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NOTES ON DAMPED MOTION AS TYPIFIED BY THE MOVING SYSTEM OF AN ELECTRICAL MEASURING INSTRUMENT

IN THE design of electrical measuring instruments where a moving system carrying a pointer gives the final indication, there are frequently requirements to produce the final deflection very rapidly or very slowly. High speed of action is sometimes required for rapidly changing phenomena, even to the point where the motion is faster than the eye can follow and photographs must be taken. The electromagnetic oscillograph is perhaps the limiting case in this regard. Conversely, there are many applications where it is required that the instrument integrate minor pulsations of certain specified frequency, such as where pulsating currents at a few cycles per second are used in signaling or, as another example, indicating the average value of the current in the pulsations from a dial telephone making a call. Further, varying degrees of damping may be required from the critically damped variety to those which may be heavily damped or, conversely, sometimes underdamped systems are best for the problem at hand.

The motion of a moving system which is spring controlled, that is, where the control torque is proportional to the angular deflection, and also where the damping associated with the system is proportional to the velocity of the motion, was first represented in the form of a differential equation by Gauss and Weber, the equation taking the form:

$$K\frac{d^2\theta}{dt^2}+G\frac{d\theta}{dt}+S\theta=0$$

where,

θ = angular deflection from the initial position, radians,

t = time from the initiation of motion, seconds,

K = moment of inertia of the system around the axis, *gm cm*²,

G = the damping coefficient, *gm cm* per radian per second,

S = spring or control constant, *gm cm* per radian.

The solution of this equation for the position of an instrument pointer at any instant of time subsequent to its start involves the use of the constants K, G and S. Classical solutions exist but the equations are quite involved.

About 1906, Mr. W. N. Goodwin, Jr., of the Weston Electrical Instrument Corporation worked out a simplified solution of this equation in view of the fact that when such a system is critically damped, that is, when its character of motion lies at the border line between just oscillatory and just overdamped, the following relation holds,

$$G_o=2\sqrt{KS}$$

where G_o is the damping coefficient for critical damping.

Mr. Goodwin discovered that if the ratio of the actual damping coefficient G to the critical damping coefficient G_o, which he designated as the specific damping coefficient, and gave it the symbol "n," is introduced into the equation as one parameter, and the undamped period as the second parameter, then the equation of motion could be reduced to a simple dimensionless form based upon two parameters.

Expressed mathematically, the value of the damping coefficient for any damping is

$$G = nG_o = 2n\sqrt{KS}$$

The use of this term "n" has materially simplified the mathematical analysis of damped moving systems as used in instruments since the parameter "n" can be derived from the common factors quite evident from a casual inspection of an instrument.

We can start with the damping factor, which is the ratio of two successive swings in opposite directions, of the moving system, or it may be considered as the reciprocal of the per cent overshoot when an impulse is applied to the instrument. The logarithmic decrement of the system, a term used quite widely in the damping of electrical oscillations, and an entirely analogous constant, is the natural logarithm of the damping factor or,

