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In This Issue

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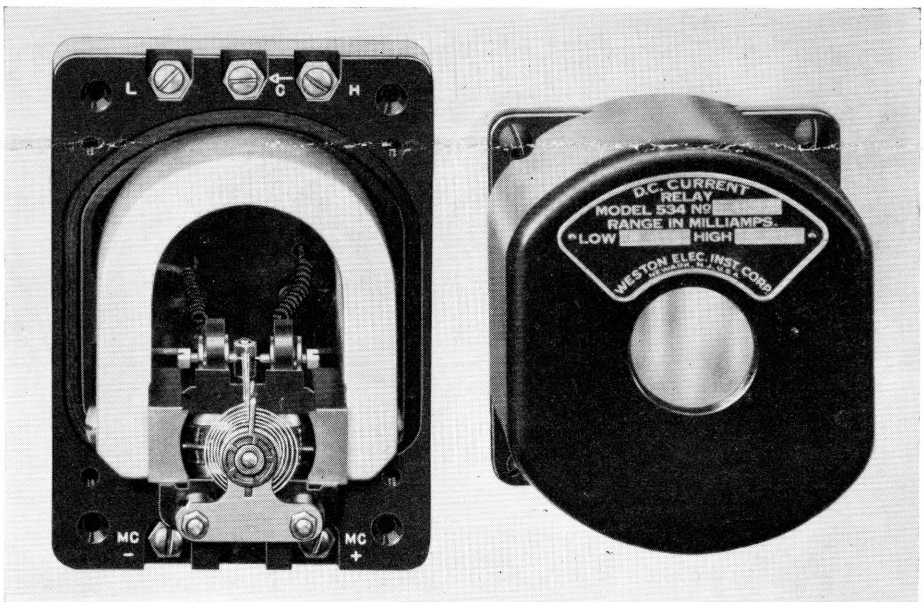
A DISCUSSION OF CONTACT MAKING RELAYS—PART I

THE RELIABILITY of a relay of the instrument type depends on the reliability of the electrical contact established by the instrument. This in turn depends on the contact material, the contact pressure, the contact shape, the type of engagement made by the contact surfaces, and the cleanliness of the contact.



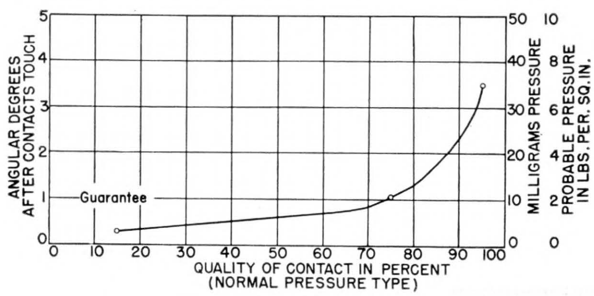
Normal Pressure Type Relay with cover removed—Weston Model 534

Appropriate contact materials may be determined for any particular relay but, in general, the contact pressure is determined by the magnitude of the variable factor upon which the control is based. In the case, for example, of a relay using normal pressure contacts, the contact pressure is in direct proportion to the current flowing through the moving coil, and where only minute currents are available in the coil, it is impossible to obtain more than a very limited contact presure. Thus relays, with normal pressure contacts, operated directly by the energy obtained from photoelectric cells, thermocouples, Wheatstone bridge circuits, etc., are rather delicate and expensive and are capable only of handling small wattages at low voltages. These relays are not ordinarily supplied to operate on less than 15 microamps or less than 50 millivolts and even at this level the contacts tend to chatter and act erratically and acceptable performance usually requires relatively large relays with a heavy main magnet, a large movable coil, etc. In addition the contacts are limited to six volt direct current circuits, and thus require six volt battery or rectifier-transformer combination, plus an auxiliary power relay to operate any 110 volt load no matter how small the load may be.

In theory, and supported by practice, the normal pressure contact relay serves best where ample energy is available for reliable contact action and where little or no vibration exists and where size, weight, and cost are acceptable. Actually the main advantage this relay possesses is that its contact arm floats freely and can "make" and "break" the fixed contact without the necessity of resetting. The largest use of non-magnetic contact relays is in fixed locations, solidly mounted on a switchboard or panel. For instance, they are used in large quantities in telephone exchanges for controlling the voltages being fed to the various relay banks and switchboards.

age back into the coil circuit under adverse conditions is considerably increased. For these reasons, electrostatic pressure type relays are rarely used.

Magnetic contact relays, a unique development in instrument relay design, provide vastly improved and dependable contact through the use of small "riders" of magnetic material affixed to the pointers and to the adjacent limiting contact or contacts. In operation, the external energy source has only to bring the two magnetized contacts within sufficient proximity to one another to allow the magnetic flux to take effect.



