Weston
ENGINEERING NOTES

VOLUME 8 MARCH 1953 NUMBER 1

In This Issue

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A NEW WESTON SENSITROL RELAY

THE original SENSITROL* relay of 1935 has been continuously improved over the intervening years, and has now been supplemented by a new design incorporating several features made possible by advances in the instrument art.

In brief, the conventional instrument mechanism used in the magnetic contact type relay, Weston Model 705, has been replaced in the new Model 723 by a core magnet system with its advantages of magnetic shielding and small space. The instrument thus becomes a CORMAG® SENSITROL relay in the parlance of the trade. Confining the mechanism to the lower half of the container has made it possible to develop a release mechanism wholly enclosed in the instrument case and which, in turn, has allowed for complete sealing of the relay against moisture, dust and other external effects. See Figure 1.

Weston relays incorporating instrument mechanisms have been made for approximately half a century. But, as the requirements for sensitivity progressed into the microampere region, contact pressures in turn dropped down to the order of a few milligrams. And with only a few milligrams pressure on a pair of contacts, occasional contact failures occurred due to contact wear and the inevitable coating of the contacts with a non-conducting layer deposited from the surrounding atmosphere, which occasionally would not conduct current at the low contact pressures available.

Figure 1—Weston Model 723 SENSITROL Relay.

About 1932, Mr. Anthony H. Lamb, vice-president, developed a magnetic contact relay which in its essential elements represents the SENSITROL relays generally. The structure consists rather simply of a fixed stationary contact which is actually a small permanent magnet, plated with silver to serve as contact material. The moving contact, in turn, is a tiny silver plated soft iron sleeve forced over the pointer tubing.

Assuming the contacts are separated, we may follow the action when the relay torque is increased until contact is to be made. As the moving contact comes within the influence of the fixed permanent magnet contacts, the attraction of the permanent magnet on the moving iron contact overcomes the spring torque and increasingly high forces accelerate the moving contact until it finally strikes the fixed contact with a considerable impact. And, because of the flexibility of the pointer, it not only strikes hard but also slides a bit. Both actions clean off the surface, initiating good contact. The contact pressure is high because of the magnetic attraction, of the order of one to two grams. This pressure, equal to that of many secondary relays, maintains the excellent contact. As a result, we have here a contact system so perfect that it will handle many of the more difficult types of circuit conditions frequently encountered in relay control problems.

The development of the core magnet system coincides with a requirement for more perfect sealing of the relays. The requirement actually derived from the desire to use relays of this type in unheated amplifier stations in the telephone network of long distance lines. Instruments and similar mechanisms so operated are frequently subjected to rather violent temperature fluctuations particularly in those portions of the country subject to storms and wide temperature cycles. Somewhat parallel with the development of ruggedized panel instruments for the military, work was started on a relay to meet these needs. Obviously, if the release mechanism were to be placed inside the case, room for it had to be made available and with the core magnet mechanism this was rather simply accomplished. A new magnetic release mechanism was designed, numerous hand-made models were tested, and ultimately a design and assembly was worked out which is not only much more compact than the older type and had a longer life—over two million operations—but also functioned on about one-quarter of the energy required in the older design. More specifically, a 6-volt release solenoid on the older Model 705 took 300 ma to operate, or 1.8 watts. With the new design, the current has been dropped down to 75 ma, 0.45 watt. Of course, both of these values have ample safety factor and will operate down to as low as 4.5 volts. The solenoid release mechanism can be seen in Figure 2.

A medium sized core magnet assembly was selected for this application, sufficiently compact to allow for an internal release mechanism, but large enough to allow for improved control forces and an increase in absolute torque for the same base sensitivity of 1 microampere. The larger moving coil may be wound with larger wire than previously used. And yet the total amount of critical materials, such as cobalt and nickel in the magnetic system, has been reduced over the prior design. The weight of the Model 705 with the conventional mechanism and the rear-extending solenoid is 13.5 ounces; this has been reduced to 11 ounces in the new design.

The older Model 705 relay was originally made only in surface type but was later offered as a flush type relay. The new Model 723 is offered only in the flush type since this mounting seems to be most desirable. Mounting dimensions are identical with the Weston Model 301 Panel Instrument, with a 3½-inch flange diameter and a body 2¾ inches in diameter. Terminals are brought out through glass-sealed solder terminal connections thus maintaining the sealed character of the relay. It is being offered in all of the various ranges for which the Model 705 was designed. In addition, it can be supplied with self-contained series resistance up to as high as 500 volts for d-c operation, as well as with a rectifier for operation on a-c. Similarly, through the use of another small self-contained rectifier, the contact resetting mechanism may be operated from a-c in which case approximately 1 watt is required. A large number of combinations are possible with either or both high or low contacts available, and with the mechanism normally at one end or at center.