$$\delta = \log\ k$$

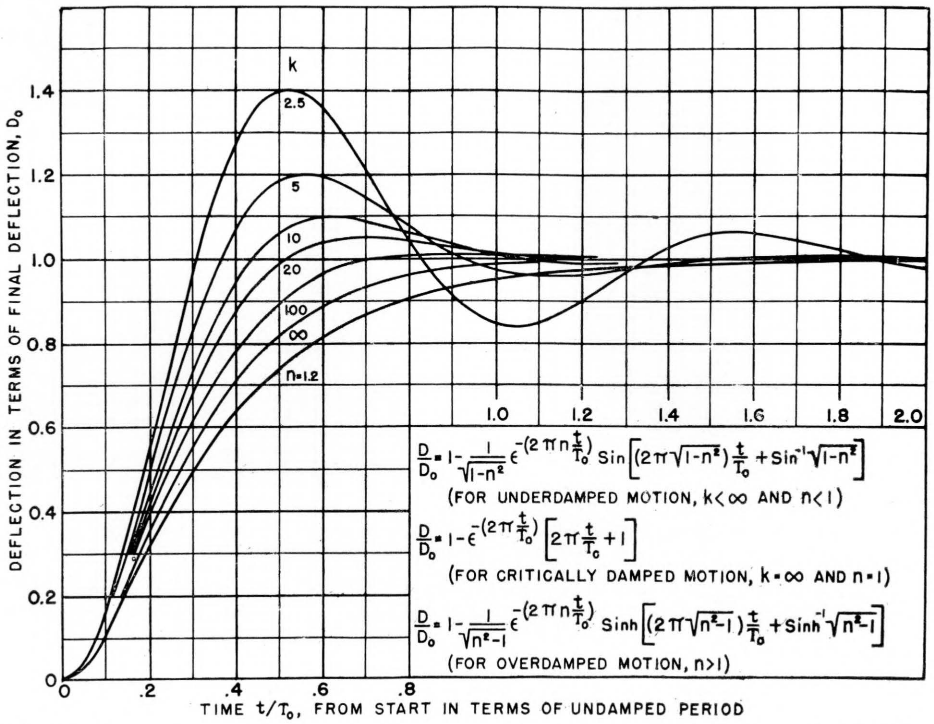
where k = damping factor

Then, for any damping less than critical, we can express "n" as

$$n = \frac{1}{\sqrt{\frac{\pi^2}{\delta^2}+1}}$$

Probably the best picture of the operation of the moving system of an instrument is obtained by plotting the pointer position with time, starting from the time a continuous unvarying torque is applied, as when a voltmeter is connected to a battery. Such a plot is shown in the figure, where, however, time, t/T_o, is in terms of the *undamped period* of the instrument, T_o, and pointer position, D/D_o, is in terms of the final steady state deflection, D_o.

To plot these curves, the differential equation above was solved by Mr. Goodwin to give values of D/D_o in terms of t/T_o and "n," the use of the latter term materially simplifying the procedure. The several solutions of the equation are given in the figure for the case of systems damped to a degree less than critical, for the critically damped condition, and where the system is overdamped. These equations were published in the *Transactions of the American Institute of Electrical Engineers,* Volume 59 for 1940, page 189, in a discussion by Mr. Goodwin of the general problem of instrument response.



Curves of damped periodic motion of an instrument pointer for any value of undamped period, T_o.

In the plot, curves are given for several different degrees of damping; corresponding values being $n = 0.28$ for $k = 2.5$, where the overshoot is 40 per cent, and for the well-damped condition $n = 0.826$ where $k = 100$, an overshoot of only one per cent. It will be noted that for the critically damped case, $n = 1$, the long equation is much simplified. In the overdamped condition, where "n" is greater than one, the equation is transformed to eliminate imaginary values through the use of hyperbolic functions.

Examination of the curves will reveal a number of rather interesting factors. It will be noted that the overdamped condition causes a very considerable time lag in the instrument response. As a matter of fact, even critical damping makes a given instrument somewhat slow in coming to its final value. Actually, an overshoot of very approximately five per cent appears to give the fastest reading, depending somewhat upon the final accuracy desired and the kind of phenomena involved. The psychological aspect is also important in that too much overshoot causes a wandering of the focus of the eyes, whereas, a moderate amount of overshoot does not appear to bother an observer. Another interesting factor is the velocity of the pointer, and it will be noted that in the overdamped condition the pointer velocity changes quite rapidly, whereas if some overshoot is allowed, the velocity of the pointer over the major part of its excursion is approximately constant. Thus, some overshoot is generally deemed desirable.

It is noted that the curves shown are all in terms of the undamped period of the instrument. Most instruments seem to work out with an adequate degree of control torque and with a period from one-half second to perhaps one second. If attempts are made to slow the instrument down by increasing the moment of inertia of the moving system, one finds that after proceeding about so far the actual weight of the moving system is such as to preclude its use on the basis that the friction developed in the bearings will give unusual errors. There is thus an upper limit to the increase of the moment of inertia unless the radius of the pointer can be increased, which means essentially making a larger instrument and one which is inherently slower.

Going in the other direction and reducing the period, instruments can, again, be worked on to a certain degree. But with a fixed magnetic system it will be found that if the period is reduced by increasing the spring control, the power required in the moving coil goes up as the fourth power of the reciprocal of the undamped period. In other words, to speed up an instrument is very costly in terms of energy associated with the instrument, even to the point where ultra-high-speed instruments may suffer from overheating of the actuating system. In practice about the fastest instrument mechanism has an undamped period of the order of one-tenth of a second and for higher speeds it is usually necessary to dispense with the pointer and use a mirror as in the now somewhat outmoded moving coil oscillograph.

But within these limits, and given sufficient energy and sufficient flexibility in other factors, it is quite possible to design for a specific requirement in terms of speed and damping although it must be appreciated that so many factors are involved in the way of wire sizes, available coil frames, and the like, that some compromises usually are required.

E. N.—No. 71 —*John H. Miller*

ORGANIZING AN ELECTRICAL INSTRUMENT STANDARDIZING LABORATORY—PART V

Power Sources and Control Equipment

THE CIRCUITS for instrument testing must be steady so that the indications of the standard and instruments under test will not fluctuate and therefore they may be accurately observed. The standards and the test instruments often have widely different periods and response so that an accurate reading on an unsteady circuit is not simply a matter of reading the pointers as they move across the calibrated marks. Circuit variations may originate either in the source of supply or in the control equipment which may be either unsuitable or badly worn.