Graphic presentation of pressure increase at contact points.

Mechanical pressure type relays overcome some of these objections since here the movable coil need only move the contact arm into the vicinity of the contact. Contact closure takes place due to the depressor bar operated by the motor. Such relays have the disadvantage of requiring a continuously running motor and are suitable for the control of only those processes or operations in which the measured factor changes but slowly, since the contact arm is depressed only at intervals, usually every 30 seconds. Depressor relays are entirely unsuitable, for example, in cases where an instantaneous alarm or control action is to follow the momentary interruption of a light beam.

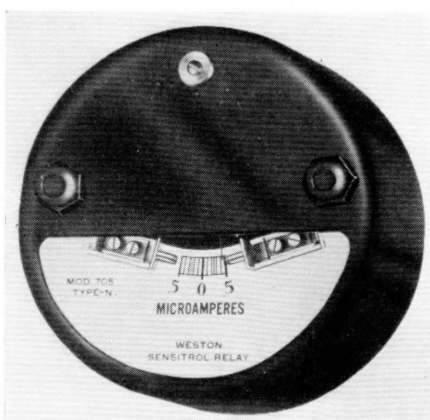
Electrostatic pressure type relays provide practically instantaneous contacting by virtue of the electrostatic attraction effected by the high voltage applied to the stationary contact. The high voltage, up to 3000 volts are used, is more than ordinarily dangerous and the likelihood of leak-

Positive contact is then established with a pressure several thousands times greater than would be obtained with non-magnetic (normal pressure) contacts.

The importance of this extra contact pressure in assuring a positive low-resistance electrical contact will be clear if it is realized that, in actual use, there is no such thing as a completely clean contact. Even if all possibility of oxidation or corrosion is eliminated by the use of pure platinum contacts, the accumulation of a film from air-borne grease, dust, moisture, etc., will take place. Even on seemingly mirror-like surfaces, its existence can be proved by resistance measurements. Apparently clean contacts are sometimes found to have resistances from hundreds of ohms to infinity even though they are in good mechanical contact. In each case, mere "touching" is not enough; it requires a "follow-up" pressure to reduce the resistance to a value approximating that of a really clean surface. Furthermore, since the relay capacity is a function of the cross-sectional area in good electrical contact, adequate unit pressure with contacts of more than needle-like dimensions is essential.

Magnetic pressure contacts, or simply, magnetic contacts offer the utmost in reliability and simplicity. By combining high sensitivity with high contact capacity, these relays are capable of providing a power transfer (amplification ratio) of some 2,500,000,000 times the energy required to actuate the moving coil of the instrument movement. This is approximately 5,000 times greater than the ratio which can be obtained if contact must be established solely by the pressure arising from the source-energy initiating the closure. It is greater than can be obtained with several stages of a radio amplifier. This type of relay can be made to close contact on as little as ¼ microampere or ¼ millivolt, while the contacts themselves will handle 10 watts at 120 volts. Relays of this type, as made by Weston under the registered trademarks "SENSITROL," have come into widespread use in the last decade.

In contrast to electronic controls the magnetic contact relay requires no continuous consumption of current, as does the heater element of a tube, nor a source of plate voltage. The relay contacts close at exact, though adjustable, values independent of



A typical relay combining high sensitivity with high contact capacity—Weston Model 705 Type N.

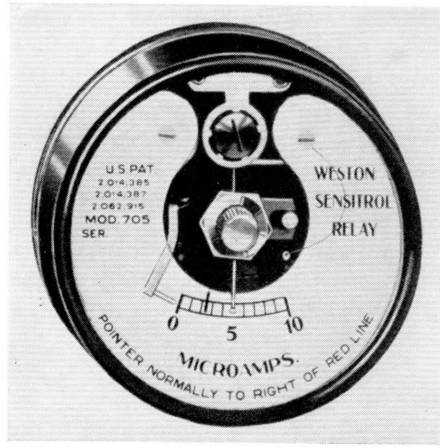
Internal view of the Weston Model 709 Illumination control unit.

line voltage or any other circuit variation, holding to a permanence of calibration which is not possible with a simple electronic amplifier. In addition, the relays will operate either 120-volt alternating or direct current secondary circuits equally well, which is not the case with grid glow tubes and some other types of electronic tubes.

Even where extreme sensitivity is not a determining factor, relays of the magnetic contact type are also frequently advised in applications in which the power available to actuate the relay is well above the limiting value for ordinary sensitive relays, in the range of 1 to 5 milliamperes, for example. Among the factors which will have considerable weight under these circumstances are the following:

1. The need for a high order of reliability when the operation of the relay takes place only at infrequent intervals. In an alarm device, for example, the relay may not be called on to operate, except for tests, for long periods of time.

