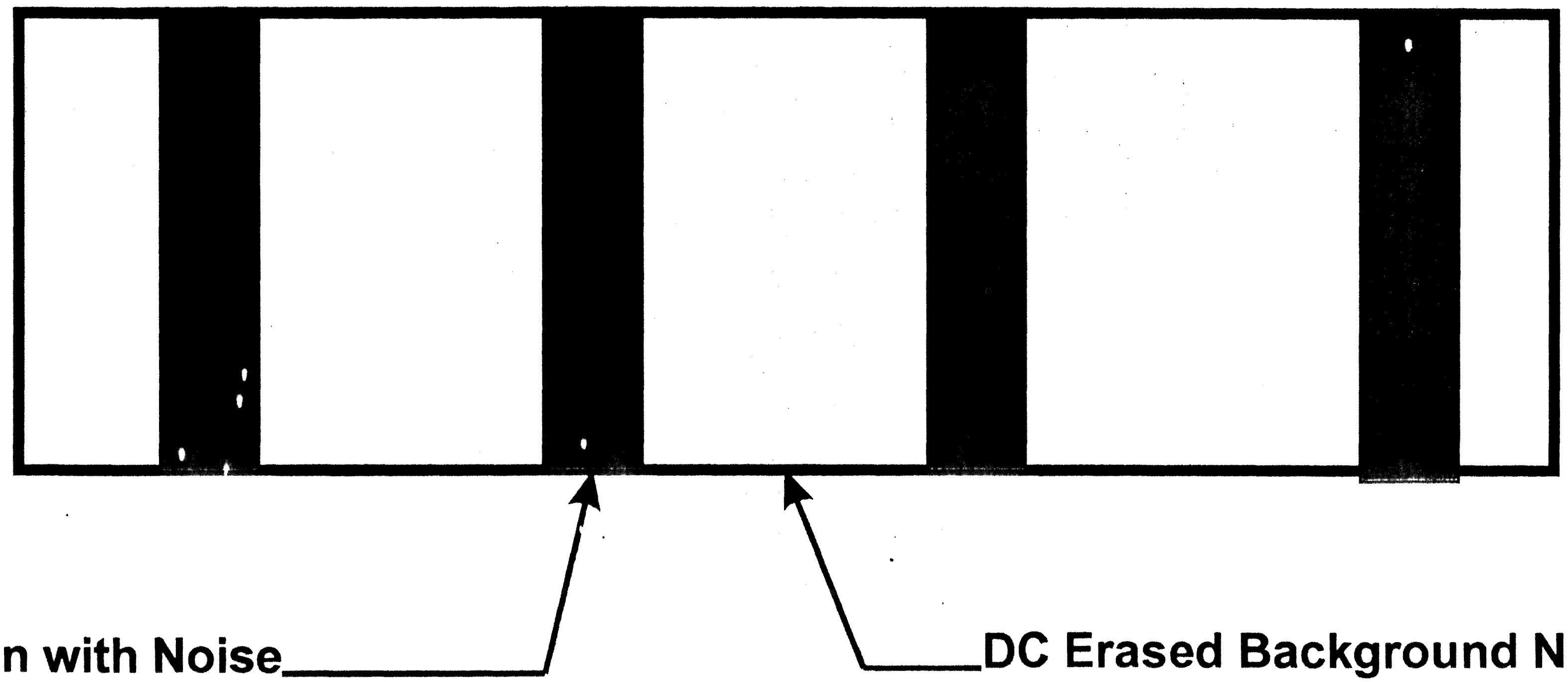
- **If we measure prototype Advanced Media with currently available ("Product") heads, how should we project performance (SNR) in a future file, which is only a paper design and for which Advanced Heads do not yet exist?**



From Nunnelley, Noise in Digital Magnetic Recording, Arnoldussen & Nunnelley editors, World Scientific 1992

T.C. Arnoldussen 12/15/97

Duty Cycle Concept
(Non-Stationarity)



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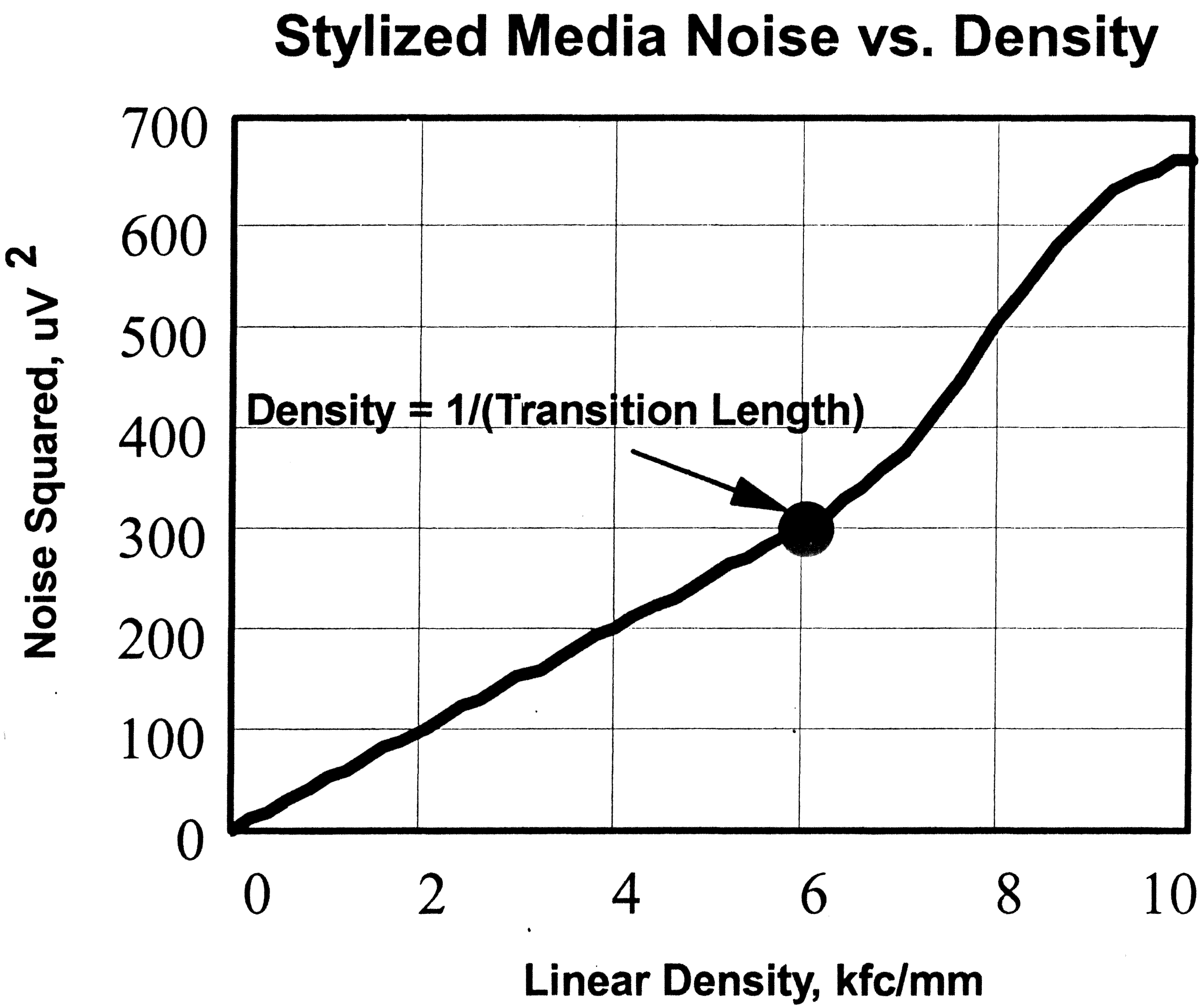

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Voltage Signal-to-Noise Ratio

Isolated Pulse - to - (total integrated) Transition Noise:

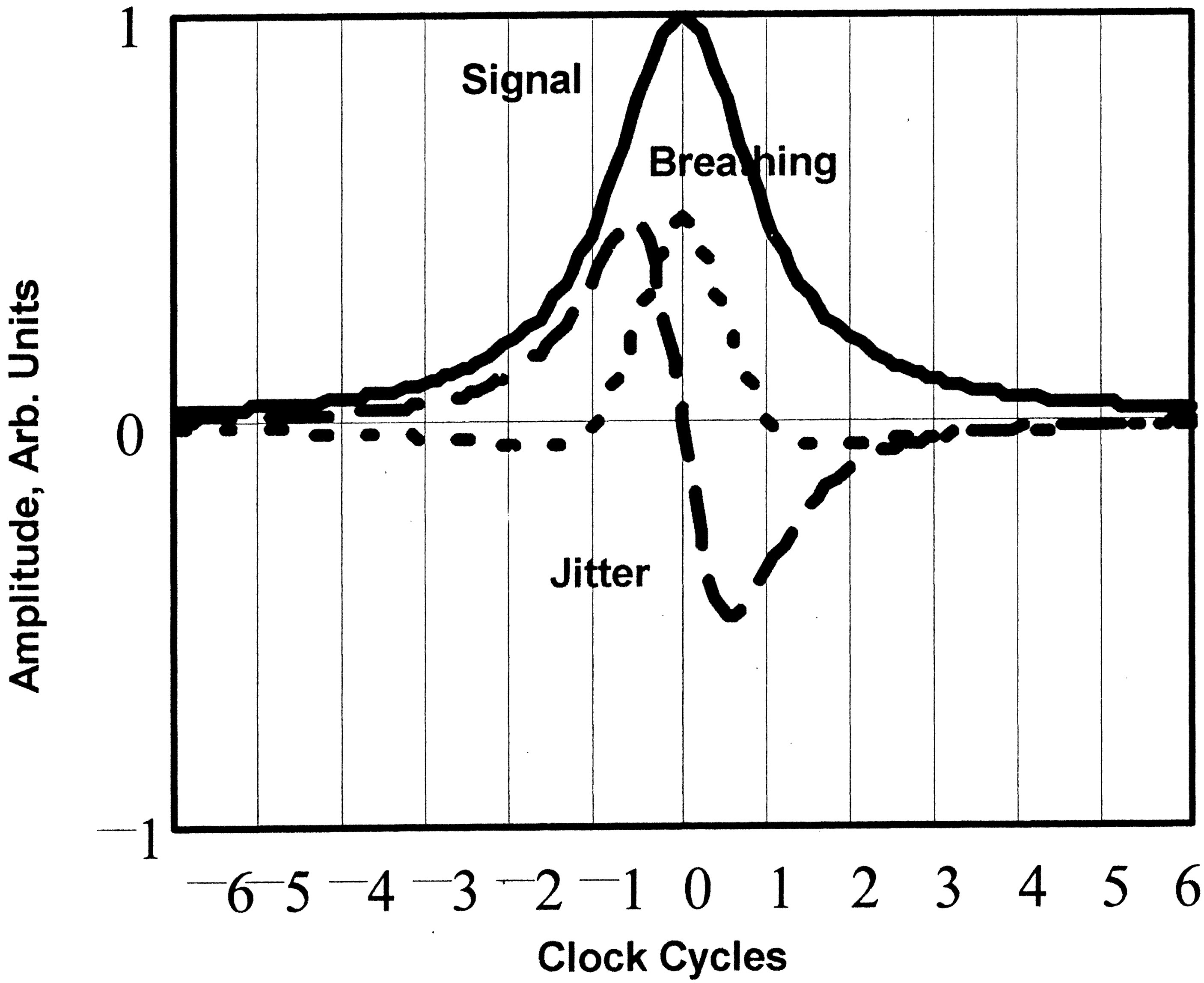
$$\left(\frac{S_0}{N}\right)^2 = \frac{16}{\pi^3}\frac{W_R}{W_C}\frac{PW50}{D_L\, a^2}$$

avg correlation grain correlation width² weights bigger grains > small grain

S_0 = Isolated Pulse Amplitude
N = RMS Noise
W_R = Read Width
W_C = Cross-track Correlation Width (Cluster Size)
PW50 = Isolated Pulse Full Width at Half Amplitude
a = Williams & Comstock Transition Parameter
D_L = Linear Transition Density = 1 / L
L = Bit Cell Length
D_T = Track Density = $1 / [\gamma W_R]$, (where $\gamma \approx 1.6$ typically)
(AD) = Areal Density = $D_T\, D_L$

T.C. Arnoldussen 1/9/98

Raw Signal and Media Noise Modes



eigen-
modes.

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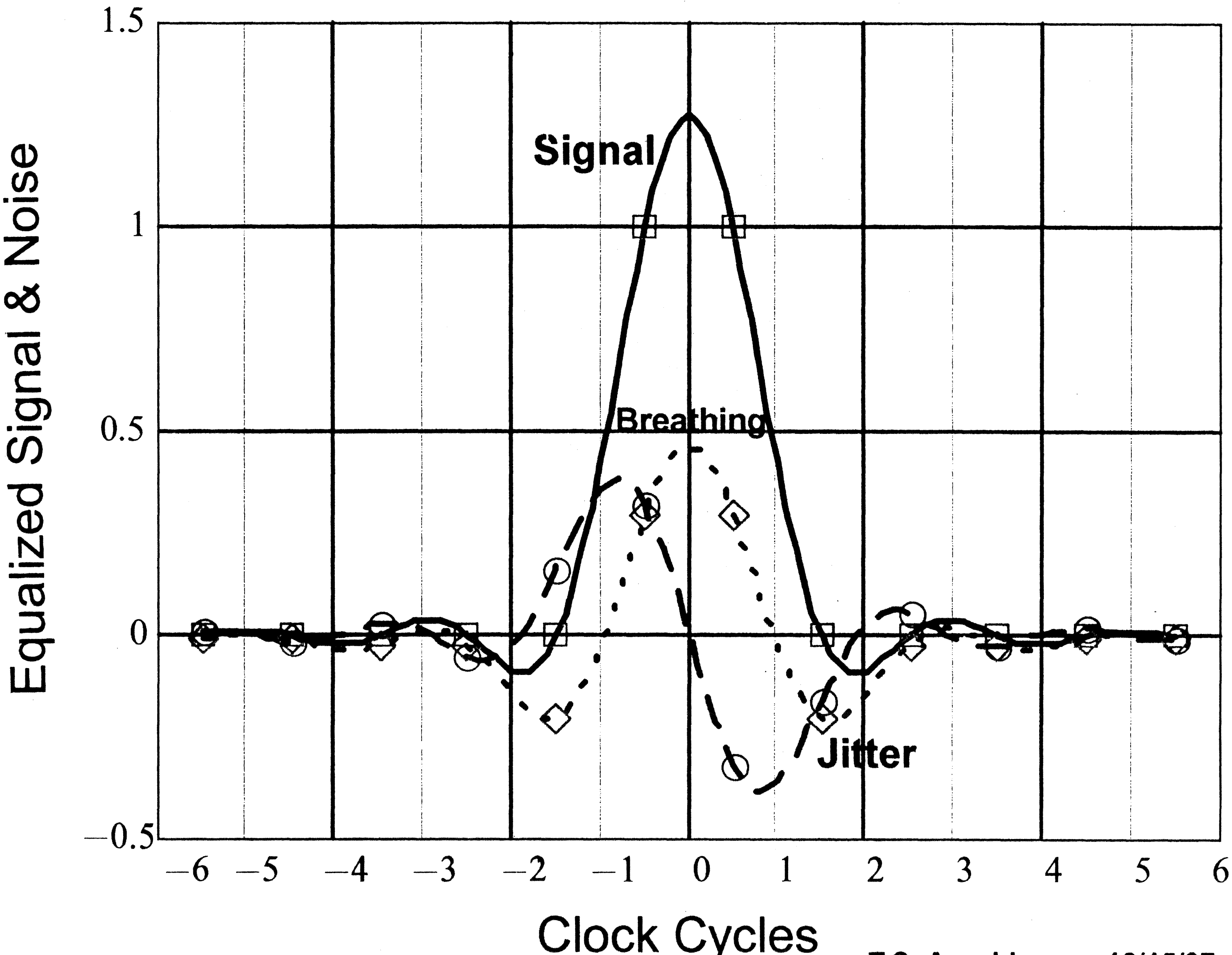

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← all equalized

– breathing & jitter correlat[ed] same after equalization

– thus all modes important.

Mean Squared PR4 Equalized Noise at x = - L/2 (signal sample = 1)

Jitter:

$$\left\langle n_{e,J}^2 \right\rangle = \frac{2 \times a \times PW50}{L^2} \times \frac{1}{\left(\frac{S_0}{\alpha_J}\right)_I^2}$$

inverse SNR

jitter extrapolated to percolation limit.