Direct Currents

A direct current voltage supply may be obtained from storage batteries or a rectifier power supply. The storage battery provides a steady unidirectional current which is beyond question as to its characteristics. The battery voltage should be 170 volts with several taps for lower voltages and possibly additional cells for voltages up to 330, 550 or 800 volts. A rectifier power supply should be suitable for all purposes, however, and it provides an output which can be varied by internal control from a low value to the maximum voltage required. The initial cost and maintenance are much lower than a storage battery, especially when high voltage is required. Such a source should be capable of supplying at least 50 milliamperes and preferably currents up to 500 milliamperes if dynamometer voltmeters are to be standardized against direct current standards. The ripple should not exceed three per cent, although a greater amount may not be harmful. The output voltage must be steady, which requires some built-in means of regulation or else it should be operated from a constant voltage stabilizing transformer which will supply a steady a-c voltage to the power supply.

Direct current for testing ammeters and milliammeters is most conveniently obtained from a storage battery. If the current required is not over 25 amperes, six-volt automobile storage batteries are easily obtained and satisfactory. For larger currents, battery manufacturers supply 300-ampere-hour storage cells. A two-volt cell is usually adequate but it may be necessary to use two cells in series if the circuit resistance is high. A charging means must be provided for the battery.

Regulation of d-c potential is easily accomplished by using tubular slidewire rheostats as illustrated in Figure 1. A good combination is a 900-ohm, a 600-ohm, and a 15-ohm, 16-inch-long rheostat connected in potentiometer arrangement across the supply line. This potentiometer rheostat draws about 170 milliamperes from the supply at 170 volts in addition to the current taken by the standard and the instrument under test. This may be too much for a small power supply. In such a case, the rheostats should be connected in series with the source and the control knob on the power supply used for the coarse adjustment and the rheostats for the fine adjustment. A switch in the source should always be opened before disconnecting anything in the circuit to avoid overloading the standard or instruments under test.

The regulation of direct current is somewhat difficult because of the wide range of the current. Carbon plate compression rheostats, such as shown in Figure 2, are almost universally used. Sometimes the carbon rheostat is shunted by a sliding wire

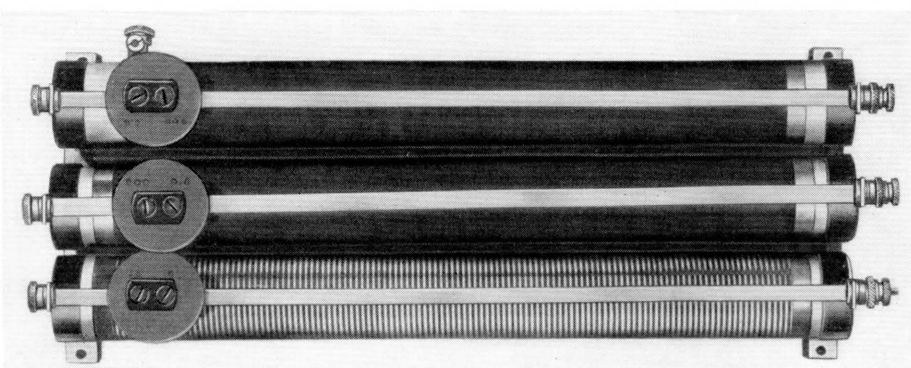


Figure 1—Slidewire Rheostats of the tubular type shown above are used in the Weston Standardizing Laboratories.

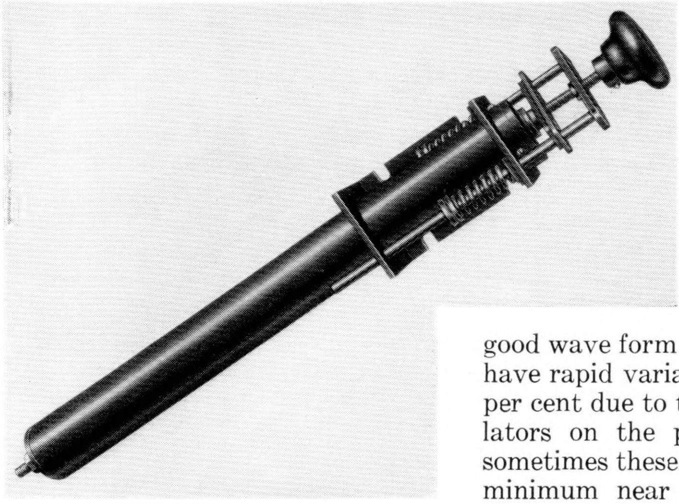


Figure 2—Carbon Plate Compression Rheostat for d-c current regulation.

tubular resistor or a switch controlled fixed resistor. The arrangement most satisfactory will have to be worked out in each laboratory. It is advisable to have on hand an assortment of adjustable tubular rheostats and small size carbon rheostats for use as requirements arise.

Alternating Current

Alternating current for checking instruments is a big problem since it must be steady and also have a good wave form. Public utility lines have rapid variations of one or two per cent due to the automatic regulators on the power system, but sometimes these variations are at a minimum near some of the load centers and such a location would provide a satisfactory and steady source.