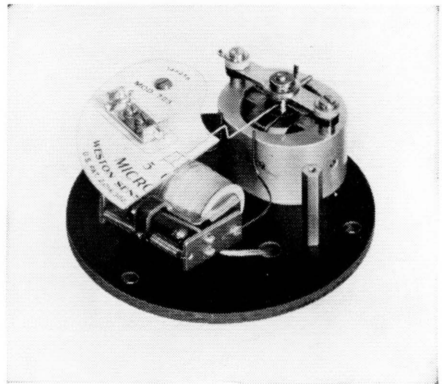

Figure 2—View of Model 723 with cover removed. Scale is cut away to show solenoid release mechanism.

While there are no plans to discontinue the Model 705 relay as such, because of its relatively large background, the advantages of the new Model 723 with its internal solenoid, its magnetic shielding and its sealed case are such that many of the users of the Model 705 will be interested in exploring this revised type and considering its use in place of the previous design.

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* SENSITROL—A registered trademark designating the magnetic contact instruments and relays manufactured by the Weston Electrical Instrument Corp.

THE INDUCTRONIC FLUXMETER

THIS is one of a series of articles describing applications of the Weston Model 1411 D-C Amplifier. As described previously (see references), this amplifier has an unusually fast response for a converter type system and may be used with feedback reactance, which in slower systems would cause instability such as phase-delay oscillation. This, in a practical sense, allows its use as an integrator having a high resolution speed and a suitably low drift rate. The amplifier is then applicable as a search-coil type of magnetic fluxmeter.

Search-Coil Fluxmeter

The classical fluxmeter comprises a search coil subjected to the magnetic flux change it is desired to measure, connected to an integrating permanent-magnet, movable-coil mechanism. In effect, the mechanism integrates the potential (E) developed in the search coil against time, and:

$$\int E dt = BAN \times 10^{-8} \text{ (volt-seconds)} \qquad (1)$$

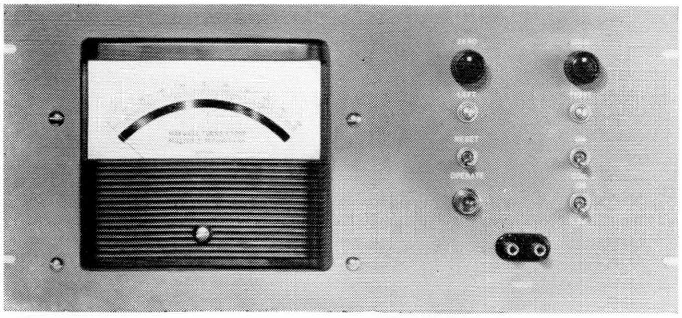


Figure 1—Weston Inductronic Fluxmeter.

Wherein B is the flux change in gausses, A is the coil area in square centimeters, and N is the number of coil turns, and the factor (BAN) is the change of total flux linkage in maxwell-turns. Thus flux measurement by the classical search-coil method is essentially a matter for integration.

Amplifier Method

The method uses a d-c amplifier with mutual inductance feedback in degenerative sense, in the manner of the illustrative circuit of Figure 3. The amplifier has a gain parameter of transfer conductance, accepting potential to deliver current, and balances applied potential by a derivative change of output current, as:

$$E = -E' = -(dI/dt)L \qquad (2)$$

which integrated gives:

$$\int E dt = IL \qquad (3)$$

wherein the constant of integration is the value (L, henrys) of the mutual feedback inductance. Thus, the change of output current (I, amperes) is a measure of the input integral. Combining (1) and (3):