2. The desirability of a positive, non-chattering response when the actuating energy builds up slowly to the level at which the relay is set to close. Illumination control units operated from photoelectric cells, for example, must act at the same level whether the change in the actuating current is sudden or very gradual, perhaps a microampere or less per hour as daylight changes, without deterioration of the contacts.



The above single fixed contact type relay may be reset manually from the front or rear.

The single limitation to the use of magnetic contact relays which seems to concern designers contemplating their application, is the necessity for resetting the relay (reseparating the magnetic contacts) after each operation. Yet, with the exception of the limited number of applications where power transfer must be necessarily continuous rather than intermittent, the problem is easily overcome.

E. N.—No. 51 —*A. H. Lamb*

NOTE: A continued discussion on contact making relays will appear in the next issue of Weston Engineering Notes.

The Editor

THERMAL PROBLEMS RELATING TO MEASURING AND CONTROL DEVICES—PART III

Introduction

THIS article will consider the problem of the heating and cooling of simple bodies under the condition that the law of cooling is not linear.

10. NON-LINEAR CONVECTION LAW.

Heating and Cooling of a Simple Body, when the Rate at which Heat is Exchanged by Convection with the Surrounding Medium is Proportional to some other Power of the Difference in Temperature than Unity.

In the previous articles, Newton's Law of cooling was assumed, namely, that the heat exchanged between a body and the surrounding medium is directly proportional to the difference in temperature between them. This law is sufficiently exact for most practical purposes and especially so for the relatively small differences in temperature encountered in measuring and control devices, with which these articles deal.

However, experimental evidence, which is supported by theoretical considerations, shows that the exchange of heat between the surface of a body and the surrounding medium is proportional to a power of the difference in temperature, which varies from 1.2 to 1.25. Then, to make the problem general, let

$$W_c = h_o(\triangle\,\theta)^n \qquad (24)$$

where,

$W_c =$ Rate of heat exchange by convection, watts

$h_o =$ Convection coefficient = rate of exchange of heat between body and medium when the difference in temperature is one deg. cent.

$\triangle\theta =$ difference in temperature at any time t.

The following analyses are given for several different conditions.

10a. HEATING OF A BODY BY THE MEDIUM.

To find the Temperature of a Simple Body at any Time after Immersion in a Medium at a Higher Temperature than that of the Body for a Non-Linear Convection Law.

Let the body be suddenly immersed in the medium which has a temperature θ_m higher than the initial temperature of the body. Then, as will be proved later, the elevation in temperature, θ, of the body at any time t, relative to the initial difference in temperature θ_m is,

$$\frac{\theta}{\theta_m} = 1 - \left[\frac{1}{(n-1)\frac{t}{t_o} + 1}\right]^{\frac{1}{n-1}} \qquad (25)$$

where t_o is the time constant, which is

$$t_o = \frac{Ms}{h_o \theta_m^{(n-1)}}. \qquad (26)$$

The heating curve corresponding to Equation (25), when $n = 1.25$ is shown as Curve B in Figure 6 in which θ/θ_m is given in terms of t/t_o.

It will be noted from Equation (26), that the time constant in this case of the non-linear convection law, is dependent upon the maximum change in temperature θ_m, whereas, in the case of the linear law previously studied, the time constant is independent of the temperature range.

Derivation of Equation (25) for Heating of a Body by the Medium.

Let the body be suddenly immersed in a medium at a temperature θ_m higher than that of the body, and let θ be the change in temperature of the body at any time t after the initial immersion. As in the linear law case, the heat transmitted to the body is absorbed by its material during its increase in temperature.

Then, the heat transmitted to the body in time dt is equal to that absorbed by it in a change in temperature $d\theta$, which expressed mathematically is,

$$h_o(\theta_m - \theta)^n dt = Ms d\theta. \qquad (27)$$

Rearranging, we have

$$(\theta_m - \theta)^{-n}\, d\theta = \frac{h_o dt}{Ms}$$

which when integrated, becomes

$$\frac{(\theta_m - \theta)^{(1-n)}}{1-n} = -\frac{h_o t}{Ms} + C. \qquad (28)$$

Now when $t = 0$, $\theta = 0$, from which

$$C = \frac{\theta_m^{(1-n)}}{1-n}$$

which inserted in Equation 28 gives, after simplification

$$\frac{\theta}{\theta_m} = 1 - \left[\frac{1}{(n-1)\left(\frac{th_o}{Ms}\right)\theta_m^{(n-1)} + 1}\right]^{\frac{1}{n-1}}. \qquad (29)$$

But the quantity $\frac{Ms}{h_o \theta_m^{(n-1)}}$ in the denominator has the dimensions of time and may be considered the time constant of the body, which let us designate t_o. Then Equation (29) becomes

$$\frac{\theta}{\theta_m} = 1 - \left[\frac{1}{(n-1)\left(\frac{t}{t_o}\right) + 1}\right]^{\frac{1}{n-1}}$$

which is Equation (25).