Breathing:

$$\left\langle n_{e,B}^2 \right\rangle = \frac{\sout{0.218}\ 0.873 \times 2 \times a \times PW50}{L^2} \times \frac{1}{\left(\frac{S_0}{\alpha_B}\right)_I^2}$$

Electronics:

$$\left\langle n_{eE}^2 \right\rangle = \left(\frac{L}{L_R}\right)^Q \times \frac{\sout{4}\ 8}{\pi^3} \times \frac{\left[\exp\left(\frac{\pi PW50}{L}\right) - 1 - \left(\frac{PW50}{L}\right)^2\right]}{\left(\frac{PW50}{L}\right)^3 \times \left[1 + \left(\frac{PW50}{L}\right)^2\right]} \times \frac{1}{\left(\frac{S_0}{\alpha_E}\right)_{REF}^2}$$

Q = 1 for constant RPM,

= 2 for constant Data Rate, At Constant Areal Density

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Bit Length to Jitter SNR

- after equal
all noise behaves
like jitter

$$\left(\frac{L^2}{\sigma^2}\right) = \left(\frac{S_0^{\,2}}{N^2}\right)_I \times \frac{L^2}{2\ \Gamma_J\ a\ PW50}$$

$$= \left(\frac{8}{\Gamma_J \times \gamma \times W_C \times (AD)}\right) \times \left(\frac{L}{(\pi\ a)^2}\right)$$

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SNR PROJECTION PROCEDURE

1. **Measure Isolated Pulse of "Qualification" (Q) disk with available "Product" (P) head. Determine $(PW50)_{Q,P}$. (Note, run T. Arnoldussen or B. Wilson linearization procedure if nonlinear MR or GMR transfer curve warrants.)**

2. **Get H_C, $M_R\delta$, S^* of Qualification Disk and g_R of Product Head.**

3. **Estimate "a" parameter and effective magnetic spacing, d_F.**

$$(i)\quad PW50 \approx \sqrt{0.5g_R^2 + 4\times(a + d_F)^2}$$

$$(ii)\quad a \approx 3.25(1.89 - 1.11S^*)\sqrt{\frac{M_R\delta d_F}{H_C}}$$

Note:

$$d_F = \sqrt{d(d + \delta)} \approx d + 0.45\delta$$

Geom. mean

T.C. Arnoldussen, IBM, April 27, 1998

SNR PROJECTION (cont'd 2)

4. **Calculate Maximum Supportable Linear Density (short of percolation) for this Disk with this Head.**

   $D_{L,\,Max,\,Q,\,P} \approx 1/(\pi a)_{Q,\,P}$

5. **Measure Integrated Media Noise, Normalized by Isolated Pulse Amplitude at or slightly below the limiting density $D_L \leq D_{L,\,Max,\,Q,\,P} \approx 1/(\pi a)_{Q,\,P}$.**

$$\left(\frac{S_0}{N}\right)^2_{D_L,Q,P} = \frac{16}{\pi^3}\frac{W_{R,P}}{W_{C,Q}}\frac{PW50_{Q,P}}{D_L a_{Q,P}^{\;2}}$$

6. **Measure microtrack profile. Full width at half height = $W_{R,\,P}$.**

SNR PROJECTION (cont'd 3)

7. **Extrapolate $(S_0/N)^2$ to $D_{L,Max,\,Q,\,P} = 1/(\pi a)_{Q,P}$ to obtain the Signal-to-Transition-Noise under measurement conditions. τ denotes "transition-only" noise.**

$$\left(\frac{S_0}{N}\right)^2_{\tau,Q,P} = \left(\frac{S_0}{N}\right)^2_{D_L,Q,P} \times D_L\,(\pi a)_{Q,P}$$

$$\left(\frac{S_0}{N}\right)^2_{\tau,Q,P} = \frac{16}{\pi^2}\frac{W_{R,P}}{W_{C,Q}}\frac{PW50_{Q,P}}{a_{Q,P}}$$

8. **From future file design, you know the Target values: $W_{R,T}$, d_T , $g_{R,T}$. Calculate $d_{F,Q,T}$, $a_{Q,T}$ and $PW50_{Q,T}$. "T" subscript denotes Target design condition for head and magnetic spacing.**

SNR PROJECTION (cont'd 4)

9. **Signal-to-Transition-Noise under future file design Target conditions is projected to be:**

$$\left(\frac{S_0}{N}\right)^2_{\tau,Q,T} = \left(\frac{S_0}{N}\right)^2_{\tau,Q,P} \times \frac{W_{R,T}}{W_{R,P}} \frac{PW50_{Q,T}}{PW50_{Q,P}} \frac{a_{Q,P}}{a_{Q,T}}$$

Transition noise only

10. **Although not generally measured, the Equalized (PR4) signal and noise determines error-rate performance. The Equalized SNR, denoted by the subscript "e," under Target and (Product head) measured conditions are related by the following, where $L_{Q,P} \geq (\pi a)_{Q,P}$ and $L_{Q,T} \geq (\pi a)_{Q,T}$ indicate the linear bit spacing for measured (with Product head) and Target conditions.**

$$\left(\frac{S_0}{N}\right)^2_{\tau,Q,T,e} = \left(\frac{S_0}{N}\right)^2_{\tau,Q,P,e} \times \frac{W_{R,T}}{W_{R,P}} \frac{L^2_{Q,T}}{L^2_{Q,P}} \frac{a^2_{Q,P}}{a^2_{Q,T}}$$

SNR PROJECTION (cont'd 5)

11. **Because the Equalized SNR is not usually measured and the *a priori* required value is subject to modeling uncertainties, we will relate the $(So/N)^2_{\tau, Q, T, e}$ and $(So/N)^2_{\tau, Q, P, e}$ to a known empirical condition:**

    The equalized SNR for the "Product" head (used to evaluate the Target media) and the "Product" disk designed to work with it, under conditions which produces an acceptable error rate (e.g., 10^{-10}). Double subscript "P, P" will denote the "Product" disk / "Product" head combination.

$$\frac{(S/N)^2_{\tau,Q,P,e}}{(S/N)^2_{\tau,P,P,e}} = \left(\frac{W_{c,P}}{W_{c,Q}}\right)\left(\frac{L_{Q,P}\, a_{P,P}}{L_{P,P}\, a_{Q,P}}\right)^2$$

where $W_{c,P}$ and $W_{c,Q}$ are cross-track correlation widths (grain size) of the Product disk and Qualification disks. This can be rewritten as:

$$\left(\frac{(S/N)^2_{\tau,Q,P,e}}{(S/N)^2_{\tau,P,P,e}}\right) = \left(\frac{(S/N)^2_{\tau,Q,P}}{(S/N)^2_{\tau,P,P}}\right) \times \left(\frac{PW50_{P,P}\, a_{Q,P}}{PW50_{Q,P}\, a_{P,P}}\right) \times \left(\frac{L_{Q,P}\, a_{P,P}}{a_{Q,P}\, L_{P,P}}\right)^2$$

SNR PROJECTION (cont'd 6)

12. Having related $(So/N)^2_{\tau, Q, P, e}$ to $(So/N)^2_{\tau, P, P, e}$ we can write the Target equalized SNR excess or deficit as

$$\left(\frac{(S/N)^2_{\tau,Q,T,e}}{(S/N)^2_{\tau,P,P,e}} \right) = \left(\frac{(S/N)^2_{\tau,Q,P}}{(S/N)^2_{\tau,P,P}} \right) \times \left(\frac{PW50_{P,P}}{PW50_{Q,P}} \right) \times \left(\frac{a_{Q,P}}{a_{P,P}} \right) \times \left(\frac{L_{Q,T}\ a_{P,P}}{a_{Q,T}\ L_{P,P}} \right)^2 \times \left(\frac{W_{R,T}}{W_{R,P}} \right)$$

This last equation is interpreted as follows. If the Product $(S/N)^2_{\tau, P, P, e}$ is also required for the Target design to obtain some acceptable error rate (e.g., 10^{-10}), then the right side of the above equation must be ≥ 1. Excess or deficit Target equalized SNR can be translated into excess or deficit areal density capability of the disk in question, subject to the constraint $L_{Q,T} \geq (\pi a)_{Q,T}$.

EXAMPLE T.C. Arnoldussen, IBM, April 27, 1998

	P-disk / P-head	**Target Design**	**Measured Q-disk / P-head**	**Projected Q-disk / T-head**
Areal Density, Gb/in^2	1.0	2.0	1.24	2.0
Linear Densiity, kbpi	133	189	165	189
Track Density, ktpi	7500	10600	7500	10600
Track Pitch, µm	3.39	2.4	3.39	2.4
Read Width, µm	2.26	1.6	2.26	1.6
Read Gap, µm	0.3	0.2	0.3	0.2
Mag. Spacing "d," nm	91	69	91	69
Hc, Oe	2800	3000	3000	3000
Mr, emu/cm^3	230	230	230	230
δ, nm	35	25	25	25
S^*	0.8	0.8	0.8	0.8
Effective Mag. Spacing "d_F," nm	107	80	103	80
W&C "a," nm	57	40	46	40
PW50, nm	390	279	365	279
PW50/πa	2.18	2.20	2.54	2.20
Wc, nm	(25)	(18)	(18)	(18)
$(So/N)^2_{\tau, P, P}$	1000 (30dB)			
$(So/N)^2_{\tau, Q, P}$			1624 (32.1dB)	
$(So/N)^2_{\tau, Q, T}$		1000 (30dB)		1000 (30dB)
$[(So/N)^2_{\tau, Q, T, e}] \div [(So/N)^2_{\tau, P, P, e}]$	1.0	1.0		1.0

SUMMARY

Outlined procedure for projecting Signal-to-RMS Noise in a future system from Media SNR Measurements made with a currently available head.

Outlined extended procedure to estimate relative post-equalization SNR in future Target design, based on known error rate performance for an existing Product head and disk combination.

Electronic noise was ignored here, but can readily be included in projecting future file design performance.

T.C. Arnoldussen, IBM, April 27, 1998

Signal-to-Noise Ratio Budgets in Disc Drive Design

Giora J. Tarnopolsky
Advanced Recording Technology
Seagate Advanced Concepts

IIST - Santa Clara University
April 27, 1998

Practitioner's Guide to Noise & SNR

❍ Acknowledgments

- H. Neal Bertram
- Eric Champion
- Gordon Hughes
- Gary Rauch
- Rajiv Ranjan
- Hans Richter
- Norm Talsoe

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Outline

Outline

❍ **Media noise properties**
- Origin
- Localization
- Parametric dependencies
- Models

❍ **Media noise power estimation**
- Estimate
- Noise budget
- Experimental verification

❍ **SNR budgets and drive design**

❍ **SNR at high recording densities. Jitter limits.**

❍ **Conclusions**

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Medium Noise Partial Bibliography

- **There is a vast literature of media noise research. A limited sample of references follows:**

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Medium Noise Partial Bibliography (cont')

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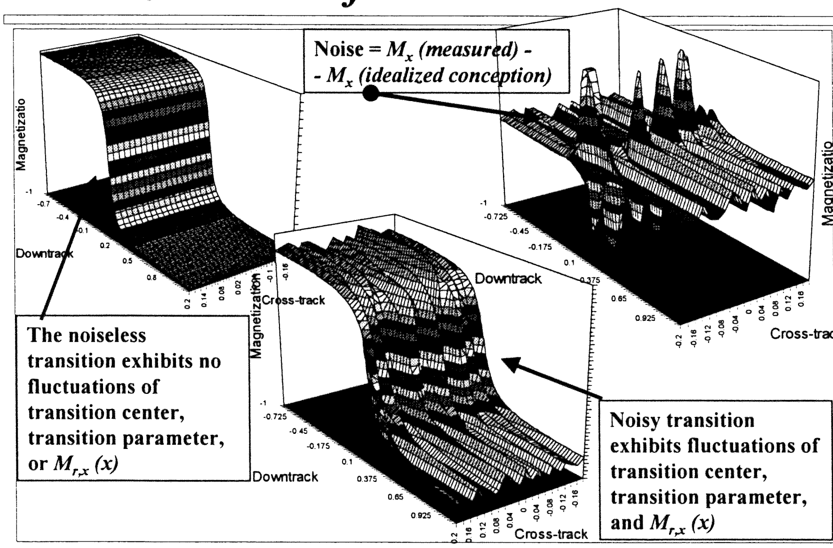
- Microtrack Model of Recorded Transitions, J. Carroselli & J. K. Wolf, IEEE Trans. Mag. vol. 32, 3917 (1996)
- Media Noise and Signal-to-Noise Ratio Estimates for High Areal Density Recording, Giora J. Tarnopolsky & P. R. Pitts, J. Appl. Phys. vol. 81, 4837 (1997)

Media Noise Properties

What is Medium Noise?

- **The reproduce voltage results from the read head sensing the "magnetic charge" of the media: - $\nabla \cdot M(x)$, the divergence of the magnetization**
- **The medium is an array of columnar magnetic grains having: irregular boundaries, dispersion in the direction of the uniaxial anisotropy axis, intergranular exchange and magnetostatic coupling, and coercivity and remanence that vary from grain to grain.**
- **Superimposed onto the intended magnetization reversals (user data) there appear the magnetization spatial fluctuations. They result in unwanted and unpredictable deviations of the reproduce voltage from ideal response.**
- **User-intended bit transitions do not match grain boundaries.**

Idealized View of Transitions & Noise



Signal & Noise Power vs. Read Width

- **The recorded "noiseless" signal is in phase across the track. Its power is proportional to the square of the track width.**

$$\text{Signal Power} = \left[\sum_{i=1}^{N} V_i(x)\right]^2 = N^2 V^2_{(\mu track)}(x) = \left(\frac{\text{Read width}}{\Delta z}\right)^2 \cdot V^2_{(\mu track)}(x)$$

- **The noise components are incoherent. The noise power is proportional to the track width.**

$$\text{Noise Power} = \left[\sum_{i=1}^{N} \delta V_i(x)\right] \cdot \left[\sum_{j=1}^{N} \delta V_j(x)\right] = \sum_{i=j}^{N} \delta V_i(x) \delta V_j(x) + \sum_{i \neq j}^{N} \delta V_i(x) \delta V_j(x) =$$

$$= N < \delta V^2(x) > = \left(\frac{\text{Read width}}{\Delta z}\right) \cdot < \delta V^2(x) >$$

- **Signal-to-noise power ratio $\propto$ Read width**

Medium Noise Characteristics

- Medium noise appears most prominently in regions where $M(x) \approx 0$
  - at the magnetic transition - hence "transition noise"
  - at the track edge
  - in the servo fields
- Once recorded, the noisy magnetization pattern does not change with time. It is read out by the magnetic channel just as the data is. Thus, the noise spectrum shows, in a spectrum analyzer, a shape proportional to the envelope of the fundamental and its harmonics
- Equalization enhances high-frequency components (generally, to slim the isolated pulse.) It thus enhances the high frequency component of the medium noise.

How is Media Noise Modeled?

- Media strips, microtracks (Arnoldussen, Carroselli & Wolf)
  - Media divided into stripes along the downtrack co-ordinate, each stripe exhibiting fluctuations in transition position and sharpness
- Micromagnetic models (Bertram and collaborators, Zhu and collaborators)
  - Media volume subdivided into regular hexagonal tiles
  - Anisotropy axis orientation, magnetostatic interactions, inter- and intra-granular exchange interactions
  - Proved that exchange-coupled grains are noisier
- Analytical models of the readback voltage (Bertram and collaborators.) Example will follow.

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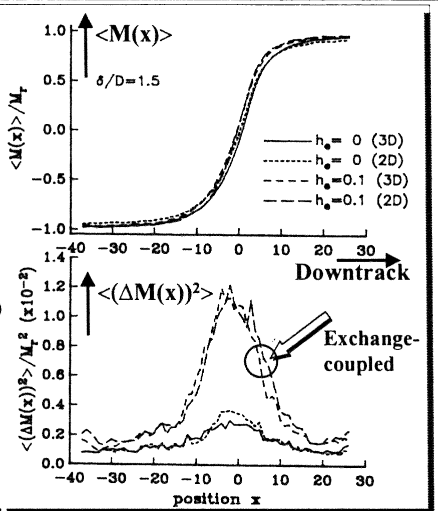
Media Noise Higher Where M(x)=0

- Ensemble mean and variance of the transition profiles for planar isotropic longitudinal films for different intergranular exchange coupling, and both 2D and 3D random easy axis orientation. A large variance occurs at the transition center. The exchange-coupled film shows much higher magnetization noise in all cases. Bertram & Zhu, IEEE Trans. Mag. vol. 27, 5043 (© IEEE 1991)
- The transition noise power P_n increases with the number of transitions per unit length, or density.
- The measurement of $P_n(density)$ affords a determination of media intrinsic noise parameters.



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Media Noise Reduction

How to reduce media noise?