A motor generator is very useful if the laboratory is large enough to justify the expense. Such a generator should be driven by a synchronous motor in order to maintain the frequency of the power supply. There should be an exciter for the fields and two generators on the same shaft are preferable and should be arranged so that the stator of one generator may be rotated to change its phase relation with respect to the other generator. This is very useful when testing wattmeters. The generators should preferably be wound for three-phase operation. The generators must be required to have a sine wave with not over three per cent distortion.

There are a number of automatic a-c voltage regulators on the market. While they hold the voltage very steady, almost all of them distort the wave form badly. Some of them are constructed to have a small amount of distortion by installing special harmonic filters in them. If a good wave form is claimed, it should be proven by a wave analyzer on each regulator with various output loads.

A-c potential circuits should be regulated with variable auto-transformers such as Variacs, Transtats, etc., as they do not distort the wave form except at the beginning of the winding. Alternating current circuits are regulated by the same type of apparatus using a step down transformer, however, to secure larger currents.

E. N.—No. 72 —*J. B. Dowden*

THERMAL PROBLEMS RELATING TO MEASURING AND CONTROL DEVICES—PART VI. DISTRIBUTION OF TEMPERATURE ALONG CURRENT CARRYING CONDUCTORS

DISTRIBUTION OF TEMPERATURE ALONG A CONDUCTOR OF ANY SHAPE CONNECTED BETWEEN TERMINALS AND HEATED BY A CURRENT PASSING THROUGH IT.

Introduction

IN THE year 1906, while engaged in the design of shunts, the author discovered that the temperature distribution along a uniform thermally short conductor connected between terminals was parabolic and was independent of the linear dimensions of the conductor, but depended solely upon the difference of potential along the conductor, and its electrical and thermal resistivities. It was found that the difference in temperature between the center of the conductor and the terminal could be expressed by the following simple relation

$$\theta_o = V^2/(8k\rho)$$

where θ_o is the difference in temperature between the center of the conductor and the terminals.

V = difference of potential between terminals in volts.

k = thermal conductivity; watts, cm., deg. C.

ρ = electrical resistivity; ohms per cm. cube.

This result was referred to in a patent(1) and in a paper(2) by the author.

Since this equation contains none of the linear dimensions of the conductor, it was surmised that it applied to a conductor of any shape and not solely to one of uniform cross-section, and, furthermore, by the use of proper co-ordinates, the temperature distribution along the conductor might also be independent of the shape of the conductor. The correctness of these assumptions was subsequently proved rigorously by the author, and these interesting facts and their results form the basis for the subject matter which follows:

17. THE GENERAL CONDUCTOR.

It will be shown in general that if the potential difference, v, between any point on the conductor carrying a direct current or low frequency alternating current, and the reference terminal is made the independent variable instead of the linear distance, then the temperature distribution along the conductor, and the corresponding equations, are the same for conductors of any shape including those with uniform cross-section. In this analysis, it is assumed (a) that the temperatures have become constant and (b) that the heat dissipated by radiation and convection is negligible relative to that conducted to the terminals. For practical purposes, however, a convection loss of as much as 10 per cent will not seriously affect the results. Let Figure 14 show a conductor of any shape connected between terminals, and for greater generality let the terminal have different temperatures, T_1 and T_2.

Then, as will be proved later, the difference in temperature between any point on the conductor and the reference terminal T_1, for a constant voltage between the terminals, is

$$\theta = \frac{V^2}{8k\rho}\left[\frac{4v}{V}\left(1-\frac{v}{V}\right)\right] + \frac{v}{V}(T_2 - T_1) \qquad (99)$$

Where:

θ = difference in temperature between any point on the conductor and the terminal T_1.

v = potential difference between the point on the conductor and the terminal T_1; volts.

T_1 and T_2 = the temperatures of the terminals.

k = thermal conductivity of the conductor, considered constant; watts, cm., deg. C.

ρ = electrical resistivity of the conductor, considered constant; ohms, cm³.

It will be noted that the first member of the right-hand side of this equation is the equation of a parabola; and the second member is the equation of a straight line which is simply the temperature gradient in terms of difference of potential, which would exist in the conductor resulting from the total fall in temperature, $T_2 - T_1$, along it, if no current were passing to heat the conductor.

The actual distribution of temperature is therefore represented by a parabola superposed upon a straight line, when using temperature and difference of potential as the co-ordinates, just as was found in the reference papers[(1) (2)] for a uniform conductor, using temperature and linear distance as co-ordinates.

