UNIVERSITY OF ILLINOIS
DIGITAL COMPUTER
STATISTICAL LIBRARY

INDEX OF STATISTICAL PROGRAMS

Programs for use on ILLIAC, the University of Illinois Electronic Digital Computer, which are of interest to persons whose research involves them in statistical operations, have been assembled and are listed on the following pages.

This is not a complete ist of those programs which conceivably might be of use to persons involved in statistical operations. Special programs designed for a single use are not included. Nor is this list a static list. New programs are added and older ones replaced as newer methods are developed. It is hoped, however, that this index will serve the purpose of acquainting users and potential users of ILLIAC with the types of analysis that currently can be accomplished with the aid of the computer.

These programs were developed by persons from several departments of the University. Many were created hastily in response to the needs of a particular research problem. It is for this reason that not all of the programs are in their final stage of development. For example, a set of written instructions available for distribution has been duplicated for only a few of these programs. It is the intention of the Digital Computer Laboratory in time to revise any which require improvement and to supply a set of written instructions for each of the programs.

This list will constitute an index for an additional auxiliary library called the Statistical Library to be added to the existing library structure of the Digital Computer Laboratory. There are, however, programs in this list from two other ILLIAC libraries. To distinguish among libraries, each program in the Statistical Library will be designated by the symbols, KSL, and an index number. When a program is classified in more than one library, both classifications will be shown. This is illustrated below:

KSL 1.11	Program 1.11 in the Statistical Library
KSL 2.00, K-8	Program 2.00, but also K-8 in the Main Library
KSL 2.10, K-4 AUX	Program 2.10, but also K-4 in the Auxiliary Library

Following the library designations and title of the program will be found a brief description, the limitations of the program, if any, and any other pertinent information.

Kern Wm. Dickman

lgr
December, 1958

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I. FACTOR ANALYSIS

A. Eigenvalues and Eigenvectors

KSL 1.00, M-18 One-Step Automatic Eigenvalue-Eigenvector Program

A real, symmetric matrix in triangular form is read into the computer. This matrix is diagonalized by performing a sequence of orthogonal transformations with each transform designed to reduce one off-diagonal element to zero. These elements will not remain zero, so the process is repeated until convergence is attained. For eigenvalues only, the order of the matrix is limited to order 40. For both eigenvalues and eigenvectors, the order is limited to 23. Programmer: Gene Golub

KSL 1.01, M-20 Eigenvalues of a Symmetric Matrix by Givens' Method

Givens' procedure first reduces the symmetric matrix to tri-diagonal form. By the use of a binary chopping procedure the eigenvalues are found. This method results in a considerable saving of machine time. This advantage is offset, however, by the fact that the data tape must be specially prepared. The order of the matrix is limited to order 128. Programmer: C.W. Gear

See also routines KSL 1.10 and KSL 1.11.

KSL 1.05, M-19 Solution of the Matrix Equation, $Ax = \lambda Bx$, Where A and B Are Symmetric and B Is Positive Definite

The routine will print the eigenvalues of B and the eigenvalues and eigenvectors of $A - \lambda B$ for matrices limited to order 19. Programmer: Gene Golub

B. Principal Axis Solutions

KSL 1.10, M-7 AUX Principal Axis Factor Analysis

This routine follows the procedure of KSL 1.09, but the eigenvectors are converted to factor loadings before printing. The order of the matrix is limited to order 23. Programmer: Jack Neuhaus

?– KSL 1.11 Principal Axis Factors

This routine will extract the largest k factors from a correlation matrix by an iterative procedure suggested by Hotelling. The associated largest k eigenvalues also are printed. Although this routine is preferred for the purpose of factor analysis, the routine will fail in the exceptional circumstance that two or more of the largest k eigenvalues are of the same size. The order of the matrix is limited to 111. Programmer: K.W. Dickman

I. FACTOR ANALYSIS

?— KSL 1.15 Transformation to Principal Axis Factors

A set of orthogonal factors which have been calculated by some other method can be transformed into a principal axis solution by this program. The solution, however, will span the factor space which is not equivalent to the test space unless there has been a complete factor extraction. The program is limited to 21 factors and 111 variables. Programmer: K.W. Dickman

C. Centroid Factor Analysis

KSL 1.20 Centroid Factors with Fixed Communalities

Following the procedures outlined by L.L. Thurstone, this program will extract centroid factors from a correlation matrix. The program is called fixed to distinguish it from other procedures wherein the communalities are altered during the course of the calculations. The largest matrix that can be factored with this program is one of 111 variables. Programmer: K.W. Dickman