10b. PROPERTY OF HEATING TIME CONSTANT FOR NON-LINEAR CONVECTION LAW.

10b$_1$. Rate of Change in Temperature.

As was found for the linear law of convection, the time required in the case of the non-linear law, for the temperature to reach the maximum value, if the the initial rate remained constant, is equal to the time constant. This is illustrated in Figure 6 by the line R, tangent to the heating curve at $t/t_o = O$. This is proved as follows. From Equation (27)

$$\frac{d\theta}{dt} = \frac{h_o}{Ms}(\theta_m - \theta)^n = \frac{h_o \theta_m^n}{Ms}\left(1 - \frac{\theta}{\theta_m}\right)^n$$

from which, since

$$Ms/h_o\theta_m^{(n-1)} = t_o$$

$$\frac{d\left(\frac{\theta}{\theta_m}\right)}{d\left(\frac{t}{t_o}\right)} = \left(1 - \frac{\theta}{\theta_m}\right)^n. \qquad (30)$$

When θ/θ_m is made zero in Equation (30), we have the initial rate

$$\frac{d\left(\frac{\theta}{\theta_m}\right)}{d\left(\frac{t}{t_o}\right)} = 1. \qquad (31)$$

This equation shows that if this initial rate continued constant, then in a time t equal to the time constant t_o, the temperature elevation θ would reach its maximum value θ_m.

10b$_2$. Temperature Change Relative to Final Change in a Time Equal to the Time Constant.

In Equation (25), if $t = t_o$, that is, $t/t_o = 1$, we have

$$\left.\frac{\theta}{\theta_m}\right]_{t=0} = 1 - \left[\frac{1}{(n-1) + 1}\right]^{\frac{1}{n-1}} \qquad (32)$$

which gives the general equation for the temperature reached in a time equal to the time constant t_o. If $n = 1.25$, which is a good value to use in practical work, then the temperature reached in a time equal to the time constant, from Equation (32) is

$$\left.\frac{\theta}{\theta_m}\right]_{t=0} = 1 - \left[\frac{1}{1.25}\right]^{\frac{1}{.25}} = 1 - \left[\frac{1}{1.25}\right]^{4} = 0.5904 \qquad (33)$$

as will also be observed by referring to Curve B in Figure 6. That is, the temperature reaches 59.04 percent of its final value in a time equal to its time constant for the non-linear law, where $n = 1.25$, whereas, for the linear-law this increase was found to be 63.2 percent.

10c. COOLING OF A SIMPLE BODY BY THE MEDIUM.

To find the Temperature of the Body at any Time after its Sudden Immersion into a Medium at a lower

Temperature than that of the Body, for a Non-Linear Convection Law.

Let the body at an initial temperature θ_m above that of the medium, be suddenly immersed into the medium, and let θ be the temperature elevation of the body at any time t after its initial immersion. Then, as will be shown, later,

$$\frac{\theta}{\theta_m} = \left[\frac{1}{(n-1)\frac{t}{t_o} + 1} \right]^{\frac{1}{n-1}} \quad (34)$$

where the time constant is

$$t_o = \frac{Ms}{h_o \theta_m^{(n-1)}}$$

which is the same as found for the heating problem. It will also be noted that Equation (34) represents a curve which is the reverse of that for Equation (25) found for the corresponding heating problem. This cooling curve is shown graphically as Curve D in Figure 6 which is Curve B reversed.

By analysis, and as is evident from the symmetry of the two curves for heating and cooling by the medium, the properties of the time constant and the rates of change in temperature, are the same for both conditions, and were given previously for the heating problem.

For purposes of comparison, the Curve C shown represents the temperature changes for the linear-law of convection. This is a dimensionless graph and is perfectly general, but in using it for practical applications, it must be remembered that the time constant t_o in general differs according to conditions.

Derivation of Equation (34) for Cooling by the Medium.

As in previous problems, the heat transferred from the body to the medium comes from the heat stored in the body. Then the heat transferred in a time dt equals the heat lost by the body by a decrease in temperature $d\theta$, that is,

$$h_o \theta^n dt = -Ms d\theta \quad (35)$$

The negative sign is used since the temperature is a decreasing function of time. Rearranging, we have,

$$\frac{d\theta}{\theta^n} = \frac{h_o}{Ms} dt$$

which when integrated, becomes

$$\frac{\theta^{(1-n)}}{n-1} = \frac{th_o}{Ms} + C.$$

The constant C can be evaluated by substituting $\theta = \theta_m$ when $t = O$. And then,

$$\frac{\theta}{\theta_m} = \left[\frac{1}{(n-1)\frac{t}{t_o} + 1} \right]^{\frac{1}{n-1}}$$

which is Equation (34).

