Weston ENGINEERING NOTES

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WESTON STANDARD CELLS AND THEIR APPLICATIONS

STANDARD cells of the cadmium type,[1] in themselves, have changed but little since they were first invented by Doctor Weston in the early 1890's. Manufacturing and control methods have been perfected and changes have been made dimensionally as well as in the type of glass used, but the fundamentals of the chemical series and their general arrangement within the cell, however, remain unchanged.

Generally speaking, the changes which have taken place have been in the housing or mounting means and in the size of the cell. Actually these changes are not consecutive changes occurring with time but rather they are variations of the original, being made currently for the various applications which have different requirements.

In discussing these various types of cells, it is preferable to start with the universally accepted basic reference of comparison, the Normal or saturated cell, then the laboratory reference Standard or unsaturated cell, the Pyrometer cell, and finally the Student cell.

The Normal or Saturated Cell

The Normal or saturated cell (Model 3, Type 7) consists of the same basic chemicals and general arrangement as the other cells of the cadmium type except for two outstanding differences:

1. The cadmium sulfate solution is saturated over the entire working range of temperatures, +4° C to +40° C, and there are cadmium sulfate crystals in the solution to insure that it remains saturated at all times.

2. The cells are not provided with a septum or mechanical means of holding the several chemicals in place during shipment. This means that these cells cannot be shipped by common carrier and must be transported by reliable messenger—to tilt them more than about 30-45 degrees from the vertical could cause the chemicals to spill from one leg of the H tube to the other after which they probably would be unstable and worthless.

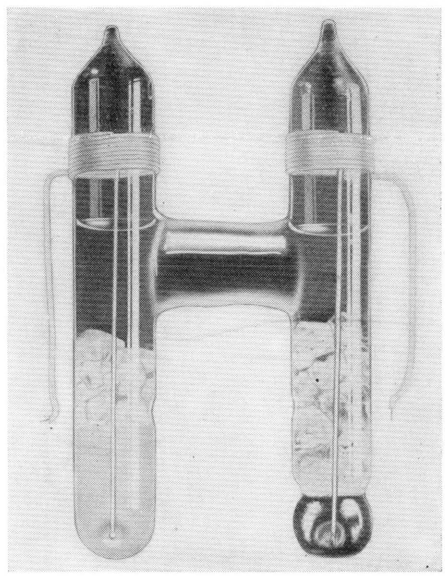

Figure 1—The Weston Normal or Saturated Cell, Model 3, Type 7.

These cells are of the smaller size with the glass tube bore being about 12½ mm (0.5 in.) in diameter. They are hermetically sealed at the top as may be seen in Figure 1. After assembly and sealing, the cells are aged for from six months to one year during which time numerous periodic readings of the emf are made and plotted.

During the early part of the aging process, the emf levels out to a constant value. After several months of verified constancy, the cells are ready for final certification.

In This Issue

John Parker, Editor

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Cells of the saturated type are usually furnished in lots of three or more and preferably five or six. They are not provided with individual cases since, being intended for use in an oil bath to control their temperature, such cases would serve little or no purpose.

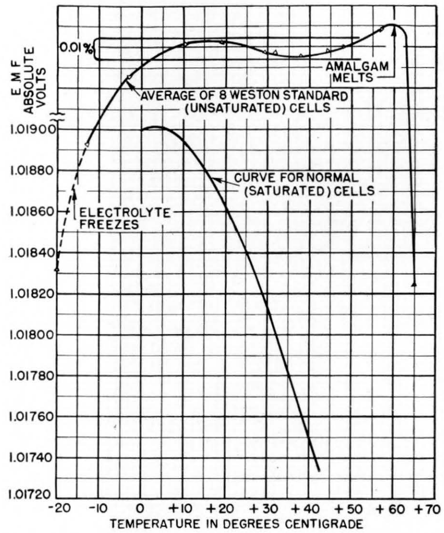

Figure 2—EMF versus Temperature for Weston Normal and Standard Cells.

Normal cells are very definitely a primary standard of the first order, intended for control purposes to standardize other working cells. To realize the high order of accuracy inherent in these cells, it is necessary to recognize their characteristics. First, they are sensitive to temperature (See Figure 2.), a change of about 50 microvolts per degree taking place in the zone from 25° C to 40° C. To offset this, these cells are intended to be immersed in an oil bath maintained at some temperature slightly over maximum room temperature, thereby affording continuous control by heating without resorting to both controlled heating and cooling. 33-35° is a satisfactory level which has not been found too difficult to maintain. The means of temperature control is a subject in itself and beyond the scope of this article. Suffice it to say, that it is not uncommon practice to hold the bath temperature to ±0.02° C corresponding to an emf change of about ±1 microvolt or 0.0001 per cent. In this regard, the advantage of a dual inertia system, whereby the sensitive element and the heating unit of the control means are not placed directly in the oil bath but either in a semi-restricted portion of it or an air bath surrounding it, should not be overlooked. When the thermal inertias of the two systems are sufficiently different, the main bath is influenced by the averaging effect and shows a marked improvement in minimizing the resulting temperature deviations. The actual temperature of the bath should be determined by a suitable thermometer[2] properly calibrated for the correct immersion length or by resistance thermometry means.