- **Achieve grains of uniform size. The reduction of the variance of columnar sizes will improve media performance, and is of more immediate concern than media thermal decay.**
  - **Underlayers: Epitaxial growth**
  - **Clean sputtering systems**
- **Reduce exchange coupling between grains**
  - **Non-magnetic material segregation to boundary**
- **Reduce the volume where noise originates**
  - **Shorter transition length**
  - **Lower $M_r\delta$**
  - **Higher H_c**

Media Noise & High-Density Recording

- "Transition jitter", σ_{jitter} = 3 to 10 nm
- For linear recording densities ≤ 1,000 kfci:
  - σ_{jitter} for given signal-to-noise ratio *SNR*?
  - (σ_{jitter} /bit length *B*) vs. *PW50*, *SNR*, and density?
  - Grain sizes and uniformity for n × 10 Gbit/in^2?
- Recorded bit boundary: shifted from ideal position
- Pulse width (*PW50*): fluctuates around track
- Peak amplitude V_0: varies bit-to-bit due to head

Media Noise Power Estimation

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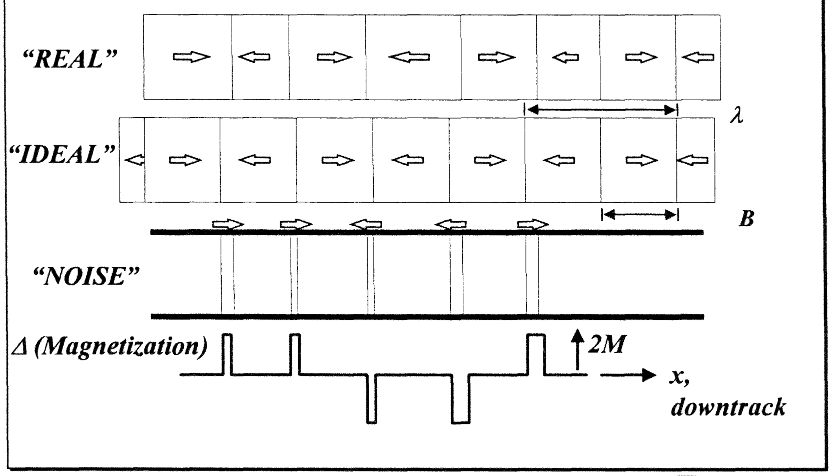
Transition Jitter: Magnetization Noise



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"Real" & Ideal Waveforms & Noise

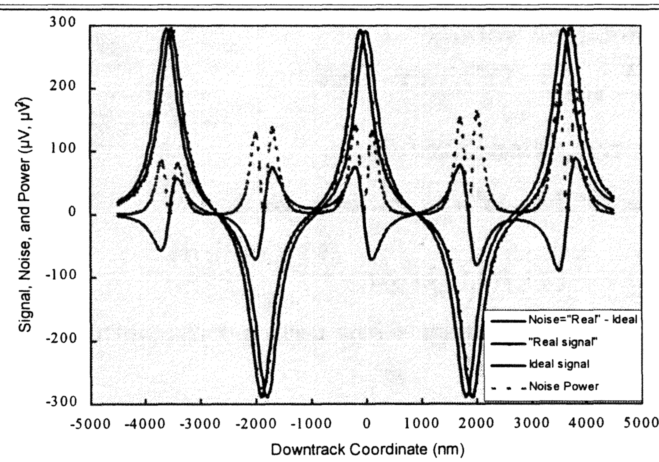

Chart shows i) an ideal noiseless trace, ii) a trace affected by transition jitter (shifted from nominal), iii) the difference between real and ideal voltages - NOISE, and iv) the noise power

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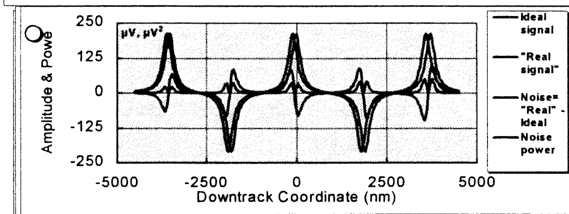
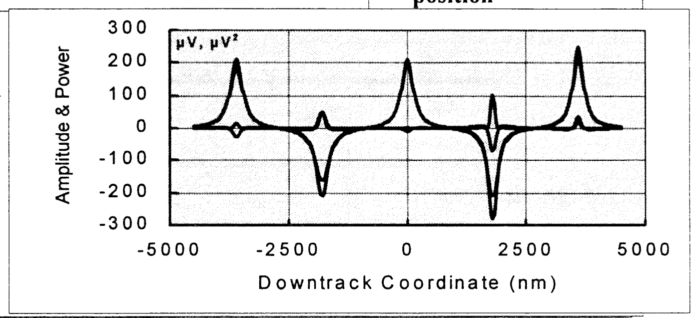
"Real" & Ideal Waveforms & Noise



← **"Jitter:" pulse center shifted from ideal position**

"PW50:" pulse width broadening or slimming from ideal (a_x fluctuations) →



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Replay Voltage

- **Replay noiseless voltage**

  $$\tilde{V}(x) = \sum_{n=-\infty}^{n=\infty} (-1)^n V_{iso}(x - nB)$$

> •Bertram & collaborators, 1987 - 1996, Noise formalism
>
> •See: Bertram: Theory of Magnetic Recording

- **Add, say, transition jitter δx_n**

  $$V(x) = \Sigma_{n=-\infty}^{n=\infty} (-1)^n V_{iso}(x - nB - \delta x_n),$$

  $$\therefore V(x) = \tilde{V}(x) + \Sigma(-1)^n \delta x_n \frac{\partial V_{iso}(x - nB)}{\partial x}$$

- **Signal = noiseless voltage + *one term per fluctuating property***
- **Assume Lorentzian pulses, compute noise power**

Noise Power Density

$$V_{iso}(x) = \frac{2A}{\pi PW50}\left(1\Big/1+\left(\frac{2x}{PW50}\right)^2\right) = V_0\left(1\Big/1+\left(\frac{2x}{PW50}\right)^2\right)$$

Lorentzian
A = area, V_0 amplitude

- Consider position fluctuations, pulse-width fluctuations, and MR head amplitude fluctuations:

  σ_{jitter} σ_{PW50} σ_{V_0}

- Noise power density at head's terminals:

$$PSD(k) = \frac{|V_{iso}(k)|^2}{B} \times \left[k^2\sigma^2_{jitter} + \frac{k^2\sigma^2_{PW50}}{4} + \frac{\sigma^2_{V_0}}{V_0^2}\right]$$

k=2π/λ, FT variable
$V_{iso}(k)$ = FT isolated pulse

Noise Power

- Noise power *density* equalized by channel transfer function *G(k)*:

$$PSD(k) = \frac{|G(k)V_{iso}(k)|^2}{B} \times \left[k^2\sigma^2_{jitter} + \frac{k^2\sigma^2_{PW50}}{4} + \frac{\sigma^2_{V_0}}{V_0^2}\right]$$

k=2π/λ, FT variable
$V_{iso}(k)$ = FT isolated pulse

- The broad-band media noise power = integral of *PSD(k)* over all frequencies:

$$P_n = \text{broad-band noise power} =$$
$$= \frac{\pi}{2}V_0^2\frac{\sigma^2_{jitter}}{B \cdot PW50} \times \left(1 + \frac{1}{4}\frac{\sigma^2_{PW50}}{\sigma^2_{jitter}}\right) + \frac{\pi\sigma^2_{V_0}PW50}{4B}$$

G. J. Tarnopolsky et al., J. Appl. Phys. vol. 81, 4837 (1997)

Media jitter scaled by bit length and PW50

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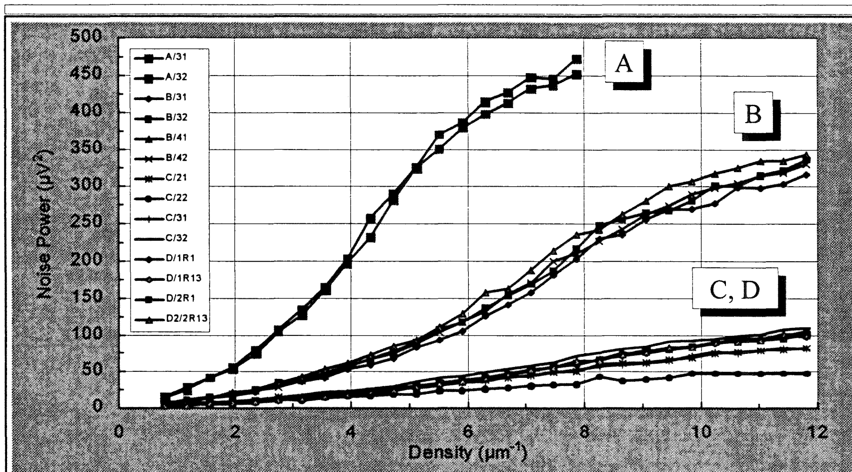
Transition Noise vs. Density: Many Alloys



- **Get σ_{jitter} from initial slopes.**

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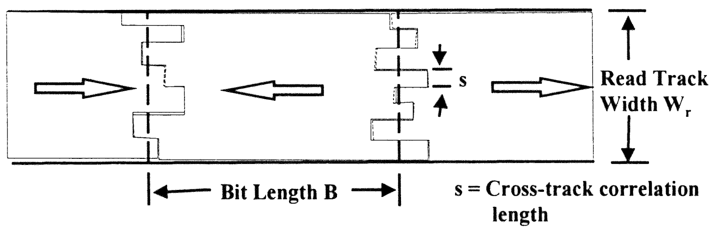
Transition Noise vs. Density: Many Alloys

Table I. Experimental Results

Disk	H_c	$M_r\delta$	V_o	PW50	OW	Iw	σ	σ_{low}	σ_{high}	#
	kOe	memu/cm²	μV	nm	dB	mA	nm	nm	nm	
A/31	2.2	1.00	383	378	39	34		8.0	12.5	
A/32	2.2	1.00	382	372	39	34		7.9	12.3	
<A>	**2.2**	**1.00**	**383**	**375**	**39**	**34**		**8.0**	**12.4**	**2**
B/31	2.5	0.70	311	295	41.6	35		5.8	9.3	
B/32	2.5	0.70	309	298		35		6.2	9.6	
B/41	2.5	0.70	305	303		35		6.2	9.9	
B/42	2.5	0.70	300	300		35		6.1	9.7	
<B>	**2.5**	**0.70**	**306**	**299**	**41.6**	**35**		**6.1**	**9.6**	**4**
C/21	2.77	0.68	226	296	36.5	45	5.4			
C/22	2.77	0.68	206	295		45	4.2			
C/31	2.77	0.68	221	294		45	5.7			
C/32	2.77	0.68	239	299		45	6.1			
<C>	**2.77**	**0.68**	**223**	**296**	**36.5**	**45**	**5.4**			**4**
<D>	**3**	**0.55**	222	**290**	**39**	**45**		3.7	**6.4**	**4**

Transition Jitter vs. Read Track Width



The number of sub-transitions is given by W_r/s. The relation between the net transition position variance and the sub-transition variance is:

$$\sigma_{jitter}^2 = \frac{s}{W_r}\,\sigma_s^2$$ (H.N. Bertram, Theory of Magnetic Recording, p. 317.)

Analytic modeling of the transition noise with a tanh transition shape yields:

$$\sigma_{jitter} = \frac{\pi^2 a}{4}\sqrt{\frac{s}{3\,W_r}}$$ (B. Slutsky and H.N. Bertram, *IEEE Trans. Magn.*, vol. 30, (no. 5), pp. 2808-2817, 1994.)

Eric Champion

SNR Budget

The Signal-to-Noise Ratio and Its Applications

- **Signal-to-Noise Ratio - a designer's tool ...**
  - **... is an *averaged*, statistical measure of goodness**
  - **... can be readily measured**
  - **... components' properties have direct, calculable SNR consequences**
  - **... ignores non-random components and system effects**
- **Bit Error Rate - the users' demand for data integrity ...**
  - **... is an *instantaneous* proof of goodness**
  - **... value depends on channel properties - a little harder to measure and even harder to implement new ideas in experiments**
  - **... components' and system properties affect BER in complex ways**

Signal-to-Noise Ratio and Bit Error Rate

- **Usefulness: the SNR correlates with the BER at the components' level, as long as the noise includes random sources only.**
- **Although BER generally correlates with SNR at the spin-stand level, it is easy to see how this correlation could break down.**

  Example: two similar disks, one with a long defect
- **In the drive, there are sources of noise that are less easily accountable by the measured (head / media / preamp) SNR, such as noise from the drive's digital circuitry or electromechanical subsystems. The (SNR - BER) correlation may seem corrupted.**
- **An *SNR value above a certain channel-dependent* SNR *threshold is a necessary*, but not sufficient, *requirement for performance.***

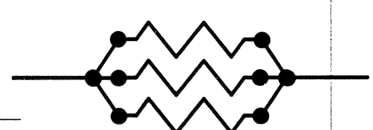
Signal and Noise Sources

- **Signal**
  - **Medium M_rT, H_c, medium thickness, overcoat, ...**
  - **Head reader: ΔR, gap length, width, overcoat, …**
  - **Head writer: saturation magnetization, field risetime, gap length, width, …**
  - **System: fly-height, TMR**
- **Noise**
  - **Medium noise: on track, erase bands, orientation ratio**
  - **Head: writer saturation effects, pole trimming; reader: Johnson noise, instabilities**
  - **Preamp: input noise voltage, sense-current noise**
  - **System: equalization, data rate (bandwidth), TMR**

SNR System Considerations

- **SNR sources add "in parallel:"**

$$SNR_{sys} = \frac{\text{Signal Power}}{\text{Incoherent Sum of Noise Powers}}$$



$$SNR_{sys} = \frac{V^2}{P_{medium} + P_J + P_{voltage} + P_{current} + \cdots}$$

$$\frac{1}{SNR_{sys}} = \frac{1}{SNR_m} + \frac{1}{SNR_J} + \frac{1}{SNR_v} + \frac{1}{SNR_c} + \cdots = \frac{1}{SNR_m} + \frac{1}{SNR_{h\&e}}$$

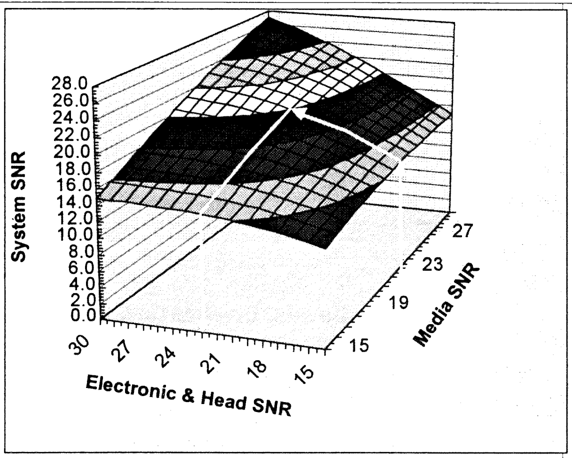
However, not all terms strictly equivalent, medium noise is colored

- **Thus,**

$$SNR_{sys} < SNR_{lowest}$$

The worst performing subsystem becomes the limiting barrier - … "media noise limited", etc. ...

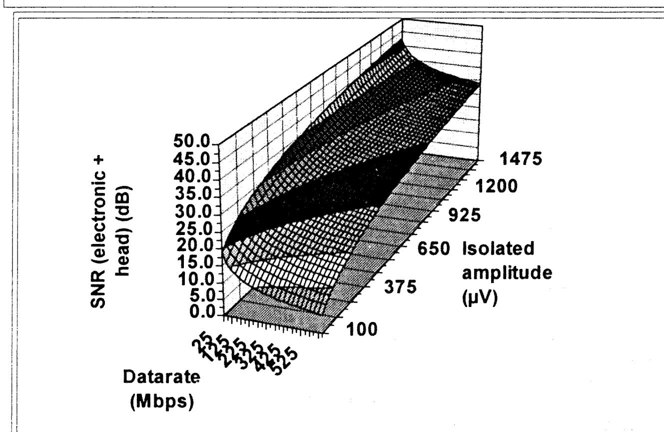
Interdependence of Subsystems' SNR



The worst performing subsystem SNR_{lowest} limits the overall system performance, $SNR_{sys} < SNR_{lowest}$

With a threshold SNR_{sys} condition, changes in subsystems' SNR affect the performance demanded of other components

Bandwidth & Head Output



The media SNR varies mildly with bandwith. The electronic SNR drops with increasing datarate. In order to maintain the necessary SNR_{sys} more performance is required elsewhere in the system.

SNR Budget: What σ_{eff} For 6 Gbit/in^2?

- 16:1 BPI/TPI Ratio: 310 kbpi x 19.4 ktpi
- Density, HF 329 kfci
- Density, MF 165 kfci
- V_{0-p} (Iso) 500 µV
- V_{0-p} (MF) 251 µV
- Radius (ID) 0.627 in
- Bandwidth 49 MHz
- Input noise voltage 0.5 nV/rt Hz
- Input noise current 20 pA/rt Hz
- Resistance 55 Ω
- B = bit length 82 nm
- PW50 ~8.4 µin

- Preamp noise 12.3 µV^2
- Current source 59.3 µV^2
- Johnson noise, $4k_BT\Delta fR$ 46.6 µV^2
- Electronics + Johnson 118.2 µV2
- SNR(E) 27.3 dB
- If 70% of the total noise is due to the media and 30% is due to electronics then σ_{eff} = 3.5 nm and the total SNR = 22 dB.
- The SNR required to achieve an on-track error rate of 10^{-7} is about 21 dB. For this case the media noise component is 382 µV^2. This leads to an effective jitter of σ_{eff} = 4.1 nm.