Where, as in the heating problem,

$$t_o = \frac{Ms}{h_o \theta_m^{(n-1)}}.$$

10d. HEATING OF A BODY FROM AN EXTERNAL SOURCE.

To find the Temperature of a Simple Body, Heated at a Constant Rate, as by an Electric Current, at any Time after the Initial Application of the Heat, when the Body is Immersed in a Cooling Medium which Exchanges Heat with the Body according to a Non-Linear Law.

Let heat be added to the body from an external source at the rate of W watts, and let the rate of heat exchange be proportional to the n^{th} power of the temperature difference, that is $h_o\theta^n$ where h_o is the same as in previous non-linear problems, and let the medium and body have the same temperature initially. The total rate of addition of heat W is the sum of the rate at which heat is dissipated to the medium, $h_o\theta^n$, and that absorbed by the body, $Ms(d\theta/dt)$, or

$$W = h_o\theta^n + Ms\frac{d\theta}{dt}. \quad (36)$$

Then,

$$\frac{d\theta}{dt} = \frac{W - h_o\theta^n}{Ms}$$

from which,

$$dt = \frac{Ms}{h_o}\left(\frac{d\theta}{\frac{W}{h_o} - \theta^n}\right). \quad (37)$$

Now $\frac{W}{h_o} = \theta_m{}^n$, where θ_m is the

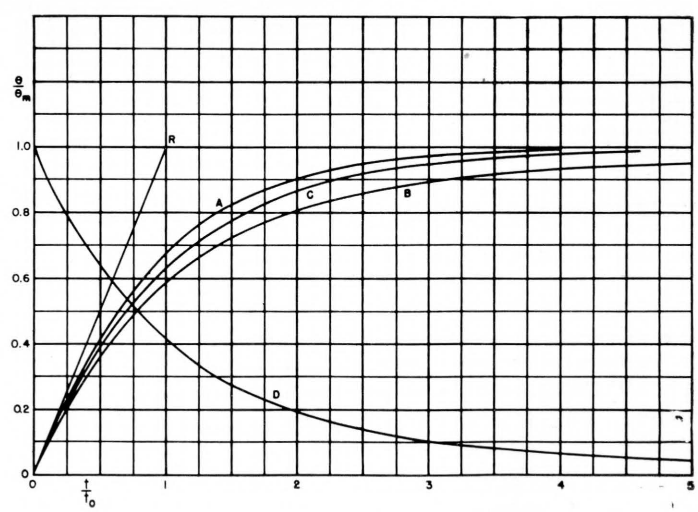


Figure 6. Heating and Cooling Curves for Non-Linear Cooling Law, $w_c = h_o\theta^n$, $n = 1.25$ when a simple body is immersed in a constant temperature medium.

CURVE A. *Temperature change of the body with time when heat is added from an external source at a constant rate.*

CURVES B AND D. *Temperature changes of the body with time after a sudden immersion in a medium differing in temperature from that of the body. Curve B, for a medium at a higher temperature, and Curve D for one at a lower temperature than that of the body.*

CURVE C. *Temperature change for the linear law $n = 1$, shown for comparison.*

Temperatures are given relative to the maximum change in temperature θ_m and time relative to the time constant t_o. Line R gives initial rate of change in temperature.

final temperature elevation, and from Equation (26), $Ms/h_o = t_o\theta_m{}^{(n-1)}$. Then substitute these values in Equation (37) and we have,

$$d\left(\frac{t}{t_o}\right) = \frac{d\left(\frac{\theta}{\theta_m}\right)}{1 - d\left(\frac{\theta}{\theta_m}\right)^n}. \qquad (38)$$

This can be integrated by changing it into a power series by dividing the denominator into unity. Then,

$$\frac{1}{1 - \left(\frac{\theta}{\theta_m}\right)^n} = 1 + \left(\frac{\theta}{\theta_m}\right)^n + \left(\frac{\theta}{\theta_m}\right)^{2n} + \left(\frac{\theta}{\theta_m}\right)^{3n} + \ldots\ldots \qquad (39)$$

This series is convergent since θ/θ_m is always less than unity.