?— KSL 1.21 Centroid Factors with Estimated Communalities

Prior to the extraction of each factor the largest correlation in absolute value from each row of the residual matrix is used to replace the communalities. Otherwise, this program is similar to KSL 1.20. The number of variables is limited to 111. Programmer: K.W. Dickman

D. Square Root or Diagonal Method of Factoring

?— KSL 1.30 Square Root Factors

This routine will extract square root factors. The factor analyst can select the test variables as pivots, or else the pivoting can be made automatic in which case the variable selected is the one with the largest absolute sum of residual correlations. The number of variables is limited to 111. Programmer: K.W. Dickman

E. Estimation of Communalities

KSL 1.50, M-10 AUX Estimation of Communalities by the Method of Maximum Likelihood

The number of factors to extract and the estimates of the communalities are determined by this routine by a chi-square test of significance. The amount of time required for this routine prohibits its use for other than theoretical investigations. The order of the matrix is limited to size 17. Programmer: Gene Golub

I. FACTOR ANALYSIS

KSL 1.51, K-7 AUX Iterative Estimation of Communalities: Principal Axis Method

In order to use this routine the number of factors, k, must be known. After each principal axis solution following the method described under KSL 1.00, communalities are calculated for k factors, and these are substituted for the previous estimates. The process is repeated iteratively until stability of the estimates is attained. The order of matrix is limited to order 18. This routine is of the kind that uses considerable machine time. It is not recommended for frequent use. Programmer: Ray Twery

?- KSL 1.52 Estimation of Centroid Communalities

The number of factors, k, must be known to use this routine. A trial run on KSL 1.20 often will reveal the approximate number of factors to extract. KSL 1.52 will extract k factors by the centroid method and estimate communalities from the residual communalities. The procedure is repeated iteratively as many times as desired. At the end of a specified number of iterations, the estimated communalities will be printed out and comparisons can be made between successive estimates. When stability is achieved, these estimates then can be used with KSL 1.20. The order of the matrix is limited to 84. Programmer: K.W. Dickman

KSL 1.53 Extraction of the Largest Element in Each Row

For large matrices some factor analysts will select as communalities estimates the largest correlation in absolute value from each row of the correlation matrix. When the matrix is in triangular form this selection is difficult and time consuming to do visually. KSL 1.53 selects these values and punches them in a form to use with routine KSL 1.20. The largest matrix that the routine will handle is of order 111. Programmer: K.W. Dickman

F. Comparisons Among Factor Sets

?- KSL 1.61 Indices of Similarity

The program calculates a matrix of cosines among a set of factor vectors. If a set of factors, F, is contained in the same space as another set, G, the cosines will indicate the extent of similarity among the factors. For example, a cosine of +1 or -1 between f_i and g_j indicates collinearity. A value of zero, on the other hand, indicates orthogonality. Similarly other values indicate the degree of similarity. There is no limit on the number of test variables. The number of factors which can be compared with one machine run is limited to 38. Programmer: Ray Twery

I. FACTOR ANALYSIS

G. Factor Scores

Factor scores currently are calculated in several steps which depend upon whether the factoring was complete or partial, whether the factors obtained are orthogonal or oblique, and upon the size of the correlation and factor matrices. A report which explains the procedures to follow under each set of conditions was prepared by Mr. John Hurley. This report entitled "Obtaining Factor Scores" has been duplicated and is available for distribution.

For the calculation of beta weights when the factors are orthogonal, see programs KSL 5.10, 5.15, 5.16, and 6.00. When the factors are oblique, see programs KSL 1.71 and 1.72. Given the beta weights, see programs KSL 1.73 or 5.00 for the calculation of the scores.