These conditions are shown graphically in Figure 14. In the left-hand diagram, the line T_1g corresponds to the temperature gradient $(1/V)\ (T_2 - T_1)$. Superposed upon this line is the curve T_1adg, resulting from the heating by the current, the ordinates of which, ab, de, and so forth, are from Equation 99, $(V^2/8k\rho)\ [(4v/V)\ (1 - v/V)]$ drawn at equal increments of v used as the independent variable. In this diagram, equal increments of distance correspond to equal increments of voltage. Along the conductor, however, equal increments of voltage are not in general spaced uniformly.

To find the actual linear distribution of temperature at any point along the conductor, the temperature ordinates for various values of v in the left-hand diagram, as cb, ca, fe, fd and so forth, are projected across to the vertical lines c_1b_1, c_1a_1, f_1e_1, f_1d_1 and so forth, drawn at distances from T_1 along the conductor corresponding to the same values of v. The location of these actual increments of v on the conductor can be determined by a millivoltmeter or other electrical means, unless the shape is such that they can be determined analytically. A curve drawn through the intersection of the projected and vertical lines gives the actual distribution of temperature along the conductor. It may be stated simply, therefore, that the temperature along conductors connected between terminals is the same for all conductors regardless of their shape, at the same relative *"electrical distances"* from a reference terminal for the same terminal temperatures.

17(a). TEMPERATURE DISTRIBUTION ALONG A GENERAL CONDUCTOR WHEN TERMINAL TEMPERATURES ARE EQUAL.

In Equation 99, if T_1 and T_2 are equal, the equation becomes

$$\theta = \frac{V^2}{8k\rho}\left[\frac{4v}{V}\left(1-\frac{v}{V}\right)\right] \qquad (100)$$

This is shown graphically in Figure 15. In this case the left-hand curve is a true parabola using the co-ordinates θ and v. When this curve is transformed into one showing the actual linear distribution along the conductor, by projecting the temperatures across to corresponding voltages, we obtain the deformed curve shown in the right-hand diagram.

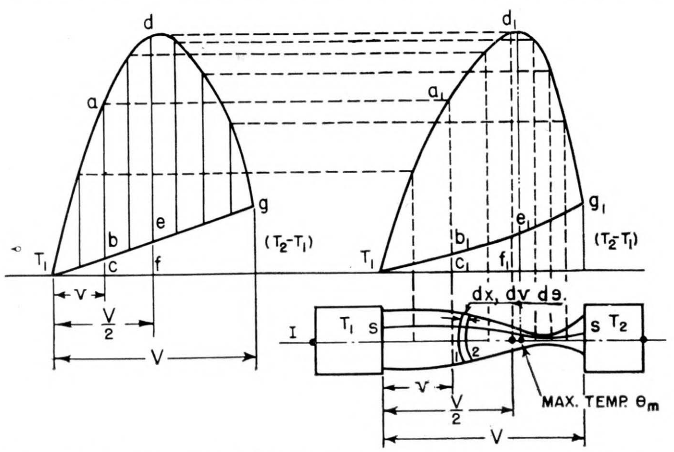


Figure 14—Steady state temperature distribution along a conductor of any shape connected between terminals having *unequal temperatures*, when the conductor is heated by an electric current passing through it, and all the heat generated is dissipated by conduction to the terminals.

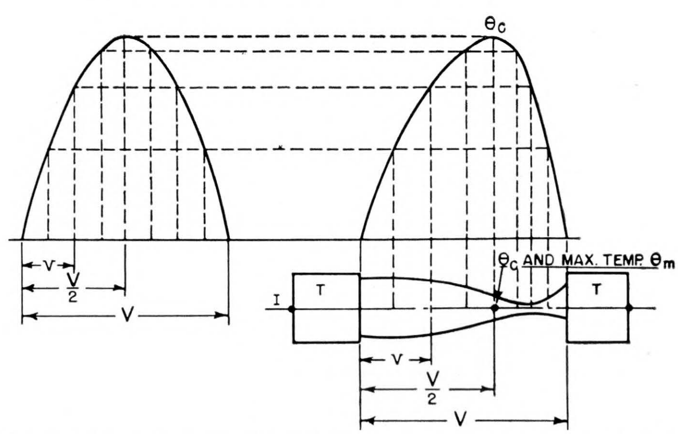


Figure 15—Steady state temperature distribution along a conductor of any shape connected between terminals having *equal temperatures*, when the conductor is heated by an electric current passing through it, and all the heat generated is dissipated by conduction to the terminals.

17(b). TEMPERATURE ELEVATION AT THE ELECTRICAL CENTER OF THE GENERAL CONDUCTOR.

At the electrical center $v = V/2$. When this value is substituted in Equation 99 we obtain

$$\theta_c = \frac{V^2}{8k\rho} + \frac{1}{2}(T_2 - T_1) \qquad (101)$$

and if the terminals have the same temperature then

$$\theta_c = \theta_o = \frac{V^2}{8k\rho} \qquad (102)$$

where θ_c = temperature elevation at the electrical center. Both of these equations are identical with those given in the reference papers(1)(2) for conductors of uniform cross-section in which the geometrical and electrical centers are the same.