Then Equation (38) becomes

$$\frac{t}{t_o} = \frac{d\left(\frac{\theta}{\theta_m}\right)}{1 - \left(\frac{\theta}{\theta_m}\right)^n} = d\left(\frac{\theta}{\theta_m}\right) + \left(\frac{\theta}{\theta_m}\right)^n d\left(\frac{\theta}{\theta_m}\right) + \left(\frac{\theta}{\theta_m}\right)^{2n} d\left(\frac{\theta}{\theta_m}\right) + \ldots\ldots \qquad (40)$$

Integrating each term, we have

$$\frac{t}{t_o} = \frac{\theta}{\theta_m} + \frac{\left(\frac{\theta}{\theta_m}\right)^{n+1}}{n+1} + \frac{\left(\frac{\theta}{\theta_m}\right)^{2n+1}}{2n+1} + \frac{\left(\frac{\theta}{\theta_m}\right)^{3n+1}}{3n+1} + \ldots\ldots \qquad (41)$$

from which values for t/t_o can be found for any value of θ/θ_m. The constant of integration is zero for the reason that $\theta = 0$ when $t = 0$. When $n = 1.25$ which is a good practical value, Equation (41) becomes

$$\frac{t}{t_o} = \frac{\theta}{\theta_m} + \frac{\left(\frac{\theta}{\theta_m}\right)^{2.25}}{2.25} + \frac{\left(\frac{\theta}{\theta_m}\right)^{3.50}}{3.50} + \frac{\left(\frac{\theta}{\theta_m}\right)^{4.75}}{4.75} + \ldots\ldots \qquad (42)$$

For values of θ/θ_m above about 0.7, this series converges rather slowly, which, for such values, makes the computation laborious. For values above 0.7, it is preferable to use numerical integration to evaluate t/t_o for various values of θ/θ_m, by measuring the area by mechanical or other means such as Simpson's Rule, under the curve corresponding to

$$f\left(\frac{\theta}{\theta_m}\right) = \frac{1}{1 - \left(\frac{\theta}{\theta_m}\right)^n}. \qquad (43)$$

In Figure 6, Curve A was determined by the use of Equation 45 to and including $\theta/\theta_m = 0.7$, and by numerical integration beyond.

10d₁. Rate of Increase in Temperature When Heat is Added at a Constant Rate.

The rate of increase in temperature at any time t is from Equation (38)

$$\frac{d\left(\frac{\theta}{\theta_m}\right)}{d\left(\frac{t}{t_o}\right)} = 1 - \left(\frac{\theta}{\theta_m}\right)^n \qquad (44)$$

The initial rate is found by placing $\theta = 0$ in Equation 44, and we have,

$$\left.\frac{d\left(\frac{\theta}{\theta_m}\right)}{d\left(\frac{t}{t_o}\right)}\right]_{\theta = 0} = 1$$

which means that if the initial rate continued constant, then for any value of n the final temperature would be reached in a time equal to the time constant, as was the case for linear-law heating and for the other forms of non-linear heating and cooling problems previously treated. This is shown as line OR in Figure 6.

10d₂. Temperature Reached in a Time Equal to the Time Constant When Heat is Added at a Constant Rate.

From the computations for the curve, it is found that in a time t equal to the time constant t_o, the temperature elevation is 67.3 per cent of its final value, as shown in Curve A in Figure 6.

It will be noted that this value is different from the value found for both heating and cooling when no heat was added from an external source. In the latter cases, as shown in Curves B and D, the corresponding value is 0.5904. This difference is obvious from physical reasoning. In the case of external heating, the loss of heat from the body to the medium is proportionately lower at the lower temperatures and, therefore, more heat is available for absorption by the body, which results in a greater rate of increase in temperature.

10e. LINEAR LAW OF CONVECTION CONSIDERED AS A SPECIAL CASE OF THE NON-LINEAR LAW.

In the equations derived for the non-linear law of convection, if n is made equal to unity as the linear law requires, then these equations all change into the corresponding equations derived for the linear-law. This is shown as follows:

Taking Equation (25) for the non-linear law of heating by the medium; rearrange and put $n = 1$. Then,

$$\left(1 - \frac{\theta}{\theta_o}\right) = \left[\frac{1}{(n-1)\frac{t}{t_o} + 1}\right]_{n=1}^{\frac{1}{n-1}} = \left(\frac{1}{1}\right)^{\infty}$$

which must be evaluated by letting n approach unity as its limit. Take the logarithm of both sides and we have

$$\log\left(1 - \frac{\theta}{\theta_m}\right) = \left.\frac{\log\left[\frac{1}{(n-1)\frac{t}{t_o} + 1}\right]}{n-1}\right]_{n=1} = \frac{0}{0}.$$

To evaluate this, differentiate numerator and denominator with respect to n, and we have

$$\log\left(1-\frac{\theta}{\theta_m}\right) = \left[\frac{-\frac{t}{t_o}}{(n-1)\frac{t}{t_o}+1}\right]_{n=1} = -\frac{t}{t_o}$$

from which

$$\frac{\theta}{\theta_m} = \left(1-\varepsilon^{-\frac{t}{t_o}}\right)$$

which is the corresponding Equation (8) for the linear case. Again, take Equation (41) for the non-linear case of heating by an external source. By placing $n = 1$ we have

$$\frac{t}{t_o} = \frac{\theta}{\theta_m} + \frac{\left(\frac{\theta}{\theta_m}\right)^2}{2} + \frac{\left(\frac{\theta}{\theta_m}\right)^3}{3} + \frac{\left(\frac{\theta}{\theta_m}\right)^4}{4} + \ldots\ldots \quad (45)$$