KSL 1.71 Hurley No. 1: Primary Factors

This program will obtain the correlations among the primary factors and the primary factor pattern. However, if the oblique rotation was obtained by the use of KSL 1.90 or 1.92, these programs include the primary factor pattern as optional output. The number of variables for Hurley No. 1 is not limited; the number of factors cannot exceed 22. Programmer: John Hurley

KSL 1.72 Hurley No. 2: Beta Weights

This program will obtain the normalized inverse of the transformation matrix, the primary factor structure, and the matrix of beta weights. The number of variables is limited to 111; the number of factors cannot exceed 22. Programmer: John Hurley

KSL 1.73 Hurley No. 3: Factor Scores

Given the measurement matrix, the beta weights, and the standard deviations, this program will obtain factor scores for either orthogonal or oblique factors. There are no practical limits to this program. Programmer: John Hurley

H. Orthogonal Rotation of Factors

KSL 1.80 Varimax

The varimax rotation procedure was devised by Dr. Henry Kaiser. The rotated factor matrix is a good approximation to orthogonal simple structure. The number of variables is limited to 127; no limits on the number of factors. Programmers: Bartky, Dickman, and Kaiser

I. FACTOR ANALYSIS

KSL 1.81, K-11 AUX Quartimax

The quartimax rotation, devised by the Messrs. Neuhaus, Wrigley and Saunders, maximizes the variance of the squared factor loadings. The routine has the characteristic that a large general factor usually becomes even larger. The number of variables multiplied by the number of factors must not exceed 746. Programmer: Jack Neuhaus

I. Oblique Rotation of Factors

KSL 1.90 Oblimax

The oblimax rotation procedure, devised by Dr. David Saunders, maximizes the factor variance such that the solution becomes a close approximation to Thurstone's oblique simple structure. The routine will print out either the reference vector structure or the primary factor pattern of both. The number of variables is limited to 111. There are no practical limits on the number of factors for the reference vector structure, but only 21 factors can be handled if the primary factor pattern is desired. Programmer: K.W. Dickman

KSL 1.91 Thurstone's Analytic Method for Simple Structure

Thurstone's procedure involves assigning weights to the transformation matrix so that the resulting factors approximate simple structure. The routine is limited to 111 variables and 18 factors. Programmers: John Hurley and K.W. Dickman

KSL 1.92 Kaiser Oblique Rotation

This oblique rotation procedure, devised by Dr. Henry Kaiser, will calculate weights to transform the factors to simple structure, but this process is repeated iteratively until convergence is attained. The number of variables is limited to 111 and the number of factors must not exceed 16. Programmer: K.W. Dickman

KSL 1.93 Trial Vector Rotation

The purpose of this routine is to transform a set of factors to a new oblique position approximately collinear with predetermined trial test vectors. The user can select clusters of variables and attempt to place the factors through these variables. The number of factors is limited to 22 and the number of variables may not exceed 127. Programmer: John Hurley

I. FACTOR ANALYSIS

?– KSL 1.94 Solution of the Matrix Equation, AT = B, for T where T Is Restricted to a Normalized Square Matrix

The user of this routine may postulate a rotated matrix B, and solve for T. The product, AT, will approximate as closely as possible the matrix B under the restraint that T is normalized by columns. Of course, there may be no solution at all which will come close to transforming A to B. This powerful routine has other uses such as finding T when the rotation has been done by a graphical method or such as matching factor patterns across studies. The number of variables or rows of A or B is limited to 136. The number of factors or columns of A or B is limited to 22. Programmer: John Hurley

?– KSL 1.95 Plotting Factor Pairs

The purpose of this routine is to photograph the plots of all factor pairs of a set. From an inspection of the plots judgments can be made for a subsequent transformation. This routine could be used to photograph scatter plots for sets of correlations provided the date were scaled to have a range consistent with the routine. Programmer: R.P. Polivka

?– KSL 1.96 Combined Rotation Routine

This program was designed to reduce the number of clerical steps in the visual rotation process. Given the former transformation, the next shift, and the original factor matrix, this routine will print the current transformation matrix, the transformed factors, the hyperplane count, the correlations among the reference vectors, and finally the next set of factor plots. The routine is limited to 111 variables. Programmer: R. Conger

KSI - 1.97 - 305 - MAXPLANE (Oblique Factor Rotation)

II. INDICES OF ASSOCIATION

A. Pearson Product-Moments

See also KSL 5.57

KSL 2.00, K-8 Product-Moment Correlations, Variance-Covariances, Means, and Standard Deviations

This routine will calculate a matrix of correlations and/or covariances and will print these either in triangular or square matrix form. The printing of the means and standard deviations also is optional. The maximum number of variables is 144. There is no limit on the number of observations. Programmer: Gene Golub

KSL 2.01 K-8 Revised to Accept Single, Positive, Unsigned Digits

The reading time for this routine is greatly reduced. Not only are the data tapes one-half as long, but also no decimal to binary conversion is necessary. The punching of IBM cards and preparation of tapes also are accelerated by the use of this routine. This routine has the same restrictions as K-8. Revised by: K.W. Dickman

KSL 2.03 - 313 Means, Standard Deviations, 3rd + 4th moments about means.