Cells of this type are periodically standardized by cross checking among themselves (the reason for having three as a minimum) or by carrying them to the Bureau of Standards for certification by comparison with their reference banks.

In actual use, these cells are seldom used directly for the routine daily comparison tests employed to standardize the laboratory standard cells; rather, a single cell of standard cell type or a single cell of the saturated type is selected as a working standard. This cell is standardized against the several cells in the primary group and the laboratory cells are standardized against this cell. The Standard cell described below is well suited for this work.

The Standard or Unsaturated Cell

The Standard cell is perhaps better known than the Normal cell, because its general characteristics conform more to industrial requirements. It is furnished suitably housed in a bakelite case (Figure 3) with binding posts for external connection. It has an accuracy of 0.01 per cent, and a temperature coefficient which is negligible. (See Figure 2.)

The cell proper employs a glass tube 25 mm (one inch) in diameter and, by virtue of the increased size and volume, holds a greater volume of chemicals. This in turn reduces its resistance to a nominal value of around 100-125 ohms, and reduces the effect of drawing a small current during the initial stages of balancing the current to zero when standardizing. When low-resistance balanced circuits are used in special applications, the low-internal resistance of the cell slightly increases the sensitivity of the circuit to small unbalanced currents. In circuits where large unbalanced currents, in the order of 100 microamps, may be encountered, the increased volume of chemicals minimizes the effect.

Standard cells (See Figures 3, 4, 5 and 6) are unsaturated at temperatures above 4° C and have, as a result, a temperature influence which for all practical purposes may be considered as negligible. It is in the order of five microvolts (0.0005 per cent) per degree Centigrade and is not a straight line function of temperature.

Figure 2 is a graph on which has been plotted the emf versus temperature for both the Normal cell and the Standard cell. The curve for the Normal cell was derived from the formula established by Wolff. The curve for the Standard cell is an average of a number of cells. Note that the curve for the Standard cells is not straight but serpentine. Also that over the range from +4° C to +40° C, the extremes do not exceed 50 microvolts (.005 per cent) from a value obtained by standardization at around 25° C. By contrast, the Normal cell would change several hundred microvolts over the same temperature range if its temperature was not controlled as previously described.

Questions are often asked as to why cells should not be used below +4° C and above +40° C. The curve explains this quite well as it can be seen that below +4° C the emf drops off quite rapidly and eventually the electrolyte freezes. Although cells have been frozen without physical damage and have subsequently re-established an emf very close to their original value, it is not expected that this is normal or reliable. Certainly some of them will break and some will not return sufficiently close to their original value to be useful. In any event, cells so treated should always be regarded with suspicion until it has been proven that they have been unimpaired. Above 40° C the reverse occurs in that at a temperature around 55° C to 60° C the amalgam melts and the emf drops off precipitously. While it is true this only happens above 55° C to 60° C, the upper limit of safety established at 40° C is insurance to allow for temperature rise which often occurs over and above the known value. This may be due to heating of the cell by radiation or from other sources unforeseen and for which allowance may not have been made.

Although Standard cells are intended primarily as a standardized reference source of emf for use in a well-balanced circuit, some current is drawn from the cell and occasionally passed through it in the reverse direction during the initial balancing operations. When such currents are less than 100 microamperes, the effect is negligible. Currents above 100 microamperes reduce the net accuracy obtainable immediately after the current has been drawn but, if the cell is allowed to stand a while, it will most likely return to the original value of emf. The answer to how much current can be drawn, how much the emf changes, how long one should wait, and what the final accuracy will be, etc., are very difficult to answer in a general way and even more so in a specific set of circumstances, because all the functions mentioned are not only non-linear and dependent on one another but, because when abused, each cell is somewhat of an individual, the actions of which are very much dependent on its past life. Evaluation of the results in terms of acceptable, good, and bad are subject to wide variations depending upon the exact use to which the cell is being put. A cell which returns to within 0.1 per cent within one hour might be acceptable for one application while on another this might be entirely unsatisfactory.

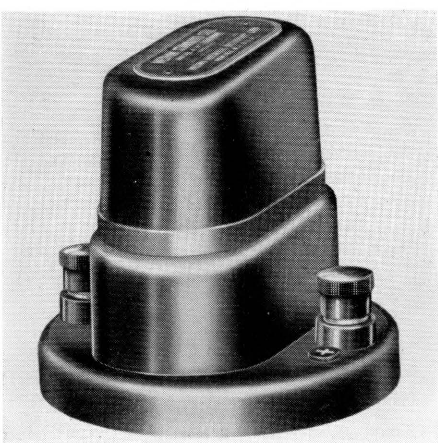

Figure 3—The Weston Standard or Unsaturated Cell, Model 4, Type 3.

A few additional remarks are in order regarding Standard cells. First, cells should not be mounted where heat will be transmitted to them by radiation such as directly under a bench lamp or in the sunshine. Also, if the ambient temperature changes rapidly as may be the case if a cell were taken from one room to another at an entirely different temperature, readings should be taken immediately before the cell has begun to change its temperature or several hours should elapse before using it. Rapid changes of cell temperature do not affect both legs of the cell equally and a temperature error may be apparent until both legs have stabilized at the same temperature for several hours. To minimize this effect, Weston Standard cells have for many years employed a bakelite case so arranged to help equalize the temperature of the air surrounding the cell. More recently, an additional inner shell of heavy copper has been added to still further minimize this effect. See Figure 4. Note that the "H" tube is "glass sealed" to provide a truly hermetic closure. The cell having the copper-thermal equalizer and the glass-tipped "H" tube was put into manufacture early in 1948. Due to the aging required, shipment of these cells began much later in the year. This cell, identified as the Model 4, Type 3, is readily distinguished from the Model 4, Type 1, cell, which it supersedes, by a deep red band around the midsection.

