Weston
ENGINEERING NOTES

VOLUME 4 APRIL 1949 NUMBER 2

ELECTRIC TACHOMETERS

SINCE the early development of the electric tachometer generator, described by W. N. Goodwin, Jr., in the WESTON ENGINEERING NOTES, Vol. 4, No. 1, many changes have been made affecting the outward appearance and detailed construction as well as the quality of this device.

Direct Current Tachometers

Although Weston Direct Current Tachometer Generators have been made with emf's higher than six volts at 1000 rpm, this value has been found most satisfactory and is still in use.

This change still further reduced the starting torque and a-c ripple, and resulted in a completely smooth turning rotor, much desired for air-speed measurements.

By magnetic treatment of the permanent magnet field, the emf of the generator is adjusted to an exact value so that the generators will be interchangeable. In addition, the total resistance of the generator is also adjusted to a definite value so that they are completely independent of the instrument circuit and may be interchanged or replaced with no loss of accuracy.

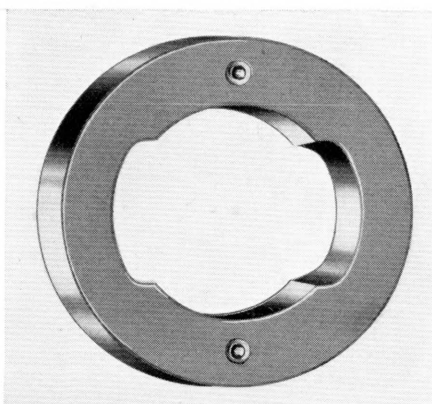

Figure 1—Circular-type magnet with integral pole pieces.

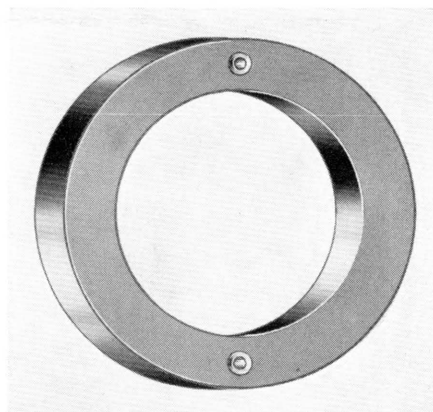

Figure 2—Ring-type magnet.

In the earlier design, the armature was rotated between pole pieces, much like those used for permanent magnet movable coil instruments. This type of construction was replaced by a circular magnet with integral pole pieces, as shown in Figure 1, reducing the number of parts, the physical dimensions, a-c ripple component and starting torque.

Another important design change introduced a ring-type magnet in which the magnetic poles are produced by charging. See Figure 2.

The brush assembly is so constructed that equal and controlled pressure is assured to each brush. Suitable alloy for brushes and commutator provide good contact and long life. While the polarity of the generated emf depends on the direction of rotation of the armature, the brushes are so positioned that its value is the same for either direction of rotation within 0.25 per cent. A generator embodying these design features is shown in Figure 3. Generators of this type have characteristics as listed in Table I.

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John Parker, Editor
E. W. Hoyer, Technical Editor

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TABLE I

Generated emf at 1000 rpm	6 volts ±¼ per cent
Emf linear with speed within	±0.1 per cent
Total resistance of generator	20 ohms ±2 per cent
Permissible current up to	50 milliamperes
Maximum speed recommended	3000 rpm
Adjustable magnetic shunt range	±4 per cent
Effect of stray field at 5 oersteds	2 per cent
Moment of inertia	142 gram centimeters^2
Number of commutator bars	12
Weight approximately	2 pounds
Effect of direction of rotation maximum	±0.25 per cent
Maximum rms value of a-c ripple	2 per cent of d-c
Starting torque less than	1 ounce-inch
Running torque at 1000 rpm	1 ounce-inch
Accuracy	±1 per cent

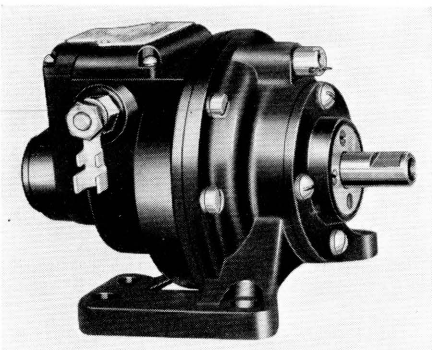


Figure 3—Weston Model 724 Type A Generator

The normal scale of an instrument used with this type of generator has uniform divisions. With suitably designed instruments, the scale may be expanded, contracted or suppressed. Speeds as low as one rpm may be indicated and measured. Figure 4 shows such a developed scale.

In adjusting an indicating instrument to this form of generator the

$$emf = \frac{6 \times rpm}{1000}$$

and the resistance of complete circuit

$$R_T = \frac{emf}{Instrument\ Current}$$

The resistance of the complete circuit includes:

1. The generator resistance.
2. The indicator mechanism resistance.
3. The indicator series resistance (usually self-contained in the indicator).