KSL 2.05, K-9 Correlations, Means, Standard Deviations, and Covariances in Logarithmic Scale

The observations are converted to logarithms (Base 10) as they are read into the machine and all further computations are done in logarithms. The number of variables is limited to 35. Programmer: Walter Jacob

KSL 2.10, K-4 AUX Analysis and Intercorrelations of Scores Based on Paired Comparisons

Whenever N groups of raters have chosen between each of the $\frac{e(e-1)}{2}$ pairs of statements formed by all possible pairing of e elements, this program can be used to find the product-moment matrix, $R_{E_iE_j}$, and the scores for each group, E_i, where E_i is defined as $E_i=\sum P_{ij}$. (P_{ij} is the proportion of raters who chose e_i in preference to e_j.) The number of elements is limited to 16. N depends upon e, but is large enough for all practical purposes. Programmer: G.C. Stone

KSL 2.15, K-5 Autocorrelations

This routine will read up to 795 observations and calculate a set of correlations, $r_{x_ix_{i+h}}$, where the lag h may be set to start and to end at any arbitrary values. Programmer: Gene Golub

II. INDICES OF ASSOCIATION

B. Multiple Regression

KSL 2.20, K-14 Multiple Regression with Transformations

This extremely versatile routine will calculate and print any part or all of the following: zero order product-moment matrix; variance-covariance matrix; means and standard deviations; multiple correlation with any of the variables chosen as independent and any other as dependent; standard error of the estimate for each R chosen; unstandardized regression weights and their standard errors for each R chosen; and the standardized regression weights and their standard error for each R.

In addition the following transformations can be made on any of the variables, X_i, before calculation is begun: X_i can be eliminated from the set; X_i can remain unchanged; X_i can be squared or cubed; the square root of X_i can be calculated; the logarithm of X_i can be calculated; or the arcsin of X_i can be calculated. The total number of variables is limited to 50. The number of non-eliminated variables for the regression analysis is limited to 22. There is no limit on the number of observations. Programmer: Gene Golub

? - KSL 2.21 Modified K-14

When several multiple R's are to be claculated on subsets of observations or variables of the same data, the modified version is preferred. The data is first stored on the drum, and then the operations are performed as described in KSL 2.20. (V+1)S must be less than 8138 where V is the number of variables and S is the sample size. Programmer: Vincent West

C. Partial Correlations

? - KSL 2.30 Higher Order Partial Correlations

This routine is in the stage of preparation. It will calculate a matrix of partial correlations given by the zero order matrix. The orders of partials will be specified by a parameter tape. There will be no practical limitations to this routine. Programmer: K.W. Dickman

D. Dichotomous Data Coefficients

KSL 2.40 Dichotomous Coefficients: Phi's or Covariances

This routine will calculate a matrix of phi's or covariances and the means and standard deviations for dichotomous data. Either a triangular or square matrix may be printed either by rows or by columns. In addition it is possible to calculate and print any submatrix from the total set of intercorrelations. The number of variables is limited to 159 and observations to 2535. Programmer: K.W. Dickman

See also KSL 5.57

II. INDICES OF ASSOCIATION

KSL 2.45 Tetrachoric Coefficients

This routine is in the stage of preparation. When finished, it will calculate a matrix of tetrachoric coefficients and print these either in triangular of square form. The number of variables is limited to 159 and the number of observations is limited to 2535. Programmer: K.W. Dickman

E. Transformations on r's

KSL 2.50 Fisher's Z Transformation: $Z = \frac{1}{2} \ln \frac{(1 + r)}{(1 - r)}$

The chief uses of Z are to be found in problems of averaging correlations and in testing the significance of differences between correlations. The routine will transform any set of signed numbers which are scaled by 10^{-1} to Z-values. Whenever a value of Z would be close to positive or negative infinity, these are bypassed by the routine and indicated by printing a dash. No restrictions are made by this routine. Programmer: K.W. Dickman

F. Chi-Square

KSL 2.60 Chi-Square for Dichotomous Data

In the dichotomous case, $\chi^2 = N \phi^2$. A dichotomous criterion can be included among a set of dichotomous test items. The routine will print the item number, the ϕ of the item with the criterion, and finally the chi-square value. Programmer: K.W. Dickman