Figure 4—Internal view of the Weston Model 4, Type 3, Standard Cell.

The Pyrometer Cell

The Pyrometer cell is chemically the same as the Standard cell but is smaller in size being 12.5 mm (0.5 inch) in diameter. It is supplied unmounted but packed in individual cartons.

These cells were developed specifically for use as a reference standard in balancing-type potentiometer circuits such as those used in potentiometric recorders, controllers and other devices using similar circuits. The cells (See Figure 5) have one leg (the + leg containing the mercurous sulphate) covered to protect it from the effect of light which might be allowed occasionally to strike the cell. This is precautionary, however, as only long exposure to light would have a noticeable effect upon the cell.

The Pyrometer cells have a nominal resistance of around 500 ohms but may be as high as 800. Although the smaller-sized cells are somewhat more affected than the larger ones when currents of the same amount are drawn from them, the same limit of 100 microamps maximum applies as the same accuracy generally is not expected of these cells. An accuracy of 0.1 per cent is generally considered quite satisfactory. Further, in actual pyrometer applications, the currents drawn are usually considerably less than 100

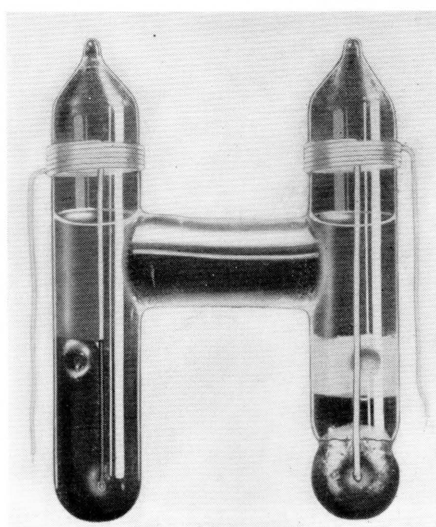

Figure 5—The Weston Pyrometer Cell, Model 3, Type 4.

microamps as balancing is very often automatic and at short intervals or at least repeated in a manner such that the current circuit requires a minimum of correction at each operation. For pyrometer applications these cells last many years before requiring replacement. These cells are identified as Model 3, Type 4.

The Student Cell

The Student cell, Weston Model 3, Type 6, is identical in most respects with the Pyrometer cell just described except that it is mounted on a metal stand with the cell fully exposed to view. It has two binding posts, one of which has a 10,000-ohm resistor in series with it to limit the current drawn to 100 microamps in case of short circuit. (See Figure 6.) The cell has a name plate which states that its emf is 1.018 volts. The actual voltage of a new cell might really be anywhere from 1.0188 to 1.0198 and with use the emf might decrease to 1.017. Since such cells may be subjected to expected but unusual usage, certifying the voltage to more than four digits serves little purpose. If the value of 1.018 is used, the student will probably be in error not more than 0.1 per cent for this reason. This is probably in keeping with the rest of the apparatus being used and is considered quite satisfactory. When higher accuracy is desired, the cell may be standardized against a Model 4 Standard cell.

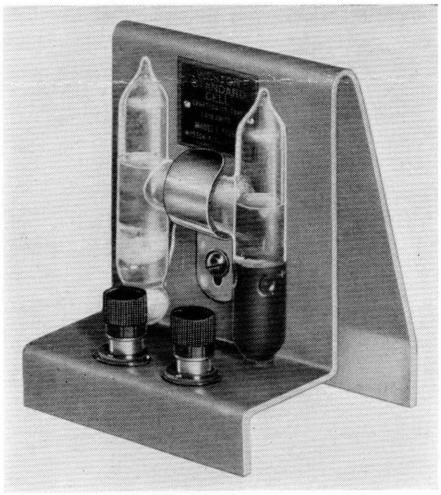

Figure 6—The Weston Student Cell, Model 3, Type 6.

Tabulation of Data

	Normal Cell	Standard Cell	Pyrometer Cell	Student Cell
Identification	Mod 3 Type 7	Mod 4 Type 3	Mod 3 Type 4	Mod 3 Type 6
Chemical Solution	Saturated	Unsaturated	Unsaturated	Unsaturated
EMF (Nominal)	1.01863	1.0190	1.0190	1.019
Normal Res., Ohms	500-800	100-150	500-800	500-800
Temp. Coefficient	App. 50 μV/deg. C	Negligible App. 0-5 μV/deg. C	Negligible App.0-5 μV/deg. C	Negligible App. 0-5 μV/deg. C
Weston Certificate	Supplied	Supplied	EMF on Label	EMF on name plate
Bureau of Standards Certificate	Available on Order	Available on Order	....	....
Mounting	None[3]	Bakelite Case	Cardboard Box[4]	Mounted on Stand
Transportation	User's Messenger	Commercial Carrier	Commercial Carrier	Commercial Carrier

