# ELECTRONIC COMPUTER SIMULATION
# OF INVENTORY CONTROL

by

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# ELECTRONIC COMPUTER SIMULATION

# OF INVENTORY CONTROL

Thompson Products has been invited here today by the American Management Association to relate to you our experiences in using an electronic computer to simulate certain phases of our inventory control. We are delighted to have this opportunity to tell you of our "new frontier".

Several years ago, Thompson Products adopted a policy of decentralization of its many plants and operations. Today the company, with general offices in Cleveland, Ohio, encompasses some 18 autonomous divisions engaged in the manufacture and sale of parts to the aircraft, automotive, industrial, and electronics industries.

The Replacement Division of Thompson Products, of which we will speak today, concerns itself solely with the distribution and sale of automotive engine and chassis parts for replacement use to some 6,000 independent automotive wholesalers, often referred to as jobbers. These wholesalers or jobbers, in turn, then sell to the independent garageman, the car dealer, and the fleet operator.

In the early days of the division, the pattern of distribution was relatively simple, in that all jobbers ordered 100% of needed material directly from the Cleveland warehouse. All that mattered was that the 10,000 part numbers, inventoried and controlled in Cleveland were sufficient to meet these demands.

As the transportation field developed and more and more cars, trucks, and buses were operated, the need to service these vehicles promptly resulted in the spotting of inventories at the source of demand. To meet this challenge, branch warehouses containing replacement parts inventories were opened by the Replacement Division throughout the United States. Today we maintain inventories in 35 company operated branch warehouses and Cleveland. Service is the "watchword" of our division. You can well imagine the impact on a centralized inventory control system that this change has wrought. Instead of controlling 10,000 part numbers in one location we are now controlling 15,000 part numbers in 35 locations, or in total, 525,000 numbers. The magnitude of our inventory control job is immense and obviously we must review all present inventory techniques in an effort to effect improvements in the reliability of supply to our jobbers. To this end, we invited the Ramo-Wooldridge Corporation, an affiliate of Thompson Products in Los Angeles, to assist us in finding a solution to this problem.

In undertaking this study, remember that our primary objective was to improve the reliability of supply to our consumer--the automotive jobber--through our branch network and yet retain centralized inventory control in

Cleveland. The first step in our program was to find an effective measure of performance of both our present system or any proposed system. Here we selected the lost sale as our yardstick. We define a lost sale as a condition where our consumer calls for a part at any of our distribution outlets and the part requested was out of stock. It was necessary to devise a system of reporting these lost sales from each of our 35 branch outlets. Then, measuring the lost sales for any given period against the total demand for the same period, we arrive at an index or percentage of demand that resulted in lost sales. Since our desire was to improve the resultant performance, a closer examination of our current system was in order.

We have described the branch distribution system as a network of branch warehouses constantly replenished from a central source at Cleveland. A single outlet of this network is shown schematically on Chart I. When demand at the branch results in a sale to a jobber, a copy of the sales slip is transmitted promptly to Cleveland for central invoicing and preparation of a replenishment shipment. Although these sales slips are transmitted daily to the central supply at Cleveland, we have found it economical to accumulate these daily demands for a week before preparing a replenishment shipment for the branch warehouse. Thus, a replenishment shipment is prepared each week for each branch warehouse in the country. Sufficient inventory is allocated to the branch to allow for the elapsed time between initial transmission of the sales slip to the receipt of the replenishment shipment at the branch, in addition to maintaining an adequate supply on hand at the warehouse itself.

In this system then, sales for each week are automatically replaced on a weekly basis. However, in spite of this somewhat routine method of replenishment, there are elements of uncertainty which complicate matters. One of the most uncertain characteristics of the system is the rate at which demand occurs at the branch. On Chart II a typical pattern of fluctuations in demand from week to week is illustrated. Even though we may forecast with reasonable certainty that the demand for this part will average about ten units per week, in any group of two or three weeks, there may be substantial deviations from this average.

Another critical area of uncertainty is the replenishment cycle time--the period between initial transmission of the sales slip to Cleveland and final receipt of the corresponding replenishment shipment at the branch. A typical pattern of successive branch replenishment cycle times, which we might experience, is presented on Chart III. Once again, although we may ordinarily expect a replenishment cycle time of about three weeks, at times it may be as long as five weeks and as short as one week. We see that demand is not steady, but will fluctuate from day to day and at the same time there will be variations in the replenishment cycle time. It is the variation in these two uncertainties that we must contend with.

