



# Weston ENGINEERING NOTES

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## MODEL 905 A-C AND D-C PORTABLE WATTMETERS

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"Westonia"

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THE Model 905 Wattmeter is the latest addition to the matched group of portable instruments consisting of the Model 901 D-C Voltmeter, Ammeter and Milliammeter, and the Model 904 Iron Vane A-C Voltmeter, Ammeter and Milliammeter. A picture of the Model 905 Wattmeter is shown in Figure 1.



Figure 1—Model 905.

The Model 905 is a single element electro-dynamometer wattmeter designed for general testing where accuracy and readability are desired. Like its matched companion instruments, it has a rated accuracy of 0.5 per cent of full scale value. The instrument features a 5.5-inch-long hand calibrated mirror scale,

EDITOR'S NOTE: Although not a mathematical or scientific analysis such as we prefer for WESTON ENGINEERING NOTES, the technical description of the new Model 905 Wattmeter appears to be worthy of publication here as information for the practicing engineer.

and a curved shadow-reducing wide-angle window which admits light from both sides to afford maximum scale visibility. The case is ribbed, and molded of bakelite. The window is a clear non-breakable plastic especially treated to eliminate electrostatic effects.

Figure 2 shows the mechanism removed from its magnetic shield. One field coil has been removed for a clearer view of the moving coil and the other field coil.

The instrument was designed to more than meet the requirements of the American Standards Association Specification C39.1-1951 for instruments of the 0.5 per cent rated accuracy class. A comparison of many of the requirements of this specification with the data from a typical Model 905 Wattmeter is shown in Table I. It should be noted that in nearly all cases the Model 905 characteristics are better than would be necessary merely to meet the requirements of the ASA specification.

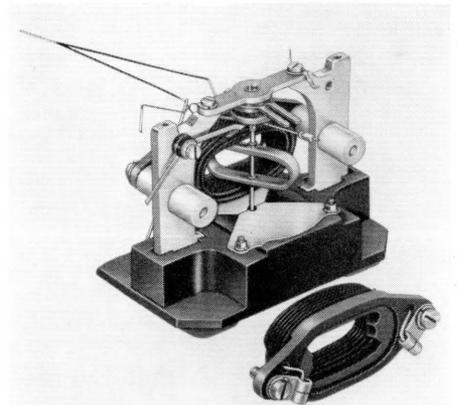


Figure 2—Model 905 mechanism with one field coil removed.

TABLE I—COMPARISON OF ASA REQUIREMENTS

Item	ASA Requirements	Model 905
Rated accuracy, per cent of full scale	0.5	0.5
Scale length, minimum	3.2"	5.5"
Damping factor, minimum	5	10
Response time, maximum	5 sec	2.5 sec
Loss, maximum:		
Potential circuit at 120 v, 60 c	5 va	3 va
Current circuit at 5 a, 60 c	2 va	0.83 va
External temperature influence, maximum	0.5%	0.25%
Sustained operation influence, maximum	0.6%	0.2%
External field influence, maximum	2.0%	1.3%
Frequency influence, maximum, 54-66 c	0.2%	Not Readable
d-c to 100 c	—	0.2%
Voltage influence, maximum, $\pm 10\%$ change	0.2%	Not Readable
Power factor influence, maximum	0.5%	0.15%

A general statement as to the electrical characteristics of the Model 905 might be that the instrument has a well damped, fast acting moving system with low losses and is well compensated for influences such as those due to wide changes in temperature, sustained high load operation, heavy external magnetic fields, and variations in frequency, voltage and power factor.

The Model 905 Wattmeter has a small sustained operation influence as compared to the value of 0.6 per cent allowed by the ASA specification. This has been achieved by means of an efficiently designed mechanism that requires very little power in both the field and moving coils for its operation. In addition, the series resistance is mounted in a compartment completely isolated from the mechanism so that the heat from the series resistance has very little effect upon the operation of the instrument.

The ASA specification states that the frequency influence shall be taken over 54 to 66 cycles and shall not exceed 0.2 per cent of full scale value. Actually, the frequency influence between d-c and 100 cycles and at any power factor between unity and 50 per cent does not exceed 0.2 per cent of full scale value on the Model 905. At 400 cycles, the error is less than 0.5 per cent of full scale value at any power factor between unity and 0.55 on the 150-volt range, and 0.70 on the 300-volt range.