[3] *Supplied mounted in carton for transportation by messenger. Intended for mounting in oil or air bath with controlled temperature.*
[4] *Cells usually removed from carton, to be mounted by user in his device.*

Transportation of Cells

Except for the Normal cell, which requires transportation by messenger as previously stated, all the other cells may be shipped by common carrier and that is the method normally used. It should be recognized, however, that packing is to a great extent an art and transportation always a hazard. When cells are packaged for shipment, they should be well supported in wadding with the first carton floated within a second one. When cells are transported to the Bureau of Standards for certification, thereby implying that an unusually high order of accuracy is desired, it is even desirable to have them carried by messenger. This should not be misunderstood to mean that the cells won't stand shipment, but rather that just as we, the manufacturers, have gone the limit to produce the cell with its high order of accuracy, so, too, the user may contribute by doing what he can to minimize the hazards of shipment.

E. N.—No. 57 —*F. X. Lamb*

[1] *See Vol. I, No. 3, June 1946,* WESTON ENGINEERING NOTES.
[2] *Tag Model 8677, Mercury in Glass Thermometer, Range 32-38° C, 1/50° C per div. 18" length, 6" to 8" immersion.*

R-C ARC SUPPRESSION OF INDUCTIVE RELAY CIRCUITS

THE use of a series-connected capacitor and resistor across a coil to damp the inductive transient surge of the coil is well known, but the design theory involved is not as generally appreciated and the following simple explanation is offered with apologies to rigorous theorists.

The diagram represents a coil, having an inductance L and a resistance R_l, operated by a sensitive primary contact which is to be protected from current and potential surge transients. The coil is shunted by a series R-C circuit, consisting of a resistor R_c, having a value:

$$R_c = R_L = R \quad (1)$$

and a capacitor C, having a value:

$$C = \frac{L}{R^2} \quad (2)$$

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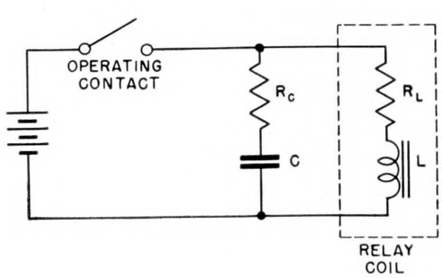

R-C Relay Surge Suppressor Circuit Diagram.

THERMAL PROBLEMS RELATING TO MEASURING AND CONTROL DEVICES—PART IV—COMPOSITE BODIES

Introduction

THE PREVIOUS chapters in this series considered thermal problems relating to a simple body, that is, one in which all parts of its mass may be assumed to be directly in contact with the surrounding heat-exchanging medium. This condition may result either when the body consists of very thin material, or when it is of any shape and its thermal resistivity is so low, relative to the thermal resistance of its contact with the medium, that the temperature gradient inside the body becomes negligible.

A composite body, as distinguished from a simple body, may be described for the purpose of this study, as one in which the thermal resistances of its various parts, either lumped or distributed, are of such magnitudes that the temperature gradients in the body are not negligible.

Composite bodies in general are too complicated to be treated analytically. However, they can often in practice be considered as consisting of two or more simple bodies having definite mutual relations, to which analytical treatment can be applied.

This chapter will consider a composite body consisting of two simple bodies related in a specified manner.

Examples of such a body are (a) thermometer elements mounted in a protecting case, such as mercury and resistance thermometers, (b) movable coils, and resistors in instrument cases, transformer windings and other similar combinations.

11. COMPOSITE BODY.

Heating and Cooling of a Combination of Two Bodies M_1 and M_2, Separated by a Thermal Resistance, of which M_1 only is in Direct Contact with the Surrounding Medium, and M_2 Exchanges Heat with the Medium only Indirectly through the Body M_1.

Figure 9 illustrates the cross section of such a combination, in which the body M_2 is enclosed by the body M_1 but separated from it by the thermal resistance in the space between them. The body M_1 is in direct contact with the surrounding medium. The bodies are shown as spheres for simplicity, but the shape is immaterial. Three conditions of heating and cooling will be considered, and the lag in the temperature of the bodies resulting when the temperature of the medium changes at a given rate.

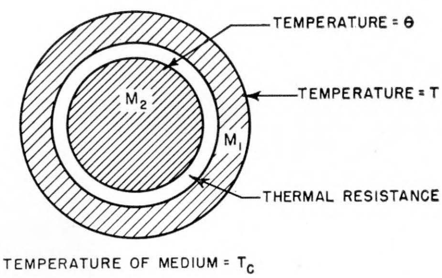

Figure 9—Diagramatic illustration of a Composite Body consisting of Two Simple Bodies M_1 and M_2 of which M_1 is in direct contact with the Heat Exchanging Medium, and M_2 is in contact with it through the Body M_1 only.

List of Symbols Used

h_1 = Rate of heat exchange between M_1 and the surrounding medium per degree difference in temperature. Watts per degree Centigrade.

h_2 = Rate of heat exchange between M_2 and M_1 per degree difference in temperature.

L = Maximum possible lag in temperature between body M_2, and the medium changing in temperature at a constant rate.

M_1 = Mass of body in direct contact with the medium; grams.

M_2 = Mass of body in contact with M_1; grams.

