UNIVERSITY OF ILLINOIS

DIGITAL COMPUTER LABORATORY

NEW COMPUTER LIBRARY ROUTINE D1-ROM-31

TITLE: Romberg integration

TYPE: closed, relocatable, mnemonic

LENGTH: 28 words

TEMPORARY STORAGE: 6 words at fixed memory locations 0-5
n words beginning at location S

FAST REGISTERS CHANGED: none

SUBROUTINES USED: none

DURATION: depends on the duration of the auxiliary routine for evaluating the function f(x). Since the auxiliary is entered $2^{n-1} + 1$ times, duration is approximately doubled when n is increased by 1.

PARAMETERS: link in M15
4 parameters which must be written in the word following the word containing the JSB instructions:

	a	address of left limit a of integration interval
	f	address of auxiliary routine
	s	address of first location of temporary storage
$2 \leq n \leq 13$	n	integration interval divided into 2^{n-1} subintervals

The right limit b of the integration interval has to be placed in the location immediately following the one which contains the left limit a.

DESCRIPTION: this subroutine computes $I = \int_a^b f(x)\,dx$ in the following way:

Compute a sequence $R_{11}, R_{21}, \ldots R_{n1}$ of approximations to I, where R_{11} is found by dividing the integration interval (a,b) into 2^{i-1} subintervals of equal length and evaluating the integral by the trapezoidal rule.

The sequence R_{i1} converges linearly to I with convergence factor 1/4, i.e.

$$\frac{R_{i,1} - I}{R_{i-1,1} - I} \approx 1/4$$

The new sequence $$R_{i2} = \frac{4R_{i,1} - R_{i-1,1}}{3}$$

also converges linearly to I, with convergence factor 1/16.

Beginning with R_{i1}, we generate different sequences R_{ij} which all converge linearly to I according to the formula

$$R_{i,j+1} = \frac{4^j R_{i,j} - R_{i-1,j}}{4^j - 1} \qquad j = 1,2 \ldots n-1$$

and arrange them as columns in a triangle:

	R_{11}				
	R_{21}	R_{22}			
	R_{31}	R_{32}	R_{33}		
	.	.	.		
	.	.	.		
	.	.	.		
	R_{n1}	R_{n2}	R_{n3}	. . .	R_{nn}
Convergence factor:	4^{-1}	4^{-2}	4^{-3}	. . .	4^{-n}

USE: The user must provide:

The left limit a and the right limit b of the integration interval, in two consecutive locations, and specify the address of a as parameter,

An auxiliary routine for evaluating the function f, which is also linked in M15,

A block of n consecutive words in memory beginning at location S.

The subroutine stores $R_{n1}/(b-a)$ $R_{n2}/(b-a)$ ... $R_{nn}/(b-a)$
in memory locations S S+1 S+n-1
and leaves the best approximation R_{nn} to the integral in the accumulator.

The choice of the parameter n (which implies that the integration interval is divided into 2^{n-1} subintervals) should depend on the behavior of the function f(x), taking into consideration that the duration is approximately doubled when n is increased by 1.

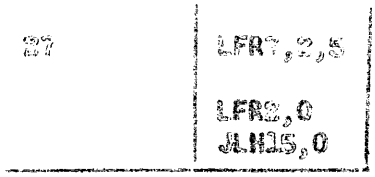
Range: $2 \leq n \leq 13$.

DATE: December 1, 1962

PROGRAMMED BY: J. Nievergelt

0		SFR2,0	
		SFR3,2,1	
		SFR4,2,2	
1			
		SFR5,2,3	
		SFR6,2,4	
2			
		ATN15,1	(M15) + 1
		LFR6,0	read parameters into F6
		SFR7,2,5	
3			
		SFN11,0	
		CAM12,2,1	(M12) = 1 - n
4		CAD8,0	left limit a
		ATN9,0	
		JSR15,0,0	f(a)
5		STR2,3	
		CAM4,2,1	$(M4) = 2^{l-1} \leftarrow 1$
		CAD8,2,1	right limit b
6			
		ATN9,0	
		JSB15,0,0	f(b)
7		ADD2,3	
		MPY10,3,2048	$S_1 = \{f(a) + f(b)\}/2$
		STR10,0	
8	(o1) →	CAD15,3	
		STR2,3	
		CAD8,2,1	right limit b
9		SUB8,0	b - a
		ATN4,0	
		DIV9,3,0	$2h = (b - a)/2^{l-1}$
10		STN4,3	
		MPY10,3,2048	negative step length -h in accumulator
		ADD8,0	
11		STR3,3	a - h = F3
		SFN4,0	
		CAM6,0	$(M6) = -k = -2^{l-1}$
	(k1) →	CAD4,3	-2h
12		ATN6,0	
		MPY9,3,0	
		ADD5,3	a - h + 2 kh

13		ATN9,0 JSB15,0,0,0	f(a - h + 2kh)
14		ADD2,3 STR2,3	
	(k) ←	CJU6,3,11R	
15		ATN4,0 DIV9,3,0	$m_i = (\Sigma f)/2^{i-1}$
		ATN10,0	
16		CAM14,0 ADD14,0 MPY10,3,2048	
17		STR2,3 ATN4,0 CAT9,3,0	$R_{i,1} = (R_{i-1,1} + m_i)/2 \rightarrow F2$
18		SIA4,0 ATN11,0 SFN12,0 CAM13,0	double 2^{j-1} in M4
19	(i) →	CAM5,0 BJF5,2 CAD9,3,1	(M5) = j ← 0 (M5) + 1
20		ADE5,0 SUB9,3,1	4^j
		STR3,3	$4^j - 1 \rightarrow F3$
21		CAD2,3 SUB14,0	
		DIV3,3	$(F2 - S_j)/(4^j - 1) + F2$
		ADD2,3	
22		ATN14,1 SFR2,0 STR2,3	(M14) + 1 F2 → S_j
	(i) ←	CJU13,1,19R	
23		ATN14,0 SFR2,0	
	(o) ←	CJU12,0,8R	
24		CSB8,1 ADD8,0	b - a
		MPY2,3	Rnn = F2 x (b - a)
25		LFR3,2,1	
		LFR4,2,2	
26		LFR5,2,3	
		LFR6,2,4	


27
LFR7,2,5
LFR2,0
JLH15,0