$$B = I(L/AN) \times 10^8 \ (gausses) \qquad (4)$$

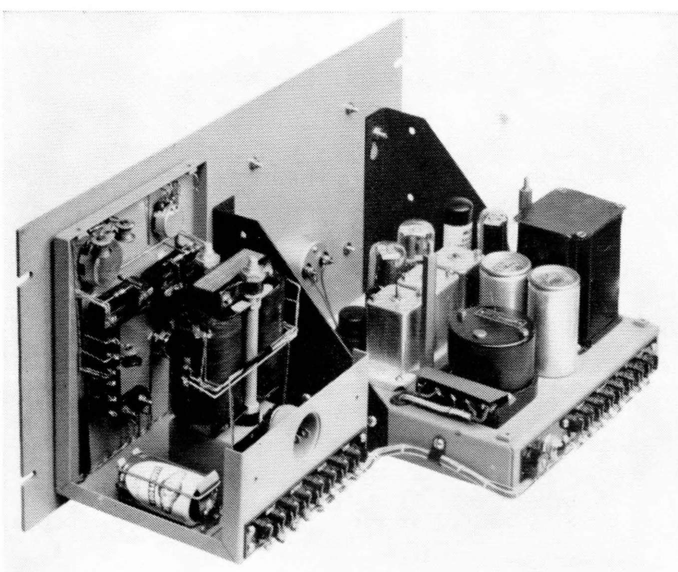


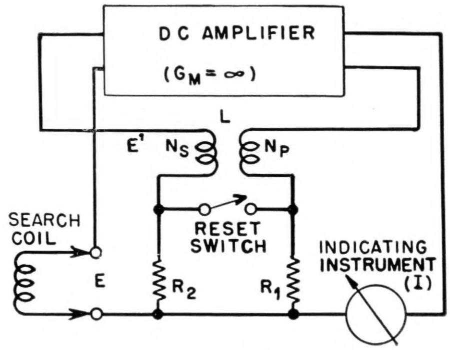
Figure 2—Rear view of Weston Inductronic Fluxmeter with covers removed to show component parts.

wherein the factor (L/AN) is a constant of calibration. The factor (AN) is a constant of the search coil.

One advantage of an amplifier method is the convenience of restoration to zero in contrast to the integrating galvanometer, which normally has little or no restoring force. When the switch in Figure 1 is closed, mutual degenerating resistance is connected into the feedback path, which causes the system to return to zero exponentially with a time constant equal to the ratio of the mutual feedback inductance and resistance (L/R). The deflection at time t, $D(t)$, relative to the initial deflection at $t=0$, D_o, is:

$$D(t)/D_o = e^{-t/(L/R)} \qquad (5)$$

and in a time equal to several time constants, the system is substantially restored to zero. In this case,



$I = \int Edt/L$

$R_1/R_2 \approx N_p/N_s$

Figure 3—Basic fluxmeter circuit.

the mutual resistance R is the parallel resistance of R_1 and R_2 when the switch is closed. For transient balance at the time of switch operation, R_1 and R_2 should have the same ratio as the reactor turns ratio, as indicated in the Figure.

Drift Factors

With purely reactive feedback coupling, the system is subject to drift with time, which must be minimized in practice. The drift influence may be classified into two factors, zero drift and gain drift, which are separately compensable.

Zero drift is caused by input displacement potential, such as a spurious thermopotential. As input potential can only be balanced by a change rate of output current, the output current will drift in response thereto in the manner of expression (2). Correction involves introduction of a balancing potential into the input circuit.

Gain drift is caused by lack of infinite transfer conductance in the amplifier. With any previously established output current level, I_o, and with zero potential applied to the input, the output current drift rate will be:

$$dI/dt = I_o/LG_m \qquad (6)$$

and so, for all values of I_o, the drift rate can be zero only when the amplifier transfer conductance (G_m) is infinite. The usual sign of the amplifier conductance is negative, so the drift will be negative and toward zero current.

However, the amplifier gain may be raised effectively to infinity by feedback means. If an additional resistive feedback conductance (G_f) is connected as mutual conductance coupling the output to the input, the over-all loop transfer conductance (G) is changed to:

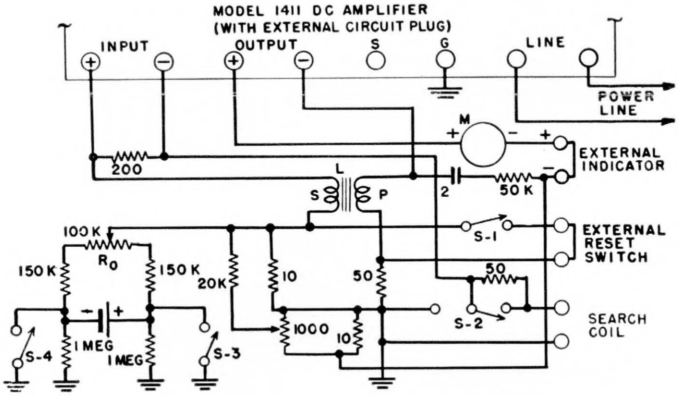
$$G = \frac{G_m G_f}{G_m + G_f} \tag{7}$$

which becomes infinite when:

$$G_f = -G_m \tag{8}$$

Thus an adjusted amount of positive feedback (regeneration) can be used to compensate for gain drift. The Model 1411 is capable of being adjusted to have an intrinsic gain beyond infinity, but in production the intrinsic gain is adjusted between infinity and about -200 mhos, assuring stable operation with any resistive feedback network. Drift compensation, therefore, requires a feedback adjustment range of 0-200 mhos (+).

Characteristically zero drift is constant for all output current levels, whereas gain drift is proportional to the level of output current. Therefore to dissociate the two influences for purposes of adjustment, zero drift is observed at zero output current, and gain drift at full-scale output current with no zero drift present. The adjustment procedure is first to compensate zero drift at zero output current, and then correct the gain drift at full-scale output current.