$P = \left[\frac{(t_1+t_2+t_m)^2}{4t_c^2}-1\right]^{1/2}$, for brevity; numeric.

r = Rate of change in the temperature of the medium; degrees per second.

s_1 and s_2 = Thermal capacities per gram per degree Centigrade of M_1 and M_2, respectively.

T = Temperature of M_1 at any time t.

T_c = Temperature of surrounding medium, constant.

T_a = Temperature at any time of a variable temperature medium.

t_1 = Thermal time constant of M_1, $= M_1s_1/h_1$; seconds.

t_2 = Thermal time constant of M_2 relative to M_1, $= M_2s_2/h_2$; seconds.

t_m = Transfer thermal time constant of M_2 relative to its heat dissipation through the outside surface of M_1, $= M_2s_2/h_1$.

t_c = $\sqrt{t_1t_2}$ for brevity; seconds.

θ = Temperature of M_2 at any time t.

θ_m = Final temperature of M_2 when heated from a source other than the medium.

θ_o = Initial temperature of the bodies.

11(a). COOLING OF A COMPOSITE BODY BY THE MEDIUM.

To Find the Temperature of the Body M_2 at any Time t after the Sudden Immersion of the Combination M_1 and M_2 into a Medium at a Lower Temperature than the Initial Temperatures of the Bodies, Assumed Initially Equal.

As is proved in a published paper by the author,[1] the elevation in temperature of the body, $(\theta - T_c)$, above that of the medium, T_c, at any time t, relative to the initial elevation, $(\theta_o - T_c)$ is

$$\frac{\theta - T_c}{\theta_o - T_c} = \frac{1}{P}\epsilon^{-\left[\left(\frac{t}{t_c}\right)\sqrt{P^2+1}\right]} \times \sinh\left[P\left(\frac{t}{t_c}\right) + \sinh^{-1}P\right] \qquad (50)$$

where

$$P = \left[\frac{(t_1+t_2+t_m)^2}{4t_c^2}-1\right]^{1/2}$$

which is a dimensionless parameter and is a function of the three time constants of the system. For the meaning of the symbols, see the list of symbols.

This is an exact equation for all values of time and thermal constants. As a result of its exponential character, however, it will be shown later that after a relatively short initial cooling time, the equation reduces to the following very simple approximate form.

$$\frac{\theta - T_c}{\theta_o - T_c} \cong \frac{P+\sqrt{P^2+1}}{2P}\,\epsilon^{-(\sqrt{P^2+1}-P)\frac{t}{t_c}} \qquad (51)$$

This equation is correct to within ¼ of 1% when

$$\left[P\left(\frac{t}{t_c}\right) + sinh^{-1} P\right]$$

in Equation (50) is equal to 3, and for greater values the error soon becomes entirely negligible.

It will be noted from Equation (51) that if the equation were correct starting from time $t=0$, then at $t=0$

$$\left.\frac{\theta - T_c}{\theta_o - T_c}\right]_{t=0} = \frac{P+\sqrt{P^2+1}}{2P} \qquad (52)$$

which means that the Equation (51) is the same as that for a simple body having a time constant

$$t_o = \frac{t_c}{\sqrt{P^2+1}-P}$$

and which starts to cool from an initial temperature elevation $(\theta - T_o)$ equal to

$$\frac{P+\sqrt{P^2+1}}{2P}$$

times the original value $(\theta_o - T_o)$.

Figure 10 shows curves of Equation (50) for various values of the parameter P, drawn on semi-log cross section paper, as a function of the dimensionless time parameter t/t_c. It will be noted, as stated above, that the curves become practically straight lines after a short time in accordance with Equation (51) and may be so drawn. These straight lines are shown extended to $t=0$, which give the initial temperature elevation, in accordance with Equation (52) which a simple body must have, to produce the same cooling curve as that for the composite body after a short initial cooling time. The initial parts of the curves, before they become straight, are shown separately to a large scale.

Derivation of Equation (51).

When the value of

$$sinh\left[P\left(\frac{t}{t_c}\right) + sinh^{-1} P\right]$$

becomes relatively large, it approaches the value

$$\frac{\epsilon^{\left[P\left(\frac{t}{t_c}\right)+sinh^{-1} P\right]}}{2} = \frac{\epsilon^{P\left(\frac{t}{t_c}\right)}}{2} \times \epsilon^{sinh^{-1} P}$$

This follows from the fact that for any hyperbolic angle x, $sinh\ x = \frac{\epsilon^x - \epsilon^{-x}}{2}$, and when x is large, ϵ^{-x} approaches zero and

$$\left. sinh\ x \right]_{x\ \text{large}} \cong \frac{\epsilon^x}{2}$$

Furthermore, from the same fundamental relation it is readily shown that the following exact relation follows,

$$\epsilon^{sinh^{-1} P} = P + \sqrt{P^2+1} \qquad (53)$$

Substituting these values in Equation (50), we obtain Equation (51).

11(b). HEATING OF A COMPOSITE BODY BY THE MEDIUM.

To find the Temperature of the Body M_2 at any Time t after the Initial Immersion of the Combination of M_1 and M_2 into a Medium at a Higher Temperature than the Initial Temperatures of the Bodies, Assumed Equal.