# BRANCH WAREHOUSE SUPPLY SYSTEM

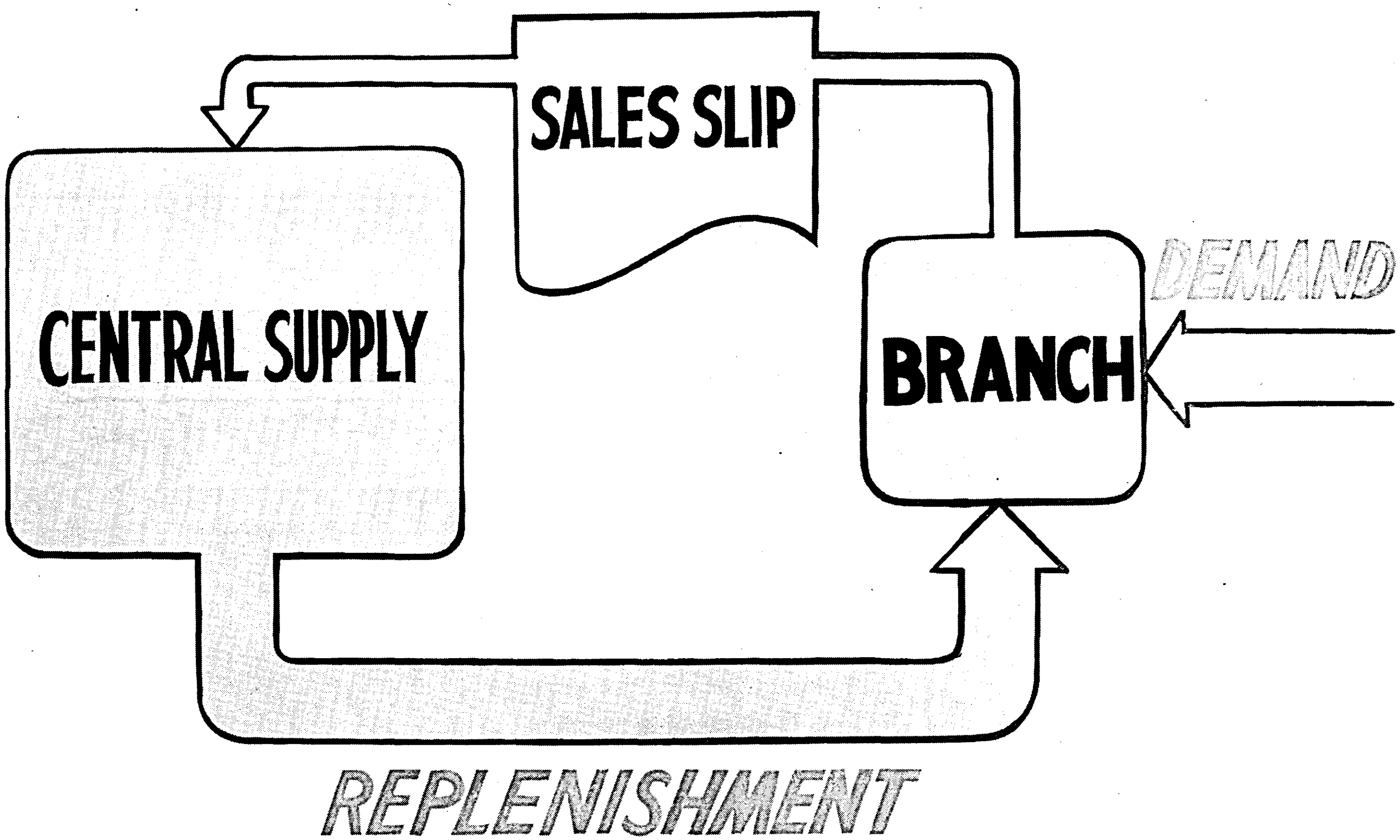




CHART I

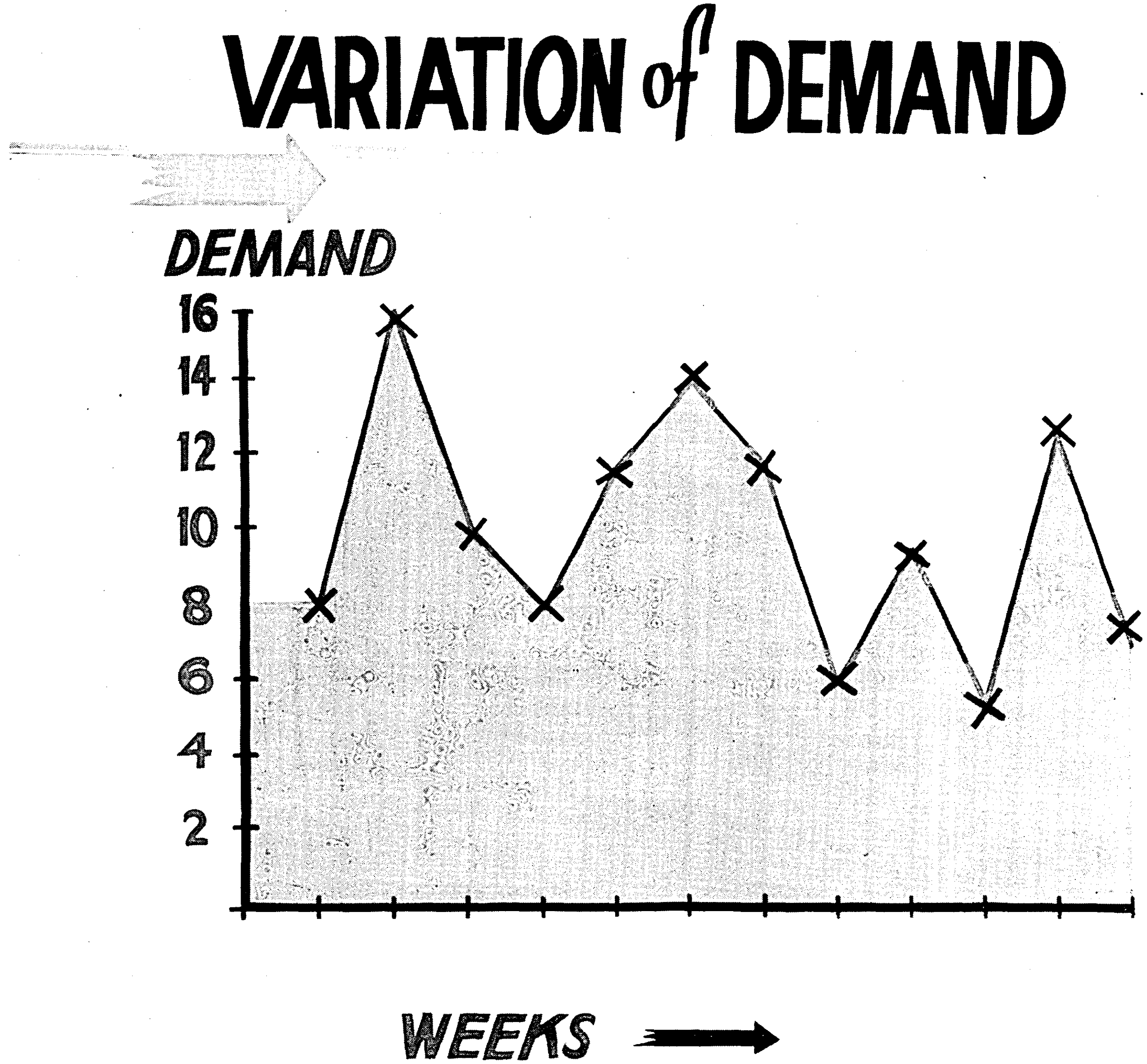




CHART II

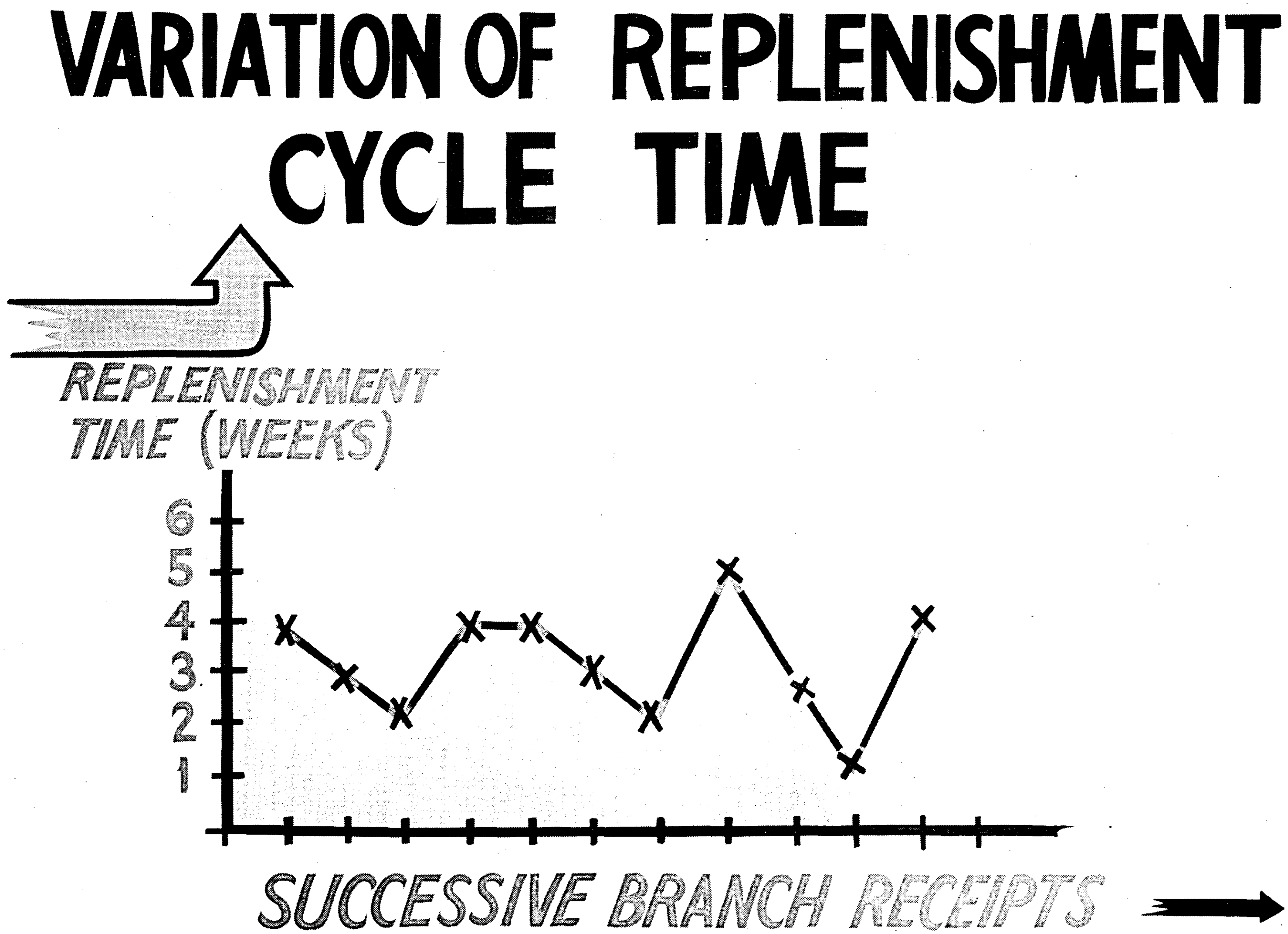
VARIATION OF REPLENISHMENT CYCLE TIME
REPLENISHMENT TIME (WEEKS)
6
5
4
3
2
1
SUCCESSIVE BRANCH RECEIPTS



CHART III

Since about 70% of our sales occur at the branches and approximately the same fraction of total inventories is carried there, the problem of increasing reliability of supply at the branch level seemed paramount to us. In the past, when lost sales have occurred, it was impractical to determine specific reasons for each lost sale. Therefore, there has been a natural tendency to fight stock-out conditions or lost sales with increased branch inventories. Obviously, there exists an optimum point in the relationship of inventory and lost sales. Until this point is clearly defined and evaluated, we must expect that in some cases, less inventory is being carried in the branch then is desired, and in other cases, far more inventory is being carried than is required to provide a high degree of reliability of supply.

Let's begin to develop now, how we might go about finding this evasive utopian point. To start with, we could collect a record of just how demand has occurred over a period of some 20 weeks. On Chart IV, each of the points indicate that during the week along the lower axis there was a total demand of so many units of the part number we are observing. For instance, the first point indicates that eight pieces were requested or sold during the first week. In the next week, we see that 17 were sold; more than double that of the preceding week, but on the following week we come down to what turns out to be the average--ten for that week. Continuing to plot demand week by week, we get the pattern illustrated on the left hand side of the chart.

Another way of looking at this variation in demand is to count the number of times in our record that each level of demand has occurred and plot this into a frequency distribution diagram as indicated on the right hand side of our chart. Here, we see that a demand of six occurred only once, although the demand of ten occurred twice, and a demand for twelve occurred three times, and so on, clear to the demand for 17 which occurred twice. Note that on the particular record that we have collected, there was no week in which we had a demand for 16.

If we continue to keep records through many, many more weeks and gradually build up the frequency diagram that we have on the right of this chart, we shall begin to see the relative frequency with which each level of demand may be expected over the long haul. On Chart V, we see the record as it might be expected over a period of several hundred weeks. We see that the average demand of ten per week will occur about 14% of the time, whereas the demand for say six units will occur about 8% of the time. With a diagram of frequencies based on this much evidence, we are well prepared to project or forecast what kind of pattern this demand may follow in the future.

