include Case 3 as well as Case 2. Studies of (9), made on the Moore School Differential Analyzer at the University of Pennsylvania, have served to substantiate the above development. Typical fundamental sets of solutions of (9) for a representative range of parameters ϵ and K are shown on Figs. 3 to 6.

VII. Conclusions

Treatment of inductance variation inside the differentiation sign, and including dissipation, gives rise to two distinct effects. One is the direct relationship with the previously studied[18] solutions of the capacitance variation in the sense that the functional expressions for capacitor charges and circuit currents are, roughly speaking, interchanged. The other, caused by dissipation, is the time varying damping, illustrated on Fig. 2.

VIII. Acknowledgments

The author wishes to express his gratitude to J. G. Brainerd for his help in preparation of this paper. The author is indebted to the Moore School of Electrical Engineering of the University of Pennsylvania for the use of the differential analyzer.

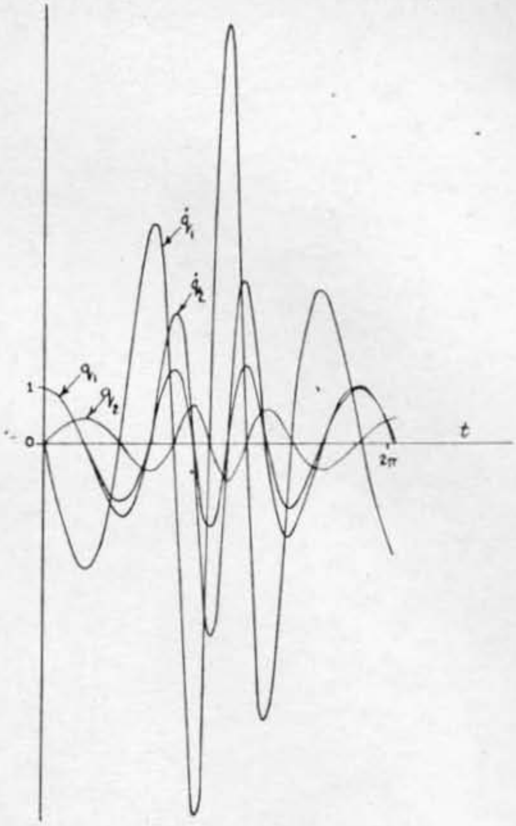

Fig. 6—Differential analyzer solutions of (9) for $K=0.7$, $\epsilon=8.0$

Some Studies of Pulse Transformer Equivalent Circuits*

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Summary—**A pulse transformer equivalent circuit is synthesized from data obtained by direct test on a typical pulse transformer. The differential equations for the equivalent circuit are solved by means of a mechanical differential analyzer for a variety of circuit constant values. Wave forms of transformer output voltage as obtained from the differential analyzer are of assistance in the design of pulse transformers.**

I. Introduction

Pulse transformers are currently finding important application in computer and radar circuits, as well as in many other devices utilizing short duration pulses. The method of analysis to be described here is based on an equivalent circuit which has been idealized to the extent that the pulse transformer which contains nonlinear, distributed parameters is represented by a circuit containing linear, lumped circuit constants. These assumptions permit a relatively simple procedure for synthesis and subsequent analysis of the equivalent circuit.

II. Synthesis of Equivalent Circuit[1]

The particular pulse transformer to be studied is treated as a four-terminal network whose basic arrangement of impedances (Z_A, Z_B, Z_C) is the T of Fig. 1. The assumption of the T configuration fixes the frequency responses of Z_A, Z_B, and Z_C for a particular transformer. These responses are obtained by measurement of the

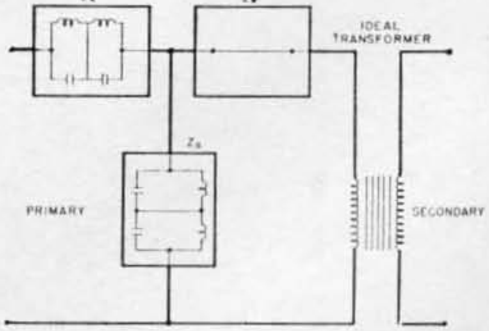

Fig. 1—Four-terminal network representing pulse transformer ($Z_B=0$).