S-1, Reset Switch, close to reset.
S-2, Search Coil Disconnect Switch.
S-3, Traverse Switch, Up, normally-open.
S-4, Traverse Switch, Down, normally open.
R_o, Zero Drift Control.
R_g, Gain Drift Control.
L, Mutual Reactor (5 henrys) ($N_p/N_s=5$)
M, Indicating Milliammeter (1 milliampere).

Circuit values indicated are typical for a range of 500,000 maxwell-turns.

Figure 4—Fluxmeter circuit, including drift and traverse controls.

Circuit Design

A complete circuit designed for operation with the Model 1411 Inductronic Amplifier is shown in Figure 4. It includes a reset switch for setting the system to zero, zero drift and gain drift adjustment controls, and two push-button switches for up and down traversing should a setting other than zero be desired. A search-coil disconnect switch is also included for changing search coils without opening the input circuit, which would release the feedback connection and cause violent random pointer deflection.

The Model 1411 Amplifier has a normal guaranteed input resolution of 5 microvolts, so the drift compensating circuits are designed to accommodate variations within this maximum. The zero drift compensating circuit supplies a current adjustable over a range of ± 0.5 microampere through a 10-ohm input circuit resistor, covering a range of ± 5 microvolts. The gain-drift adjustment must accommodate the undegenerated transfer conductance of the Model 1411 which varies in production adjustment to a minimum of -200 mhos. This is compensated by a pi network adjustable from zero to 0.005 ohm as regenerating feedback resistance for gain-drift adjustment. While Model 1411 Amplifiers are capable of being adjusted to a condition of higher-than-infinite gain, they are not normally so adjusted in production, so no gain-drift adjustment in the negative feedback region is provided.

The critical element of calibration stability is the mutual reactor. The reactor used is mechanically similar to the Weston reactors used in frequency meter resonant circuits, and has an air gap adjustable by large adjusting nuts securely locked in position after adjustment. The primary and secondary windings are divided and balance wound on each leg of the reactor to minimize stray flux linkage and to reduce leakage reactance.

Binding post connections are provided for external indicating instruments or recorders and for external reset. The equipment is sometimes used to measure the developed flux in permanent magnet structures during charging, and the relatively large stray field in the vicinity of the charging equipment prohibits proximity of the fluxmeter. In such cases, remote location is necessary, with an external indicating instrument and reset switch at the point of operation. For recording, a 1-milliampere direct-writing instrument is suggested. The resistance of recorders of this type is usually well below the maximum burden resistance recommended for the Model 1411 Amplifier (5,000 ohms).

Damping and Input Rate Change

While the Model 1411 Amplifier has a response speed that is high for amplifiers of the converter type, it will develop phase shift in the audio range, and may normally be expected to oscillate when coupled back by pure reactance. Oscillation would occur at a frequency that would not be apparent even with a high-speed indicating instrument, but gain would suffer. It is thus necessary to damp the reactor to effect a feedback phase delay sufficient to prevent the over-all loop gain from becoming infinite at any possible frequency. The reactor is damped by a series R-C circuit across the primary winding and the zero-reset network.

However, this is a transient shunt load upon the output current, and effects the ability of the system to balance input potential above a certain maximum. This corresponds to a maximum rate of change of search-coil flux which must be recognized when measuring sharp changes.

Assuming that the R-C time constant of the damping network is large with respect to the duration of the input transient, it is reasonably valid to assume that the maximum balancing potential that can be developed in the mutual reactor originates in the flow of output current through the damping resistor. This appears in the secondary circuit as:

$$E'_{(max.)} = I_{(max.)} R_s (N_s/N_p) \qquad (9)$$

Wherein N_s and N_p are the number of secondary and primary turns on the reactor, respectively, and R_s is the damping resistor. $I_{(max.)}$ is the maximum steady-state output current that can be delivered to a load resistance equal to R_s. This is somewhat involved, but an acceptable rule of thumb is to assume a value of 5 volts for the factor $I_{(max.)}R_s$.

The maximum balancing speed in terms of scale lengths may be stated by combining (9) with (1):

$$\frac{E'_{(max.)}}{\int Edt\ (full\text{-}scale)} = \frac{I_{(max.)} R_s N_s/N_p}{I\ (full\text{-}scale)\ L}\ (scale\text{-}lengths/second) \qquad (10)$$

The result usually exceeds a speed of 100 scale-lengths/second.