SNR Estimates and Trends for Higher Areal Density

SNR Estimate

- **Peak amplitude of linear superposition of Lorentzian pulses**

$$V_{peak}(1/B) = V_0 \times \frac{\pi\cdot\frac{PW50}{2B}}{\sinh\left(\pi\cdot\frac{PW50}{2B}\right)} = V_0\times\frac{(\pi U/2)}{\sinh(\pi U/2)}$$

U=channel density

Comstock & Williams (1973)

- **SNR = (Peak mid-frequency signal power)/(broad band noise power of high-frequency fundamental)**

$$SNR = \frac{V_{peak}^2(1/2B)}{P_n(1/B)} = \frac{\left(\frac{2B\cdot PW50}{\pi\sigma_{jitter}^2}\right)}{\left[1+\frac{1}{4}\frac{\sigma_{PW50}^2}{\sigma_{jitter}^2}+\frac{1}{2}\frac{\sigma_{V_0}^2}{V_0^2}\cdot\left(\frac{PW50}{\sigma_{jitter}}\right)^2\right]}\times\frac{(\pi U/4)^2}{\sinh^2(\pi U/4)}$$

1T, 1/(2T) hi-freq, "1/(4T)" mid-freq

Could lump all noise sources into σ_n [Spectrum analyzer]

Resolution of Lorentzian Waveform

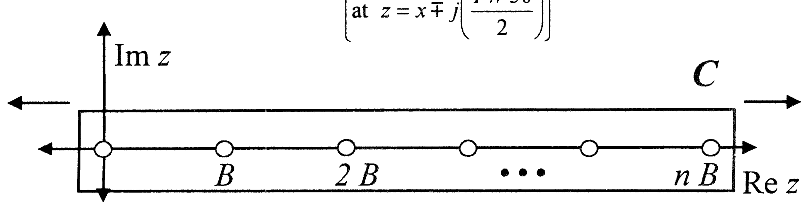
- **A rainy afternoon in the complex plane**

$$\frac{1}{2\pi j}\oint_C \frac{\pi}{B}\frac{V_{iso}(x-z;PW50)}{\sin\left(\frac{\pi z}{B}\right)}dz=$$

R. L. Comstock & M. Williams, IEEE Trans. Magn., MAG-9(3), 342 (1973)

Morse & Feshbach, Vol. I, Ch. 4, PP. 378 - 414

$$= \sum_{n=-\infty}^{\infty}(-1)^n V_{iso}(x-nB;PW50) + \left\{\begin{array}{l}\text{residues of}\\ V_{iso}(x-z;PW50)\\ \text{at } z = x\mp j\left(\frac{PW50}{2}\right)\end{array}\right\} = 0$$



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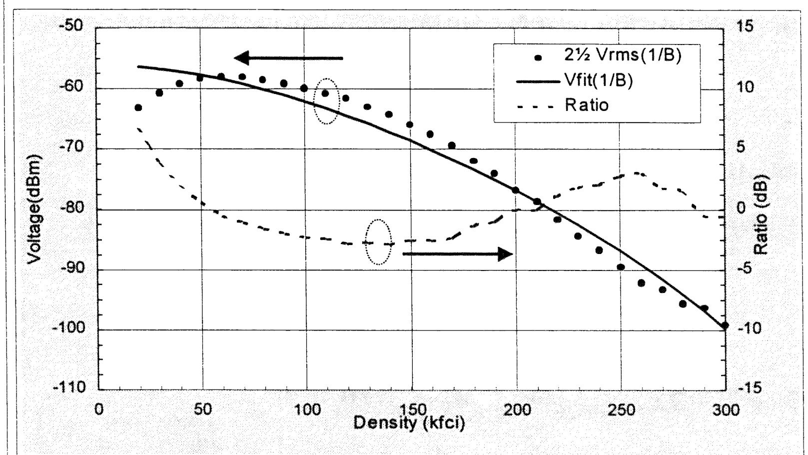
TAA Formula vs. Experiment



- **From 30 to 300 kfci, data & fit differ by < 3 dB**

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SNR Estimate

- ***SNR*** **= (Peak *mid-frequency* signal power)/(broad band noise power of *high-frequency* fundamental)**

1T, 1/(2T) hi-freq, "1/(4T)" mid-freq

$$SNR = \frac{V_{peak}^2(1/2B)}{P_n(1/B)} = \frac{\left(\dfrac{2B \cdot PW50}{\pi \sigma_{jitter}^2}\right)}{\left[1 + \dfrac{1}{4}\dfrac{\sigma_{PW50}^2}{\sigma_{jitter}^2} + \dfrac{1}{2}\dfrac{\sigma_{V_0}^2}{V_0^2} \cdot \left(\dfrac{PW50}{\sigma_{jitter}}\right)^2\right]} \times \frac{(\pi U/4)^2}{\sinh^2(\pi U/4)}$$

Could lump all noise sources into σ_n [Spectrum analyzer]

SNR Estimate (cont...)

- **SNR definition and formula:**

$$SNR = \frac{2B \cdot PW50}{\pi \sigma_n^2} \times \frac{(\pi U/4)^2}{\sinh^2(\pi U/4)}$$

- **Media & head noise < total system noise**
- **Use P_n = fraction α of total noise P_{system} , $\alpha = 3/4 < 1$**

$$SNR_{sys} = \alpha \frac{2B \cdot PW50}{\pi \sigma_n^2} \times \frac{(\pi U/4)^2}{\sinh^2(\pi U/4)}$$

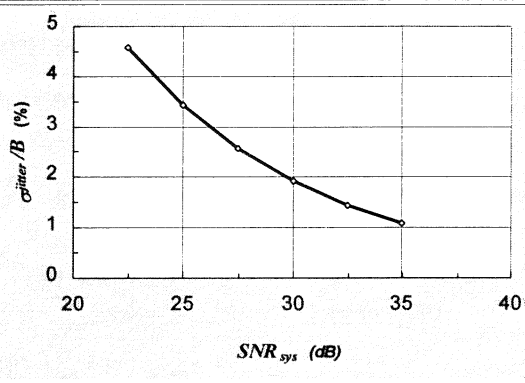
- **For U = 2.5, $\sigma_{PW50} = \sigma_{jitter}/4$, $\sigma_{V0} = 0$,**

$$SNR_{sys}(U = 2.5) \cong 0.5\alpha\left(B / \sigma_{jitter}\right)^2$$

At constant channel density, ratio (σ_{jitter}/bit length) is function of *SNR* alone

Ratio Transition Jitter/Bit Length vs. SNR_{sys}

- **If the head gap scales with density, U = constant**
- **Jitter/Bit = *f(SNR)* only**
- **To maintain any given SNR, σ_{jitter} must scale with B**



- **Example: 500 kfci, B = 50 nm, PW50 = 125 nm, σ_{jitter} = 1.5 nm @ 26 dB**

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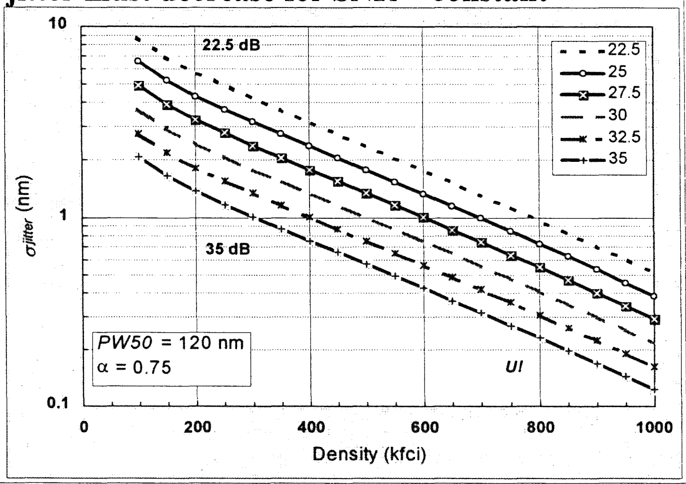
Jitter vs. Density, Various System SNR's

- Media jitter must decrease for SNR = constant



Giora J Tarnopolsky

IIST - Santa Clara University
1998 SNR & Disc Drive Performance Symposium

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Information, the way you want it.

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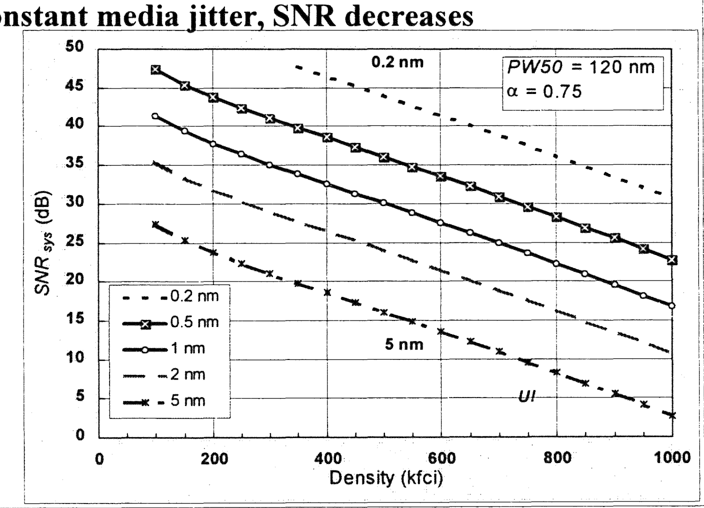
System SNR, Various Media Jitter Values

- At constant media jitter, SNR decreases



Giora J Tarnopolsky

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1998 SNR & Disc Drive Performance Symposium

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Information, the way you want it.

• 3-7 nm jitter for todays media

Areal Density Estimators

- H. Neal Bertram has postulated an expression for media jitter,

$$\sigma^2_{jitter} = \left(\frac{\pi}{2}\right)^4 a_x^2 \frac{s}{3(RW)}$$

Cross track correlation length

B. Slutsky & H. N. Bertram, IEEE Trans. Mag. vol. 30, 2808 (1994)

which allows to link the SNR values derived here to properties of the media (a_x) and of the recording system (*RW*).

a_x = transition parameter
***s* = cross-track correlation length**
***RW* = head's read width**

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Outline

Outline

- **Media noise properties**
  - Origin
  - Localization
  - Parametric dependencies
  - Models
- **Media noise power estimation**
  - Estimate
  - Noise budget
  - Experimental verification
- **SNR budgets and drive design**
- **SNR at high recording densities. Jitter limits.**
- **Conclusions**

Media Noise & SNR for High Density Recording

- **Expressions for:**
  - **Noise Power Spectral Density**
  - **Total Noise Power**
  - **SNR(mid-freq/broad band HF noise)**
- **For constant channel density, ratio (jitter/bit length) depends only on SNR, not on density**

$$SNR_{sys}(U = 2.5) \cong 0.5\alpha\left(B / \sigma_{jitter}\right)^2$$

- $\sigma_{jitter} / B = 3\%$ **@ 26 dB SNR, 1.5 nm for 500 kfci**
- **At 1000 kfci, transition jitter < 1 nm. Difficult to accomplish. Thermal & coercivity issues.**

S. Charap et al., TMRC '96

Conclusions

- **Media noise power unambiguous index of media performance. As a function of commonly measured properties,**

$$P_n = \text{broad-band media noise power} = \frac{\pi}{2} V_0^2 \frac{\sigma^2_{effective}}{B \cdot PW50} \propto$$

$$\propto \left[M_r \delta \text{ \& head effiency}\right]^2 \frac{(\text{effective jitter})^2}{\text{magnetic bit length } x \; PW50}$$

- **Signal-to-noise ratio**

$$SNR_{sys} = \alpha \frac{2B \cdot PW50}{\pi \sigma^2_{effective}} \times \frac{(\pi U / 4)^2}{\sinh^2(\pi U / 4)}$$

G. J. Tarnopolsky et al., J. Appl. Phys. vol. 81, 4837 (1997)

- ***SNR_{sys}* above threshold is *necessary condition* for BER performance. Powerful design tool.**

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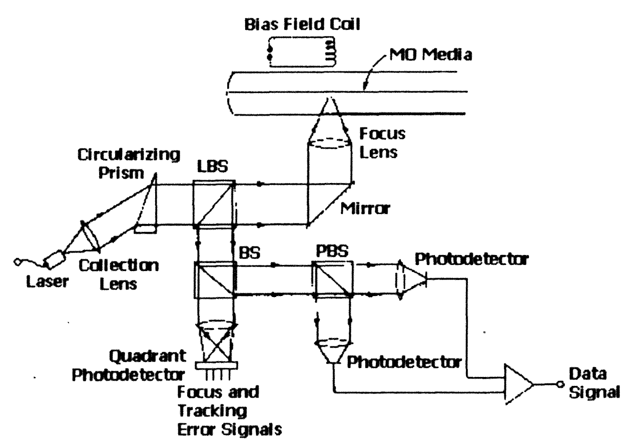
Signals & Noise in Optical Recording

Rob Lynch
April 27, 1998

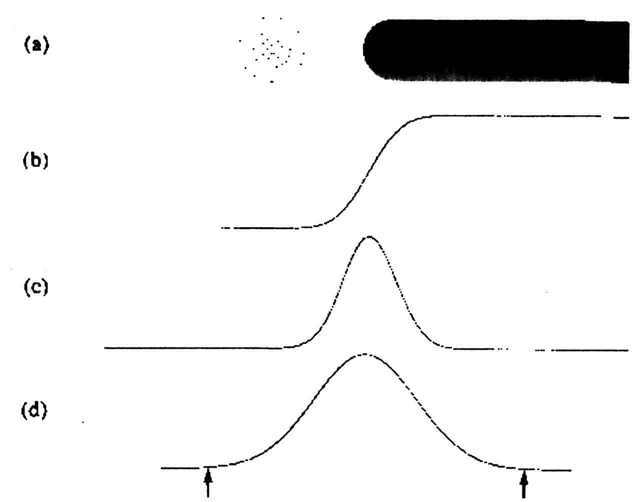
Generic MO Head



Maximizing Signal

- Signal ∝ laser power × Kerr rotation
- materials
- read power dependence
- detection methods

MO Signal Formation

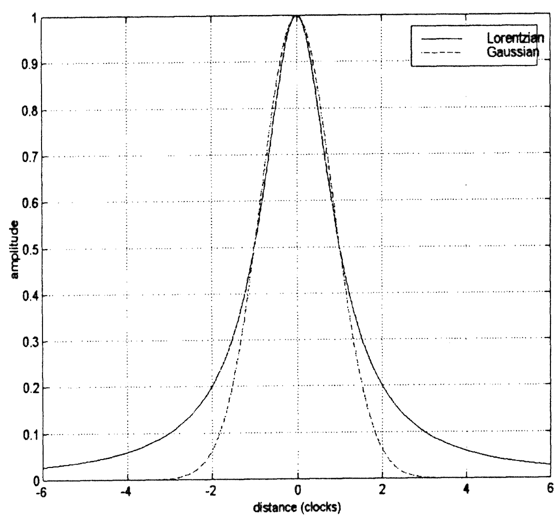


X-site
Impulse response
Fourier X-form
[channel X-fer function]

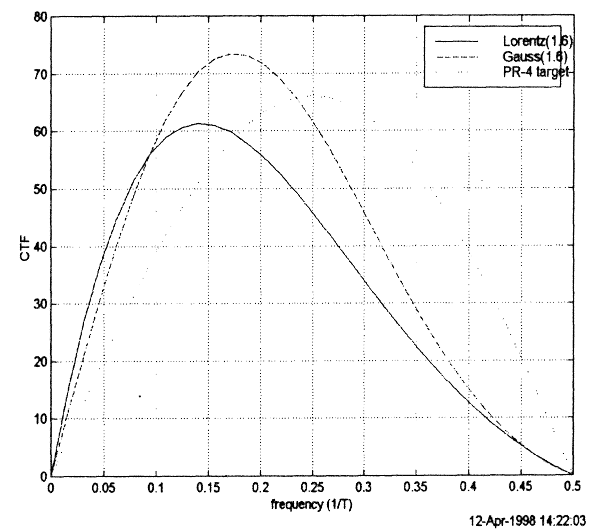
- mixed NRZ & NRZI data (factor of 2 error)

- optical channel passes DC.