As shown in the reference paper,[1] the increase in temperature of the body M_2, $(\theta - \theta_o)$, above its initial value, θ_o, at any time, relative to its final change in temperature, $(T_c - \theta_o)$ is

$$\frac{\theta - \theta_o}{T_c - \theta_o} = 1 - \frac{1}{P}\epsilon^{-(\sqrt{P^2+1})\frac{t}{t_c}} \times sinh\left[P\left(\frac{t}{t_c}\right) + sinh^{-1} P\right] \qquad (54)$$

It will be noted that the second part of the right-hand member of this equation is identical with the right-hand member of Equation (50) found for the problem of cooling by the medium. The same curves, therefore, given in Figure 10 apply to the present case also, except that the ordinates correspond to values

$$1 - \frac{\theta - \theta_o}{T_c - \theta_o}$$

An approximate equation corresponding to Equation (51) also applies to this case as follows:

$$\frac{\theta - \theta_o}{T_c - \theta_o} \cong 1 - \frac{P+\sqrt{P^2+1}}{2P}\,\epsilon^{-(\sqrt{P^2+1}-P)\frac{t}{t_c}} \qquad (55)$$

11(c). ADDITION OF HEAT AT A CONSTANT RATE.

To find the Temperature of the Body M_2 in which Heat is Added at a Constant Rate from a Source

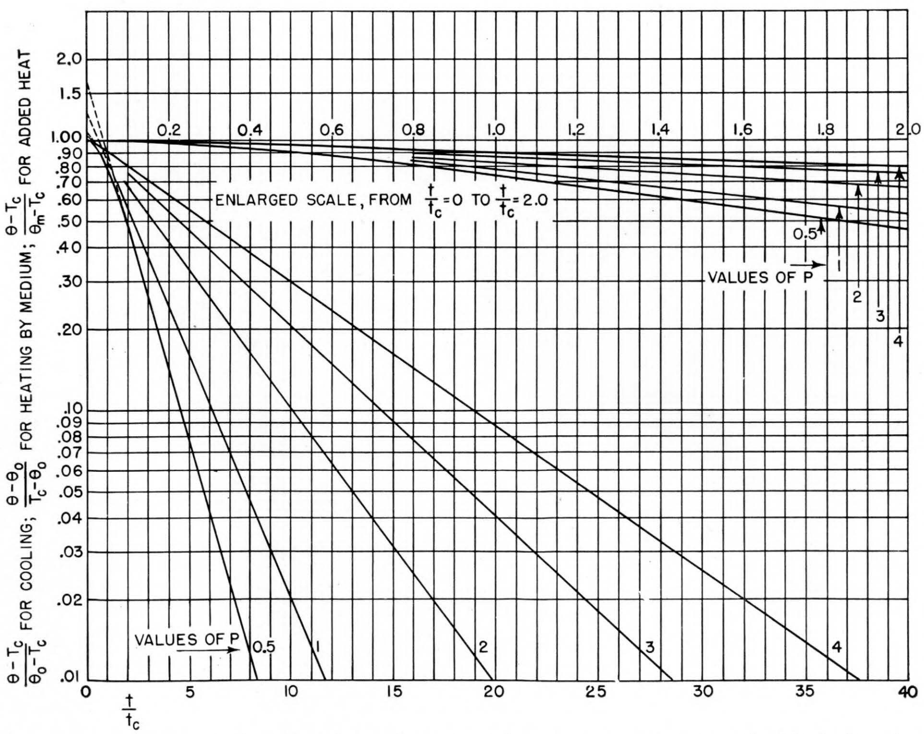

Figure 10—Temperature Elevation above the Surrounding Medium, relative to the Final Elevation, of a Body M_2 dependent for its Heating and Cooling upon the Body M_1 immersed in the Heat Exchanging Medium, at any Time after Cooling or Heating begins. Curves are given for various values of P, and as a function of t/t_c.

Other than the Medium, at any Time after the Initial Application of the Heat, while the Combination M_1 and M_2 are immersed in a Cooling Medium.

This problem was not included in the reference paper[1] as it was not pertinent to the subject. It will therefore, be shown later, that the increase in temperature of M_2, $(\theta - T_c)$ at any time t, relative to the final change in temperature $(\theta_m - T_c)$ is

$$\frac{\theta - T_c}{\theta_m - T_c} = 1 - \frac{1}{P}\epsilon^{-(\sqrt{P^2+1})\frac{t}{t_c}} \times sinh\left[P\left(\frac{t}{t_c}\right) + sinh^{-1} P\right] \quad (56)$$

The approximate equation corresponding to Equation (51) applying to this case is

$$\frac{\theta - T_c}{\theta_m - T_c} \simeq 1 - \frac{P + \sqrt{P^2+1}}{2P}\epsilon^{-(\sqrt{P^2+1}-P)\frac{t}{t_c}} \quad (57)$$

where $(\theta_m - T_c)$ is the final increase in temperature of M_2.

As the right-hand sides of these equations are the same as those of Equations (54) and (55), the same curves shown in Figure 10 can be used also for this case, except that the ordinates give values of

$$\frac{\theta - T_c}{\theta_m - T_c}.$$

Derivation of Equation (56).

The rate W at which heat is generated in M_2 must equal the rate at which heat is absorbed by the body M_2, plus the rate of heat transfer to the body M_1.