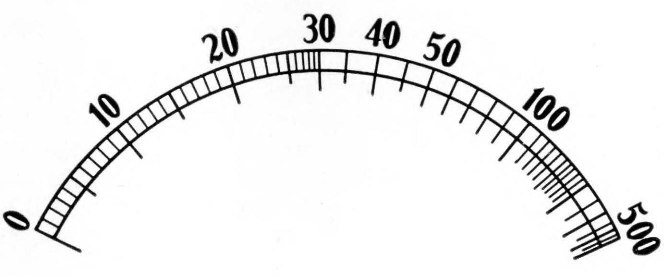


Figure 4—A typical scale showing expanded and contracted divisions.

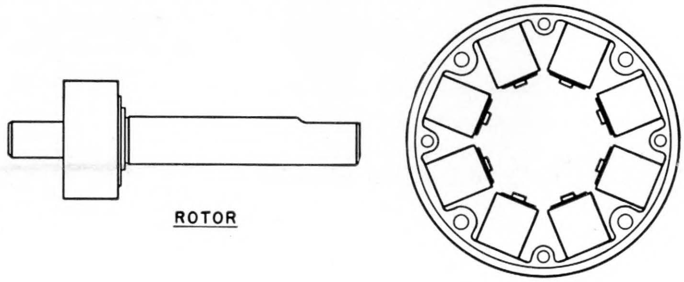
Alternating Current Generators

By transferring the rectification from the generator to the indicating instrument, considerable design change can be effected. This is accomplished by removing the commutator and brushes in the generator and inserting a rectifier into the instrument circuit. It is then possible to revolve a magnet instead of a winding, which contributes considerably to the ruggedness of the generator. Figure 5 shows the essentials of an a-c generator embodying this design.

The rotor consists of a cylinder of high coercive magnetic alloy brazed to a shaft. It is therefore statically and dynamically balanced. The effect of temperature on its magnetic strength is corrected by a means attached to the rotor. Since the magnet is a cylinder, it may be charged without regard to its position.

Figure 6 shows a stator assembly used for charging the magnet. After the magnet is charged, it is treated magnetically for stability and to an exact flux density in the air gap. This results in a stable magnet and an exact emf for a given speed and also controls the reactance of the stator within reasonable limits. The stator consists of a silicon steel laminated structure on which eight coils are mounted. The total resistance of these coils is adjusted to an exact value so that these generators may be interchangeable. For most speed ranges the total resistance of the indicating instrument, including its rectifier, is sufficiently high as to allow the generators and indicators to be interchangeable. Generators of this type have characteristics as listed in Table II. A typical Weston a-c generator is the Model 758 Type A illustrated in Figure 8. The basic types of the Model 758 are Type A, Type J and Type K, each of which has parts affixed to form variations as shown in Figures 8, 9 and 10.

Figure 5—Illustrating the essential parts of an a-c generator.



A typical circuit for the a-c generator and indicator is shown in Figure 7. The generator contains resistance, r_1, and reactance, x_1; the indicator contains the d-c mechanism with rectifier, r_2, and the series resistance, r_3.

Although the generator reactance is small, it is well to take it into account when calculating the instrument adjustment.

If we assume a full scale speed of 1800 rpm with an instrument which requires 0.010 ampere alternating current, the impedance of the complete circuit is

$$Impedance = \frac{emf}{Current} = \frac{18}{0.010} = 1800\ ohms$$

At this frequency and current, the reactance equals 400 ohms. Then the circuit resistance will be

$$R_T = \sqrt{1800^2 - 400^2}$$
$$= 1755\ ohms$$

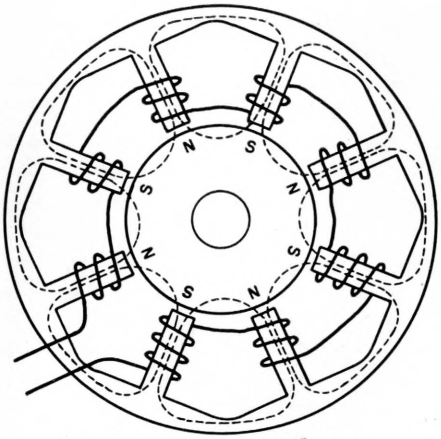


Figure 6—A stator assembly used for charging the magnet.

and 1755 − 100 (generator resistance) = 1655 as the total resistance of the instrument which includes movable coil, rectifier and series resistance.

The importance of the reactance is apparent and since it varies because of frequency and current, it must be taken into account when adjusting or calibrating. A satisfactory method is to adjust and calibrate the instrument directly with the generator. The wave form is substantially sinusoidal, showing about one per cent third harmonic as a maximum.

The choice of using the alternating current or the direct current tachometer system is influenced by the characteristics of the system and the application for which it is desired.

Accuracy: Either generator is rated at one per cent accuracy and will be usually found well within this value. The basic d-c instruments, panel mounting, are also rated at one per cent accuracy. An extra one-half per cent is added to the alternating current system for the rectifier.