While the standard cataloged instruments are designed for use at d-c and 25-125 cycles, the instruments can also be supplied frequency compensated for d-c and

25-1,000 cycles, with a frequency influence of less than 0.25 per cent of full scale value over the entire frequency range and from unity to 50 per cent power factor.

The voltage influence in the ASA specification is limited to a test of  $\pm 10$  per cent variation from the nominal voltage rating, and an error of 0.2 per cent of full scale is allowed. The Model 905 Wattmeter is an air core electro-dynamometer, and as such the voltage influence is negligible for any value up to its maximum rating. In other words, the potential circuit can be used at any voltage provided the maximum ratings in potential and current are not exceeded in their respective circuits.

The ASA specification permits a power factor error of 0.5 per cent over a range of unity to 0.5 power factor. The Model 905 Wattmeter can be used at any power factor from zero to unity with an error due to power factor of less than 0.2 per cent at 60 cycles. Both the power factor and frequency influence are kept small by an efficient design of moving and field coils and by keeping eddy currents to a minimum in the mechanism parts.

The standard instrument is designed to provide an overload rating of 100 per cent on the current circuits and 50 per cent on the potential circuits based on the nominal voltage and current rating. These overload features can be utilized when taking measurements on low power factor circuits and full scale deflection can be obtained at 50 per cent power factor. At lower power factor, such as 10 to 20 per cent, the instrument can be supplied

specially adjusted so that full scale deflection will be obtained at 25 per cent power factor.

The instrument is normally supplied with two potential ranges and one or two current ranges. The voltage is limited to self-contained ranges up to 300 volts maximum, with higher ranges supplied in an external multiplier. The maximum self-contained current range is 50 amperes.

The Model 905 Wattmeter can also be supplied with a sturdy leather case for greater protection of the instrument, which was particularly designed for field work, utility service trucks, factory maintenance, and similar work where protection of the instrument is desirable. This case is designed in such a fashion that all electrical connections can be made to the instrument without removing the instrument from the case. In addition, the front flap, which covers the dial window, can be unsnapped and bent back so that the instrument dial is exposed. This feature permits complete use of the instrument in its carrying case. The wattmeter in its carrying case is shown in Figure 3.

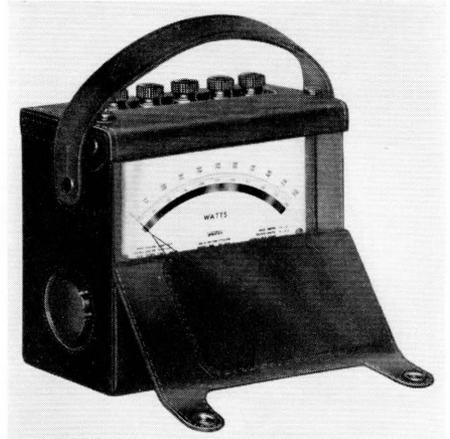


Figure 3—Weston Model 905 Wattmeter in carrying case.

In summary, the Model 905 Wattmeter is an accurate, easily read, long scale instrument which more than meets the requirements of the ASA Specification 39.1-1951. It is suitable for laboratory, shop and field work and, where greater protection is desired, it can be supplied with a leather case which need not be removed when using the instrument.

E. N.—No. 99

—R. F. Estoppey.

## THERMAL PROBLEMS RELATING TO MEASURING AND CONTROL DEVICES—PART X

### Temperature Distribution in a Circular Disk or Sector Under Given Conditions of Heating and Cooling

#### Introduction

IN PREVIOUS chapters, the temperature distribution in uniform conducting strips connected between terminals was considered. In that case, the lines of heat flow are parallel whereas in the circular disk now under consideration they are radial.

In some applications of thermometry, a circular disk is used for the thermometer element, mounted on a surface such as the outer surface of aircraft, to measure surface or outside temperature. Such a disk may also be used as a radiation meter to measure radiant power incident upon it.

The following analysis is for the purpose of determining the temperature distribution in a disk under given conditions.

#### 32. DETERMINE THE TEMPERATURE AT ANY PART OF A CIRCULAR DISK OR SECTOR HEATED UNIFORMLY AT A CONSTANT RATE, WHICH EXCHANGES HEAT BY CONVECTION WITH THE SURROUNDING MEDIUM AND BY CONDUCTION TO A HEAT-ABSORBING CONDUCTOR CONNECTED TO ITS RIM.