Performance

The circuit constants of a typical design including an indicating instrument of 0.25 per cent accuracy class are indicated in Figure 4. The range and performance characteristics are listed below:

Range: 500,000 maxwell-turns. (5 millivolt-seconds).
Zero-Drift Adjustment: ±500 maxwell-turns/sec.
Gain-Drift Adjustment: ±500 maxwell-turns/sec./scale length.
Zero-Drift/Microvolt: 100 maxwell-turns/sec.
Zero-Restoration Time Constant: 0.5 sec. approx.
Traverse Rate (Up and Down): 15,000 maxwell-turns/sec. (approximate).
Maximum Input Rate of Change: 10^8 maxwell-turns/sec. (1 volt).
Output Current Range: 1 milliampere into 4,000 ohms max.
Search-Coil Resistance: 50 ohms maximum.
Reactor Inductance (Mutual): 5 henrys, $N_p/N_s = 5$.
Damping Circuit: 50,000 ohms and 2 mfd.

Figures 1 and 2 show an Inductronic Fluxmeter designed for measuring the flux distribution about the magnetic circuit of a small instrument magnetic system. The drift rate is sufficiently low to move the search coil around the system and measure the included flux at several stations to determine the leakage flux loss about the circuit.

References:

Electrical Engineering, Vol. 70, No. 10, Pages 893-898, "A Sensitive Instrument Converter, the Induction Galvanometer."
WESTON ENGINEERING NOTES, Vol. 6, No. 1, April, 1951, "The Weston Model 1411 Inductronic D-C Amplifier."
Weston Circular B-31-A.

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THERMAL PROBLEMS RELATING TO MEASURING AND CONTROL DEVICES—PART IX—CONTINUED

Steady State Distribution of Temperature on Various Shapes of Conductors Connected to Terminals

Introduction

THIS section, which is the second section of Part IX, will continue the analysis of the steady state distribution of temperature along and the heat conduction in uniform conductors, and will give some practical applications.

29. TEMPERATURE DISTRIBUTION IN A CONDUCTOR OF UNIFORM CROSS SECTION CONNECTED TO TERMINALS WHEN NO HEAT IS ADDED OR REMOVED OTHER THAN THAT EXCHANGED WITH THE TERMINALS AND THE SURROUNDING MEDIUM.

In this case, the conductor may be of any material (either an electrical or simply a heat-conducting material) and be, for example, any uniform bar or strip supported in air between heat-absorbing devices as terminals which may or may not be at air temperature or at equal temperatures. For this condition, w and, therefore, θ_o are zero, and if this value is substituted in Equation (185), the equation for the temperature of the conductor above ambient at any position x becomes

$$\theta = \frac{T_1 \sinh (L-x)u + T_2 \sinh xu}{\sinh Lu} \qquad (195)$$

The mid-point temperature, θ_c, above ambient under the same condition is found by substituting $\theta_o = 0$ in Equation (186) from which we find that

$$\theta_c = \frac{T_1 + T_2}{2 \cosh (Lu/2)} \qquad (196)$$

which shows that the temperature at the mid-point above ambient, under the conditions given, is equal to the mean of the terminal temperatures divided by the hyperbolic cosine of half the thermal length of the conductor.

30. WESTON COMPENSATED THERMOAMMETER.

Figure 26 illustrates the principle of the compensated thermocouple ammeter, which is designed primarily for the measurement of alternating currents at radio frequencies.[1] The heating conductor, *h.c.*, is connected between two terminals having temperatures T_1 and T_2 above ambient. Compensating conductors, *c.c.*, are thermally connected to but electrically insulated from the terminals at their ends by thin sheets of mica or other material. A thermocouple, *t.c.*, has its "hot" junction connected to the center of the heating conductor. Its "cold" junctions are thermally connected to the center of the compensating conductors, and it is convenient to use these conductors as a means for electrical connection to the indicating instrument.

The thermocouple then generates an emf. proportional to the difference in temperature between the centers of the heating and compensating conductors, which is dependent solely upon the heating effect of the current to be measured, and independent of terminal and ambient temperatures. This is shown as follows:

If the "cold" junctions of the couple were connected directly to the indicating instrument, then the instrument indications would be proportional to the difference in temperature between θ_c at the center of the heating conductor and the indefinite temperature of the instrument. As shown by Equation (188), θ_c depends not only upon the current to be measured, corresponding to the potential difference V, but is affected also by the temperature of the terminals T_1 and T_2, and by the ambient temperature upon which θ_c is based.

Now, if the compensating conductors are so designed that their thermal lengths ($L_c u_c$) are equal to the thermal length of the heating conductor, Lu, then the temperature, T_c, at the center of the compensating conductors is given by Equation (196). If then the "cold" junctions of the couple are connected to the centers of the compensating conductors, the emf. generated will be proportional to the difference in temperature, $\theta_c - T_c$, between the centers of the heating and compensating conductors, as given by the difference between Equations (188) and (196), that is,

$$(\theta_c - T_c) = \frac{V^2}{8k\rho}\left[\frac{2\left(1 - \frac{1}{\cosh Lu/2}\right)}{(Lu/2)^2}\right] \tag{197}$$

which shows that the instrument indications are entirely independent of ambient and terminal temperatures and, further, for a given set of instrument parameters, they depend solely upon the square of the voltage, and therefore upon the square of the current.

