

UNIVERSITY OF ILLINOIS
DIGITAL COMPUTER

LIBRARY ROUTINE M 18 - 213

TITLE One-Step Automatic Eigenvalue-Eigenvector Program

TYPE Entire Program

ACCURACY Usually 10 or 11 decimal places

DURATION (a) 20 seconds to input program
(b) $n^2/250$ to $n^2/40$ seconds (depending on number of digits) to input elements of matrix
(c) $n^3/200$ seconds per iteration. Most matrices require from 4 iterations ($n = 3$) to 7 iterations ($n = 40$) for convergence.
(d) $n^2/2$ seconds to punch results.

METHOD OF USE (1) Read program tape into memory in usual way. If no read in error has been committed machine will stop on 20 (03L)₁₆.
(2) Place data tape in reader and restart machine.
(3) After computation has been completed, results will be punched out for printing. If only the eigenvalues are computed, then they are printed in a single column. If the eigenvalues and eigenvectors are computed, then following each eigenvalue will be the corresponding eigenvector.
(4) The machine stops on a 20 (03L)₁₆. A new problem can be begun by repeating step 2.

PUNCHING OF THE DATA To compute the eigenvalues or the eigenvalues and eigenvectors of the real symmetric matrix

A_{ij} $i = 1, \dots, n; j = 1, \dots, n$ proceed as follows:

(a) Scale the matrix so that

$$\sum_{i,j=1}^n A_{ij}^2 < 1/2.$$

(b) Punch the elements of (A_{ij}) for $i \geq j$, row by row, as a sign digit followed by up to twelve decimal digits.

(c) Element A_{nn} is followed by an N or J. If followed by an N, then the eigenvalues and eigenvectors are computed. If followed by a J, then only the eigenvalues are computed.

(d) The last character is followed by a sexadecimal character p which determines the number of decimal digits to be printed. The character p can assume the values 1,2,3,..., 9, K, or S where K = 10 and S = 11.

CAPACITY

All the eigenvalues and eigenvectors can be found for a matrix of order 23. However, all the eigenvalues of a matrix of order 40 can be computed.

INTERNAL CHECKS

During the input and operation of the program a number of checks are made. If they fail, the machine stops. A list of the locations and reasons for failure is given below

<u>LOCATION</u>		<u>REASON FOR FAILURE</u>
<u>Decimal</u>	<u>Sexadecimal</u>	
908	38N	Sum check on program tape ₂ fails
8	008	Data tape fails to have $\frac{n_2+n}{2}$ elements
23	017	Drum failure
33	021	Drum failure
71	047	Drum failure
79	04L	Drum failure
168	OK8	Arithmetic failure in Routine M-4. This can be caused by incorrect scaling.

This program is essentially a combination of M-4, P-2, N-2, and X-8. It replaces M-5.

DATE	Re: April 3, 1957
PROGRAMMED BY	Gene Golub
APPROVED BY	<i>D.E. Muller</i>

LOCATION	ORDER		NOTES	PAGE 1
	Library Routine X-1	Decimal Order Input		
	00 3K			
	00 F			
	00 203F			
	00 F			
	00 2560F			
	00 800K			
0	41 202F			
	L1 39F	by 2; from 3		
1	L4 202F			
	40 202F		Compute sum check	
2	F5 0L			
	40 0L			
3	L0 10L			
	32 0L			
4	00 1F			
	L5 39F	by 6; from 8		
5	86 11F			
	00 S4	by 7	Store Routine M-4 on drum	
6	F5 4L			
	42 4L			
7	F5 5L			
	40 5L			
8	L0 11L			
	32 4L			
9	26 999F		Read in another part of program	
	00 F			
10	N1 202F			
	L1 202F			
11	06 11F			
	00 164S4			
12	41 0F			
	L1 39F	by 4; from 5		

LOCATION	ORDER		NOTES	PAGE 2
13	L4 OF 40 OF		Compute sum check	
14	F5 12L 42 12L			
15	L0 33L 32 12L			
16	00 1F L5 39F	by 18; from 20		
17	86 11F 00 164S4	by 19;	Store routine on drum	
18	F5 16L 40 16L			
19	F5 17L 40 17L			
20	L0 34L 32 16L			
21	00 1F L5 0F		Store sum check	
22	86 11F 00 225S4		on drum	
23	41 39F L1 5F	by 25; from 26		
24	L4 39F 40 39F		Compute sum check	
25	F5 23L 42 23L			
26	L0 35L 32 23L			
27	00 1F L5 5F	by 29; from 31		
28	86 11F 00 226S4	by 30	Store routine on drum	
29	F5 27L 42 27L			