Comparison of Pulse Shapes

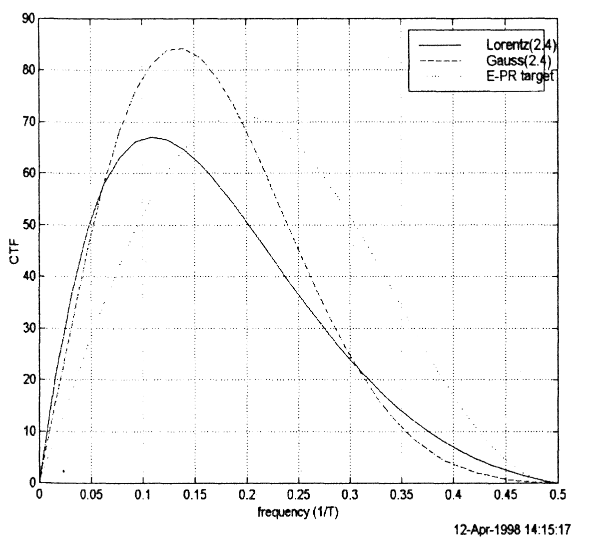


CTF Comparison
at PW50/Tc=1.6



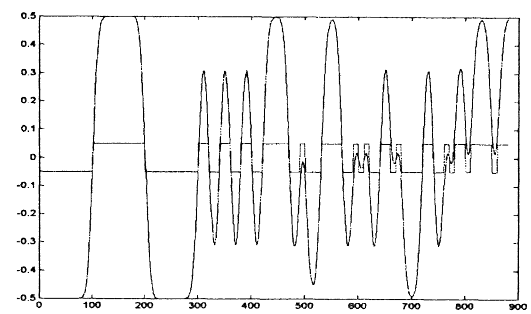
Can't run MO channel density up like magnetic recording (Gaussian vs Lorentzian)

CTF Comparison
at PW50/Tc=2.4



Signal of Lorentzian @ higher freq. it's then have more capacity despite SNR equivalence

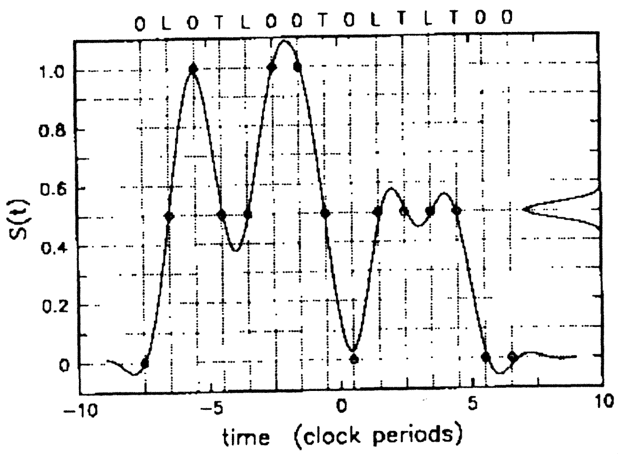
Noise-free MO Signal



- Signal contains DC
- Fundamental shape is Gaussian

Different
optimum channel.
∟ PR1 & PR2

PR-I Equalized Signal



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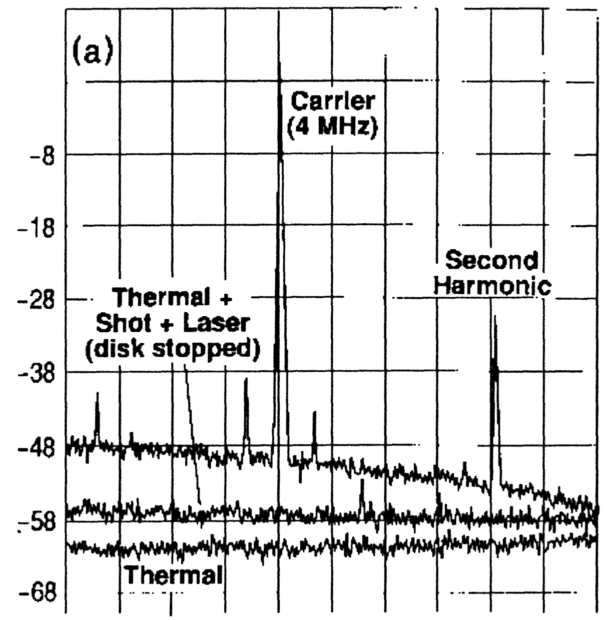
Noise Sources



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Minimizing Noise

- reduce laser noise
- balanced photodetection
- good electrical design
- thermal design of disk layers
- suppress media noise

Other Impairments

- Writing noise
- Bloom
- Read Crosstalk

looks a lot like signal

Signal-to-Noise Ratio Budgets in Disc Drive Design

Giora J. Tarnopolsky
Advanced Recording Technology
Seagate Advanced Concepts

IIST - Santa Clara University
April 27, 1998

Media Noise & High-Density Recording

- "Transition jitter", σ_{jitter} = 3 to 10 nm
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  - σ_{jitter} for given signal-to-noise ratio *SNR*?
  - (σ_{jitter} /bit length *B*) vs. *PW50*, *SNR*, and density?
  - Grain sizes and uniformity for n × 10 Gbit/in^2?
- Recorded bit boundary: shifted from ideal position
- Pulse width (*PW50*): fluctuates around track
- Peak amplitude V_0: varies bit-to-bit due to head

Media Noise Power Estimation

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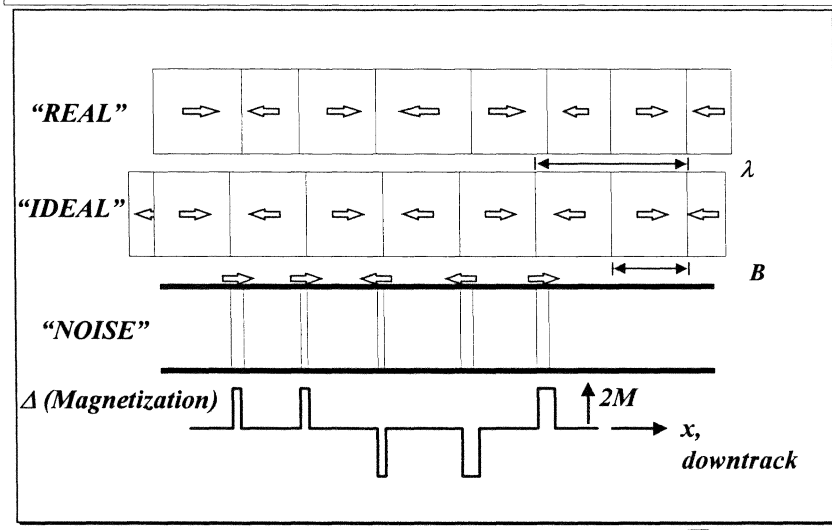
Transition Jitter: Magnetization Noise



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"Real" & Ideal Waveforms & Noise

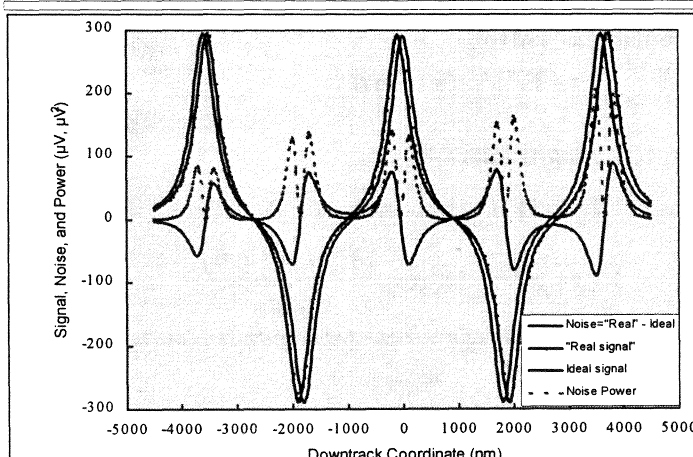

Chart shows i) an ideal noiseless trace, ii) a trace affected by transition jitter (shifted from nominal), iii) the difference between real and ideal voltages - NOISE, and iv) the noise power

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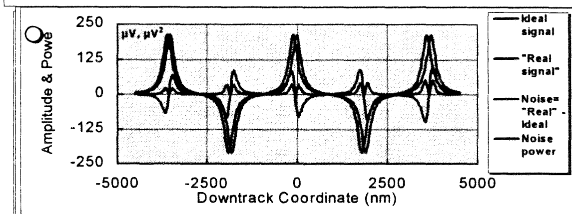
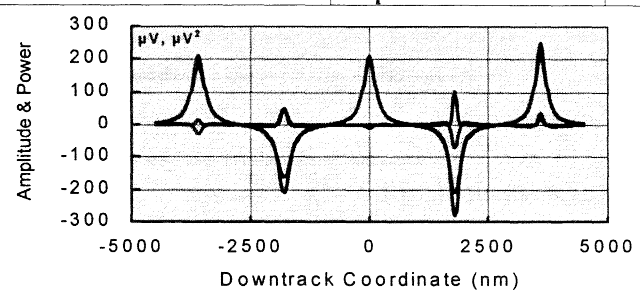
"Real" & Ideal Waveforms & Noise



← **"Jitter:" pulse center shifted from ideal position**

"PW50:" pulse width broadening or slimming from ideal (a_x fluctuations) →



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Replay Voltage

- **Replay noiseless voltage**

$$\tilde{V}(x) = \sum_{n=-\infty}^{n=\infty} (-1)^n V_{iso}(x - nB)$$

- **Add, say, transition jitter** δx_n

$$V(x) = \Sigma_{n=-\infty}^{n=\infty} (-1)^n V_{iso}(x - nB - \delta x_n),$$

$$\therefore V(x) = \tilde{V}(x) + \Sigma(-1)^n \delta x_n \frac{\partial V_{iso}(x - nB)}{\partial x}$$

- **Signal = noiseless voltage +** ***one term per fluctuating property***
- **Assume Lorentzian pulses, compute noise power**

•Bertram & collaborators, 1987 - 1996, Noise formalism

•See: Bertram: Theory of Magnetic Recording

Noise Power Density

$$V_{iso}(x) = \frac{2A}{\pi PW50}\left(1\Big/1+\left(\frac{2x}{PW50}\right)^2\right) = V_0\left(1\Big/1+\left(\frac{2x}{PW50}\right)^2\right)$$

Lorentzian
A = area, V_o amplitude

- **Consider position fluctuations, pulse-width fluctuations, and MR head amplitude fluctuations:**

  σ_{jitter} σ_{PW50} σ_{V_0}

- **Noise power density at head's terminals:**

$$PSD(k) = \frac{|V_{iso}(k)|^2}{B} \times \left[k^2\sigma^2_{jitter} + \frac{k^2\sigma^2_{PW50}}{4} + \frac{\sigma^2_{V_0}}{V_0^2}\right]$$

k=2π/λ, FT variable
$V_{iso}(k)$ = FT isolated pulse

Noise Power

- **Noise power *density* equalized by channel transfer function *G(k)*:**

$$PSD(k) = \frac{|G(k)V_{iso}(k)|^2}{B} \times \left[k^2\sigma^2_{jitter} + \frac{k^2\sigma^2_{PW50}}{4} + \frac{\sigma^2_{V_0}}{V_0^2}\right]$$

k=2π/λ, FT variable
$V_{iso}(k)$ = FT isolated pulse

- **The broad-band media noise power = integral of *PSD(k)* over all frequencies:**

$$P_n = \text{broad} - \text{band noise power} =$$

$$= \frac{\pi}{2}V_0^2 \frac{\sigma^2_{jitter}}{B \cdot PW50} \times \left(1 + \frac{1}{4}\frac{\sigma^2_{PW50}}{\sigma^2_{jitter}}\right) + \frac{\pi\sigma^2_{V_0} PW50}{4B}$$

G. J. Tarnopolsky et al., J. Appl. Phys. vol. 81, 4837 (1997)

Media jitter scaled by bit length and PW50

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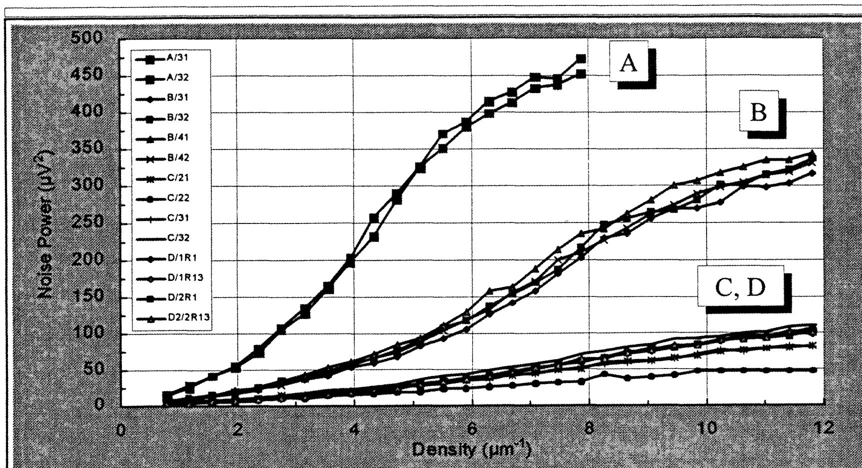
Transition Noise vs. Density: Many Alloys



- Get σ_{jitter} from initial slopes.

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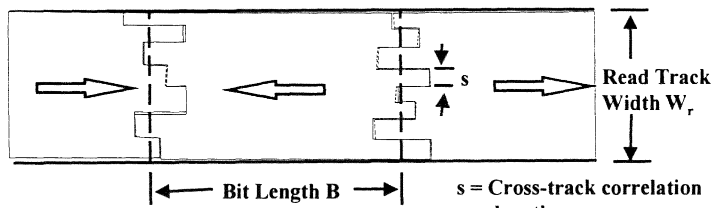
Transition Noise vs. Density: Many Alloys

Table I. Experimental Results

Disk	H_c	$M_r\delta$	V_o	PW50	OW	Iw	σ	σ_{low}	σ_{high}	#
	kOe	$\frac{\text{memu}}{\text{cm}^2}$	μV	nm	dB	mA	nm	nm	nm	
A/31	2.2	1.00	383	378	39	34		8.0	12.5	
A/32	2.2	1.00	382	372	39	34		7.9	12.3	
<A>	**2.2**	**1.00**	**383**	**375**	**39**	**34**		**8.0**	**12.4**	**2**
B/31	2.5	0.70	311	295	41.6	35		5.8	9.3	
B/32	2.5	0.70	309	298		35		6.2	9.6	
B/41	2.5	0.70	305	303		35		6.2	9.9	
B/42	2.5	0.70	300	300		35		6.1	9.7	
<B>	**2.5**	**0.70**	**306**	**299**	**41.6**	**35**		**6.1**	**9.6**	**4**
C/21	2.77	0.68	226	296	36.5	45	5.4			
C/22	2.77	0.68	206	295		45	4.2			
C/31	2.77	0.68	221	294		45	5.7			
C/32	2.77	0.68	239	299		45	6.1			
<C>	**2.77**	**0.68**	**223**	**296**	**36.5**	**45**	**5.4**			**4**
<D>	**3**	**0.55**	222	**290**	**39**	**45**		**3.7**	**6.4**	**4**

Transition Jitter vs. Read Track Width



s = Cross-track correlation length

The number of sub-transitions is given by W_r/s. The relation between the net transition position variance and the sub-transition variance is:

$$\sigma^2_{jitter} = \frac{s}{W_r}\,\sigma^2_s$$

(H.N. Bertram, Theory of Magnetic Recording, p. 317.)

Analytic modeling of the transition noise with a tanh transition shape yields:

$$\sigma_{jitter} = \frac{\pi^2 a}{4}\sqrt{\frac{s}{3W_r}}$$

(B. Slutsky and H.N. Bertram, *IEEE Trans. Magn.*, vol. 30, (no. 5), pp. 2808-2817, 1994.)

Eric Champion

SNR Budget

The Signal-to-Noise Ratio and Its Applications

- Signal-to-Noise Ratio - a designer's tool ...
  - ... is an *averaged*, statistical measure of goodness
  - ... can be readily measured
  - ... components' properties have direct, calculable SNR consequences
  - ... ignores non-random components and system effects
- Bit Error Rate - the users' demand for data integrity ...
  - ... is an *instantaneous* proof of goodness
  - ... value depends on channel properties - a little harder to measure and even harder to implement new ideas in experiments
  - ... components' and system properties affect BER in complex ways

Signal-to-Noise Ratio and Bit Error Rate

- Usefulness: the SNR correlates with the BER at the components' level, as long as the noise includes random sources only.
- Although BER generally correlates with SNR at the spin-stand level, it is easy to see how this correlation could break down.