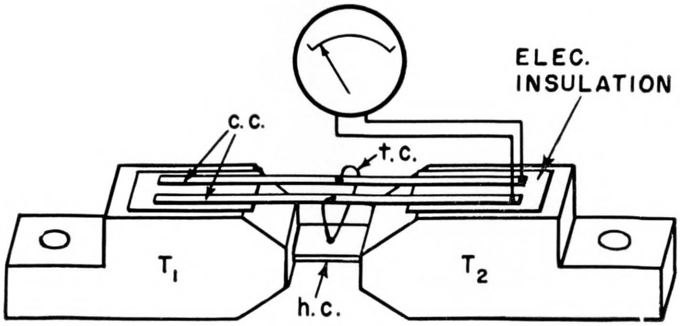

Figure 26—Weston Compensated Thermoammeter. H.c. is the heating conductor through which the current passes; c.c. are the compensating conductors; and t.c. is the thermocouple.

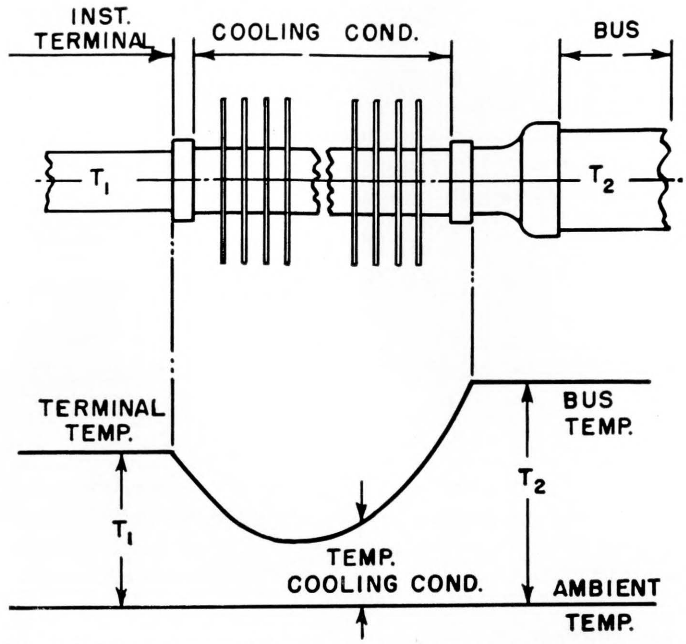

Figure 27—A cooling conductor connected between terminals having different temperatures, designed to cool the lower temperature terminal, and to prevent heat reaching it from the higher temperature terminal. The diagram below the conductor illustrates the temperature distribution.

The same general method can be employed to compensate for the effect of ambient and terminal temperatures in series and bridge type thermocouple heating elements and in radiation measuring instruments.

31. THE RATE AT WHICH HEAT IS CONDUCTED AT THE TERMINALS TO OR FROM A CONDUCTOR OF UNIFORM CROSS SECTION CONNECTED BETWEEN THEM.

The rate at which heat is conducted across any section of the conductor is equal to the product of the area of the cross section by the thermal conductivity and by the temperature gradient at that section. Then the rate at which heat is conducted across any section in the positive direction, which in Figure 24 is in the direction of increasing values of x is

$$H = -Ak\frac{d\theta}{dx} \tag{198}$$

The sign is negative to correspond to the physical fact that the heat flow is in the direction of diminishing temperature.

By differentiating Equation (185) with respect to x and substituting the value of $d\theta/dx$ thus found in Equation (198), we have the rate of heat conduction across any section of the conductor in the positive direction,

$$H = -Aku\left[\frac{(\theta_o - T_1)\cosh (L-x)u - (\theta_o - T_2)\cosh xu}{\sinh Lu}\right] \tag{199}$$

Then the rate H_1 at which heat is conducted in the positive direction at the terminal T_1, that is from the terminal to the conductor, is found by setting $x=0$ in the general Equation (199), from which

$$H_1 = -Aku\left[\frac{(\theta_o - T_1)\cosh Lu - (\theta_o - T_2)}{\sinh Lu}\right] \qquad (200)$$

Likewise the rate at which heat is conducted in the positive direction at T_2, that is from the conductor to the terminal, is found by making $x=L$ in Equation (199) then

$$H_2 = -Aku\left[\frac{(\theta_o - T_1) - (\theta_o - T_2)\cosh Lu}{\sinh Lu}\right] \qquad (201)$$

It is seen from physical reasoning and from the equation, that when the terminals are at a lower temperature than the conductor, the flow of heat at terminal T_1 is negative, that is, toward the terminal, and at T_2 it is positive, that is, from the conductor to the terminal. In some cases, however, the conductor may be at a lower temperature than the terminals, or the terminals may have greatly differing temperatures, under which conditions the direction of the heat flow may be opposite to that given above. In any case, however, the proper direction will be clearly indicated by the sign of H as determined by the equations.