LOCATION	ORDER		NOTES	PAGE 3
30	F5 28L 40 28L			
31	L0 36L 32 27L			
32	26 999F 00 F		Read in another part of program	
33	N1 0F L1 100F			
34	06 11F 00 225S4			
35	N1 39F L1 39F			
36	06 11F 00 261S4			
37	41 114F L1 82F	by 39; from 40		
38	L4 114F 40 114F			
39	F5 37L 40 47L			
40	L0 47L 32 37L			
41	00 1F L5 82F	by 43; from 45		
42	86 11F 00 261S4	by 44	Store routine on drum	
43	F5 41L 40 41L			
44	F5 42L 42 42L			
45	L0 48L 32 41L			
46	20 63F 00 F			

LOCATION	ORDER		NOTES	PAGE 4
47	N1 114F L1 114F			
48	06 11F 00 294S4 00 39K Library Routine M-4		Change OFF OFF in word 129 of M-4 to FF 28F FF 28F	
0	26 800N 00 5K 50 S3	by 60; from 73	Read in data	
1	50 L 26 77L			
2	40 OF L3 4F			
3	32 3L FF 27F		Test to see if enough elements have been read in	
4	L3 OF 32 5L	from 3	Eigenvalues or Eigenvalues and Eigenvectors?	
5	L5 30L 26 6L			
6	L5 31L 40 19L	from 4 from 5		
7	L5 3F L0 28L			
8	42 29L 00 20F			
9	46 19L 81 4F		Read in print parameter	
10	00 20F 46 18L			
11	41 2F 85 11F		Read eigenvalue routine off drum.	
12	00 S4 32 12L			
	40 (34L)			

LOCATION	ORDER		NOTES
13	L4 2F 40 2F		
14	F5 12L 40 12L		
15	F5 11L 40 11L		
16	L0 32L 36 11L		
17	L3 2F 32 18L		Has routine been read off drum correctly?
18	FF (0)F 92 167F	by 10	
19	50 ()F 50 19L	by 6, 8	
20	26 34L 41 2F		Perform computations
21	85 11F 00 164S4	from 26 by 25	Read next routine off drum
22	32 22L 40 34L	by 24	
23	L4 2F 40 2F		
24	F5 22L 40 22L		
25	F5 21L 40 21L		
26	L0 33L 36 21L		
27	L3 2F 36 34L		
28	FF 30F 00 1F		
29	80 F 00 (n)F	by 7	

LOCATION	ORDER		NOTES
30	JO F 50 19L		
31	50 F 50 19L		
32	05 11F 00 164S4		
33	05 11F 00 226S4		
34	41 5F L5 18L	by 22; from 27	
35	46 42L 46 48L		Print parameters.
36	L5 38L 42 41L		
37	L5 29L 50 29L		
38	74 29L S5 S3		
39	10 1F L4 38L		
40	40 6F 92 131F	from 57	
41	92 515F L5 ()F	by 55	
42	50 ()F 50 42L	by 35	
43	26 77L L1 19L		Enter print routine
44	36 54L L5 5F		
45	L4 6F 42 47L		
46	92 131F 41 7F		

LOCATION	ORDER	NOTES	PAGE 7
47	92 131F	from 53	
	L5 ()F	by 45,50	
48	50 ()F	by 35	
	50 48L		
49	26 77L		
	L5 47L		
50	L4 29L		
	42 47L		
51	92 515F		
	F5 7F		
52	40 7F		
	L0 29L		
53	36 47L		
	92 131F		
54	F5 41L	from 44	
	F4 5F		
55	42 41L		
	F5 5F		
56	40 5F		
	L0 29L		
57	32 40L		
	92 551F		
58	20 58L		Stop; prepare to begin
	41 2F		new problem
59	85 11F	from 64	
	00 226S4	by 63	
60	32 60L		
	40 0L	by 62	
61	L4 2F		
	40 2F		
62	F5 60L		
	40 60L		
63	F5 59L		
	40 59L		
64	L0 75L		
	36 59L		

LOCATION	ORDER	NOTES	PAGE 8
65	L3 2F 32 66L		
66	FF 31F 41 3F	from 65	
67	85 11F 00 261S4	from 72 by 71	
68	36 68L 40 77L	by 70	
69	L4 3F 40 3F		
70	F5 68L 42 68L		
71	F5 67L 40 67L		
72	L0 76L 36 67L		
73	L3 3F 36 5F		
74	FF 32F 00 F		
75	05 11F 00 261S4		
76	05 11F 00 294S4 00 82K		
	Library Routine P-2 52 26 812N 00 82K		
	Library Routine (Revised N-2) 00 900K		
	Library Routine X-7 142 26 837N		Change FF order here to FF 26F