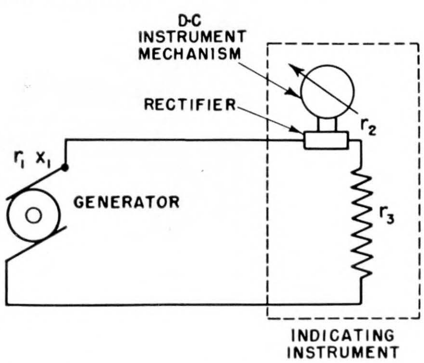


Figure 7—A typical circuit diagram for the a-c generator and indicator.

TABLE II

Generated emf at 1000 rpm	10 volts ± ¼ per cent
Emf linear with speed to 4 per cent up to	5000 rpm
Maximum speed recommended	5000 rpm
Total resistance of generator	100 ohms ± 0.3 per cent
Copper portion of total resistance	35 ohms
Permissible current up to	50 milliamperes
Effect of stray fields	Nil
Moment of inertia	110 gram centimeters^2
Weight	1.5 to 3 pounds
Reactance at 1800 rpm and 10 milliamperes	400 ohms
Starting torque	3 ounce-inches
Running torque at 1000 rpm	3 ounce-inches
Frequency at 1800 rpm	120 cycles/sec.
Accuracy	± 1 per cent

Speed Range: 3000 rpm is recommended as the maximum speed for the d-c generator. Slightly higher speeds may be used but require special treatment of the brushes and commutator. Speeds as low as one rpm can be indicated and measured.

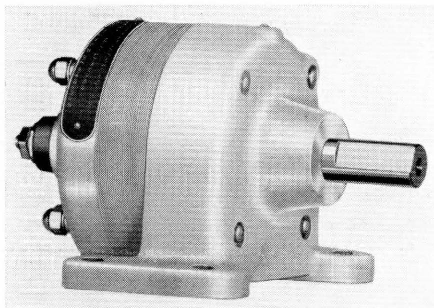


Figure 8—The Weston Model 758 Type A, a-c generator.

The maximum speed of the a-c generator is listed as 5000 rpm, but it may be used up to 25,000 rpm for short intervals. Where these high speeds are to be sustained, suitable lubrication of bearings must be provided. A speed of 500 rpm at full scale deflection of the indicator is recommended as the low limit because the lower frequencies of the alternating current generated would cause fluctuations of the indicator pointer.

Ruggedness: The simplicity of the rotor in the a-c generator with the absence of the brushes and commutator is a very decisive factor where vibration of any form is present.

Torque: The d-c generator is rated at one ounce-inch for starting but may be readily made for a lower amount. The more rugged bearings in the a-c generator bring this requirement up to about three ounce-inches. The running torque will be in about the same proportion.

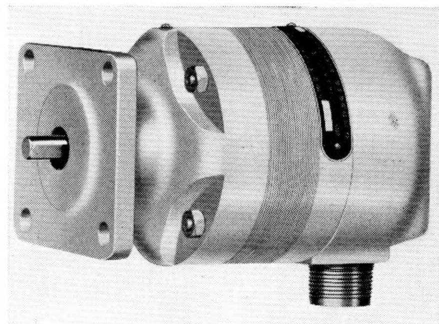


Figure 9—The Weston Model 758 Type K-6—square pad mounting.

Resistance: Because of the low resistance of the d-c generator, 20 ohms, as against 100 ohms plus reactance for the a-c generator, the potential drop in the d-c generator will be much lower.

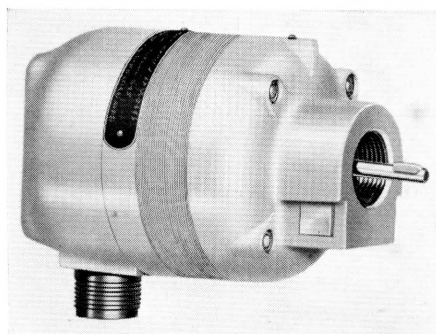


Figure 10—The Weston Model 758 Type J-5—screw type mounting.

Servicing: Except for an application of lubricating oil each 4000 running hours, the a-c generator requires no servicing. The ball-bearing construction of the d-c generator reduces the servicing of this part to about once in two years. It is, however, recommended that the brushes and commutator be inspected and cleaned at least once each year.

E. N.—No. 63 —*A. H. Wolferz*

ORGANIZING AN ELECTRICAL INSTRUMENT STANDARDIZING LABORATORY—PART IV

Power Measurements

THE MEASUREMENT of power is of considerable importance since it is more closely related to cost than to potential and current which are usually concerned with control. While wattmeters are not as numerous as voltmeters and ammeters, they are widely used for testing the efficiency and power requirements of motors and generators and for core-loss measurements in transformers, as well as in power generating stations where they are used to determine the loading and power requirements.