However, to collect and manually plot and digest this mass of data seemed almost insurmountable in terms of the length of time involved. It was decided at this point to turn to the computer to simulate our actual operations. In order to provide a familiar example of how the computer was able to take such a collection of data and use it to simulate actual operations, a device of

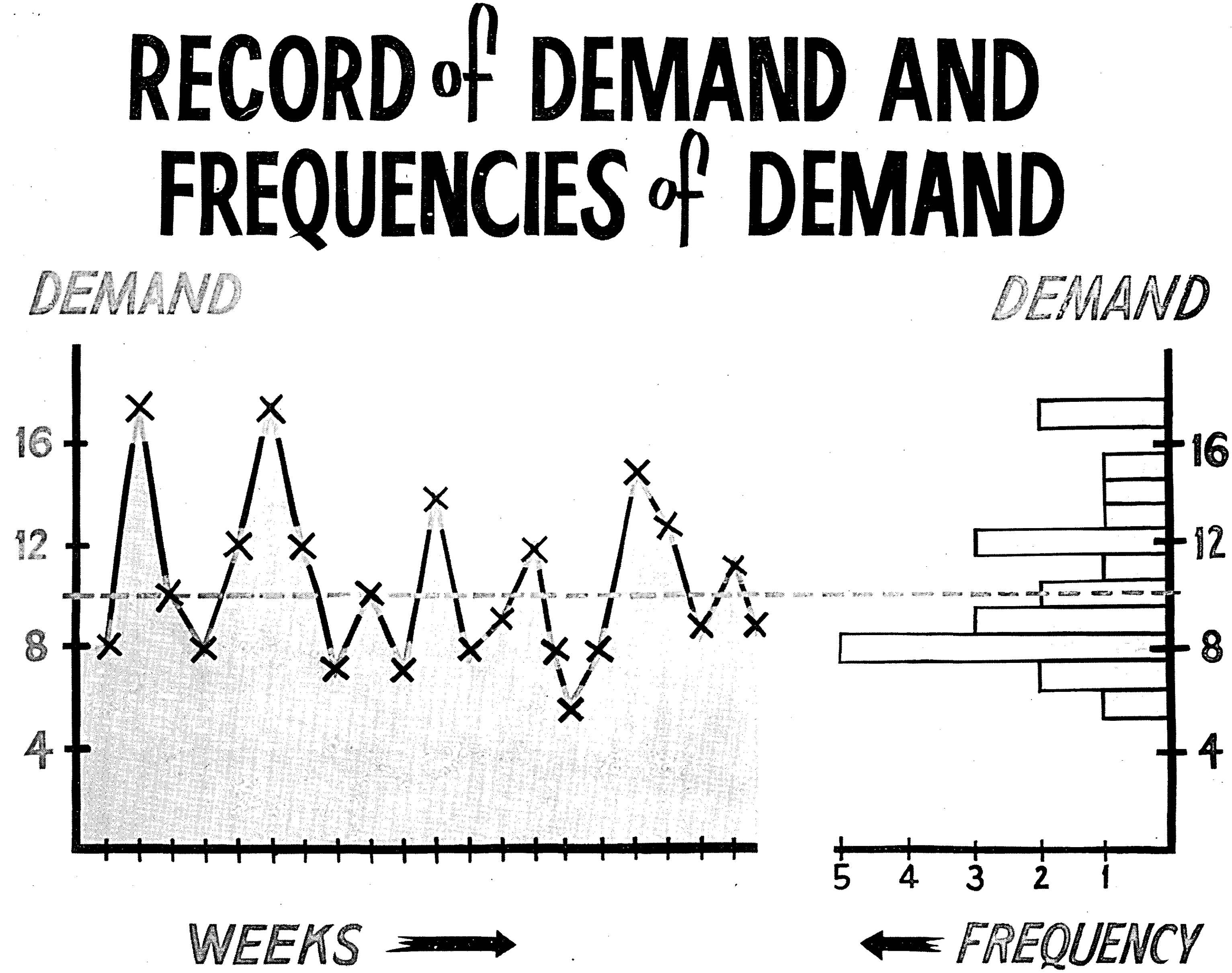
RECORD of DEMAND AND FREQUENCIES of DEMAND
DEMAND
DEMAND
16
12
8
4
16
12
8
4
5 4 3 2 1
WEEKS
FREQUENCY

