

UNIVERSITY OF ILLINOIS
DIGITAL COMPUTER

LIBRARY ROUTINE 85 - 231

By D. B. Gillies

TITLE $\frac{1}{32}$ Natural Logarithm (D.O.I. or SADOI)

TYPE Closed functional subroutine

NUMBER OF WORDS 36

TEMPORARY STORAGE 0,1,2,3

ACCURACY $\pm 2^{-38}$ if $y > 0$

DURATION 8.3 to 14.3 Millisec, depending on how much scaling is required.

USE Enter with $y > 0$ in A, link in Q. When link is obeyed, A has $\frac{1}{32} \ln y$
If $y \leq 0$, the machine will stop at 3'L on FF 02L (sexadecimal) with Q = y.

METHOD Define $z = 2^n y$, where $\frac{1}{2} \leq z < 1$,
and $x = 2 \left(\frac{z - \frac{1}{\sqrt{2}}}{z + \frac{1}{\sqrt{2}}} \right)$

Then $\frac{1 + \frac{1}{2} x}{1 - \frac{1}{2} x} = \sqrt{2} z$ is an algebraic identity.

$\ln \sqrt{2} z = 2 \left(\frac{x}{2} + \frac{1}{3} \left(\frac{x}{2} \right)^3 + \frac{1}{5} \left(\frac{x}{2} \right)^5 + \dots \right)$

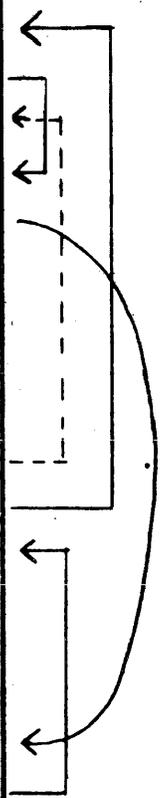
This expression is replaced by an abbreviated power series $P(x) = x + C_3 x^3 + C_5 x^5 + C_7 x^7 + C_9 x^9$ with a maximum error of less than 10^{-11} for $\frac{1}{2} \leq z < 1$.

Then $\frac{1}{32} \ln y = \frac{1}{32} (-n \ln 2 - \frac{1}{2} \ln 2 + P(x))$

lgr

RT: 10/29/58 DATE February 3, 1957 CODED BY <i>D. B. Gillies</i> CHECKED BY <i>D. E. Muller</i>
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LOCATION	ORDER	NOTES
0	00 K(S5) 40 1F	store y
1	K5 F 42 28L 41 F	plant link clear location for $2^{-5} \ln(y)$
2	50 1F	y to Q for scaling
3	S5 F 36 4L	y if $y < 0$ stop
4	FF 47F L0 8L	
5	36 10L 00 6F	is scaled $y' < 2^{-6}$ if so form $2^6 y'$
6	L1 29L L4 F 40 F	and subtract $\frac{6}{32} \ln 2$ from $2^{-5} \ln y$
7	32 3L 22 2L	if $y = 0$, $2^{-5} \ln y$ overflows eventually, jump to stop loop
8	01 4033F L5 30L	$2 y'$ in Q approximate constant 2^{-6}
9	L4 F 40 F	subtract $\frac{1}{32} \ln 2$ from $2^{-5} \ln y$
10	SJ F 36 8L	if $y' < \frac{1}{2}$ jump back.
11	S5 F 50 5L	A has $z = y' = \text{scaled } y; Q=0$
12	L0 31L 10 1F	$\frac{1}{2} (z - \frac{1}{\sqrt{2}})$
13	L4 31L 40 1F	$N(1) = \frac{1}{2}(z + \frac{1}{\sqrt{2}})$
14	L0 31L 00 1F	$z - \frac{1}{\sqrt{2}}$, small number in Q
15	66 1F S5 F	
16	40 1F 7J 1F	$N(1) = \frac{2(z - \frac{1}{\sqrt{2}})}{(z + \frac{1}{\sqrt{2}})} = x$



LOCATION	ORDER	NOTES
17	40 2F 50 2F	$N(2) = x^2$
18	7J 32L L4 33L	$C_9 x^2$
19	40 3F 50 2F	$C_9 x^2 + C_7$
20	7J 3F L4 34L	
21	40 3F 50 2F	$C_9 x^4 + C_7 x^2 + C_5$
22	7J 3F L4 35L	
23	40 3F 50 1F	$C_9 x^6 + C_7 x^4 + C_5 x^2 + C_3$
24	7J 3F 40 3F	
25	50 2F 75 3F	$C_9 x^7 + C_7 x^5 + C_5 x^3 + C_3 x$
26	L4 1F 10 4F	$P(x) = x + C_3 x^3 + C_5 x^5 + C_7 x^7 + C_9 x^9$
27	F4 30L 10 1F	$\frac{1}{32} (-\ln \sqrt{2} + P(x))$
28	L4 F 22 (link)F	$-\frac{m}{32} \ln 2 (= \frac{1}{32} \ln y)$
29	00 F 00 71,449,067,324 F	Obey link $\frac{6}{32} \ln 2$
30	00 F 00 1,087,603,449,889F	$= \frac{1}{32} \ln 2$
31	00 F 00 388,736,063,997 F	$\frac{1}{\sqrt{2}}$
32	00 F 00 255,111,138 F	$C_9 = .0004 \quad 6404 \quad 4457$
33	00 F 00 1,225,163,409 F	$C_7 = .0022 \quad 2855 \quad 8603$
34	00 F 00 6,872,049,879 F	$C_5 = .0125 \quad 0018 \quad 5911$
35	00 F 00 45,812,982,352 F	$C_3 = .0833 \quad 3332 \quad 9444$