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[1] T. E. Shea, "Transmission Networks and Wave Filters," D. Van Nostrand Co., Inc., New York, N. Y.; 1929.

primary and then the secondary frequency characteristics for both open- and short-circuit condition on the transformer.

The test data obtained from the transformer (Fig. 2) indicate that Z_B is very nearly zero. Therefore, for equivalence, Z_A should display the same response as that seen from the primary winding with the secondary

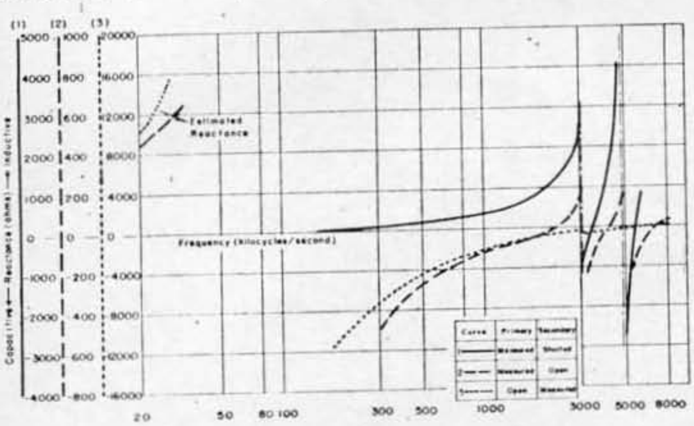

Fig. 2—Frequency characteristics of a typical pulse transformer (test data).

short circuited (i.e., the two anti-resonant and two series resonant points of curve 1). The network representing Z_A in Fig. 1 is capable of producing this response. Correspondingly, the network representing Z_C in Fig. 1 is obtained from consideration of Fig. 2, curve 3 which is the transformer's response as seen from the secondary with the primary open. Hence the circuit of Fig. 1 will exhibit the frequency response characteristics of the actual transformer.

The approximation is made that the circuits representing Z_A and Z_C could be replaced by single LC parallel combinations. The validity of this step is questionable where frequencies above 4.0 Mc appear (see Fig. 2). The Fourier frequency spectrum of a single pulse exhibits an appreciable amplitude over a frequency interval extending from $-(10/2\epsilon)$ to $+(10/2\epsilon)$ where ϵ is the pulse duration. For a 1-μ sec pulse duration then, $10/2\epsilon = 5$ Mc. Hence neglecting all components above 4.0 Mc might introduce serious error. Assuming, however, that this simplification is permissible, the equivalent L (T with the impedance $Z_B = 0$) is shown in Fig. 3. An equivalent pulse generator and load (referred to the primary) has been added. Capacitor C_2 is assumed to contain any load capacitance; values of generator and load resistance (R_1 and R_2) may be modified to include transformer copper loss and core loss, respectively.

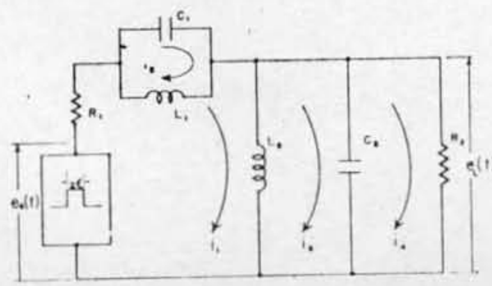

Fig. 3—Assumed equivalent circuit of pulse transformer.

III. Analysis of Equivalent Circuit[2]

Analysis of this equivalent circuit is based on its conventional loop equations which follow:

$$e_{in}(t) = R_1 i_1 + L_1 \frac{d}{dt}(i_1 - i_2) + L_2 \frac{d}{dt}(i_1 - i_3) \quad (1)$$

$$0 = L_1 \frac{d}{dt}(i_2 - i_1) + \frac{1}{C_1}\int i_2 dt \quad (2)$$

$$0 = L_2 \frac{d}{dt}(i_3 - i_1) + \frac{1}{C_2}\int (i_3 - i_4) dt \quad (3)$$

$$0 = R_2 i_4 + \frac{1}{C_2}\int (i_4 - i_3) dt \quad (4)$$

where $e_{in}(t)$ is a voltage pulse of magnitude E_0 and duration ϵ.