CHART IV

# MAKING A ROULETTE WHEEL TO REPRESENT FREQUENCIES OF DEMAND

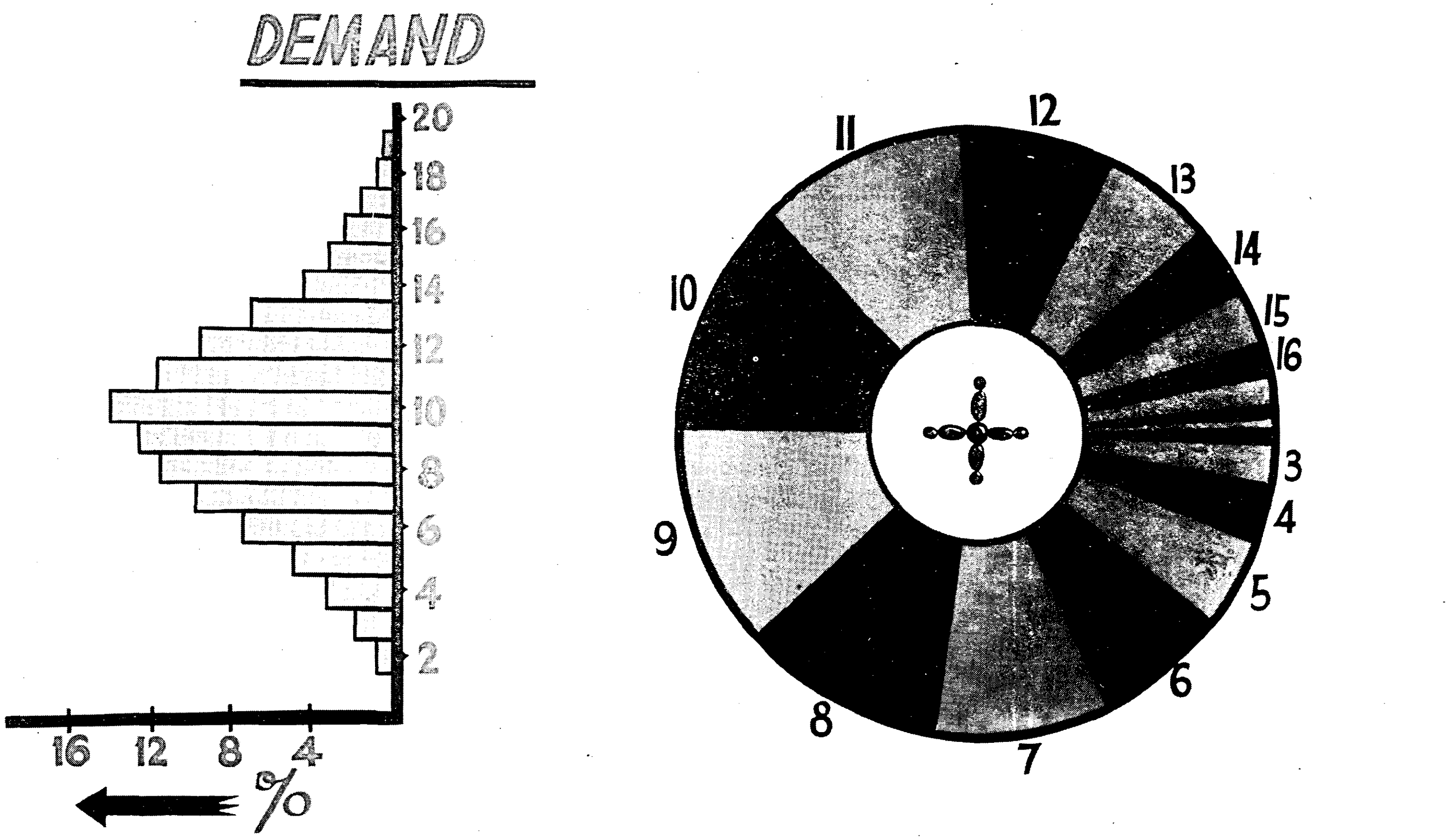




CHART V

questionable background but familiar to most of us, assumes an unexpectedly useful roll. As you can see, we are going to use a roulette wheel; not the kind that is faithful only to the house in Las Vegas, but one which will serve our needs, one that is very likely to win for us. Instead of dividing the circumference of the roulette wheel into equal intervals and then setting the odds for the house to win, we are going to divide our roulette wheel into unequal intervals. In fact, we propose to divide it into intervals which are exactly proportional to the various bars on the frequency diagram on the left. Imagine now, that we took the little bar on the top of the frequency diagram opposite number 19 and then took the bar from opposite 18, laid them side by side and then continued down the diagram laying each succeeding bar to the left of the line of the bars above them. Having completely arranged all of these bars into a single line and in the sequence of going, say from the top down to the bottom of the diagram we could twist this long line of bars around into a circle and we would find the circumference of the circle divided into exactly the same proportions that we want our roulette wheel to have. We have done this on Chart V so that the amount of space on the wheel which has been reserved for number 10 for instance, is proportional to the amount of space reserved for say number 19 in exactly the same ratio as indicated on the frequency diagram on the left. We can well imagine that were we to use this roulette wheel in the same way as our friends in Las Vegas spin their roulette wheels and count the number of times that the ball stops at each of the intervals indicated around this diagram, we could plot them as we have on the frequency diagram on the left, eventually, building up a chart which looks something like the diagram we already have. This is because, on a large number of trials, we would expect the ball to stop more frequently on 9 than say on eight or twelve and certainly much more frequently than on two or three where only a small amount of space has been reserved on the wheel.

We have, as yet, completed only a portion of our task. We must construct still another roulette wheel. You will remember that when we were talking about the diagram of the branch warehouse replenishment system, we mentioned the replenishment cycle time also fluctuates. To study this, it will also help to collect a record of experience. On Chart VI we once again have a record of some 20 odd receipts. This time we are plotting for each receipt the total elapsed time in weeks between the initial transmission of the packing sheet to the central source of supply, and the ultimate receipt of the corresponding goods at the branch warehouse. In the case of the first receipt, we see that it took four weeks to execute the entire replenishment cycle. In the next instance, it took only three weeks, and this was followed by a faster performance of two weeks. But in the case of the next shipment, there was once again a replenishment cycle time of four weeks, and so on.

Once again now, just as we did for our record of demand, we can translate this record of replenishment cycle times into a frequency diagram. On the right of the chart we have indicated the frequency with which each replenishment cycle time represented on our record has occurred. A cycle time of one week occurred in two cases. Three weeks occurred in eight instances, and so on.