KSL 2.61 Chi-Square for a Set of Frequency Tables

This routine will read any number of frequency tables and print the chi-square value for each table. It will also print the expected values associated with each table if desired. The total frequency for each table must not exceed 1,000,000,000. There are no practical limits for the size of the tables, nor do successive tables need to be the same size. Programmer: K.W. Dickman

G. D-Statistic

KSL 2.70 D-Statistic

The statistic D, the distance between profiles, is defined as:

$$D_{ij} = [\sum_{v} (X_{vi} - X_{vj})^2]^{1/2}$$

Suppose that there m descriptions of a person where each description consists of the responses on a set of items. This program will calculate all of the m(m-1)/2 inter-indices for each person. The number of items per description may not exceed 100 and the number of descriptions is limited to 12. There is no limit to the number of persons. Programmer: Ray Twery

II. INDICES OF ASSOCIATION

?- KSL 2.75 Data Tape Checking for D Statistic

The purpose of this routine is to check tapes which have been prepared for use with KSL 2.70. Programmer: Ray Twery

H. Pattern Analysis

?- KSL 2.90 Agreement Scores, Form A

This program will calculate and print in page form a matrix of agreement scores among a set of persons. An agreement score is merely a tally of the number of items on which two persons agree. The items should be coded as 2 for a positive response, 1 for a negative response, and 0 for an omitted item. The number of persons that can be handled by the program depends upon the number of items; for example, for 114 items, 146 persons can be handled, and for 58 items, the maximum number of persons is 293. Programmer: K.W. Dickman

?- KSL 2.91 Ranked Agreement Scores, Form F

This program will calculate a matrix of agreement scores, but only an arbitrary number of high agreements will be printed in rank order for each person. Programmer: K.W. Dickman

?- KSL 2.93 Agreement Patterns, Form G

This program, starting with the first person and continuing until the final person, will form joint agreement patterns and scores in the following manner: the person who agrees highest with "i", say person "j", will be found; next the joint pattern "ij", will be formed; then the person who agrees highest with "ij", say "k", will be selected and the pattern "ikj" will be formed; this continues until the joint score drops below some predetermined value, and then the person numbers and joint scores will be printed. The process is repeated with the next person. The limits fpr this program are the same as for Form A. Programmer: K.W. Dickman

?- KSL 2.94 Agreement Patterns, Form H

This program first finds the highest agreement score among all the persons. Additional persons are selected as long as the joint score multiplied by the number of persons in the group is increased. When the score no longer increases, the persons and joint scores are printed, and the joint pattern of responses is removed from the measurement matrix forming a residual measurement pattern matrix. The process continues by again finding the highest agreement score in the residual matrix, repeating the above steps resulting in a second residual matrix. Whenever the agreement scores become lower than some predetermined value, the process is at an end. Programmer: K.W. Dickman

II. INDICES OF ASSOCIATION

?- KSL 2.95 Multiple Alternative Item Conversion

This program will convert multiple alternative items to dichotomously coded items acceptable for all of the programs in this section. The number of alternatives per item, moreover, need not be consistent from item to item. Thus, by the use of this program, all of the programs in this section become general. The maximum number of alternatives per item is 9. Programmer: K.W. Dickman

?- KSL 2.96 Data Tape Checking for Pattern Analysis

This routine will detect any errors made in the preparation of data tapes for use with routines in this section. Programmer: K.W. Dickman

III. ANALYSIS OF VARIANCE AND COVARIANCE

A. Integrated and General Programs

KSL 3.00, K-13 Analysis of Variance by the Method of Fitting of Constants

This program will print the required information for the analysis of variance: gross sums of squares and cross products; constants; inverse matrix; and accounted for sums of squares and cross products. The advantage of this routine is its generality. It will accept data for any type of design including the analysis of covariance provided each observation is represented by a row of coefficients indicating the relationship of the observation to the constants to be fitted. There is no limit on the number of observations. The number of variables is dependent upon the design. Programmer: Gene Golub

B. Completely Randomized Design

KSL 3.10 Analysis of Variance for Completely Randomized Design

This program will print the treatment means, the general mean, and a table which shows the sums of squares, mean sums of squares, and degrees of freedom associated with total, treatment, and error. In addition the F ratio is calculated and printed. For any treatment the number of observations is limited to 300 and the total number should not exceed 2000. Programmer: Walter Jacob

C. Randomized Block Design

KSL 3.20 Randomized Complete Block Design

This program will print the treatment means, the general mean, and a table which shows the sums of squares, mean sums of squares, and degrees of freedom associated with each partition of the total. In addition the F ratio and standard deviation of the mean is printed. The number of replications is limited to 100. The total number of observations should not exceed 2000. Programmer: Walter Jacob

The following programs, written by Dr. Walter Jacob, are all variations of KSL 3.20. They differ from KSL 3.20 only in the section where the data is read into the computer.