Standardizing one-half and one per cent class wattmeters require one-quarter per cent class electrodynamometer wattmeters as standards. If one-quarter per cent class wattmeters or rotating standards are to be tested, it is necessary to have a one-tenth per cent class Laboratory Standard indicating wattmeter. Such an instrument can be standardized against the basic standards on direct current and used with equal accuracy on alternating current when necessary. The standard indicating wattmeter should have voltage ranges such as 75-150-300 volts and current ranges of 0-2.5 and 0-5 amperes with corresponding wattage scales of 187.5 to 1500 watts.

High-grade current transformers such as one having ranges of 10-20-50-100-200-300-400-600-1200 to 5 amperes should be provided for higher ranges where required. This transformer can be the same one recommended for use with the standard a-c ammeter (Reference W.E.N., Volume 4, No. 1).

Figure 1—Weston Model 310 Single Phase Wattmeter—Accuracy ¼ of 1 per cent.

When it is necesary to test wattmeters on direct current, the polarity of the potential and current should be reversed at each check point and the mean of the reversed readings taken as the true value.

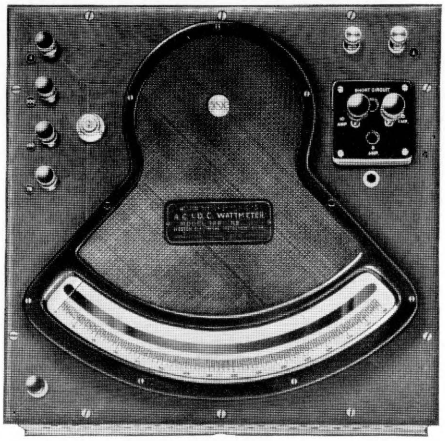


Figure 2—Weston Model 326 Single Phase Wattmeter—Accuracy 1/10 of 1 per cent.

Frequency Measurements

The precise measurement of the frequency of alternating current is not often required. It may, however, be important to such activities as a public utility or a radio manufacturer although in the former case it is low frequency and in the latter case high frequency which are measured with very different apparatus.

Low frequencies ranging from 25 to 1000 cycles may be measured with a single or multiple-range indicating frequency meter within an accuracy of one-half of one per cent. This is adequate for instrument testing and for isolated plants the output of which is not intended for the operation of synchronous clocks. Higher accuracy, perhaps within 0.1 per cent, may be obtained by using the generator or a synchronous motor to actuate a revolution counter. Time is measured by an accurate, easily read timepiece such as a stop watch. Another standard is a temperature-compensated self-oscillating tuning fork which is an accurate source of a single frequency that may be used for comparison purposes.

High frequencies may be measured by comparing them with a standardized variable radio oscillator which may be standardized against a self-contained quartz crystal or against the standard frequencies broadcast by the National Bureau of Standards. Licensed radio broadcasting stations are also an accurate source of frequencies since their frequency must be controlled within 0.002 per cent according to FCC regulations.

Resistance Measurements

Resistance is generally measured on a Wheatstone bridge which is suitable for measuring resistances from approximately 1 to 1,000,000 ohms although the accuracy of measurement at the extremes of this range is quite poor due to contact resistance, leakage, and loss of galvanometer sensitivity. A reliable Wheatstone bridge capable of measurements within 0.1 per cent is recommended. A sensitive spotlight or reflecting galvanometer should be used with it. A higher precision Wheatstone bridge with an ac-

Figure 3—Weston Model 339 Frequency Meter—Accuracy ½ of 1 per cent.

curacy of 0.05 per cent may be procured if the higher cost can be justified.

Resistances less than one ohm should be measured on a Kelvin Double Bridge, which eliminates the resistance of the connections. If there are not many low resistance measurements to be made, a Kelvin Double Bridge may not be justified. Low resistances can be measured by applying a known current through the unknown resistance and measuring the potential drop across it with a potentiometer or a millivoltmeter if allowance is made for the current drawn by the millivoltmeter.

Resistances higher than one megohm may be measured by connecting a microammeter in series with the unknown resistance and applying a known potential across the combination. The resistance is calculated from this data by means of Ohm's Law. The resistance of the microammeter should be subtracted from the total resistance to obtain the value of the resistance under measurement. While this method is moderately accurate, it depends on the accuracy of the voltage source and the milliammeter. Where it is necessary to check high resistances at relatively high operating voltages, for example, series resistors for high voltage plate supply voltmeters, a bridge network may be used very successfully. This method has been described in WESTON ENGINEERING NOTES, Volume 1, No. 6, and reference should be made thereto.

E. N.—No. 64 —*J. B. Dowden*

MEASUREMENT OF TELEVISION SCREEN BRIGHTNESS

THE Weston Photronic* Photocell is often used to measure the screen brightness of television picture tubes, and with a suitable instrument can be quite effective for this purpose. The pulsed nature of the emitted light, however, requires some precautions not necessary in normal photometric measurements to procure maximum accuracy.