  Example: two similar disks, one with a long defect
- In the drive, there are sources of noise that are less easily accountable by the measured (head / media / preamp) SNR, such as noise from the drive's digital circuitry or electromechanical subsystems. The (SNR - BER) correlation may seem corrupted.
- An *SNR value above a certain channel-dependent SNR threshold is a necessary*, but not sufficient, *requirement for performance.*

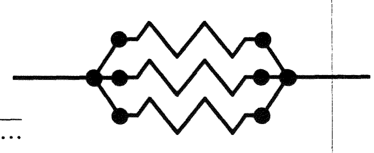
Signal and Noise Sources

- **Signal**
  - **Medium M_rT, H_c, medium thickness, overcoat, ...**
  - **Head reader: ΔR, gap length, width, overcoat, ...**
  - **Head writer: saturation magnetization, field risetime, gap length, width, ...**
  - **System: fly-height, TMR**
- **Noise**
  - **Medium noise: on track, erase bands, orientation ratio**
  - **Head: writer saturation effects, pole trimming; reader: Johnson noise, instabilities**
  - **Preamp: input noise voltage, sense-current noise**
  - **System: equalization, data rate (bandwidth), TMR**

SNR System Considerations

- **SNR sources add "in parallel:"**

$$SNR_{sys} = \frac{\text{Signal Power}}{\text{Incoherent Sum of Noise Powers}}$$



$$SNR_{sys} = \frac{V^2}{P_{medium} + P_J + P_{voltage} + P_{current} + \cdots}$$

$$\frac{1}{SNR_{sys}} = \frac{1}{SNR_m} + \frac{1}{SNR_J} + \frac{1}{SNR_v} + \frac{1}{SNR_c} + \cdots = \frac{1}{SNR_m} + \frac{1}{SNR_{h\&e}}$$

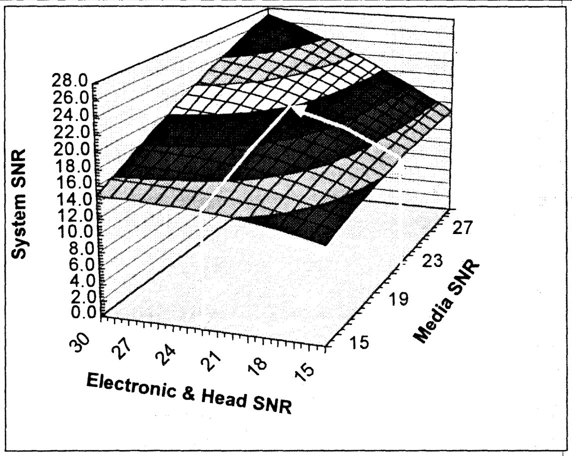
However, not all terms strictly equivalent, medium noise is colored

- **Thus,**

$$SNR_{sys} < SNR_{lowest}$$

The worst performing subsystem becomes the limiting barrier - ... "media noise limited", etc. ...

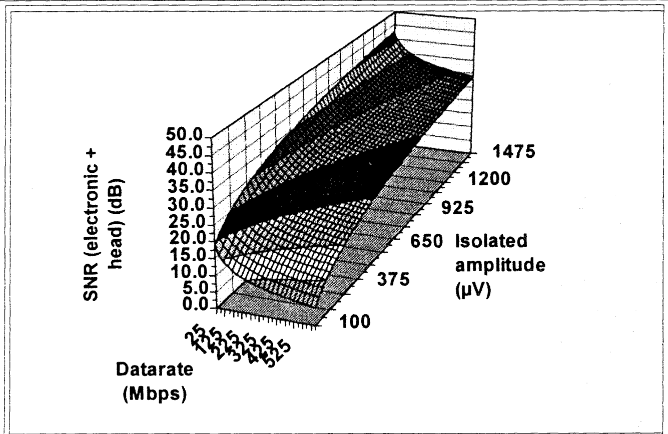
Interdependence of Subsystems' SNR



The worst performing subsystem SNR_{lowest} limits the overall system performance, $SNR_{sys} < SNR_{lowest}$

With a threshold SNR_{sys} condition, changes in subsystems' SNR affect the performance demanded of other components

Bandwidth & Head Output



The media SNR varies mildly with bandwith. The electronic SNR drops with increasing datarate. In order to maintain the necessary SNR_{sys} more performance is required elsewhere in the system.

SNR Budget: What σ_{eff} For 6 Gbit/in^2?

- 16:1 BPI/TPI Ratio: 310 kbpi x 19.4 ktpi
- Density, HF 329 kfci
- Density, MF 165 kfci
- $V_{0\text{-}p}$ (Iso) 500 µV
- $V_{0\text{-}p}$ (MF) 251 µV
- Radius (ID) 0.627 in
- Bandwidth 49 MHz
- Input noise voltage 0.5 nV/rt Hz
- Input noise current 20 pA/rt Hz
- Resistance 55 Ω
- B = bit length 82 nm
- PW50 ~8.4 µin

- Preamp noise 12.3 µV^2
- Current source 59.3 µV^2
- Johnson noise, $4k_BT\Delta fR$ 46.6 µV^2
- Electronics + Johnson 118.2 µV2
- SNR(E) 27.3 dB
- If 70% of the total noise is due to the media and 30% is due to electronics then σ_{eff} = 3.5 nm and the total SNR = 22 dB.
- The SNR required to achieve an on-track error rate of 10^{-7} is about 21 dB. For this case the media noise component is 382 µV^2. This leads to an effective jitter of σ_{eff} = 4.1 nm.

SNR Estimates and Trends for Higher Areal Density

SNR Estimate

- **Peak amplitude of linear superposition of Lorentzian pulses**

$$V_{peak}(1/B) = V_0 \times \frac{\pi \cdot \frac{PW50}{2B}}{\sinh\left(\pi \cdot \frac{PW50}{2B}\right)} = V_0 \times \frac{(\pi U/2)}{\sinh(\pi U/2)}$$

U=channel density

Comstock & Williams (1973)

- **SNR = (Peak mid-frequency signal power)/(broad band noise power of high-frequency fundamental)**

$$SNR = \frac{V_{peak}^2(1/2B)}{P_n(1/B)} = \frac{\left(\frac{2B \cdot PW50}{\pi \sigma_{jitter}^2}\right)}{\left[1 + \frac{1}{4}\frac{\sigma_{PW50}^2}{\sigma_{jitter}^2} + \frac{1}{2}\frac{\sigma_{V_0}^2}{V_0^2}\cdot\left(\frac{PW50}{\sigma_{jitter}}\right)^2\right]} \times \frac{(\pi U/4)^2}{\sinh^2(\pi U/4)}$$

1T, 1/(2T) hi-freq, "1/(4T)" mid-freq

Could lump all noise sources into σ_n [Spectrum analyzer]

Resolution of Lorentzian Waveform

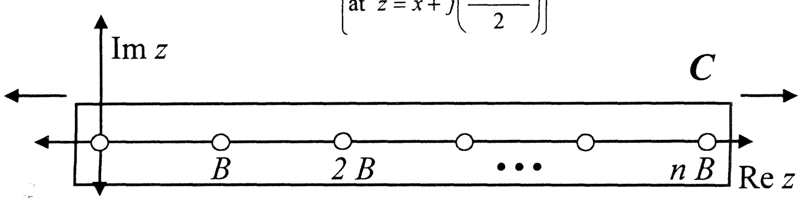
- **A rainy afternoon in the complex plane**

$$\frac{1}{2\pi j}\oint_C \frac{\pi}{B}\frac{V_{iso}(x-z;PW50)}{\sin\left(\frac{\pi z}{B}\right)}dz =$$

R. L. Comstock & M. Williams, IEEE Trans. Magn., MAG-9(3), 342 (1973)

Morse & Feshbach, Vol. I, Ch. 4, PP. 378 - 414

$$= \sum_{n=-\infty}^{\infty}(-1)^n V_{iso}(x-nB;PW50) + \left\{ \begin{array}{l} \text{residues of} \\ V_{iso}(x-z;PW50) \\ \text{at } z = x \mp j\left(\frac{PW50}{2}\right) \end{array} \right\} = 0$$



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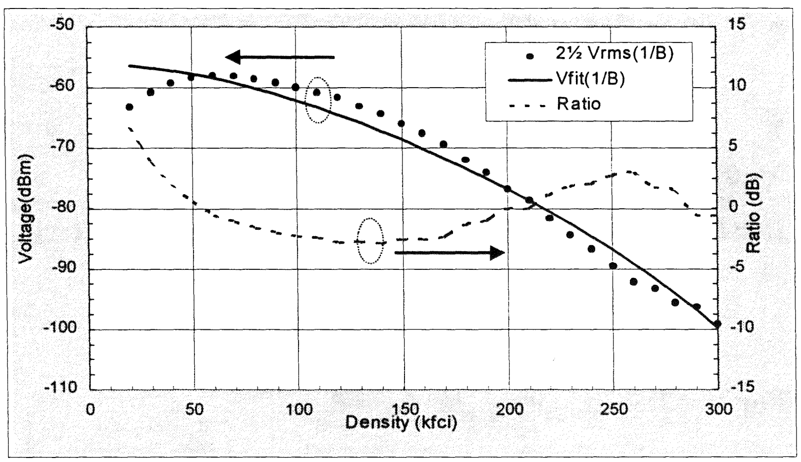
TAA Formula vs. Experiment



- From 30 to 300 kfci, data & fit differ by < 3 dB

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SNR Estimate

- ***SNR*** **= (Peak *mid-frequency* signal power)/(broad band noise power of *high-frequency* fundamental)**

1T, 1/(2T) hi-freq,
"1/(4T)" mid-freq

$$SNR = \frac{V_{peak}^2(1/2B)}{P_n(1/B)} = \frac{\left(\dfrac{2B \cdot PW50}{\pi\sigma_{jitter}^2}\right)}{\left[1 + \dfrac{1}{4}\dfrac{\sigma_{PW50}^2}{\sigma_{jitter}^2} + \dfrac{1}{2}\dfrac{\sigma_{V_0}^2}{V_0^2} \cdot \left(\dfrac{PW50}{\sigma_{jitter}}\right)^2\right]} \times \frac{(\pi U/4)^2}{\sinh^2(\pi U/4)}$$

Could lump all noise sources into σ_n [Spectrum analyzer]

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SNR Estimate (cont...)

- **SNR definition and formula:**

$$SNR = \frac{2B \cdot PW50}{\pi \sigma_n^{\ 2}} \times \frac{(\pi U / 4)^2}{\sinh^2(\pi U / 4)}$$

- **Media & head noise < total system noise**
- **Use P_n = fraction α of total noise P_{system} , $\alpha = 3/4 < 1$**

$$SNR_{sys} = \alpha \frac{2B \cdot PW50}{\pi \sigma_n^{\ 2}} \times \frac{(\pi U / 4)^2}{\sinh^2(\pi U / 4)}$$

- **For U = 2.5, $\sigma_{PW50} = \sigma_{jitter}/4$, $\sigma_{V0} = 0$,**

$$SNR_{sys}(U = 2.5) \cong 0.5\alpha\left(B / \sigma_{jitter}\right)^2$$

At constant channel density, ratio (σ_{jitter}/bit length) is function of *SNR* alone

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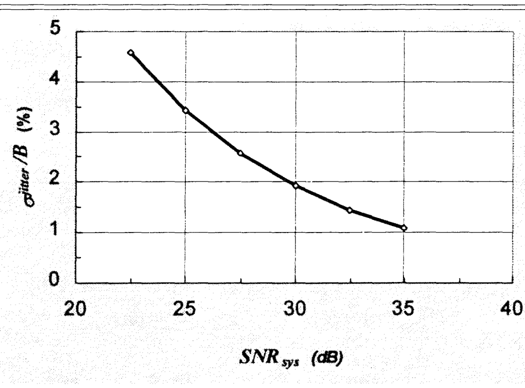
Ratio Transition Jitter/Bit Length vs. SNR_{sys}

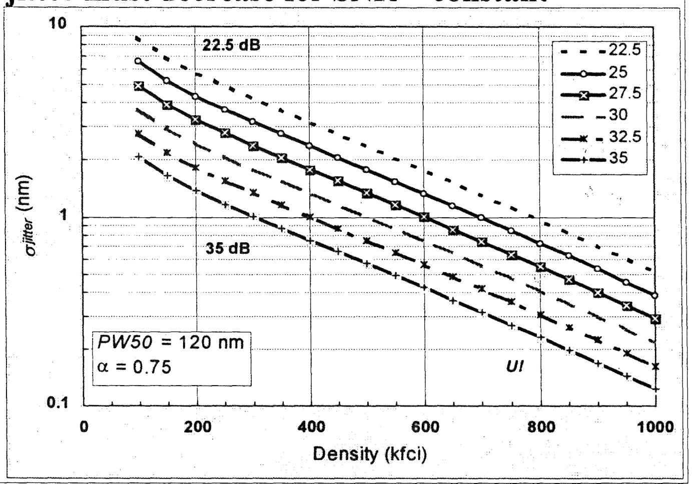
- **If the head gap scales with density, U = constant**
- **Jitter/Bit = *f(SNR)* only**
- **To maintain any given SNR, σ_{jitter} must scale with B**



- **Example: 500 kfci, B = 50 nm, PW50 = 125 nm, σ_{jitter} = 1.5 nm @ 26 dB**

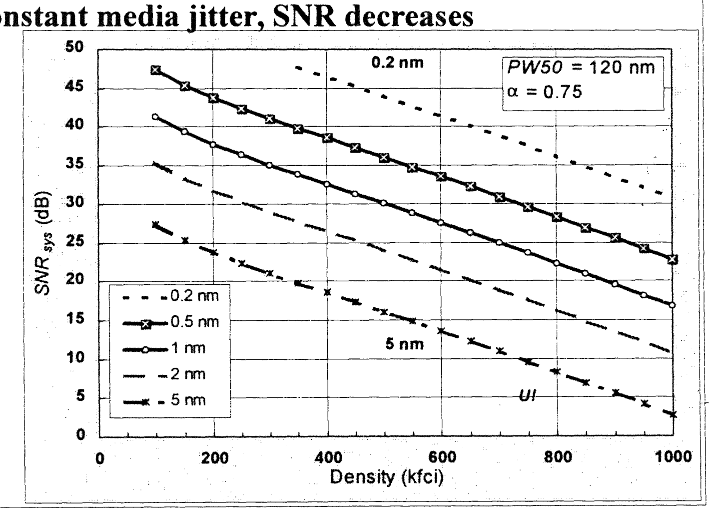
Jitter vs. Density, Various System SNR's

- Media jitter must decrease for SNR = constant



System SNR, Various Media Jitter Values

- At constant media jitter, SNR decreases



Signal-to-Noise Ratio Budgets in Disc Drive Design

Giora J. Tarnopolsky
Advanced Recording Technology
Seagate Advanced Concepts

IIST - Santa Clara University
April 27, 1998

Practitioner's Guide to Noise & SNR

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- H. Neal Bertram
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- Rajiv Ranjan
- Hans Richter
- Norm Talsoe

Outline

Outline

- Media noise properties
  - Origin
  - Localization
  - Parametric dependencies
  - Models
- Media noise power estimation
  - Estimate
  - Noise budget
  - Experimental verification
- SNR budgets and drive design
- SNR at high recording densities. Jitter limits.
- Conclusions

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Medium Noise Partial Bibliography

- **There is a vast literature of media noise research. A limited sample of references follows:**

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- Noise in Digital Magnetic Recording, T. C. Arnoldussen & L. L. Nunnelley, Editors (1992)
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Medium Noise Partial Bibliography (cont')

- Noise of Interacting Transitions in Thin-Film Recording Media, J.-G. Zhu, IEEE Trans. Mag. vol. 27, 5040 (1992)
- Experimental Studies of Noise Autocorrelation in Thin Film Media, G. Herbert Lin & H. Neal Bertram, IEEE Trans. Mag. vol. 29, 3697 (1993)
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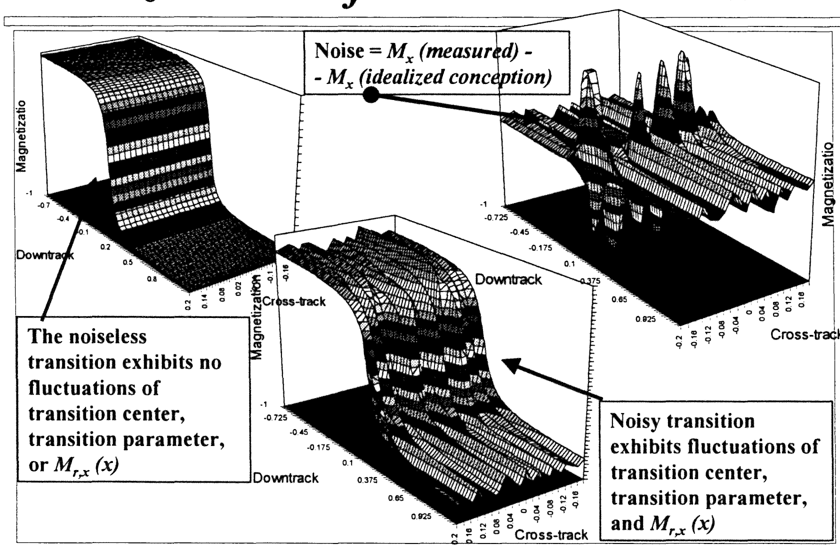
- Transition Noise Analysis of Thin-Film Magnetic Recording Media , B. Slutsky & H. Neal Bertram, IEEE Trans. Mag. vol. 30, 2808 (1994)
- Transition Noise Spectral Measurements in Thin Film Media, G. Herbert Lin & H. Neal Bertram, IEEE Trans. Mag. vol. 30, 3987 (1994)
- Microtrack Model of Recorded Transitions, J. Carroselli & J. K. Wolf, IEEE Trans. Mag. vol. 32, 3917 (1996)
- Media Noise and Signal-to-Noise Ratio Estimates for High Areal Density Recording, Giora J. Tarnopolsky & P. R. Pitts, J. Appl. Phys. vol. 81, 4837 (1997)

Media Noise Properties

What is Medium Noise?