KSL 3.21 Randomized Complete Block Design with Read In for Handling Conversion (XC)

KSL 3.22 Randomized Complete Block Design with Read In for Handling Conversion (XY). Replications Limited to 50.

KSL 3.23 Randomized Complete Block Design with Read In for Handling Conversion (XYC). Replications Limited to 49.

III. ANALYSIS OF VARIANCE AND COVARIANCE

KSL 3.24 Randomized Complete Block Design with Read In for Handling Conversion (XYZ). Replications Limited to 33.

KSL 3.25 Randomized Complete Block Design with Read In for Handling Conversion (XYZC). Replications Limited to 33.

KSL 3.26 Randomized Complete Block Design with Read In for Handling Conversion (-XY + X)C. Replications Limited to 49.

KSL 3.27 Randomized Complete Block Design with Read In for Handling Conversion ($10^{-1}Y/X$). Replications Limited to 50.

KSL 3.28 Randomized Complete Block Design with Read In for Handling Conversion ($10^{-1}X/C$). Replications Limited to 99.

KSL 3.29 Randomized Complete Block Design with Read In for Handling Conversion $10^{-1}(X - Y)/X$. Replications Limited to 50.

D. Rectangular Lattice Design

KSL 3.60 Analysis of Variance for Rectangular Lattice Design

This routine can be used for any rectangular lattice design of size 15 x 16 or smaller with 3 replications only. The print out which is completely labeled will show the sums of squares and mean sums of squares for each source of variation. The treatment means and F ratio are also printed out. This routine is general in that any combination of 4 transformations and 10 conversions can be made on the observations. For example, the observations can first be transformed to logarithms and then the conversion $10^{-1}X/C$ can be made. Programmer: Walter Jacob

IV. OTHER STATISTICAL PROCEDURES

A. Miscellaneous

KSL 4.00 Frequency Distributions

This routine will read sets of n signed integers and form frequency charts for each set. The lower boundary and the interval between categories are specified by parameters. Alongside the charts will be printed bar graphs. The width of each bar graph is automatically adjusted so an not to **exceed** the width of the teletype paper. The routine imposes no restrictions. Programmer: K.W. Dickman

KSL 4.05 Ranked Numbers

This routine will read a group of sets of numbers and extract from each set a number of the largest positive and smallest negative values. These will be printed in rank order with a number indicating their position in the set. Any number of sets may be ranked, but the number within a set is limited to 700. Programmer: K.W. Dickman

KSL 4.10 Standard Score Transformation

Before a set of raw scores can be compared or combined in any manner, it is often necessary to transform them to the same scale with the same mean and same standard deviation; that is, to standard scores. The mean and standard deviation of the transformed data are specified as parameters. There is no limit on the number of observations, but the number of variables is limited to 145. Programmer: John Hurley

KSL 4.20 Construct Samples from a Normal Distribution with Specified Means, Covariances, and Serial Trend

The purpose of this routine is to generate samples from a multivariate normal population with specified parameters in order to study the effects of sampling under various sets of conditions. The variables will be correlated in the population, but not necessarily in the sample, as specified by the parameters. Positive or negative trend can be introduced in any or all of the variables. The number of variables, n, and the sample size, S, can vary according to the following inequality:

$$n^2 + 9n + 2nS \leq 1500$$

Programmer: K.W. Dickman

IV. OTHER STATISTICAL PROCEDURES

B. Least Squares

7 – KSL 4.40, K-3 Least Squares

The purpose of this routine is to fit a set of N weighted experimental points by the best polynomial of the form,

$$Y = \sum_s \frac{1}{2} a_s X^s \qquad (s = 0, 1, \ldots (n - 1)$$

K-3 produces a set of simultaneous linear equations which can be solved by using the linear equation solver, KSL 6.00 (L-7). If the weights are unequal, N is limited to 291. If the weights are equal, the maximum number of points is N = 436. A frequent use of this routine by statisticians will be when the polynomial reduces to a straight line, $Y = \frac{1}{2}(a_0 + a_1 X)$.
Programmer: R. Rubenstein