Figure 30 illustrates such a disk. To make the analysis more general, consider a sector as shown having an angle  $\phi$ . The heat may be applied, for example, by radiant energy.

Let  $w$  = rate of application of heat to the surface per unit area; watts/cm.<sup>2</sup>

$h$  = rate of exchange of heat with the surrounding medium per unit area and per degree temperature; watts/cm.<sup>2</sup>/deg. C.

$k$  = thermal conductivity of disk material; watts cm./deg. C.

$\theta$  = temperature above ambient at any radius  $x$ .

$R$  = radius of disk in centimeters.

$b$  = thickness of disk in centimeters.

$T_o$  = temperature of heat-absorbing ring above ambient.

$\theta_o = w/h$  is the temperature above ambient which would result if all heat were dissipated by convection and none by conduction to the rim.

Then, as will be shown later, the temperature at any part of the disk above ambient is

$$\theta = \frac{w}{h} \left[ 1 - \frac{I_o(x\sqrt{h/(kb)})}{I_o(R\sqrt{h/(kb)})} \right] + T_o \left[ \frac{I_o(x\sqrt{h/(kb)})}{I_o(R\sqrt{h/(kb)})} \right] \quad (203)$$

where  $I_o(x\sqrt{h/(kb)})$  etc. are modified Bessel functions of zero order and the first kind. Some tables give values of  $J_o(iz)$  which is the same as  $I_o(z)$ , where  $i = \sqrt{-1}$ .

It is also shown that the temperature distribution is independent of the angle  $\phi$  of the sector, so that the temperature distribution is the same for the entire disk as for any sector of it.

These Bessel functions may be found in tables readily available.<sup>1</sup> However, if one wishes to compute them, they may be obtained by the use of the following series:

$$I_o(z) = 1 + (z/2)^2 + \frac{(z/2)^4}{(2!)^2} + \frac{(z/2)^6}{(3!)^2} + \frac{(z/2)^8}{(4!)^2} + \dots \quad (204)$$

in which for the present case,  $z = x\sqrt{h/(kb)}$   $\Big]_{x=0 \text{ to } R}$

If  $z$  is so large that Equation (204) becomes unwieldy, the following asymptotic series may be used to a close approximation.

$$I_o(z) \approx \frac{\epsilon^2}{\sqrt{2\pi z}} \left[ 1 + \frac{1^2}{1!(8x)} + \frac{1^2 \times 3^2}{2!(8x)^2} + \dots \right] \quad (205)$$

where  $\epsilon$  = basis of natural logarithms.

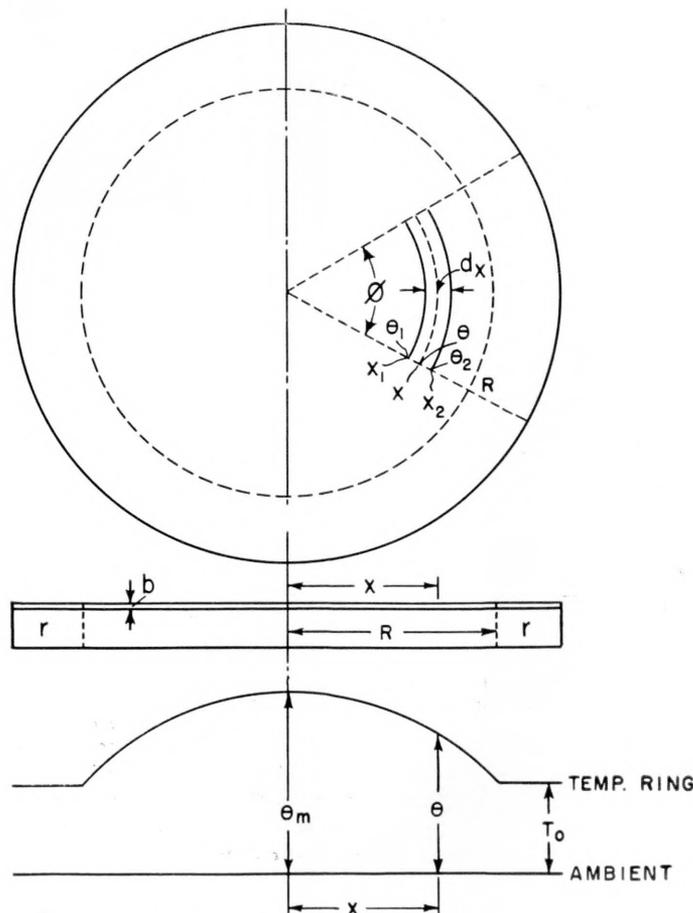


Figure 30—Circular disk thermally connected at its rim to a heat-absorbing ring  $r$ . The diagram below the disk illustrates the distribution of temperature.