The rate at which heat is absorbed by M_2 is $M_2 s_2 (d\theta/dt)$, and the rate that heat is transferred to M_1 is $(\theta - T)h_2$. Then,

$$W = M_2 s_2\left(\frac{d\theta}{dt}\right) + (\theta - T)h_2 \quad (58)$$

Also, the rate at which heat is absorbed by M_1 plus the rate at which heat is transferred from M_1 to the cooling medium, must equal the rate of heat transfer from M_2 to M_1. Thus,

$$(\theta - T)h_2 = M_1 s_1\left(\frac{dT}{dt}\right) + (T - T_c)h_1 \quad (59)$$

These equations may be written

$$\frac{W}{h_2} = \frac{M_2 s_2}{h_2}\left(\frac{d\theta}{dt}\right) + (\theta - T) \quad (60)$$

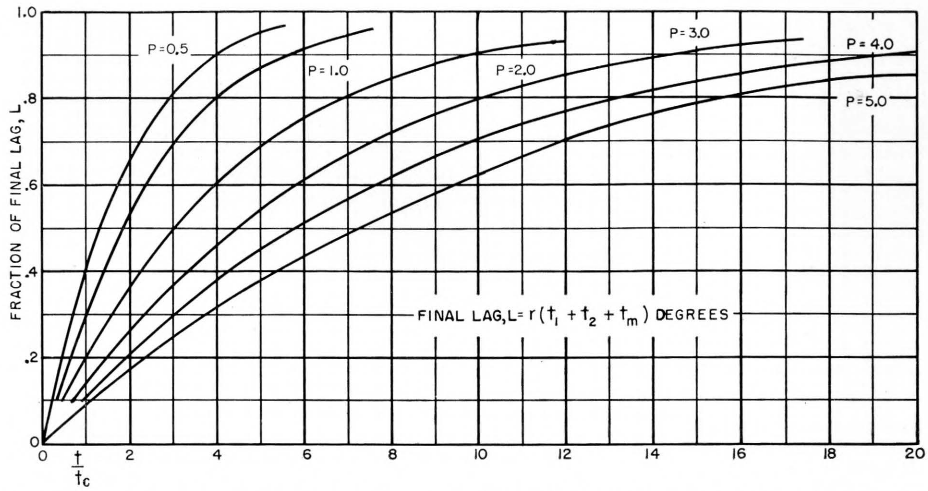

Figure 11—Lag in Temperature at any Time of a Body M_2 Heated or Cooled through a Second Body M_1 in contact with a Medium which changes in Temperature at a Constant Rate. Curves are given for various values of P as functions of t/t_c and of the final value of the Lag.

and

$$(\theta - T)\frac{h_2}{h_1} = \frac{M_1 s_1}{h_1}\left(\frac{dT}{dt}\right) + (T - T_c) \quad (61)$$

Now, $M_1 s_1/h_1$ and $M_2 s_2/h_2$ have the dimensions of time, and for brevity are here designated by t_1 and t_2 respectively, which are time constants. Also let $\sqrt{t_1 t_2} = t_c$.

Substitute these values in Equations (60) and (61) and eliminate dT/dt from Equation (61) by substituting its value found by differentiating Equation (60) with respect to t, we have after simplification

$$\frac{d^2\theta}{dt^2} + \left(\frac{t_1 + t_2 + t_m}{t_1 t_2}\right)\frac{d\theta}{dt} + \frac{\theta - \left(\frac{W}{h_2} + \frac{W}{h_1} + T_c\right)}{t_1 t_2} = 0 \quad (62)$$

where the transfer time constant $t_m = M_2 s_2/h_1$.

This equation is the same as that found in the reference paper[1] for the condition of cooling by the medium, except that the parameter T_c in the latter is replaced by $(W/h_2 + W/h_1 + T_c)$. We may therefore write down immediately the final solution by making this substitution in Equation (50), which affects the left-hand member only, then in Equation (50),

$$\frac{\theta - T_c}{\theta_o - T_c} \text{ becomes } \frac{\theta - \left(\frac{W}{h_2} + \frac{W}{h_1} + T_c\right)}{\theta_o - \left(\frac{W}{h_2} + \frac{W}{h_1} + T_c\right)} \quad (63)$$

It should be remembered that this substitution is not the result of equality, but simply of mathematical similarity, and that having T_c in both quantities is incidental.

Now the initial temperatures of the bodies, θ_o, is the same as the temperature of the medium T_c. Therefore, $\theta_o - T_c = 0$, and the quantity (63) becomes,

$$\frac{\theta - \left(\frac{W}{h_2} + \frac{W}{h_1} + T_c\right)}{\frac{W}{h_2} + \frac{W}{h_1}} = \frac{\theta - T_c}{\frac{W}{h_2} + \frac{W}{h_1}} - 1 \quad (64)$$

but W/h_2 is the final difference in temperature between M_2 and M_1, and W/h_1 is the difference in temperature between M_1 and the medium. Therefore, $W/h_2 + W/h_1$ is the final increase in the temperature of M_2 above that of the medium.

Let this be $(\theta_m - T_c)$, where θ_m is the final temperature of M_2.

Then substituting these values in Equation (50) we obtain Equation (56) as given.

11(d). LAG IN TEMPERATURE CHANGES.

To find the Lag in the Change in Temperature of the Body M_2 when the Medium in which the Combination M_1 and M_2 is Immersed, Changes in Temperature at a Constant Rate.