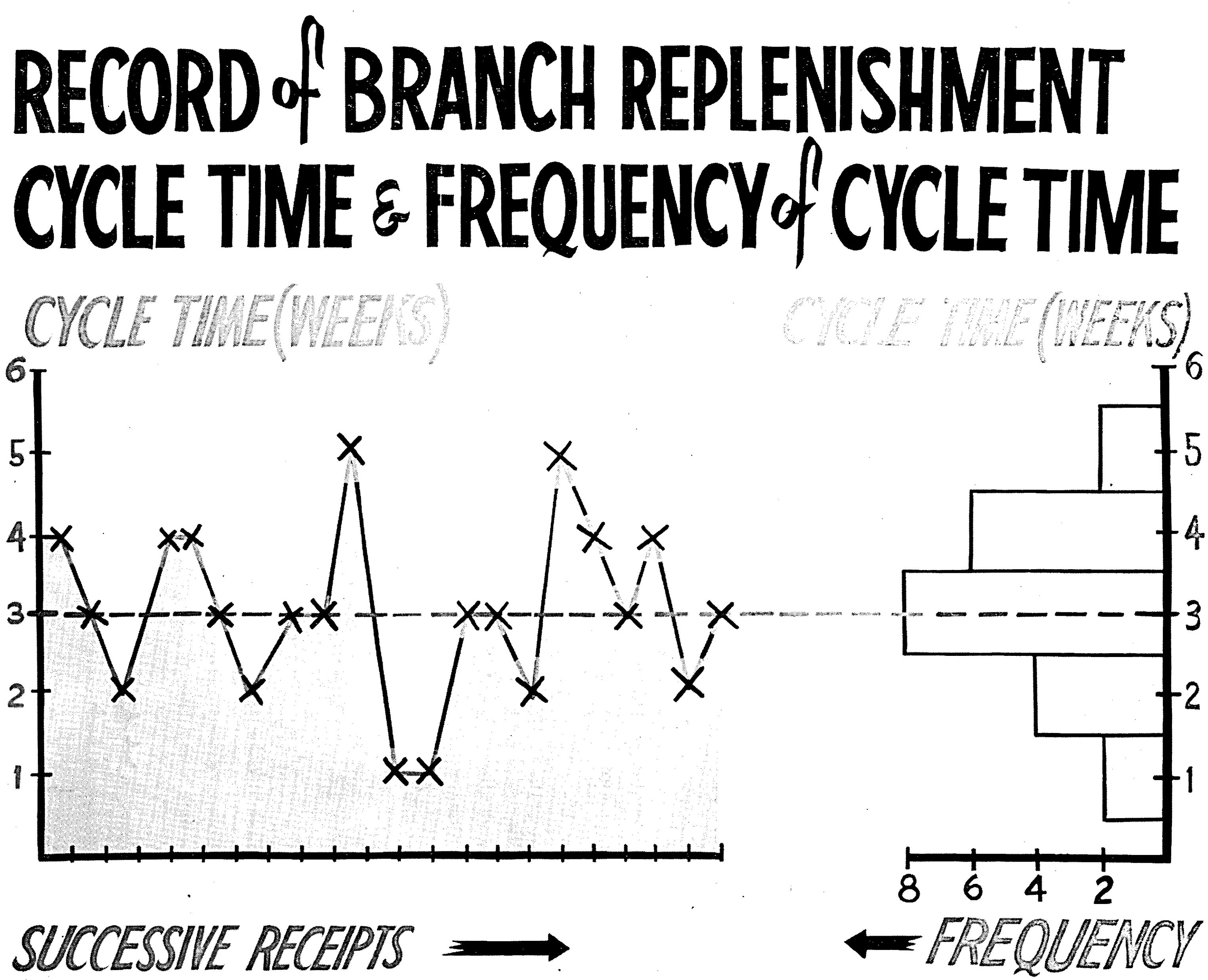
RECORD of BRANCH REPLENISHMENT
CYCLE TIME & FREQUENCY of CYCLE TIME
CYCLE TIME (WEEKS)
CYCLE TIME (WEEKS)
6
5
4
3
2
1
8 6 4 2
SUCCESSIVE RECEIPTS
FREQUENCY

CHART VI

Imagine now that once again we could collect a substantial record of these events and develop considerable confidence in the relative frequency with which we might expect each of these replenishment cycle periods to occur. In just the same manner that we developed a roulette wheel for the distribution of frequencies for demand, we could design another roulette wheel following the diagram of frequencies for replenishment cycle time. Then, with these two roulette wheels before us, it will be possible to simulate operation of a branch warehouse replenishment system, so that it will have a striking resemblance to the pattern of events as they actually occur.

On Chart VII, we show a single branch replenishment system again, but this time we introduce dynamic variability in demand and replenishment cycle time by placing the roulette wheels we have designed in the system. This gives us a very useful model of the processes that we want to investigate. Let's proceed now to operate the model, in order that we may study the behavior of important characteristics of the system. Previously, we had decided that one of the most important things we would like to know is what effect different levels of inventory will have upon the reliability of supply to customers. For the sake of an example, let's assume that we have an average weekly demand expectation of ten per week for some part we want to study. Also, let's use the replenishment cycle time expectations that we developed for our roulette wheel. Finally, for the sake of this experiment, let's see what kind of lost sales might result if we have an initial inventory of 47 units in stock at the selected branch.

Now we spin the two roulette wheels simultaneously and record the numbers that win on this spin of the wheels. Let's set our record up as we have on Chart VIII so that opposite week number one we enter values that came up on the first spin of the wheel, that is, a demand of eight and a replenishment cycle time for that eight units of four weeks. Opposite this, we enter a week-ending inventory of 39 which is simply a reduction of the starting inventory by the eight units which were sold during the first week. We could then go on down through the weeks spinning our roulette wheels and obtaining pairs of numbers just as they have been entered here on the chart. Then, if we also keep track of just when each of the replenishment shipments for the demands that have occurred are to arrive at the branch, we would get a record of inventory fluctuation as indicated in the fourth column of the chart.

During week number seven, we register our first lost sale. Here, we indicate that an inventory of ten units was available at the start of the week, but there were no replenishment shipments arriving that week and there was a demand of twelve, resulting in lost sales of two units.

At this point we have experienced a demand for 84 units in total for the first seven weeks, but have been unable to fill the demand for two of those units, resulting in a lost sales performance index of 2.4% through the first seven weeks.