17(c). LOCATION OF THE MAXIMUM TEMPERATURE ALONG THE GENERAL CONDUCTOR.

To find the location of the maximum temperature along the conductor, differentiate Equation 99 with respect to v and equate the result to zero, and we find that the maximum value of θ is located at the point on the conductor where

$$\frac{v}{V} = \frac{1}{2} + \frac{T_2 - T_1}{8\theta_o} \qquad (103)$$

in which $\theta_o = V^2/8k\rho$.

This shows that the maximum temperature elevation is located at an *"electrical distance"* $V(T_1 - T_2)/8\theta_o$ beyond the electrical center where $v = V/2$.

If the terminals have the same temperature, that is $T_2 - T_1 = 0$, then the maximum temperature, from Equation 103, is located at $v/V = 1/2$, which is the electrical center.

17(d). VALUE OF MAXIMUM TEMPERATURE ELEVATION.

The value of the maximum temperature elevation θ_m above that of the terminal T_1 is determined by substituting the value of v/V from Equation 103 in Equation 99, and we have after simplification, the maximum temperature

$$\theta_m = \frac{V^2}{8k\rho}\left[1 + \frac{T_2 - T_1}{4\theta_o}\right]^2 \qquad (104)$$

where,
$\theta_o = V^2/8k\rho$.

Example:

If $T_2 - T_1 = 10$ *deg.*, and $\theta_o = 33$ *deg.* above the temperature of the terminal T_1, then from Equation 103 the maximum temperature elevation is located at $v = (1/2 + 10/264)V = 0.538V$. That is at a point where the difference of potential between the point and terminal T_1 is 0.538 times the voltage between terminals, or $0.038\ V$ beyond the electrical center, where it would be located if $T_1 = T_2$.

The maximum temperature elevation in this example is from Equation 104

$$\theta_m = \theta_o\left[1 + 10/(4 \times 33)\right]^2 = 1.158\ \theta_o$$

or 1.158 times the elevation which would result if $T_2 = T_1$.

18. EXPERIMENTAL METHODS OF DETERMINING THE LOCATION OF THE MAXIMUM TEMPERATURE ELEVATION ON THE CONDUCTOR.

In special cases where the conductor has some simple geometrical shape, the temperature distribution and the location of the maximum temperature can be determined analytically. In general, however, the shapes are too complicated to be so treated.

The principles given, however, suggest two very simple methods of making this determination experimentally. (a) By means of a voltmeter or millivoltmeter determine the point on the conductor carrying a suitable constant current, between which and a terminal the difference of potential is one-half (1/2) of the total voltage drop between the terminals. The position thus found will be the location of the maximum temperature, provided the terminals have the same temperature and, as shown previously, very close to it even if they have slightly unequal temperatures. (b) Connect a potentiometer, for example a slidewire type, to the terminals in parallel with the conductor. With a suitable current passing through the conductor, which need not be constant, set the potentiometer to its mid position, and by moving the galvanometer lead terminal along the conductor, a point will be found at which the potentiometer is balanced. This point is the location of the maximum temperature.

The temperature distribution may be found by setting the potentiometer for equal voltage increments and finding the corresponding points on the conductor, knowing for these points that the distribution is parabolic. The equal voltage increments on the potentiometer slidewire correspond to the equal increments on the left-hand diagram in Figures 14 and 15, whereas the equal voltages, but unequal distance along the conductor, correspond to the right-hand diagrams in these figures.

19. CONDUCTORS HAVING UNIFORM CROSS-SECTIONS.

In this special case of the general conductor, the difference of potential along the conductor is directly proportional to the distance x from the terminal T_1. The total potential difference between terminals is therefore proportional to the total length of the conductor L, from which it follows that $v/V = x/L$. If this is substituted in Equation 99 we have

$$\theta = \frac{V^2}{8k\rho}\left[4\frac{x}{L}\left(1 - \frac{x}{L}\right)\right] + \frac{x}{L}(T_2 - T_1) \qquad (105)$$

which is equivalent to that given in the reference paper(2) for uniform conductors. Likewise all other equations given for the general conductor apply to the uniform conductor, using linear distances instead of *"electrical distances"* if v/V is replaced by x/L.

Example, Circular Disc Conductor:

As an example of a conductor having a shape which permits analytical treatment, let us find the position of maximum temperature on a sector of or a complete circular disc, carrying a current, having one terminal at its center and the other around the rim. The purpose of this might be, for example, to determine the best locations of a thermocouple junction for measuring purposes.

In Figure 16, the sector and the complete disc are shown in full and dotted lines respectively.

Let A = angle of sector = 2π for complete disc; radians.
b = uniform thickness of disc.

T_1 and T_2 are terminals, having inside and outside radii R_1 and R_2 respectively, and for simplicity, assume that the terminals have the same temperature.