C. Limited Information Estimation

KSL 4.50 Limited Information Estimation, Single Equation

The LISE program will read a number of triangular covariance matrices and store these on the drum. A set of parameters will be read which specify which variables to extract for the total set and which to extract for the endogenous set and which to extract for the exogenous set. The routine will then operate successively on each of the covariance matrices to calculate the estimates of the endogenous and exogenous variables together with their standard errors. There are 9300 locations available for the store of the covariance matrices. The largest order that can be handled after extraction is 14. Both the endogenous and exogenous sets are limited to order 6.
Programmer: K.W. Dickman

D. Linear Programming

KSL 4.80, M-15 Linear Programming

This program can be used for maximization problems or for minimization problems of the following type:
Given a linear functional $Z = C_1\lambda_1 + C_2\lambda_2 + \cdots + C_n\lambda_n$
to be maximized subject to the constraints

$$A_{11}\lambda_1 + A_{12}\lambda_2 + \cdots + A_{1n}\lambda_n \leq b_1$$
$$A_{m1}\lambda_1 + A_{m2}\lambda_2 + \cdots + A_{mn}\lambda_n \leq b_m$$

or to be minimized subject to the constraints

$$\begin{array}{ccccccc} A_{11}\lambda_1 & + & A_{12}\lambda_2 & + \cdots + & A_{1n}\lambda_n & \geq & b_1 \\ \vdots & & \vdots & & \vdots & & \vdots \\ A_{m1}\lambda_1 & + & A_{m2}\lambda_2 & + \cdots + & A_{mn}\lambda_n & \geq & b_m \end{array}$$

For further discussion of this routine, see the written instructions for M-15. Programmer: L. Isaacson

V. MATRIX OPERATIONS

A. Matrix Multiplication

KSL 5.00, M-21 Matrix Multiplication

If matrix A is of size i by j and B of size j by k, this routine will form the product AB = C provided jk is less than 10,240 and j is less than 413. The routine is designed to prevent overflow in an element of C by scaling this particular element. Thus, unless the output tape is to be used in a subsequent operation, no attention need be given to scaling. Programmer: R.P. Polivka

KSL 5.01 Mass Production Matrix Multiplication with Rescaling

The purpose of this routine is to multiply a series of small matrices, $A_pB_p = C_p$, and to rescale the products. If the size of B is k by j, and if p is the number of the matrices, the limit of this routine is $pjk < 824$. Programmer: K.W. Dickman

KSL 5.02 Multiplication of a Matrix by its Transpose

This routine will read a matrix, A, and form the product, A'A, or the product AA'. The advantage in using this routine is that it avoids the necessity of transposing either A or A' prior to the use of KSL 5.00. This routine is currently in preparation. Programmer: K.W. Dickman

B. Matrix Inversion

KSL 5.10, M-13 Complete Linear Matrix Equation Solver and General Matrix Inversion

This routine solves the linear matrix equation, A X = B, where A is an n by n non-singular matrix, and B is an n by m matrix. The magnitudes of n and m are governed by the relationship:

$$nm + n + m + n(n+1)/2 \leq 842.$$

In case B is the identity matrix, the inverse A is obtained. Programmer: W.L. Frank

KSL 5.15 Routine to Prepare Data Tapes for M-13

An operation frequently encountered in obtaining factor scores is to solve the equation R B = F where R is symmetric and in triangular form. This routine will read the triangular matrix, R, and the rectangular matrix, B, and punch the augmented matrix, R:B, in the form required for M-13. Programmer: John Hurley

V. MATRIX OPERATIONS

KSL 5.16 Routine to Rescale the Results of M-13

The results of routine M-13 consist of the columns of X with a scaling value at the end of each column. In order to use the matrix, X, for subsequent operations, it is necessary to remove the scaling value, and also to scale each column by the same power of 10. The purpose of KSL 5.16 is to read a number of such matrices, and to scale all of them by the same predetermined power of 10. There are no restrictions as to the size of the matrices or the number of them. Programmer: K.W. Dickman

C. Addition and Subtraction of Matrices

KSL 5.20 Matrix Addition or Subtraction

If the matrices, A and B, conform, this routine will either add or subtract a series of matrices, $A_p \pm B_p = C_p$. The results, C_p, can be printed in rectangular form, or, if the C_p are symmetric, they can be printed in triangular form. The set of A_p are stored on the drum and the total number of elements cannot exceed 10,240. Programmer: K.W. Dickman