# BRANCH WAREHOUSE SUPPLY SYSTEM

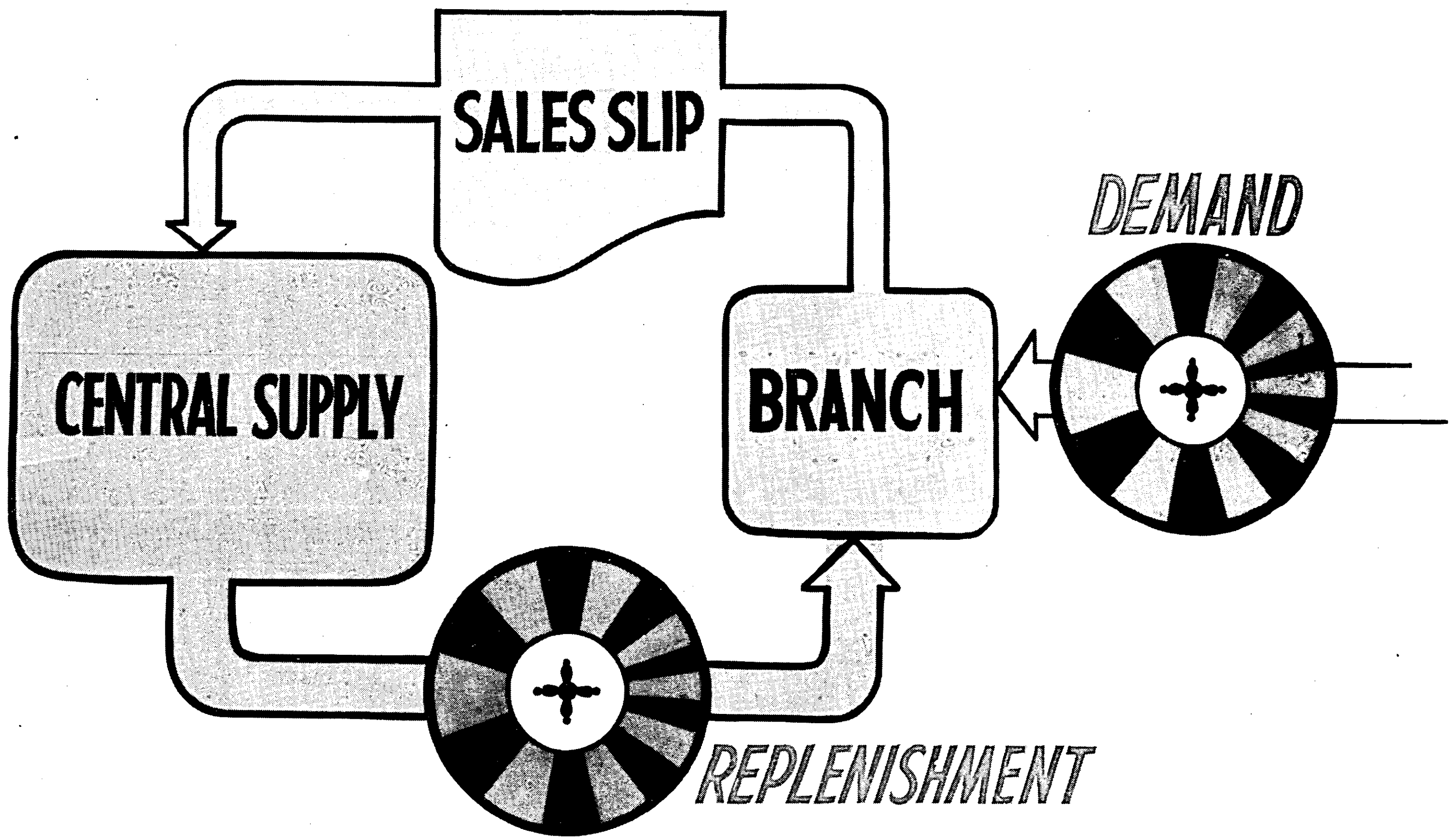


CHART VII

# INITIAL RESULTS of OPERATING THE SYSTEM

| WEEK | DEMAND | REPLEN. TIME | REPLEN. QTY | INVENTORY | LOST SALES | % |
|---|---|---|---|---|---|---|
| | | | | 47 | | |
| 1 | 8 | 4 | 0 | 39 | | |
| 2 | 17 | 3 | 0 | 22 | | |
| 3 | 10 | 2 | 0 | 12 | | |
| 4 | 8 | 4 | 0 | 4 | | |
| 5 | 12 | 4 | 35 | 27 | | |
| 6 | 17 | 3 | 0 | 10 | | |
| 7 | 12 | 2 | 0 | 0 | 2 | 2.4% |
| 8 | 7 | 3 | 8 | 1 | | |



CHART VIII

As we proceed as recorded on Chart IX, we experience no more out-of-stock conditions until we hit the 20th week, at which time we start out with four units in stock. Apparently we get no receipts that week, but have a demand for eleven units, resulting in lost sales of seven units. Of course, through the 20th week then, we have experienced lost sales of nine units and the aggregate demand in those weeks has been 213, resulting in a performance index of 4.2% of total demand going unfilled.

Now, you see, we have run this example to 21 weeks and we have already observed a rather substantial fluctuation in the percentage of total demand which may result in lost sales. Of course, at the beginning it was 0%, then after seven weeks it was 2.4% and now after twenty weeks it is 4.2%. In order to determine what the average expectation of lost sales over the long haul might be for this case, we might have to continue to operate the model and analyze the results for several hundred or perhaps several thousand weekly steps. At whatever point we found that the lost sales percentage no longer fluctuated appreciably, we would have established a single point describing the relationship between lost sales and a starting inventory of 47 units.

A natural question, of course, would be "What would the lost sales have been had I started with a different inventory." Obviously, we could run through a long series of such weekly observations again and establish a new number representing expected lost sales as a per cent of demand for some other starting inventory. In fact, if we continued this process for several different figures, we could obtain enough data to establish a trend showing the relationship between inventory and lost sales for the average demand that we have selected under a replenishment cycle pattern which applies to the system. Were we to do this, we could ultimately develop a curve as shown on Chart X. For example, we might have started with an inventory of 50 units as shown on this chart, and after spinning our roulette wheels long enough, we could have established the ratio of 6% indicated on the chart. Then, we might have tried again for an inventory of say 80 units and established that the expected lost sales would be about 1% of demand. Continuing in this manner, we would have determined any of the other points on the curve and ultimately we should have sufficient data to construct a curve representing a wide range of inventory figures with corresponding lost sales expectations.

So far, however, we have conducted our work here at the conference around an average demand of just ten units per week. Clearly, for any given part number the demand may change from one season to the next; there will be an over-all increasing and decreasing pattern for the lifetime of the part; and we are concerned with several thousand different part numbers, each with its own expected rate of demand. So, in order to give us guide lines throughout the entire range of expected demand rates for all articles carried in inventory, we need a series of curves like the one just completed. However, even in the

# CONTINUING RESULTS *of* OPERATING *the* SYSTEM

| WEEK | DEMAND | REPLEN. TIME | REPLEN. QTY. | INVENTORY | LOST SALES | % |
|---|---|---|---|---|---|---|
| 13 | 9 | 3 | 7 | 33 | | |
| 14 | 12 | 3 | 0 | 21 | | |
| 15 | 6 | 2 | 7 | 22 | | |
| 16 | 8 | 5 | 9 | 23 | | |
| 17 | 15 | 4 | 18 | 26 | | |
| 18 | 13 | 3 | 0 | 13 | | |
| 19 | 9 | 4 | 0 | 4 | | |
| 20 | 11 | 2 | 0 | 0 | 7 | 4.2 |
| 21 | 9 | 3 | 28 | 19 | | |