From symmetry we know that the current and heat flow stream lines are radial, and the equipotential and isothermal lines are circles concentric with the disc.

As shown previously, the maximum temperature occurs at a position from which to a terminal the resistance is one-half the total resistance. The elementary resistance across a circular ring, da wide, and at any radius a is

$$\frac{\rho da}{Aab}$$

where ρ = resistivity of the disc material. The resistance, therefore, from the terminal T_1 to any radius a is then

$$\int_{R_1}^{a} \frac{\rho da}{Aab} = \frac{\rho}{Ab} \log \frac{a}{R_1}$$

The total resistance between T_1 and T_2 is then

$$\frac{\rho}{Ab} \log \frac{R_2}{R_1}$$

If a_m is the radius to which the resistance is one-half the total, then,

$$\frac{\rho}{Ab} \log \frac{a_m}{R_1} = \frac{1}{2} \frac{\rho}{Ab} \log \frac{R_2}{R_1}$$

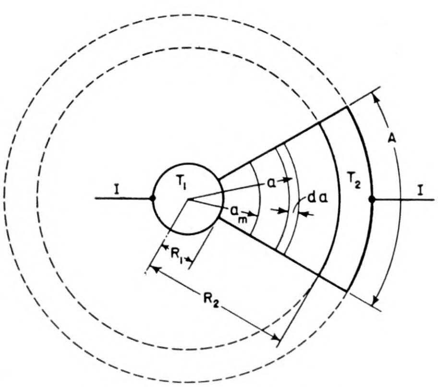


Figure 16—Location of the maximum temperature in a circular disc or a sector thereof, having a circular terminal at its center, and a ring terminal at its rim, when heated by a current passing through the disc between terminals. Maximum temperature lies at a radium $a_m = \sqrt{R_1R_2}$.

from which

$$a_m = \sqrt{R_2 R_1}$$

which shows that the maximum temperature occurs at a radius which is the mean proportional between the radii of the terminals, and is independent of the angle of the sector, or of the material.

As an example, in a disc having a radius to the outer terminal of one inch, and a center terminal radius of ⅛ inch, the maximum temperature lies at the radius

$$a_m = \sqrt{1 \times 1/8} = 0.354 \text{ in.}$$

or 0.229 inch from the center terminal.

20. DERIVATION OF THE GENERAL EQUATION 99.

In Figure 14, let SS be a stream line and lines 1 and 2 traces of the equipotential and isothermal surfaces at any point on the conductor at a distance dx apart.

Since the laws of heat flow and current flow in conductors carrying direct current or low frequency alternating current are mathematically the same, the stream lines are the same for both, and the equipotential and isothermal surface are identical; therefore:

Let the difference in temperature, and the difference of potential across dx, be $d\theta$ and dv respectively, and let v be the independent variable of which x and θ are functions.

Let i = current density at the stream line; amperes per unit area.
v = difference of potential from terminal T_1 to the equipotential surface at any point on the conductor; volts.
a = area of the elementary tube of current and heat flow at the stream line.

In the elementary tube of flow, the rate at which heat is conducted across the surface 1 in the positive direction is $-ak(d\theta_1/dx)$, and that across surface 2 is $-ak(d\theta_2/dx)$. The difference between these two rates of heat flow is equal to the rate at which heat is generated between the two surfaces in this tube having an area a, which is equal to product of the current by the voltage drop or $ai\ dv$; then

$$-ak\left(\frac{d\theta_2}{dx}\right) + ak\left(\frac{d\theta_1}{dx}\right) = ai\ dv \quad (106)$$

But the temperature gradient at the surface 2 is equal to that at surface 1 plus the change in gradient which takes place between them, that is

$$\frac{d\theta_2}{dx} = \frac{d\theta_1}{dx} + \frac{d}{dx}\left(\frac{d\theta}{dx}\right)dx$$

Substitute this in Equation 106 and we obtain

$$ak\left(\frac{d^2\theta}{dx^2}\right)dx = -ai\ dv \quad (107)$$

Now, $dv = ia \times$ (electrical resistance of elementary tube dx long) and the resistance of this tube is $(\rho dx)/a$. Then $dv = ia(\rho dx)/a = i\rho dx$, and therefore

$$dx = \frac{dv}{i\rho} \quad (108)$$

Substitute the value for dx from Equation 108 in Equation 107, and we have

$$aik\ \rho\left(\frac{d^2\theta}{dv^2}\right)dv = -ai\ dv, \text{ which} \quad (109)$$

$$\text{becomes } k\rho\left(\frac{d^2\theta}{dv^2}\right) = -1$$

which gives us an equation in terms of temperature and voltage. Integrating Equation 109 twice yields

$$\theta = -\frac{v^2}{2k\rho} + c_1 v + c_2 \quad (110)$$

where c_1 and c_2 are constants of integration which can be evaluated by knowing that at the terminal T_1, $\theta = 0$ and $v = 0$, and at terminal T_2, $\theta = (T_2 - T_1)$ and $v = V$. Substituting these values in Equation 110 we obtain

$$c_2 = 0,$$

and $c_1 = [V^2/(2k\rho) + (T_2 - T_1)](1/V)$

Then Equation 110 becomes

$$\theta = \frac{V^2}{8k\rho}\left[\frac{4v}{V}\left(1 - \frac{v}{V}\right)\right] + \frac{v}{V}(T_2 - T_1)$$

which is Equation 99.