D. Transposition

KSL 5.30 Matrix Transposition and/or Scaling

This routine will transpose a matrix of size r by c where neither r nor c exceeds 814 and where r x c does not exceed the drum capacity, 10,240. The results can be scaled at the same time by the following powers of 10: +2, +1, +0, -1, -2. It is also possible to use this routine for scaling without transposition. Programmer: John Hurley and K.W. Dickman

E. Deletion and Dilatation of Matrices

KSL 5.50 Delete Any Rows and Any Columns from a Set of Rectangular Matrices

This routine will read a set of rectangular matrices, and punch out a set of smaller matrices with rows and columns deleted as specified by the parameter tape. Programmer: K.W. Dickman

KSL 5.51 From Square to Triangular Form with or without Deletions

This routine will reduce a square, symmetric matrix of order n to order n-k and punch the resulting matrix in triangular form. The k variables to be deleted are specified by a parameter tape, and k may be any value less than 200 including zero. The value of n must be less than 644. Programmer: K.W. Dickman

V. MATRIX OPERATIONS

KSL 5.52 - 322 Matrix Interleafer (SADOI)

KSL 5.55 Triangular Form to Square

A matrix in triangular form not exceeding order 42 can be restored to square, symmetric form by the use of this routine. Programmer: K.W. Dickman

KSL 5.56 Triangular Form to Rectangular Form

It is sometimes necessary to perform operations on selected columns of a matrix which may be on tape in triangular form. The purpose of this routine is to extract complete columns from a matrix in triangular form of order not exceeding 111. The number of columns selected is not limited, so that it is possible to restore the entire square matrix. Programmer: K.W. Dickman

KSL 5.57 Correlations in Page Form

Whenever a matrix is symmetric, both space and computer time are conserved if the matrix is punched onto tape in triangular form. This form is optional output for routines KSL 2.00, 2.01, 2.05, 2.40, and others. The purpose of this routine is to read data in triangular form, and punch it back onto tape in page form. Each row and each column is correctly labeled. There are no practical limitations on the use of this routine. Programmer: K.W. Dickman

F. Normalization

KSL - 5.60

This routine reads a matrix by rows, normalizes by columns (i.e., sums of squares of the elements equals one), and prints the matrix either by rows or by columns. If the matrix is of size r by c, this routine is limited to the case where rc is less than 840. Programmer: K.W. Dickman

G. Summing Elements of a Matrix

KSL 5.70 Row Sums and Column Sums

The sums of elements across rows and down columns can be found by using this routine. If r is the number of rows, and c the number of columns, the limitations of this routine are such that 2 r + c is less than 901. This routine can be used to check the symmetry of a large matrix. Programmer: Jack Neuhaus

V. MATRIX OPERATIONS

KSL 5.71 Sums of Squares, Rows and Columns

The sums of squares of the elements of a matrix, by rows, by columns, and total sums of squares are calculated by this routine. If r is the number of rows, and c is the number of columns, the maximum size matrix that can be used with this program is governed by the relationship: 2 r + c must be less than 724. Programmer: K.W. Dickman

H. Data Tape Checking

KSL 5.90 Checking Matrices

The purpose of this routine is to check data tapes. The routine will read the first row of a matrix and compare all subsequent rows for number of elements, number of digits in each element, and for the terminating symbol. If there is any disagreement with the first row, the routine will print the row number and the type of error. This routine will also check in the event that the first row consists of single, unsigned elements. An additional option is that the routine can be used to check triangular matrices. The number of elements in the first row cannot exceed 700. Programmer: K.W. Dickman

VI. USEFUL ALGEBRAIC ROUTINES

KSL 6.00, L-7 Automatic Linear Equation Solver

The purpose of this routine is to solve a set of simultaneous linear equations. The maximum number of equations is 143. Programmer: D.E. Muller

KSL 6.10, J-2 Roots of a Polynomial

This program uses an iterative method to calculate all the roots of a polynomial, $f(x) = a_0x^n + a_1x^{n-1} + \ldots + a_n = 0$, where the coefficients a_i may be real or complex numbers. Programmer: D.E. Muller

KSL 6.20, M-12 Matrix Triangularization and Computation of its Determinant

Given any number of properly scaled square matrices of order up to 40, this routine will successively upper triangularize each matrix and compute its determinant. Programmer: W.L. Frank