- **The reproduce voltage results from the read head sensing the "magnetic charge" of the media: - $\nabla \cdot M(x)$, the divergence of the magnetization**
- **The medium is an array of columnar magnetic grains having: irregular boundaries, dispersion in the direction of the uniaxial anisotropy axis, intergranular exchange and magnetostatic coupling, and coercivity and remanence that vary from grain to grain.**
- **Superimposed onto the intended magnetization reversals (user data) there appear the magnetization spatial fluctuations. They result in unwanted and unpredictable deviations of the reproduce voltage from ideal response.**
- **User-intended bit transitions do not match grain boundaries.**

Idealized View of Transitions & Noise



Signal & Noise Power vs. Read Width

- **The recorded "noiseless" signal is in phase across the track. Its power is proportional to the square of the track width.**

$$\text{Signal Power} = \left[\sum_{i=1}^{N} V_i(x)\right]^2 = N^2 V^2_{(\mu track)}(x) = \left(\frac{\text{Read width}}{\Delta z}\right)^2 \cdot V^2_{(\mu track)}(x)$$

- **The noise components are incoherent. The noise power is proportional to the track width.**

$$\text{Noise Power} = \left[\sum_{i=1}^{N} \delta V_i(x)\right] \cdot \left[\sum_{j=1}^{N} \delta V_j(x)\right] = \sum_{i=j}^{N} \delta V_i(x) \delta V_j(x) + \sum_{i \neq j}^{N} \delta V_i(x) \delta V_j(x) =$$

$$= N < \delta V^2(x) > = \left(\frac{\text{Read width}}{\Delta z}\right) \cdot < \delta V^2(x) >$$

- **Signal-to-noise power ratio ∝ Read width**

Medium Noise Characteristics

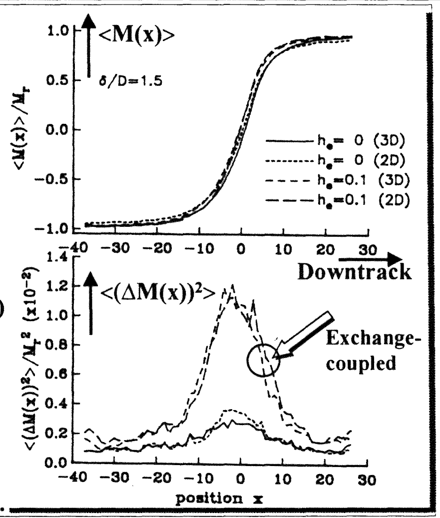
- Medium noise appears most prominently in regions where $M(x) \approx 0$
  - at the magnetic transition - hence "transition noise"
  - at the track edge
  - in the servo fields
- Once recorded, the noisy magnetization pattern does not change with time. It is read out by the magnetic channel just as the data is. Thus, the noise spectrum shows, in a spectrum analyzer, a shape proportional to the envelope of the fundamental and its harmonics
- Equalization enhances high-frequency components (generally, to slim the isolated pulse.) It thus enhances the high frequency component of the medium noise.

How is Media Noise Modeled?

- Media strips, microtracks (Arnoldussen, Carroselli & Wolf)
  - Media divided into stripes along the downtrack co-ordinate, each stripe exhibiting fluctuations in transition position and sharpness
- Micromagnetic models (Bertram and collaborators, Zhu and collaborators)
  - Media volume subdivided into regular hexagonal tiles
  - Anisotropy axis orientation, magnetostatic interactions, inter- and intra-granular exchange interactions
  - Proved that exchange-coupled grains are noisier
- Analytical models of the readback voltage (Bertram and collaborators.) Example will follow.

Media Noise Higher Where M(x)=0

- **Ensemble mean and variance of the transition profiles for planar isotropic longitudinal films for different intergranular exchange coupling, and both 2D and 3D random easy axis orientation. A large variance occurs at the transition center. The exchange-coupled film shows much higher magnetization noise in all cases. Bertram & Zhu, IEEE Trans. Mag. vol. 27, 5043 (© IEEE 1991)**
- **The transition noise power P_n increases with the number of transitions per unit length, or density.**
- **The measurement of P_n*(density)* affords a determination of media intrinsic noise parameters.**



Media Noise Reduction

How to reduce media noise?

- **Achieve grains of uniform size. The reduction of the variance of columnar sizes will improve media performance, and is of more immediate concern than media thermal decay.**
  - **Underlayers: Epitaxial growth**
  - **Clean sputtering systems**
- **Reduce exchange coupling between grains**
  - **Non-magnetic material segregation to boundary**
- **Reduce the volume where noise originates**
  - **Shorter transition length**
  - **Lower $M_r\delta$**
  - **Higher H_c**

Media Noise & High-Density Recording

- "Transition jitter", σ_{jitter} = 3 to 10 nm
- For linear recording densities ≤ 1,000 kfci:
  - σ_{jitter} for given signal-to-noise ratio *SNR*?
  - (σ_{jitter} /bit length *B*) vs. *PW50*, *SNR*, and density?
  - Grain sizes and uniformity for n × 10 $Gbit/in^2$?
- Recorded bit boundary: shifted from ideal position
- Pulse width (*PW50*): fluctuates around track
- Peak amplitude V_0: varies bit-to-bit due to head

Media Noise Power Estimation

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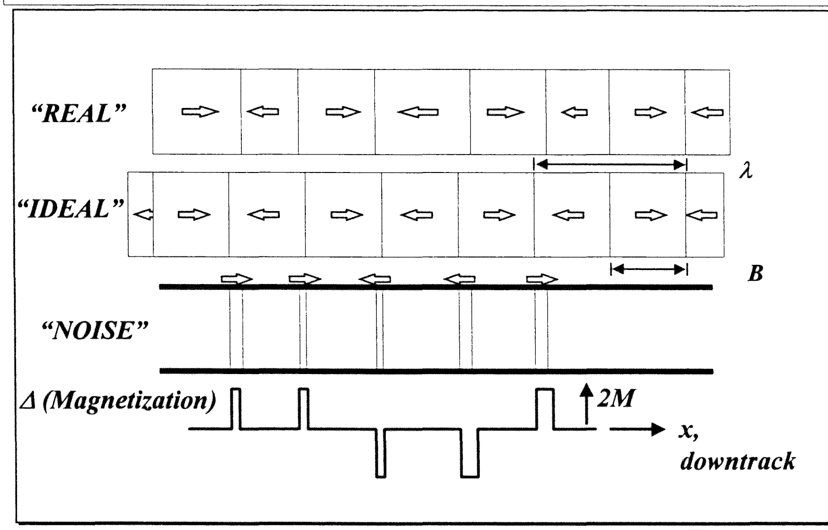
Transition Jitter: Magnetization Noise



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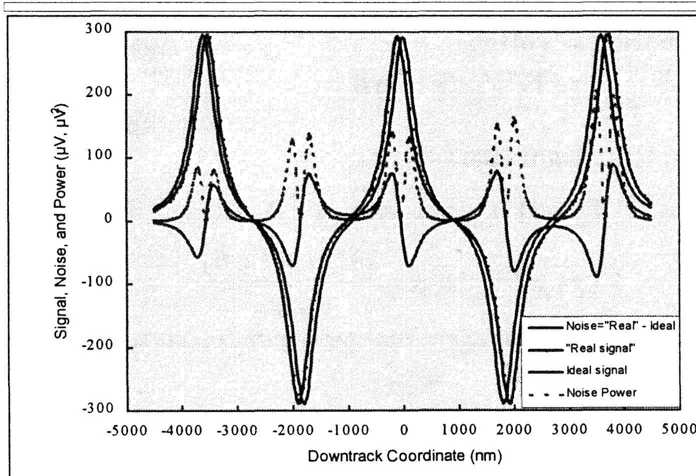
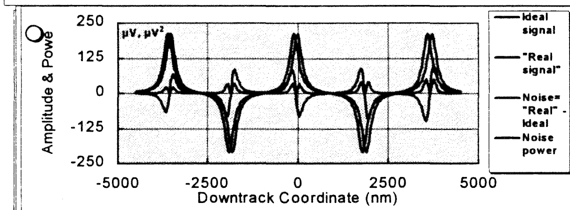


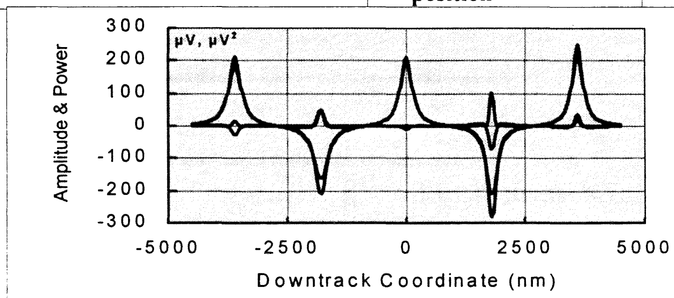
Chart shows i) an ideal noiseless trace, ii) a trace affected by transition jitter (shifted from nominal), iii) the difference between real and ideal voltages - NOISE, and iv) the noise power

"Real" & Ideal Waveforms & Noise



← "Jitter:" pulse center shifted from ideal position

"PW50:" pulse width broadening or slimming from ideal (a_x fluctuations) →



Replay Voltage

- **Replay noiseless voltage**

$$\tilde{V}(x) = \sum_{n=-\infty}^{n=\infty} (-1)^n V_{iso}(x - nB)$$

•Bertram & collaborators, 1987 - 1996, Noise formalism
•See: Bertram: Theory of Magnetic Recording

- **Add, say, transition jitter δx_n**

$$V(x) = \Sigma_{n=-\infty}^{n=\infty} (-1)^n V_{iso}(x - nB - \delta x_n),$$

$$\therefore V(x) = \tilde{V}(x) + \Sigma(-1)^n \delta x_n \frac{\partial V_{iso}(x - nB)}{\partial x}$$

- **Signal = noiseless voltage +** ***one term per fluctuating property***
- **Assume Lorentzian pulses, compute noise power**

Noise Power Density

$$V_{iso}(x) = \frac{2A}{\pi PW50}\left(1\Big/1+\left(\frac{2x}{PW50}\right)^2\right) = V_0\left(1\Big/1+\left(\frac{2x}{PW50}\right)^2\right)$$

Lorentzian
A = area, V_0 amplitude

- **Consider position fluctuations, pulse-width fluctuations, and MR head amplitude fluctuations:**

  σ_{jitter} σ_{PW50} σ_{V_0}

- **Noise power density at head's terminals:**

$$PSD(k) = \frac{|V_{iso}(k)|^2}{B} \times \left[k^2\sigma_{jitter}^2 + \frac{k^2\sigma_{PW50}^2}{4} + \frac{\sigma_{V_0}^2}{V_0^2}\right]$$

k=2π/λ , FT variable
$V_{iso}(k)$ = FT isolated pulse

Noise Power

- **Noise power *density* equalized by channel transfer function *G(k)*:**

$$PSD(k) = \frac{|G(k)V_{iso}(k)|^2}{B} \times \left[k^2\sigma_{jitter}^2 + \frac{k^2\sigma_{PW50}^2}{4} + \frac{\sigma_{V_0}^2}{V_0^2}\right]$$

k=2π/λ , FT variable
$V_{iso}(k)$ = FT isolated pulse

- **The broad-band media noise power = integral of *PSD(k)* over all frequencies:**

$$P_n = \text{broad-band noise power} =$$

$$= \frac{\pi}{2}V_0^2\frac{\sigma_{jitter}^2}{B\cdot PW50}\times\left(1+\frac{1}{4}\frac{\sigma_{PW50}^2}{\sigma_{jitter}^2}\right) + \frac{\pi\sigma_{V_0}^2 PW50}{4B}$$

G. J. Tarnopolsky et al., J. Appl. Phys. vol. 81, 4837 (1997)

Media jitter scaled by bit length and PW50

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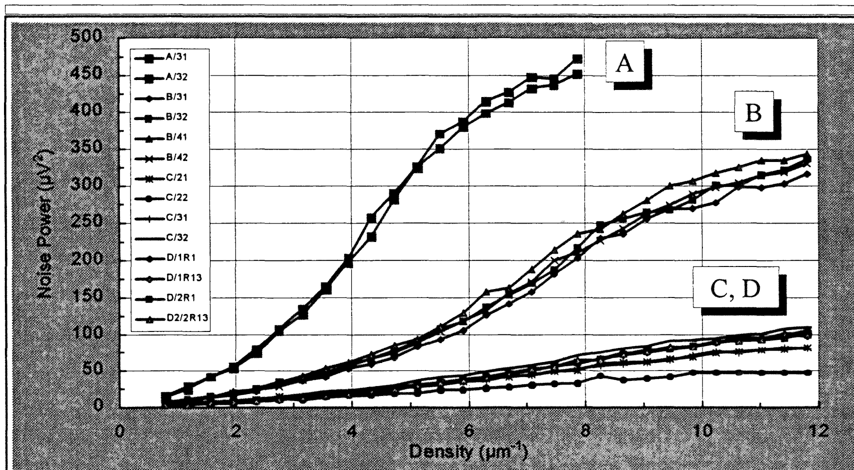
Transition Noise vs. Density: Many Alloys



- **Get σ_{jitter} from initial slopes.**

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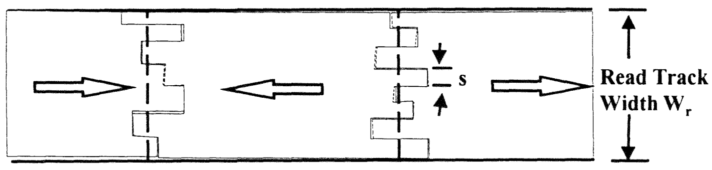
Transition Noise vs. Density: Many Alloys

Table I. Experimental Results

Disk	H_c	$M_r\delta$	V_o	PW50	OW	Iw	σ	σ_{low}	σ_{high}	#
	kOe	memu/cm²	μV	nm	dB	mA	nm	nm	nm	
A/31	2.2	1.00	383	378	39	34		8.0	12.5	
A/32	2.2	1.00	382	372	39	34		7.9	12.3	
<A>	**2.2**	**1.00**	**383**	**375**	**39**	**34**		**8.0**	**12.4**	**2**
B/31	2.5	0.70	311	295	41.6	35		5.8	9.3	
B/32	2.5	0.70	309	298		35		6.2	9.6	
B/41	2.5	0.70	305	303		35		6.2	9.9	
B/42	2.5	0.70	300	300		35		6.1	9.7	
<B>	**2.5**	**0.70**	**306**	**299**	**41.6**	**35**		**6.1**	**9.6**	**4**
C/21	2.77	0.68	226	296	36.5	45	5.4			
C/22	2.77	0.68	206	295		45	4.2			
C/31	2.77	0.68	221	294		45	5.7			
C/32	2.77	0.68	239	299		45	6.1			
<C>	**2.77**	**0.68**	**223**	**296**	**36.5**	**45**	**5.4**			**4**
<D>	**3**	**0.55**	**222**	**290**	**39**	**45**		**3.7**	**6.4**	**4**

Transition Jitter vs. Read Track Width



|← Bit Length B →|

s = Cross-track correlation length

The number of sub-transitions is given by W_r/s. The relation between the net transition position variance and the sub-transition variance is:

$$\sigma^2_{jitter} = \frac{s}{W_r}\sigma^2_s$$

(H.N. Bertram, Theory of Magnetic Recording, p. 317.)

Analytic modeling of the transition noise with a tanh transition shape yields:

$$\sigma_{jitter} = \frac{\pi^2 a}{4}\sqrt{\frac{s}{3W_r}}$$

(B. Slutsky and H.N. Bertram, *IEEE Trans. Magn.*, vol. 30, (no. 5), pp. 2808-2817, 1994.)