If a photocell of usual area is placed directly upon a screen face of large area, the peak intensity will be many times larger than the indicated average intensity as the scanned spot traverses the included portion of the screen area. The screen persistence is not sufficient to provide any degree of effective averaging. The photocell has some measure of unavoidable nonlinearity in its illumination/output characteristic, which is influenced by load resistance, to weight the averaging effect of the photocell and its instrument. This causes a proportional error which will lower the indicated value relative to that obtained from a steady light source, for example, a standardized lamp used for primary calibration.

The effect is demonstrable by placing the photocell against a screen of uniform brightness; the indicated brightness will increase as the cell is brought away from the screen until the sighting angle begins to miss the edges of the screen raster.

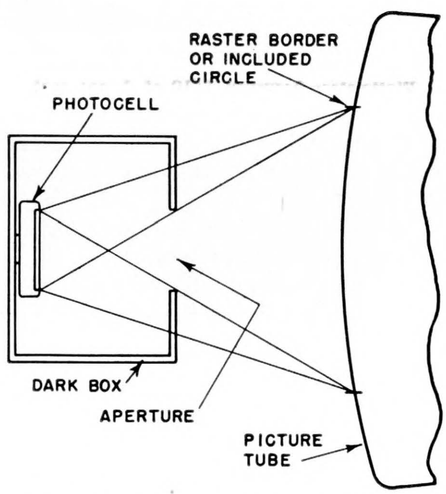


Figure 1—A suggested sighting arrangement with aperture.

To avoid severe pulsing, the cell should be located a distance from the screen, at least the order of the screen diameter, and equipped with baffling or a lens system to limit the sighting angle within the raster frame. When properly baffled, the system responds to true brightness of the included scene. A suggested structure is shown in Figure 1.

The vertical frame scanning will still pulse the cell to a minor degree and a slight further improvement can be obtained by shunting the photocell with a capacitor having an impedance that is low with respect to the meter resistance at the scanning frequency of 60 cps.

The Weston Photographic Exposure Meter is inherently a brightness meter and is equipped with a lens-baffle system to restrict the angle of sight. This is recommended where a small instrument of nominal accuracy is satisfactory. Or, alternatively, a Model 594 Photocell, connected to an appropriate instrument, may be equipped with the lens-baffle used in the Photographic Exposure Meter to avoid the necessity for any external baffle structure.

The Weston Photographic Exposure Meter is calibrated in candles per square foot. If brightness in foot-lamberts is desired, the exposure meter reading may be multiplied by π (3.14).

**PHOTRONIC—A registered trade-mark designating the photoelectric cells and devices manufactured exclusively by Weston Electrical Instrument Corp.*

W. E. N.—No. 65 —*R. W. Gilbert*

Publication Dates Changed

Effective with this issue, *Weston Engineering Notes* will be published every three months. Issues will appear in January, April, July and October.

THERMAL PROBLEMS RELATING TO MEASURING AND CONTROL DEVICES—PART V

The following article is a discussion on the cyclic pulsation in the temperature of a conductor carrying an alternating current, or a full wave rectified alternating current with applications to the resulting oscillation of the pointers of thermal instruments.

Introduction

IN THERMAL instruments, such as thermo-ammeters and milliammeters, having thermocouples connected to heating conductors, or vacuum thermocouples, the indications depend upon the temperature of the conductor resulting from the heat produced in it by the passage of the current. When the current is alternating or rectified alternating current, the rate at which heat is generated in the conductor pulsates in amount during each cycle, and causes a fluctuation in the temperature of the conductor, which in turn results in a corresponding oscillation of the instrument pointer. For frequencies commonly used in practice, the fluctuation is so small that it is negligible. However, in special cases of relatively low frequencies, the temperature pulsation and the resulting pointer oscillation may be appreciable, and it is the object of this study to determine their magnitudes.

12. PULSATION IN TEMPERATURE OF A CONDUCTOR PRODUCED BY AN ALTERNATING CURRENT OR FULL WAVE RECTIFIED ALTERNATING CURRENT.

It is well known from a-c theory, that when a sinusoidal alternating current, or full wave rectified alternating current, is passed through a conductor, the rate at which heat is generated varies from zero to a maximum, and back to zero twice during each cycle. This results from the fact that the rate at which heat is generated at any instant is proportional to the square of the current at that instant.

This is illustrated graphically in Figure 12 in which the a-c wave is shown as a full line, and if it is a rectified wave, the rectified part is shown dotted.

After the current has been applied for a relatively short time, the mean temperature which the conductor assumes is of such a value that the average rates of heat production and dissipation are equal.

The actual temperature then fluctuates above and below the mean temperature twice during each cycle of the applied current. This is illustrated in Figure 12 which shows the a-c wave, the resulting heating wave, and conductor-temperature wave and their phase relations.

It will be shown later that this temperature fluctuation is sinusoidal, having a frequency twice that of the alternating current, and that its value at any time, relative to the mean temperature, is

$$\frac{\theta}{\theta_o}=\frac{1}{\sqrt{1+16\pi^2(t_of_o)^2}}\,sin(4\pi f_ot-\beta_o) \quad (68)$$

where,

θ = the deviation of the conductor temperature from its mean value at any time t.