Now it is well known that since $\frac{\theta}{\theta_m}$ is less than unity, Equation (45) is the expansion of $-\log\left(1-\frac{\theta}{\theta_m}\right)$. Then from Equation (45)

$$\log\left(1-\frac{\theta}{\theta_m}\right) = -\frac{t}{t_o} \quad (46)$$

from which

$$1-\frac{\theta}{\theta_m} = \varepsilon^{-\frac{t}{t_o}}$$

or

$$\frac{\theta}{\theta_m} = \left(-\varepsilon^{-\frac{t}{t_o}}\right)$$

which is the corresponding Equation (2) for the linear case.

10f. COMPARISON OF RESULTS OF LINEAR AND NON-LINEAR LAWS OF CONVECTION.

It is important to consider the differences between the time constants and the temperature curves for the linear and non-linear convection laws.

The time constant for the linear-law from Equation (1) is,

$$t_o = \frac{Ms}{h} \quad (47)$$

where the convection coefficient h for the linear-law differs from the coefficient h_o for the non-linear law as given in Equation (24).

In the case where the exchange of heat between the body and the surrounding medium is actually linear, that is, exactly proportional to the difference in temperature, then the value of the convection coefficient h can be determined experimentally by measuring the rate at which heat is exchanged for any measured temperature difference, and dividing this rate by the temperature difference to obtain the rate per degree, as the definition of h requires. The heat may be applied electrically or by other measurable methods. The value for h thus determined will be the same regardless of the temperature difference used in making the test. When, however, the convection law is actually non-linear, but is assumed linear as an approximation, then the value determined experimentally for the convection coefficient h, will depend upon the temperature difference used in making the test.

We shall now find the magnitude of the effect of the non-linear law upon the measurement of the convection coefficient h under the assumption that the law is linear.

The true heat exchange rate, from Equation (24), is

$$W_c = h_o\theta^n$$

where W_c is the measured heat rate, and θ the measured difference in temperature between the body and the medium.

Under the assumption, as an approximation, that the law is linear, the value for the convection coefficient for the linear-law, in watts per degree, is

$$h = \frac{W_c}{\theta} = \frac{h_o\theta^n}{\theta} = h_o\theta^{(n-1)} \quad (48)$$

which shows that the value of h thus determined varies with the $(n-1)$ power of the difference in temperature. In practice n is approximately 1.25, then $(n-1) = 0.25$, so that h would vary as the fourth root of the temperature difference.

In practice, it is the maximum temperature reached under given conditions that is usually the most important result to be computed. For this reason it is preferable to determine the value of h for the average maximum difference in temperature encountered in practice. Let us call this θ_m.

If the coefficient for linear convection, h, is determined from tests made for the maximum change in temperature, θ_m, in the heating test discussed above, then we have the interesting result that the computed time constants for the linear and non-linear conditions will be equal. This is shown as follows: From Equation (48)

$$h = h_o\theta_m^{(n-1)}$$

or

$$h_o = \frac{h}{\theta_m^{(n-1)}}. \quad (49)$$

When this value for h_o in terms of h, is substituted in Equation (26) for the time constant for the non-linear law, we obtain

$$t_o = \frac{Ms}{h_o\theta_m^{(n-1)}} = \frac{Ms\theta_m^{(n-1)}}{h\theta_m^{(n-1)}} = \frac{Ms}{h}$$

which is the time constant for the linear law given in Equation (47).

10g. PRACTICAL APPLICATIONS.

To use the equations previously given it is necessary to know either the time constant t_o by direct measurement or the convection coefficient h or h_o, together with the mass M of the body, and its heat capacity per gram, *s*. For the latter method, measured values of the non-linear convection coefficient, h_o, are given below for some specific shapes of bodies.

STRIPS.

The curve in Figure 7 gives the non-linear convection coefficient h_o, for still air, for various widths of simple strip material mounted horizontally in its lengthwise direction, and with the surfaces vertical. The values given consider losses from convection only and not from heat lost by conduction to terminals, if any. The tests were made at a temperature difference of 50 deg. C. and the values in the curve are based upon the assumption that the convection rate is proportional to the 1.25th power of the difference in temperature, which tests have substantiated.

The values are given in watts per inch length of wire, per $\theta^{1.25}$ and were computed from tests made at a temperature difference of 100°C. As in the case of strips, the values are for convection losses only. Tests at other temperatures showed that the 1.25th power law is substantially correct.