In order to explore the effects on the output wave shape of changing the circuit parameter values, a number of solutions corresponding to several sets of values of circuit constants were desired. Solutions obtained by techniques of operational calculus would have required an excessive expenditure of man hours of computing. For this reason the problem was solved by means of the University of California mechanical differential analyzer. This computer was selected because of its ability to produce many solutions rapidly, inexpensively, and with the desired accuracy. Equations (1) to (4) were solved in dimensionless form in order to facilitate the generalization of results. A selection of representative solutions (output curves) obtained from the differential analyzer appears in Figs. 4 and 5.

IV. Conclusions

The various effects on the output wave form for changes in the circuit parameters (see Figs. 4 and 5) are of special interest in determining design criteria. Existing discrepancies between the output wave shape of the transformer tested, and the wave shape obtained from analysis of the equivalent circuit are directly traceable to the simplifying assumptions made in arriving at the equivalent circuit of Fig. 3. It is difficult to obtain simple, accurate, equivalent circuits of devices which contain distributed parameters when consideration of a wide frequency range is necessary. In particular, the representation of Z_A and Z_C by single LC combinations is valid only if frequencies above the first resonant point are negligible in the circuit. Further, the assumed linearity of all circuit elements may produce discrepancies under certain conditions.

It is over these considerations that a compromise must always be made between ease of analysis and accuracy of results to determine the most suitable choice

[2] "The Differential Analyzer of the University of California," University Press, University of California, Los Angeles, Calif., 1947.

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of equivalent circuit. Greater precision is obtainable by selecting more complicated impedance structures for Z_A, Z_B, and Z_C and/or by treating certain elements (primarily L_2) as nonlinear. However, the output wave shapes of the actual transformer and those obtained from the equivalent circuit of Fig. 3 (employing measured values of circuit constants) show sufficiently good agreement for most practical purposes.

The desirability of employing a computer is immediately evident from a comparison of the time required to solve the circuit equations by means of the Fourier transform and differential analyzer methods.

Twenty-five hours were required to obtain a solution for a single set of parameters by the Fourier transform method whereas the average time required by the differential analyzer was less than twenty minutes per solution. More complicated equations which would of necessity result from a more complicated equivalent circuit and the introduction of nonlinear elements would greatly increase the advantage of a computer solution over other methods.

Acknowledgments

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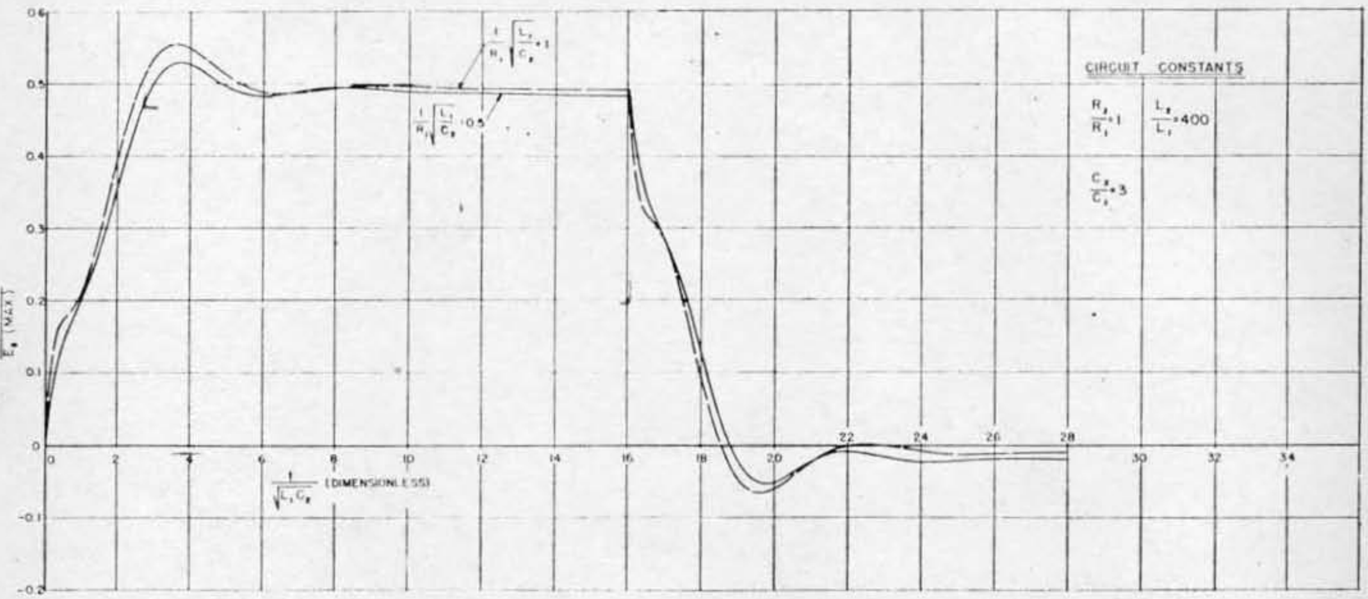