Eric Champion

SNR Budget

The Signal-to-Noise Ratio and Its Applications

- **Signal-to-Noise Ratio - a designer's tool ...**
  - **... is an *averaged*, statistical measure of goodness**
  - **... can be readily measured**
  - **... components' properties have direct, calculable SNR consequences**
  - **... ignores non-random components and system effects**
- **Bit Error Rate - the users' demand for data integrity ...**
  - **... is an *instantaneous* proof of goodness**
  - **... value depends on channel properties - a little harder to measure and even harder to implement new ideas in experiments**
  - **... components' and system properties affect BER in complex ways**

Signal-to-Noise Ratio and Bit Error Rate

- **Usefulness: the SNR correlates with the BER at the components' level, as long as the noise includes random sources only.**
- **Although BER generally correlates with SNR at the spin-stand level, it is easy to see how this correlation could break down.**

  Example: two similar disks, one with a long defect
- **In the drive, there are sources of noise that are less easily accountable by the measured (head / media / preamp) SNR, such as noise from the drive's digital circuitry or electromechanical subsystems. The (SNR - BER) correlation may seem corrupted.**
- **An *SNR value above a certain channel-dependent SNR threshold is a necessary*, but not sufficient, *requirement for performance.***

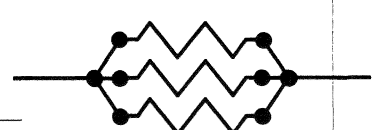
Signal and Noise Sources

- Signal
  - Medium M_rT, H_c, medium thickness, overcoat, ...
  - Head reader: ΔR, gap length, width, overcoat, …
  - Head writer: saturation magnetization, field risetime, gap length, width, …
  - System: fly-height, TMR
- Noise
  - Medium noise: on track, erase bands, orientation ratio
  - Head: writer saturation effects, pole trimming; reader: Johnson noise, instabilities
  - Preamp: input noise voltage, sense-current noise
  - System: equalization, data rate (bandwidth), TMR

SNR System Considerations

- SNR sources add "in parallel:"

$$SNR_{sys} = \frac{\text{Signal Power}}{\text{Incoherent Sum of Noise Powers}}$$



$$SNR_{sys} = \frac{V^2}{P_{medium} + P_J + P_{voltage} + P_{current} + \cdots}$$

$$\frac{1}{SNR_{sys}} = \frac{1}{SNR_m} + \frac{1}{SNR_J} + \frac{1}{SNR_v} + \frac{1}{SNR_c} + \cdots = \frac{1}{SNR_m} + \frac{1}{SNR_{h\&e}}$$

However, not all terms strictly equivalent, medium noise is colored

- Thus,

$$SNR_{sys} < SNR_{lowest}$$

The worst performing subsystem becomes the limiting barrier - … "media noise limited", etc. ...

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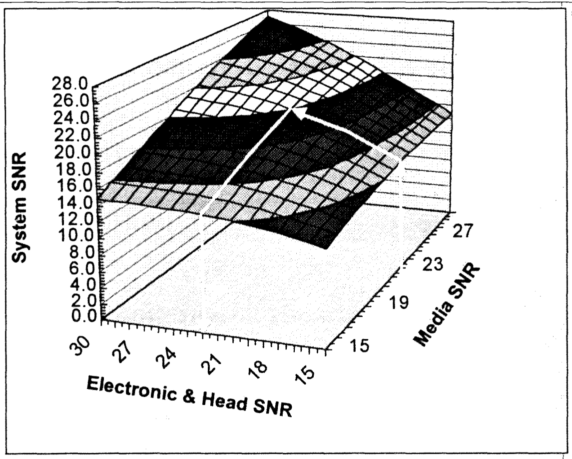
Interdependence of Subsystems' SNR



The worst performing subsystem SNR_{lowest} limits the overall system performance, $SNR_{sys} < SNR_{lowest}$

With a threshold SNR_{sys} condition, changes in subsystems' SNR affect the performance demanded of other components

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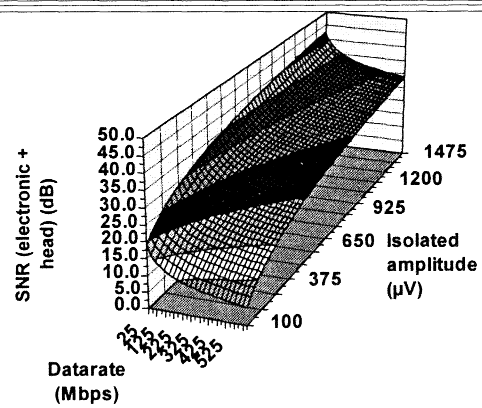
Bandwidth & Head Output



The media SNR varies mildly with bandwith. The electronic SNR drops with increasing datarate. In order to maintain the necessary SNR_{sys} more performance is required elsewhere in the system.

SNR Budget: What σ_{eff} For 6 Gbit/in^2?

- **16:1 BPI/TPI Ratio: 310 kbpi x 19.4 ktpi**
- **Density, HF 329 kfci**
- **Density, MF 165 kfci**
- **V_{0-p} (Iso) 500 µV**
- **V_{0-p} (MF) 251 µV**
- **Radius (ID) 0.627 in**
- **Bandwidth 49 MHz**
- **Input noise voltage 0.5 nV/rt Hz**
- **Input noise current 20 pA/rt Hz**
- **Resistance 55 Ω**
- **B = bit length 82 nm**
- **PW50 ~8.4 µin**

- **Preamp noise 12.3 µV^2**
- **Current source 59.3 µV^2**
- **Johnson noise, $4k_B T\Delta f R$ 46.6 µV^2**
- **Electronics + Johnson 118.2 µV2**
- **SNR(E) 27.3 dB**
- **If 70% of the total noise is due to the media and 30% is due to electronics then σ_{eff} = 3.5 nm and the total SNR = 22 dB.**
- **The SNR required to achieve an on-track error rate of 10^{-7} is about 21 dB. For this case the media noise component is 382 µV^2. This leads to an effective jitter of σ_{eff} = 4.1 nm.**

SNR Estimates and Trends for Higher Areal Density

SNR Estimate

- **Peak amplitude of linear superposition of Lorentzian pulses**

$$V_{peak}(1/B) = V_0 \times \frac{\pi \cdot \frac{PW50}{2B}}{\sinh\left(\pi \cdot \frac{PW50}{2B}\right)} = V_0 \times \frac{(\pi U/2)}{\sinh(\pi U/2)}$$

U=channel density

Comstock & Williams (1973)

- **SNR = (Peak mid-frequency signal power)/(broad band noise power of high-frequency fundamental)**

$$SNR = \frac{V_{peak}^2(1/2B)}{P_n(1/B)} = \frac{\left(\frac{2B \cdot PW50}{\pi \sigma_{jitter}^2}\right)}{\left[1 + \frac{1}{4}\frac{\sigma_{PW50}^2}{\sigma_{jitter}^2} + \frac{1}{2}\frac{\sigma_{V_0}^2}{V_0^2} \cdot \left(\frac{PW50}{\sigma_{jitter}}\right)^2\right]} \times \frac{(\pi U/4)^2}{\sinh^2(\pi U/4)}$$

1T, 1/(2T) hi-freq, "1/(4T)" mid-freq

Could lump all noise sources into σ_n [Spectrum analyzer]

Resolution of Lorentzian Waveform

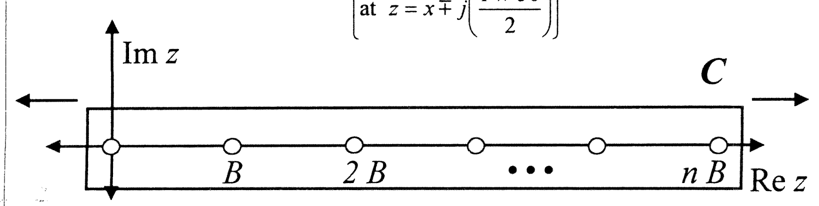
- **A rainy afternoon in the complex plane**

$$\frac{1}{2\pi j}\oint_C \frac{\pi}{B}\frac{V_{iso}(x-z;PW50)}{\sin\left(\frac{\pi z}{B}\right)}dz =$$

R. L. Comstock & M. Williams, IEEE Trans. Magn., MAG-9(3), 342 (1973)

Morse & Feshbach, Vol. I, Ch. 4, PP. 378 - 414

$$= \sum_{n=-\infty}^{\infty}(-1)^n V_{iso}(x-nB;PW50) + \left\{\begin{matrix}\text{residues of}\\ V_{iso}(x-z;PW50)\\ \text{at } z = x \mp j\left(\frac{PW50}{2}\right)\end{matrix}\right\} = 0$$



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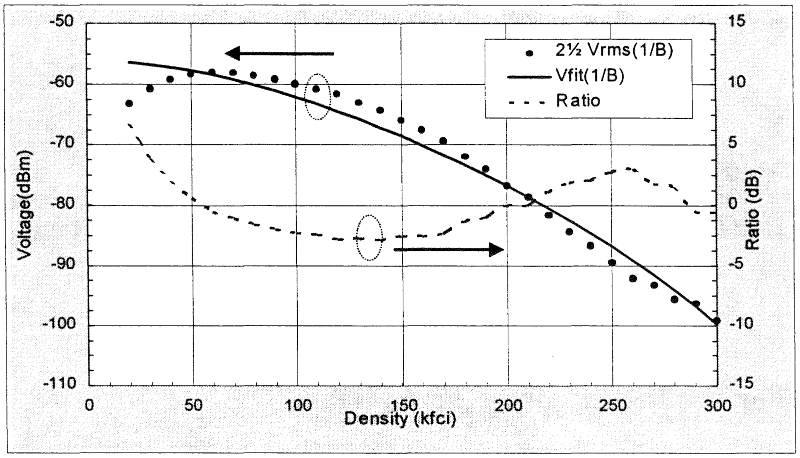
TAA Formula vs. Experiment



- **From 30 to 300 kfci, data & fit differ by < 3 dB**

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SNR Estimate

- ***SNR*** **= (Peak *mid-frequency* signal power)/(broad band noise power of *high-frequency* fundamental)**

1T, 1/(2T) hi-freq,
"1/(4T)" mid-freq

$$SNR = \frac{V_{peak}^2(1/2B)}{P_n(1/B)} = \frac{\left(\dfrac{2B \cdot PW50}{\pi\sigma_{jitter}^2}\right)}{\left[1+\dfrac{1}{4}\dfrac{\sigma_{PW50}^2}{\sigma_{jitter}^2}+\dfrac{1}{2}\dfrac{\sigma_{V_0}^2}{V_0^2}\cdot\left(\dfrac{PW50}{\sigma_{jitter}}\right)^2\right]} \times \frac{(\pi U/4)^2}{\sinh^2(\pi U/4)}$$

Could lump all noise
sources into σ_n
[Spectrum analyzer]

SNR Estimate (cont...)

- **SNR definition and formula:**

$$SNR = \frac{2B \cdot PW50}{\pi \sigma_n^{\,2}} \times \frac{(\pi U / 4)^2}{\sinh^2(\pi U / 4)}$$

- **Media & head noise < total system noise**
- **Use P_n = fraction α of total noise P_{system} , $\alpha = 3/4 < 1$**

$$SNR_{sys} = \alpha \frac{2B \cdot PW50}{\pi \sigma_n^{\,2}} \times \frac{(\pi U / 4)^2}{\sinh^2(\pi U / 4)}$$

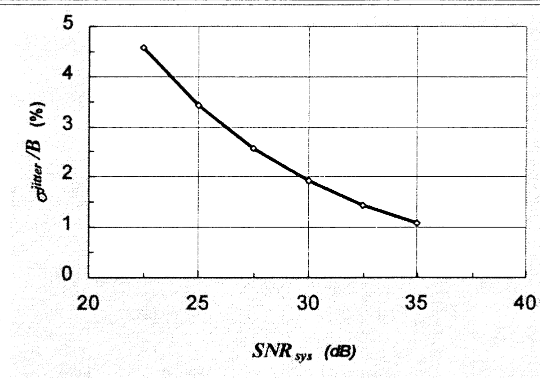
- **For U = 2.5, $\sigma_{PW50} = \sigma_{jitter}/4$, $\sigma_{V0} = 0$,**

$$SNR_{sys}(U = 2.5) \cong 0.5\alpha \left(B / \sigma_{jitter}\right)^2$$

At constant channel density, ratio (σ_{jitter}/bit length) is function of *SNR* alone

Ratio Transition Jitter/Bit Length vs. SNR_{sys}

- **If the head gap scales with density, U = constant**
- **Jitter/Bit = *f(SNR)* only**
- **To maintain any given SNR, σ_{jitter} must scale with B**



- **Example: 500 kfci, B = 50 nm, PW50 = 125 nm, σ_{jitter} = 1.5 nm @ 26 dB**

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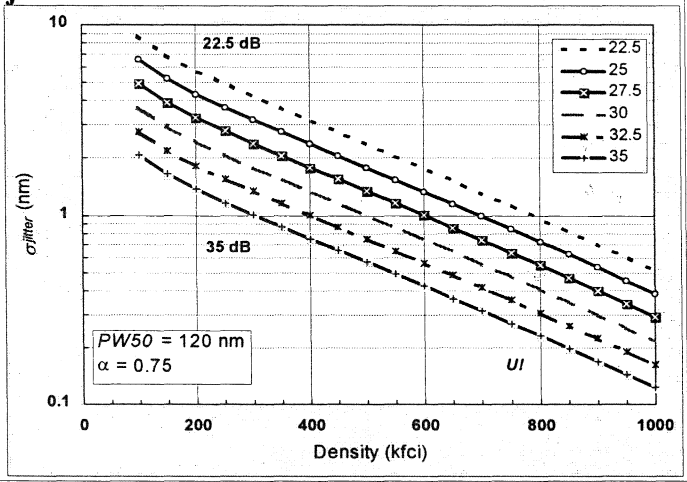
Jitter vs. Density, Various System SNR's

- Media jitter must decrease for SNR = constant



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1998 SNR & Disc Drive Performance Symposium

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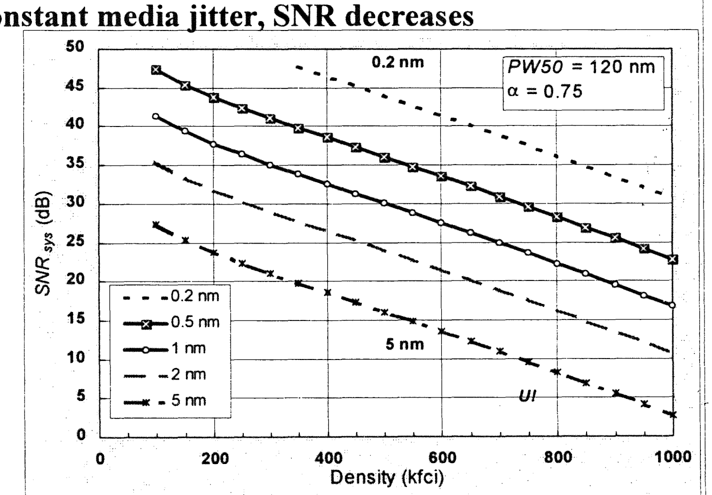
System SNR, Various Media Jitter Values

- At constant media jitter, SNR decreases



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Areal Density Estimators

- H. Neal Bertram has postulated an expression for media jitter,

$$\sigma^2_{jitter} = \left(\frac{\pi}{2}\right)^4 a_x^2 \frac{s}{3(RW)}$$

Cross track correlation length (s)

B. Slutsky & H. N. Bertram, IEEE Trans. Mag. vol. 30, 2808 (1994)

which allows to link the SNR values derived here to properties of the media (a_x) and of the recording system (RW).

a_x = transition parameter
s = cross-track correlation length
RW = head's read width

Outline

Outline

- Media noise properties
  - Origin
  - Localization
  - Parametric dependencies
  - Models
- Media noise power estimation
  - Estimate
  - Noise budget
  - Experimental verification
- SNR budgets and drive design
- SNR at high recording densities. Jitter limits.
- Conclusions

Media Noise & SNR for High Density Recording

- Expressions for:
  - Noise Power Spectral Density
  - Total Noise Power
  - SNR(mid-freq/broad band HF noise)
- For constant channel density, ratio (jitter/bit length) depends only on SNR, not on density

$$SNR_{sys}(U=2.5) \cong 0.5\alpha\left(B/\sigma_{jitter}\right)^2$$

- $\sigma_{jitter}/B = 3\%$ @ 26 dB SNR, 1.5 nm for 500 kfci
- At 1000 kfci, transition jitter $\leq$ 1 nm. Difficult to accomplish. Thermal & coercivity issues.

S. Charap et al., TMRC '96

Conclusions

- Media noise power unambiguous index of media performance. As a function of commonly measured properties,

$$P_n = \text{broad-band media noise power} = \frac{\pi}{2}V_0^2\frac{\sigma^2_{effective}}{B\cdot PW50} \propto$$

$$\propto [M_r\delta \text{ \& head effiency}]^2 \frac{(\text{effective jitter})^2}{\text{magnetic bit length } x\ PW50}$$

- Signal-to-noise ratio

$$SNR_{sys} = \alpha\frac{2B\cdot PW50}{\pi\sigma^2_{effective}} \times \frac{(\frac{\pi}{2}U/4)^2}{\sinh^2(\pi U/4)}$$

G. J. Tarnopolsky et al., J. Appl. Phys. vol. 81, 4837 (1997)

- SNR_{sys} above threshold is *necessary condition* for BER performance. Powerful design tool.