Location of Maximum or Minimum Temperature on the Conductor

The maximum or minimum temperature lies at the value of x at which $d\theta/dx=0$, that is, where no heat is transferred across the corresponding section. Then, if H in Equation (199) is made zero, we find

$$\tanh xu\Big]_{\substack{\text{max. or}\\ \text{min.}}} = \frac{\cosh Lu - \dfrac{T_2-\theta_o}{T_1-\theta_o}}{\sinh Lu} \qquad (202)$$

from which xu may be found.

The actual value of the maximum or minimum temperature may be computed by substituting the value of xu found by Equation (202), in Equation (185).

Practical Applications

Assume a circuit carrying current into which an instrument or other device is to be connected. Assume further that for reasons of necessity the bus tubing must operate at a temperature of 100 degrees centigrade above an ambient of 35 degrees centigrade, whereas the instrument terminals are limited to an increase of 40 degrees centigrade. One practical method of accomplishing this is to connect the bus tubing and the instrument by means of a cooling conductor so designed that it will transmit no heat from the bus to the instrument, but actually provide additional cooling for the instrument. Finned copper tubing is suggested as one of the most efficient forms of cooling conductor.

The part of the circuit of interest is shown in Figure 27. The diagram below the conductor illustrates the desired form of temperature distribution, in which the conductor temperature is lower than that of the terminal it is intended to cool.

Figure 28 shows the form and dimensions of two finned tubular conductors, and of a smooth tube without fins for comparison, on which careful heating tests were made. Figure 29 shows the results of these tests in the form of curves giving the rate of dissipation of heat solely by convection and radiation from the conductor to the surrounding still air in watts per inch length of conductor, as a function of the temperature of the conductor above that of the air. It will be noted that the experimental values lie remarkably close to a straight line when plotted on log-log paper. The equations of the curves determined by the method of least squares are also given in Figure 29, and are designated by the same letters as the corresponding forms of conductors in Figure 28.

For the application being considered, we will use the conductor designated *a*, as it is the most efficient of the group.

Thermal Constants of the Tube

The theoretical equations given are based upon the linear law of cooling. Although the actual cooling law is not linear for the conductors, it may be considered so for practical purposes within the narrow limits of temperature in the problem. Assume 40 degrees centigrade as an average temperature, then from equation *a* in Figure 29 the approximate linear convection coefficient is

$$h = \frac{h_o}{40} = 0.00945 \times 40^{0.439} = 0.0477$$

watt per degree centigrade per cm. length. For practical purposes we may assume this to be uniformly distributed along the length, although actually lumped at the fins. From the dimensions given in Figure 28(a), the cross sectional area is computed to be $A = 1.053\ cm.^2$

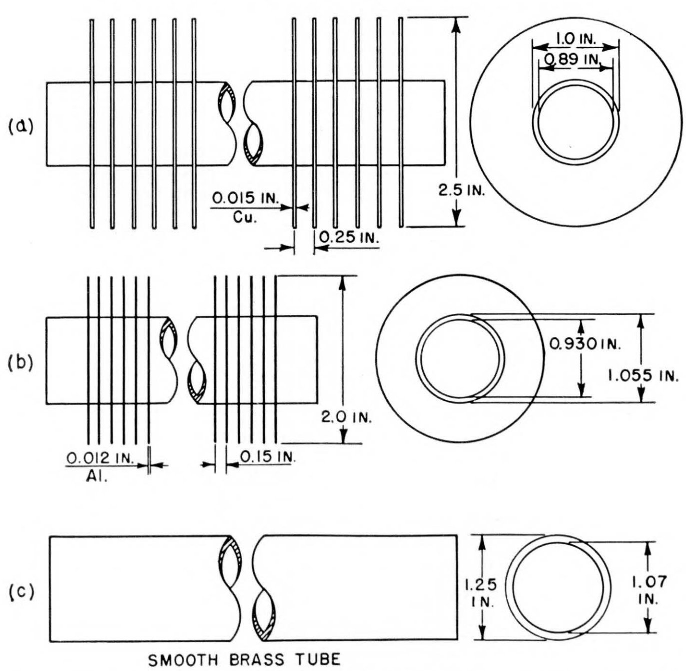

Figure 28—Two examples of finned tubing, and of a smooth tube, without fins, for comparison.