E. N.—No. 73 —*W. N. Goodwin, Jr.*

References:

(1) *W. N. Goodwin, Jr., U. S. Patent No. 1,407,147; 1916.*
(2) *W. N. Goodwin, Jr.*, The Compensated Thermocouple Ammeter Trans. A.I.E.E., Vol. 55; Page 23, 1936.

TAGLIABUE PRODUCTS MANUFACTURED BY WESTON

The C. J. Tagliabue Corp. (N. J.), a subsidiary of Weston, is a name familiar throughout industry. Its history dates back to the year 1769 when TAG, which is the company trade name, sold thermometers to explorers, inventors and prominent manufacturers. Since that time, the TAG line has been expanded to include hydrometers and thermometers, made in accordance with the standards of the American Petroleum Institute and the American Society for Testing Materials; pyrometers, recording and control apparatus for temperature and pressure; and devices for testing petroleum products, and for determining the moisture content of grain, lumber and other materials.

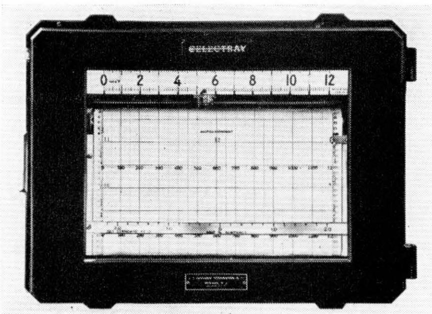
The Tagliabue organization was formerly located in Brooklyn, New York, and in January, 1948, when ownership was transferred to Weston, the entire manufacturing and administration departments were moved to Newark, New Jersey, and integrated with the corresponding departments at Weston. With the local activity completely integrated, it was then necessary to combine the field offices and, at the present time, field representation for the TAG line is being consolidated with Weston Representatives and District Offices located in principal cities throughout the United States.

A few of the many Tagliabue products now manufactured by Weston are illustrated here to give the reader some idea of the scope of the line. More detailed information dealing with the technical characteristics and use of these instruments will be covered in future issues of WESTON ENGINEERING NOTES.

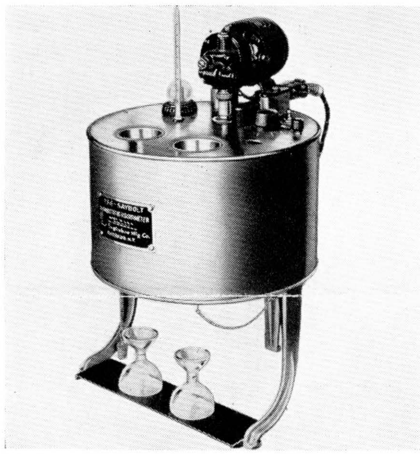
Until recently the Weston line of products was limited to indicating instruments for measuring electrical quantities and temperature. Now the line is extended to include the functions of recording and controlling. The manufacture of all three classes of instruments in one plant places Weston in a unique position to offer a more complete instrumentation service to industry.

—Editor

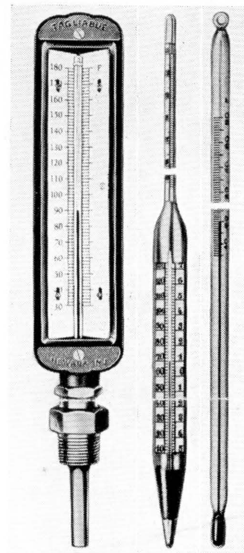
CELECTRAY Indicating Potentiometer Controller



CELECTRAY Recording Potentiometer



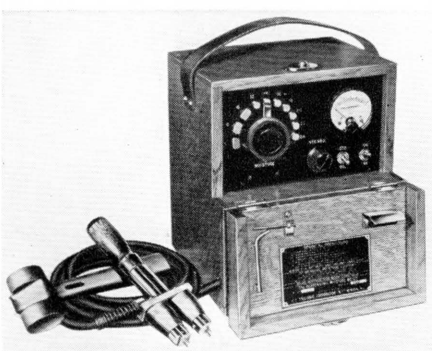
Thermostatic Viscosimeter



Thermometers and Hydrometers

Temperature or Pressure-Indicating Controller

Temperature or Pressure Recorder



Moisture Tester for Lumber

Printed in U.S.A.