θ_o = the mean temperature of the conductor.

t_o = time constant of the conductor; seconds.

f_o = frequency of the alternating current; cycles/sec.

h = rate of heat dissipation from conductor; watts/deg. C. per unit length.

$\beta_o = tan^{-1}(4\pi t_o f_o)$ = phase angle of temperature wave.

For a conductor cooled directly by the surrounding medium, as given in Equation 1, Part 1,

$$t_o=\frac{h}{Ams}$$

where,

A = cross-sectional area of conductor; cm^2.

m = the density of the conductor material; gram/cm^3.

s = the heat capacity in joules per gram per degree C.

For a conductor cooled by conduction to the terminals, as will be shown later in this series,

$$t_o=\frac{L^2ms}{\pi^2k}$$

where,

L = length of conductor between terminals.

k = thermal conductivity of conductor material.

From a practical standpoint, it is the maximum value of the temperature fluctuation which is of the most interest, and this occurs when $sin\,(4\pi f_ot-\beta_o)=1$. Then from Equation (68), the maximum temperature deviation relative to the mean temperature is,

$$\frac{\theta_m}{\theta_o}=\frac{1}{\sqrt{1+16\pi^2(t_of_o)^2}}. \quad (69)$$

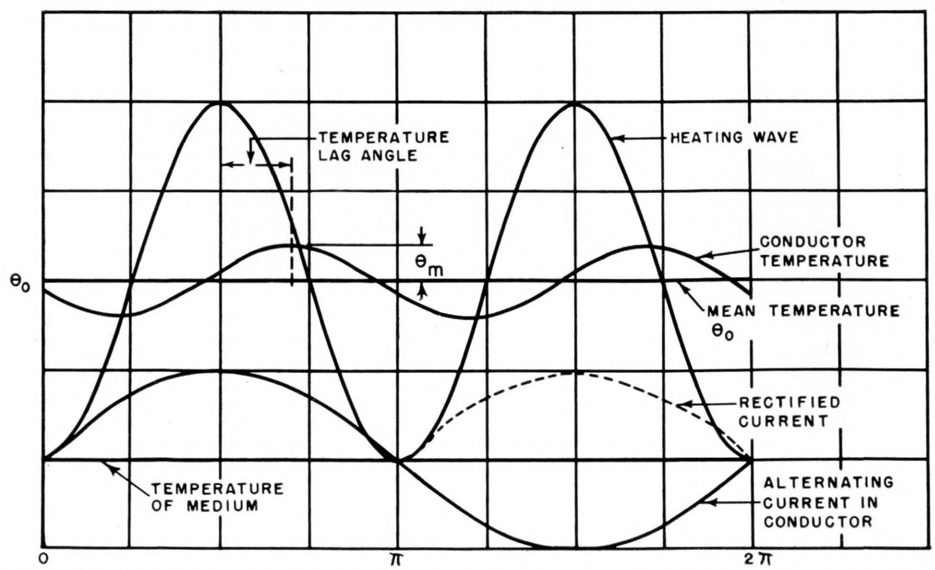


Figure 12—Temperature pulsation in a conductor heated by alternating current or full wave rectified current.

If the product of the time constant and the a-c frequency is equal to or greater than 1, which is usually the case in practice, then the 1 in the denominator of Equation 69 may be neglected with an error of less than 0.4 per cent, and the equation reduces to the following simple form,

$$\frac{\theta}{\theta_o} \simeq \frac{1}{4\pi t_o f_o}. \qquad (70)$$

Example:

Find the maximum deviation in temperature from its mean temperature and relative to it, of the conductor in a vacuum thermocouple having a time constant $t_o = 1.5$ seconds, when carrying an alternating current having a frequency of 3 cycles per second. Then from Equation 70,

$$\frac{\theta}{\theta_o} = \frac{1}{4\pi \times 1.5 \times 3} = 0.0177$$

that is, 1.77 per cent deviation. The phase angle may be of some interest and is computed from Equation 68;

$$\beta_o = tan^{-1}(4\pi t_o f_o) = tan^{-1}(4\pi \times 1.5 \times 3) = tan^{-1}\,56.5$$

or

$$\beta_o = 89 \text{ degrees}$$

that is nearly in quadrature. It will be noted that even if $t_o f_o$ is as low as 1, the phase angle is higher than 85 degrees, so that in most practical cases, the temperature fluctuations are roughly in quadrature with the heating wave.

The following are the approximate time constants of a few types of heating elements, to which that of an ordinary 50 millivolt Weston shunt is given for comparison.