CHART IX

# BRANCH INVENTORY vs. LOST SALES FOR AVERAGE DEMAND of TEN PER WEEK

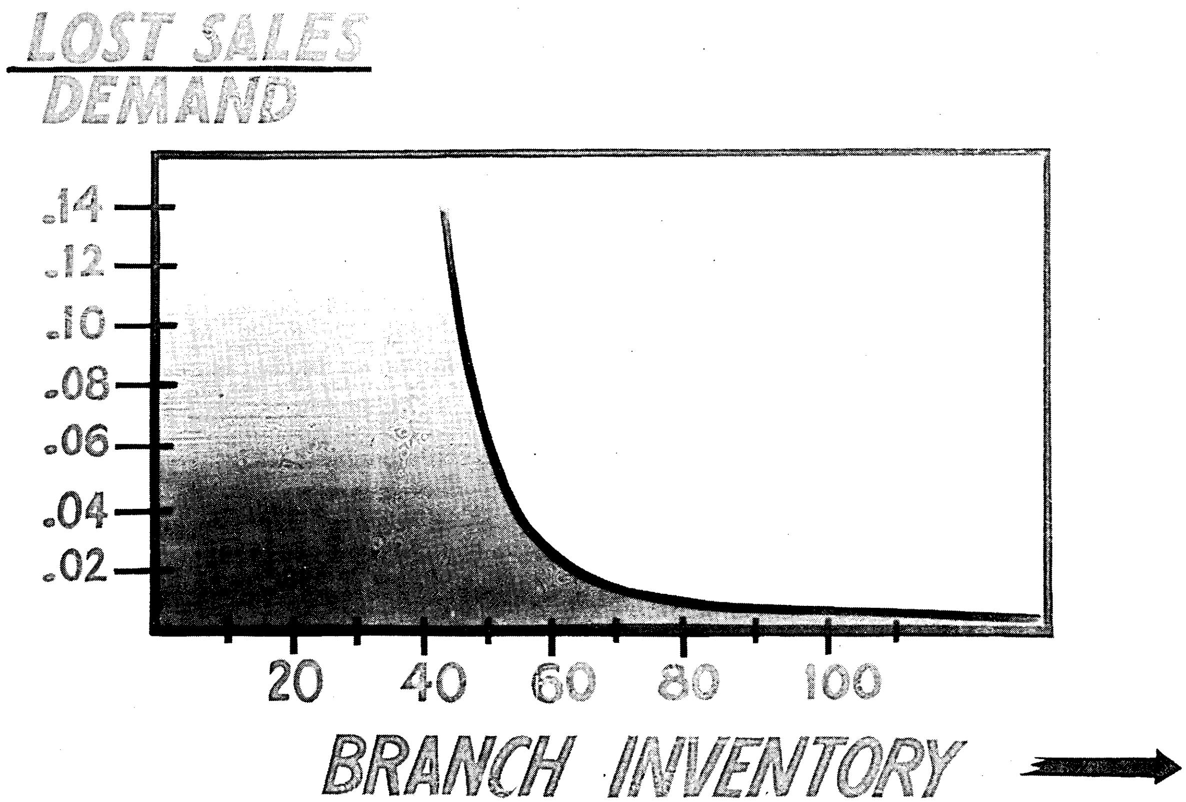


CHART X

somewhat oversimplified case that we have described here, it is estimated that several man years would have to be expended to complete adequate computations. In the model that was actually studied, it was necessary to include other characteristics of the system, which greatly increased the computational requirements. Fortunately, we had already decided to use computer simulation techniques and it was practical for us to take a rather liberal attitude about such complications.

So, a model of our branch replenishment supply system, quite similar to that which we have described here, but somewhat more complicated was programmed as an application for the Univac Scientific Computer in the computational laboratories of our affiliate, The Ramo-Wooldridge Corporation, in Los Angeles. The system was operated on the computer until several points for each of seven curves had been generated. Of course, each of these several points was carefully selected to give us the best possible indication of the shape of the curve to which each applied. To give you some idea of the magnitude of the computational task here, it was found necessary to run up to about 10,000 replenishment cycles for some of the points to get a stable indication of the lost sales expectations at any given level of demand and inventory. Each one of these several thousand cycles involved about 100 program instructions to the computer. Altogether, it is estimated that the computer executed about 30 million instructions in the course of the simulation study. Careful analysis of the data, as it was generated, indicated that with seven basic curves determined by the computer, additional intermediate curves could be fitted in very efficiently by manual methods.

Design and translation of the model into computer language involved less than six man-months of work, and then the complete simulation process was completed with the use of slightly less than two hours computer time. Thus, in less than two hours, the computer provided us with data equivalent to nearly 200 years experience.

Now that the computer simulation work has been completed, the next step is to test the validity of the results. We are now in this stage of the entire program. Representative part numbers and branches from our nationwide network have been selected and placed on a system of inventory control involving the use of the data obtained from the simulation study. The results of this phase are as yet inconclusive. We expect, however, to realize three major benefits from this understanding of the relationship of lost sales and inventory. First, we expect to be able to operate with reduced inventory investment. Second, we expect to improve profits through decreased costs of inventory, and the attainment of higher sales volume as a result of a reduction in lost sales. Thirdly, we anticipate an improved competitive position through improved customer service.

A number of by-products of our study have already been generated. These are both interesting and important. The experiment has required us to re-examine and improve our forecasting techniques and this will have an impact on the rate at which obsolescence is generated. It is now easier to identify the cause of the lost sale. This has led to a greater concentration on the problem of stock-outs in our central supply at Cleveland, and as a consequence, we are already realizing the benefits of improved inventory techniques in this area of operation. Our sharper focus on reliability of supply problems has stimulated us to analyze our records more extensively. In turn, management now receives more and better organized data on which to render decisions regarding the investment in inventory as related to lost sales.

This is as much of the story as can be told today. We feel that we have discovered a new and exciting technique for solving many present and future business problems. Our experience thus far make us enthusiastic to the point that it is not difficult for any of us to imagine the many problems which may finally yield to this powerful new tool of management.