From these curves, the values of h also, assuming a linear law as an approximation, can be determined for any desired difference in temperature, by use of Equation (46); for example, for a maximum assumed temperature elevation of 50 deg. C.

$$h = h_o\theta^{n-1} = h_o\theta^{1/4} =$$
$$h_o \times 50^{1/4} = 2.66\ h_o$$

which for a strip one inch wide, for example, would be h, in watts per sq. inch per degree cent., $= 2.66 \times 0.00335 = 0.00892$. It must be remembered that whereas the values of the convection

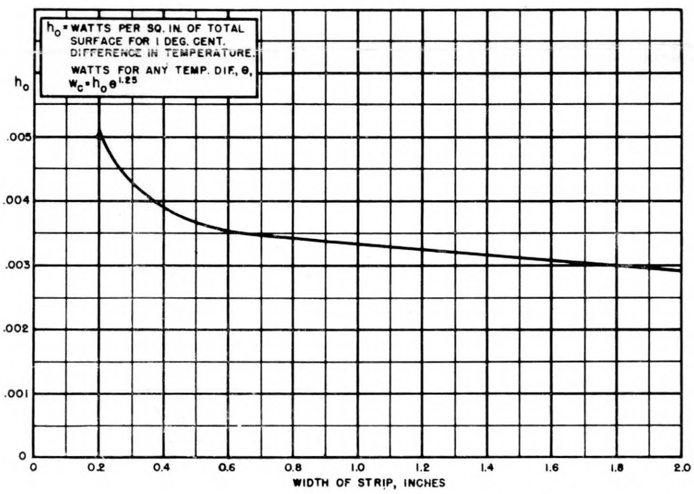


Figure 7. Convection coefficient, h_o for a single strip mounted horizontally with its surfaces vertical, or for a relatively large diameter tube with its axes vertical, in still air, in watts per sq. in. for 1 deg. cent. difference in temperature. Watts loss for any temperature difference θ, is $W_c = h_o\theta^{1.25}$.

The values given for h_o are in watts per square inch of total superficial area, including edges, per $\theta^{1.25}$.

These values given for strips also apply to relatively large diameter tubes mounted with their axes vertical, in which both the interior and external surfaces must be considered, if exposed.

WIRES.

Figure 8 gives values of the non-linear convection coefficient, h_o for various diameters of wires mounted horizontally in still air.

coefficients shown in the curves are given in watts per sq. inch or inch length, h and h_o are actually the total heat rate in watts associated with the surfaces of the mass M used in the equations.

TUBES AND RODS.

Tubes and rods of the order of ¼ inch in diameter are frequently encountered in heating problems. The convection coefficient for such a body subject to the 1.25th power cooling law, in still air is $h_o = 0.00353$ watt per sq. inch, for one deg. cent. difference in temperature, which it will be

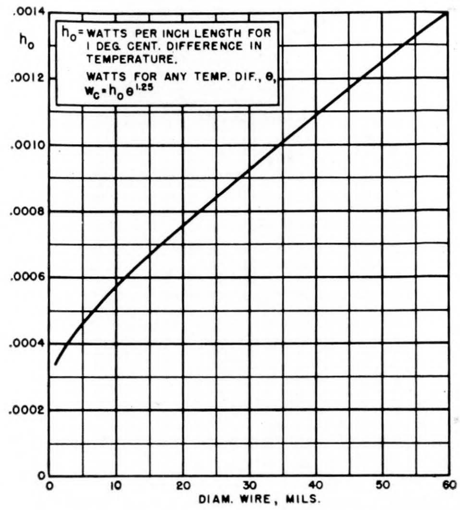


Figure 8. Convection coefficient *h* for single wires mounted horizontally in still air, in watts per inch length for 1 deg. cent. difference in temperature. Watts loss for any temperature difference θ, is $W_c = h_o\theta^{1.25}$.

noted is equal to that from a 0.6 in. strip. In still water, the coefficient is 0.264 watt per sq. in. for one deg. centigrade, for such bodies.

Data are not available for the convection coefficients in still water, for all of the other shapes given for air, but as a reasonably close approximation for use in practice, the coefficients for still water may be taken as 75 times the values given for still air.

E. N.—No. 52 —*W. N. Goodwin, Jr.*

CORRECTION NOTICE

Your attention is called to an error which appeared in Part II of the article, "Thermal Problems Relating to Measuring And Control Devices" which was published in the April 1948 issue of Weston Engineering Notes, Volume 3, Issue No. 2. The first sentence in the last paragraph on page six reads as follows: "In the problem, a heating period of *10* seconds was assumed." This sentence should read: "In the problem, a heating period of *1.0* second was assumed."

The *Editor*

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