Device	*Time Constant,* t_o
50 mv Weston Manganin Shunt	47.0 Seconds
Weston Thermo-Ammeter Heating Element	0.2 Second
Weston Vacuum Thermocouple, Metal Conductor	1.6 Seconds
Weston Vacuum Thermocouple, Carbon Conductor	0.5 Second*

**This may vary considerably depending upon the character of the carbon.*

13. DERIVATION OF EQUATION 68, FOR TEMPERATURE PULSATION.

From a-c theory we know that the power, and therefore the heat wave, may be considered as consisting of a constant component upon which is superposed a sinusoidal a-c wave of twice the frequency of the alternating current. We shall confine our attention to the a-c component.

In addition to the symbols previously used, let

f = frequency of alternating heat wave, $= 2f_o$.

$\omega = 2\pi f$.

W = crest value of the rate at which heat is generated per unit length of conductor $= I_m^2 r$ watts.

I_m = crest value of the alternating current.

r = resistance of conductor per unit length.

Then, $W/2$ = crest value of the deviation in the rate of heat production from the mean value. The deviation in the rate of any time t then is,

$$\frac{W}{2} \sin \omega t. \qquad (71)$$

The difference between the rate at which heat is exchanged with the surrounding medium at any time and the corresponding rate at the mean temperature is $h\theta$. The rate at which heat is absorbed or released by the material at any time is

$$Asm\left(\frac{d\theta}{dt}\right). \qquad (72)$$

Then the heat generated equals the heat exchanged with the medium plus the heat absorbed or released by the material in changing its temperature, or

$$\frac{W}{2} \sin \omega t = h\theta + Ams\frac{d\theta}{dt}$$

or

$$\frac{W}{2Ams} \sin \omega t = \frac{h\theta}{Ams} + \frac{d\theta}{dt}. \qquad (73)$$

For brevity, let

$$\frac{h}{Ams} = \frac{1}{t_o} = a, \text{ and } \frac{W}{2Ams} = b. \qquad (74)$$

Then Equation (73) becomes

$$\frac{d\theta}{dt} + a\theta = b \sin \omega t. \qquad (75)$$

To integrate this let $d/dt = p$, an operator, which when substituted in Equation (75) gives

$$p\theta + a\theta = b \sin \omega t.$$

Then,

$$\theta = \frac{b \sin \omega t}{p + a}.$$

Operate upon numerator and denominator by $(p - a)$ and expand, then we have

$$\theta = \frac{b(p \sin \omega t - a \sin at)}{p^2 - a^2}. \qquad (76)$$

Performing the operations indicated we have

$$p \sin \omega t = \omega \cos \omega t.$$

Operating again by p gives,

$$p^2 \sin \omega t = -\omega^2 \sin \omega t.$$

therefore,

$$p^2 = -\omega^2.$$

Substitute these values in Equation (76) and then

$$\theta = \frac{b(a \sin \omega t - \omega \cos \omega t)}{\omega^2 + a^2}.$$

This can be simplified by multiplying and dividing by $\sqrt{a^2+\omega^2}$, from which by the usual transformation, we have

$$\theta = \frac{b}{\sqrt{a^2+\omega^2}} \sin (\omega t - \beta) \qquad (77)$$

where,

$$\beta = tan^{-1}\omega/a = tan^{-1} 2\pi f t_o.$$

Substitute the values for a, b and ω and we have

$$\theta = \frac{W/2h}{\sqrt{4\pi^2(t_o f)^2+1}} \sin (2\pi f t - \beta). \quad (78)$$

But $W/2h = \theta_o$, the mean temperature of the conductor. Then Equation (78) becomes

$$\theta = \frac{\theta_o}{\sqrt{4\pi^2(t_o f)^2+1}} \sin (2\pi f t - \beta). \quad (79)$$

This gives the deviation in temperature at any time from the mean temperature, in terms of the frequency f of the heating wave, which is double the frequency of the alternating current.

This equation expressed in terms of the frequency f_o of the alternating current then is

$$\theta = \frac{\theta_o}{\sqrt{1+16\pi^2(t_o f_o)^2}} \sin (4\pi f_o t - \beta_o)$$

where,

$$\beta_o = tan^{-1} 4\pi t_o f_o$$

which is Equation 68.

From these temperature equations we shall be able in the next section of this analysis to determine the resulting oscillations of the pointer of an instrument dependent upon the fluctuating temperature.

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(*Part V to be continued*)

CORRECTION NOTICE

YOUR ATTENTION is called to an error which appeared in the article "Thermal Problems Relating to Measuring and Control Devices—Part IV—Composite Bodies" published in the December, 1948, issue of WESTON ENGINEERING NOTES. Mr. William Kegelman of the Philadelphia Electric Co. called the author's attention to the fact that a term was omitted from Equation 56 and consequently from Equation 57 as a result of an oversight in applying the same boundary conditions to this problem as were applied to a previous problem.

In determining the constants of integration, the initial rate of increase in temperature was assumed zero, which is obviously incorrect as is evident from Equation 60, and also from purely physical reasoning.