Fig. 4—Typical output curves from mechanical differential analyzer, showing the effect of a varying $1/R_1\sqrt{(L_1/C_2)}$.

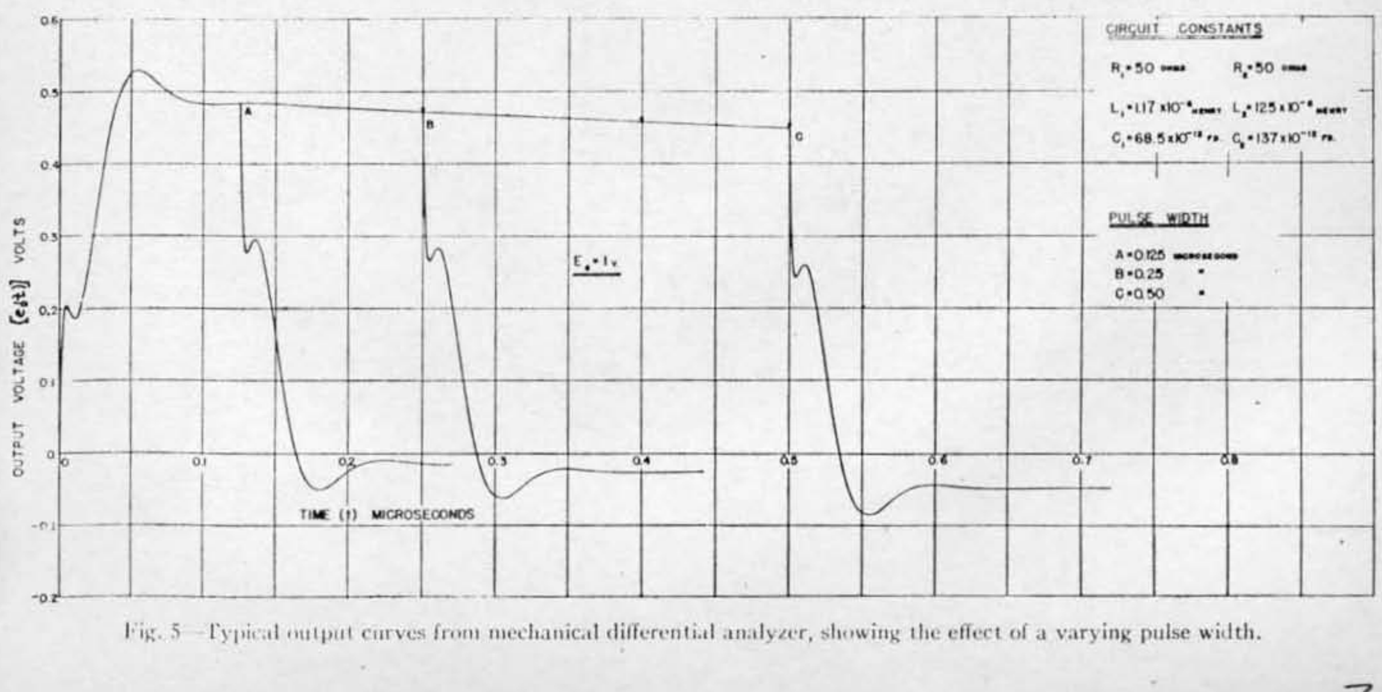

Fig. 5—Typical output curves from mechanical differential analyzer, showing the effect of a varying pulse width.

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