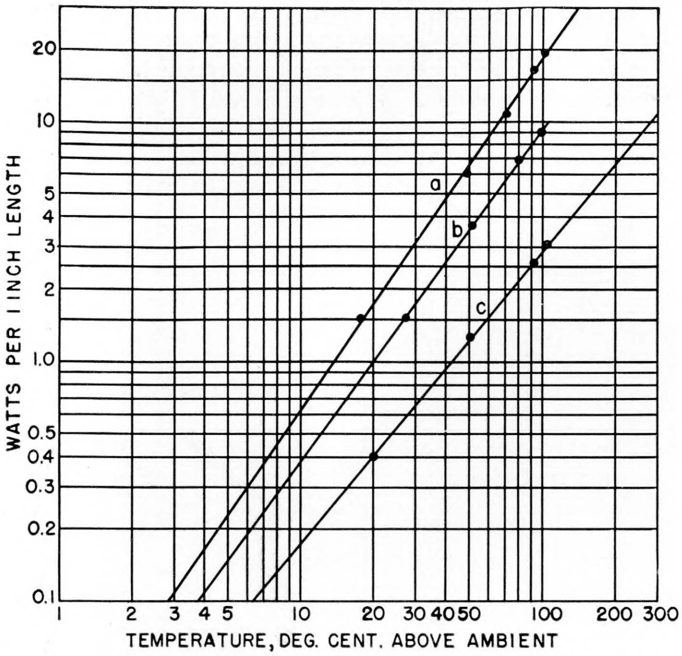

Figure 29—Rate of dissipation of heat by convection and radiation, from the conductors shown in Figure 28, for various values of their temperature elevation above ambient, in still air. Ambient = 26 degrees centigrade. The equations of the three curves in watts per inch length as a function of temperature are,

a; $h_o = 0.0240 \times \theta^{1.4390}$

b; $h_o = 0.01633 \times \theta^{1.3751}$

c; $h_o = 0.01076 \times \theta^{1.2162}$

The thermal conductivity of copper is $k = 3.81$ watts per cm.[2] for a temperature gradient of 1 degree centigrade per cm. Then the thermal length per cm. length is

$$u = \sqrt{h/(Ak)} = \sqrt{0.0477/(1.053 \times 3.81)} = 0.109 \text{ hyperbolic radians.}$$

Let us assume that 10 inches is the practical limit for the length of the cooling conductor. We wish to determine the rate at which heat can be conducted away from terminal T_1, connected to the instrument under the assumed conditions of terminal temperatures, and while carrying an alternating current of 100 amperes.

Let the a-c resistance of the cooling tube at the frequency used be 5×10^{-5} ohm per cm. length. Then the power generated per cm. length of the tube at 100 amperes is $w = I^2R = 10{,}000 \times 5 \times 10^{-5} = 0.5$ watt per cm., and therefore θ_o, which is the temperature increase which would result if no heat were dissipated by conduction is

$$\theta_o = w/h = 0.5/0.0477 = 11.2 \text{ degrees centigrade.}$$

The thermal length $Lu = 10$ in. $\times 2.54 \times 0.109 = 2.77$ hyperbolic radians. Then from Equation (200), the rate at which heat is conducted away from the terminal T_1 by the conductor is

$$H_1 = 1.053 \times 3.81 \times 0.109 \left[\frac{(40-11.2)\cosh 2.77 - (100-11.2)}{\sinh 2.77}\right] = 7.84 \text{ watts}$$

To determine the efficiency of the 10-inch cooling conductor, it may be compared with the transfer of heat which would result if the conductor were infinitely long. By making L = infinity in Equation (200), we obtain

$$H_1\Big]_{L=\infty} = Aku\,(T_1 - \theta_o) = 1.053 \times 3.81 \times 0.109 \times (40-11.2) = 12.6 \text{ watts}$$

which shows that the 10-inch cooling conductor has an efficiency of 62 per cent.

Location of Minimum Temperature on Cooling Conductor

From Equation (202), at the position of minimum temperature, we have

$$\tanh xu\Big]_{\min.\,\theta} = \frac{\cosh 2.77 - \dfrac{100-11.2}{40-11.2}}{\sinh 2.77} = 0.62$$

Then $xu\Big]_{\min.\,\theta} = 0.725$ hyperbolic radians from T_1

and $x\Big]_{\min.\,\theta} = 0.725/u = 0.725/0.109 = .65$ cm. $= 2.62$ inches from the terminal T_1.

Minimum Temperature of Cooling Conductor

In Equation (185), substitute $u = 0.725$ as the location in hyperbolic radians of the minimum temperature and we have

$$\theta\,(\text{min.}) = 11.2 + \frac{28.8 \sinh(2.77 - 0.725) + 88.8 \sinh 0.725}{\sinh 2.77} = 33.8$$

degrees centigrade above ambient temperature.

Reference:

[1] *W. N. Goodwin, Jr.* The Compensated Thermocouple Ammeter. Trans. A.I.E.E., Page 25, Vol. 55, 1936.

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