To correct the material in this article, two parts of the text and the designation "For Added Heat" in Figure 10 should be deleted and new material substituted as follows:

1. Delete material from and including Equation 56, to "Derivation of Equation 56," and substitute the following:

$$\frac{\theta - T_c}{\theta_m - T_c} = 1 - \frac{1}{P}\epsilon^{-\frac{t}{t_c}\sqrt{P^2+1}} \times \left[\sinh\left(P\frac{t}{t_c} + \sinh^{-1}P\right) - \frac{t_c}{t_2+t_m}\sinh P\frac{t}{t_c}\right] \qquad (56)$$

The corresponding approximate equation applying to this case is,

$$\frac{\theta - T_c}{\theta_m - T_c} \simeq 1 - \frac{(P+\sqrt{P^2+1}) - \frac{t_c}{t_2+t_m}}{2P}\epsilon^{-\frac{t}{t_c}\left(\sqrt{P^2+1}-P\right)} \qquad (57)$$

where $(\theta_m - T_c)$ is the final increase in temperature of M_2.

2. Delete material beginning "This equation is the same, etc.," following Equation 62, and ending at section 11(d), and substitute the following:

In Equation 62, $W/h_2 + W/h_1$ is the final difference in temperature between M_2 and the medium. Therefore, $W/h_2 + W/h_1 + T_c = \theta_m$, and $(\theta - \theta_m)$ may be substituted for the numerator of the last term in Equation 62.

This equation was integrated in the writer's reference paper,[1] and after putting the time constants in terms of P, the solution is

$$\theta - \theta_m = \epsilon^{-\frac{t}{t_c}\sqrt{P^2+1}} \times \left[a\epsilon^{P\frac{t}{t_c}} + b\epsilon^{-P\frac{t}{t_c}}\right]$$

Now, since fundamentally,

$$\epsilon^{P\frac{t}{t_c}} = \cosh P\, t/t_c + \sinh P\, t/t_c$$

and, $\epsilon^{-P\frac{t}{t_c}} = \cosh P\, t/t_c - \sinh P\, t/t_c$

we have

$$\theta - \theta_m = \epsilon^{-\frac{t}{t_c}\sqrt{P^2+1}} \times \left[A \cosh P\, t/t_c + B \sinh P\, t/t_c\right]$$

where $A = a + b$, and $B = a - b$ are constants to be determined.

When $t = 0$, $\theta = T_c = T$; then from Equation 60, at $t = 0$, we have

$$\frac{d\theta}{dt} = \frac{W}{h_2 t_2}$$

To express this in terms of the maximum change in temperature $\theta_m - T_c$, multiply W/h_2t_2 by $\theta_m - T_c$ and divide it by $(W/h_1 + W/h_2)$, since they are equal, and also express time in terms of t_c, and we have

$$\left.\frac{d\theta}{d\left(\frac{t}{t_c}\right)}\right]_{t=0} = \frac{\frac{W}{h_2 t_2}(\theta_m - T_c)t_c}{W/h_1 + W/h_2}$$

Divide numerator and denominator by W, and multiply them by $m_2 s_2$ and we have

$$\left.\frac{d\theta}{d(t/t_c)}\right]_{t=0} = \frac{\frac{m_2 s_2}{h_2 t_2}(\theta_m - T_c)t_c}{\frac{m_2 s_2}{h_1} + \frac{m_2 s_2}{h_2}} = \frac{(\theta_m - T_c)t_c}{t_2 + t_m}$$

When these boundary conditions are substituted in the equation for $\theta - \theta_m$, and in its derivative after differentiating it, and are solved for A and B we have

$$A = -(\theta_m - T_c)$$

and

$$B = (\theta_m - T_c)\left[\frac{t_c}{t_2 + t_m} - \sqrt{P^2+1}\right]$$

which when substituted in the equation for $\theta - \theta_m$ yields

$$\theta - \theta_m = -\frac{\theta_m - T_c}{P}\left[\left(P \cosh P\frac{t}{t_c} + \sqrt{P^2+1}\sinh P\frac{t}{t_c}\right) - \frac{t_c}{t_2+t_m}\sinh P\frac{t}{t_c}\right]$$

Changing this in the usual manner to a function of $\sinh P\, t/t_c$ and use the identity $\theta - \theta_m = \theta - T_c - (\theta_m - T_c)$, we have

$$\frac{\theta - T_c}{\theta_m - T_c} = 1 - \frac{1}{P}\epsilon^{-\frac{t}{t_c}\sqrt{P^2+1}} \times \left[\sinh\left(P\frac{t}{t_c} + \sinh^{-1}P\right) - \frac{t_c}{t_2+t_m}\sinh P\frac{t}{t_c}\right]$$

which is Equation (56).

It will be noted from this equation that the temperature-time response curve of the composite body, when heat is added directly to the inner unit, is equal to that when heat is added from the surrounding medium given in Equation 54 plus a term which is a function of time and of the rate at which heat is added.

E. N.—No. 67 —*W. N. Goodwin, Jr.*

[1]*Response Time and Lag of a Thermometer Element Mounted in a Protecting Case. Trans. A.I.E.E., Page 665, Vol. 64, 1945.*