Let T_a = temperature of the medium at any time t, and r = rate of change in the temperature of the medium in degrees per second. Then as shown in the author's reference paper,[1] the lag in tem-

Reference 1: Response Time and Lag of a Thermometer Element Mounted in a Protecting Case. Trans. A.I.E.E., Page 665, Vol. 64, 1945.

perature at any time, of the body M_2, $(T_a - \theta)$, behind the temperature of the medium, relative to the maximum possible lag L, is

$$\frac{T_a - \theta}{L} = 1 - \frac{1}{2P\sqrt{P^2+1}} \epsilon^{-(\sqrt{P^2+1})\frac{t}{t_c}} \times$$

$$\sinh\left[P\left(\frac{t}{t_c}\right) + \sinh^{-1} 2P\sqrt{P^2+1}\right] \quad (65)$$

$$L = 2rt_c\sqrt{P^2+1} = r(t_1+t_2+t_m) \text{ degrees} \quad (66)$$

which is the lag after an infinite time. For decreasing temperatures, the lag equation is the same as Equation (65) except that $(T_a - \theta)/L$ becomes $(\theta - T_a)/L$. Figure 11 shows curves based upon Equation (65) giving the lag in temperature of M_2 in the composite body, for various values of P relative to the maximum possible lag L.

It will be noted that the maximum possible lag L is simply the product of the rate r by the sum of the three time constants of the composite body.

As referred to in Part I of these articles, the maximum lag in a simple body can be found by assuming that the bodies M_1 and M_2 are reduced to one simple body of mass M_1+M_2, by eliminating all thermal resistance between them, in which case t_2 becomes zero. Then,

$$t_1 + t_m = \frac{M_1 s}{h_1} + \frac{M_2 s}{h_1} = (M_1+M_2)\frac{s}{h_1} = \frac{Ms}{h_1} = t_o$$

where M is the combined mass of the simple body and t_o its time constant. Then Equation (66) becomes, for the simple body

$$L = r(t_1+t_m) = rt_o \quad (67)$$

Composite Bodies Consisting of Three Simple Bodies

The solution of the problem of a composite body consisting of three simple bodies thermally related in a manner similar to the two simple bodies just studied is theoretically possible in a similar manner. However, it involves a third order differential equation containing combinations of six time constants, three of which are independent, and the final equation would become very cumbersome for practical use.

Thermal Resistance:

The concept of thermal resistance is often more useful in solving thermal problems than that of thermal conductivity usually employed. This is especially true when materials of differing conductivities are placed in series, as is the case in the problem of two bodies just discussed.

As a result of this resistance concept, the law of thermal conduction is exactly similar to Ohm's Law in electric conduction through resistors. In the electric circuit, the rate of transfer of electricity in amperes through a resistor is equal to the difference in potential across it divided by its resistance. That is $I = E/R$. In the thermal circuit, the rate of transfer of heat in watts through a thermal resistor is equal to the difference in temperature across it, divided by its thermal resistance, that is

$$W = \frac{T_2 - T_1}{R}$$

where R represents the thermal resistance.

The rate of heat transfer for one degree Centigrade is then $W = 1/R$. In the problems studied in this article, the rate of heat transfer between M_2 and M_1 was given as h_2. Therefore, $1/R = h_2$ and the thermal resistance between them is $R = 1/h_2$.

The unit of thermal resistance is the thermal ohm, and is defined as that thermal resistance which will conduct heat through it at a rate of one watt, when the difference in temperature across it is one degree Centigrade. Thermal resistivity corresponds to electric resistivity, and is the thermal resistance between opposite faces of a centimeter cube.

To give some idea of the magnitudes of thermal resistivities, values in thermal ohms are given for some common materials; copper, 0.26; iron, 1.48; constantan, 4.42; manganin, 4.55; glass, 95.5; paper, 792.

The thermal resistance of the surface contact of a body with the medium is $R = 1/h$. The value of h, for air at room temperatures, for a strip one inch wide, is given on Page 8 of Part III of this series in WESTON ENGINEERING NOTES for August, 1948, as 0.00892 watt per square inch per one degree Centigrade. Then the corresponding thermal contact resistance is $R = 1/0.00892 = 112$ thermal ohms per square inch.

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ARC SUPPRESSION

(Continued from page 4)

Rearranging expression (2) gives:

$$RC = \frac{L}{R} = \text{time constant} \quad (3)$$

so the time constants of the branches are equal. Also, from expression (2):

the total resistance =

$$2R = 2\sqrt{\frac{L}{C}}, \quad (4)$$

by resonant circuit theory and the circuit, L, C and $2R$, is critically damped.

This combination of matched time constants and critical damping means that the impedances of the branches are numerically equal but of opposite reactive sign, therefore, the reactances cancel and the combined circuit is purely resistive at all frequencies. As a pure resistance the circuit cannot develop transients. The presented resistance is R.

The speed of response, however, is still limited by inductance, and can only be improved by increasing R_L to reduce the time constant, which increases R_c and decreases C as design values.

In most cases C will be large and can best be a low-voltage electrolytic capacitor. But if the ideal is inconvenient, any R-C combination will improve the transient situation, provided the time constants of the branches are equal; neglect requirement (1) but observe:

$$R_c C = \frac{L}{R_L} \quad (5)$$

as a concession design.

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