**32(a). Temperature at Center.**

In practical cases it is usually desired to know the temperature at the center of the disk above ambient. To determine this, set  $x=0$  in Equation (203) and we have the temperature at the center.

$$\theta_m = \frac{w}{h} \left[ 1 - \frac{1}{I_0(R\sqrt{h/(kb)})} \right] + \frac{T_o}{I_0(R\sqrt{h/(kb)})} \quad (206)$$

This equation is similar in form to Equation (186) given in Section 25(a), Part IX, for a uniform conductor connected between terminals, namely

$$\theta_m = \theta_o \left[ 1 - \frac{1}{\cosh\left(\frac{L}{2} u\right)} \right] + \frac{T_o}{\cosh\left(\frac{L}{2} u\right)}$$

in which  $\theta_o = w/h$ ,  $L/2$  corresponds to  $R$ ,  $u = \sqrt{h/(kb)}$ , and the terminals have the same temperature  $T_o$ .

The curves in Figure 31 give the ratios of the temperature  $\theta_m$  above ambient at the center of the disk, to the temperature above ambient,  $\theta_o = w/h$ , which would result if all the heat were dissipated by convection and none by conduction to the rim, as a function of  $R\sqrt{h/(kb)}$ . For a comparison, a similar curve is given for a uniform conductor connected between terminals, as a function of  $\frac{L}{2} \sqrt{h/(kb)}$ . For these curves, it is assumed that the rim and terminals are at ambient temperature, that is,  $T_o = 0$ . These curves show that the temperatures in the disk are lower than in the strip for the same conditions.

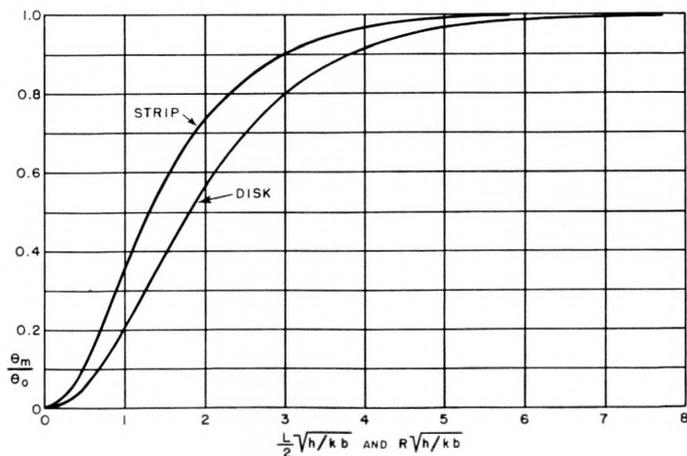


Figure 31—Ratio,  $\theta_m/\theta_o$ , of the temperature,  $\theta_m$ , above ambient at the center of the disk heated uniformly at a constant rate and cooled by convection and conduction, to the temperature above ambient  $\theta_o = w/h$  which would result if all the heat were dissipated by convection and none by conduction, as a function of  $R\sqrt{h/(kb)}$ . A similar curve is shown for a uniform conductor connected between terminals for comparison, as a function of  $\frac{L}{2} \sqrt{h/(kb)}$ . In these curves, it is assumed that ring and terminals are at ambient temperature,  $T_o = 0$ .

**32(b). The Case Where No Heat Is Added From an External Source.**

Under this condition,  $w = 0$ , and from Equation (203),

the temperature above ambient at the center of the disk becomes

$$\theta_m = \frac{T_o}{I_0(R\sqrt{h/(kb)})} \quad (207)$$

This shows that in general, when no heat is applied, the disk does not have the same temperature as the medium, which will cause an error when it is to be used as a thermometer, since it is affected by the temperature of the rim. In fact, Equation (207) is the error resulting if the disk is used as a thermometer element.

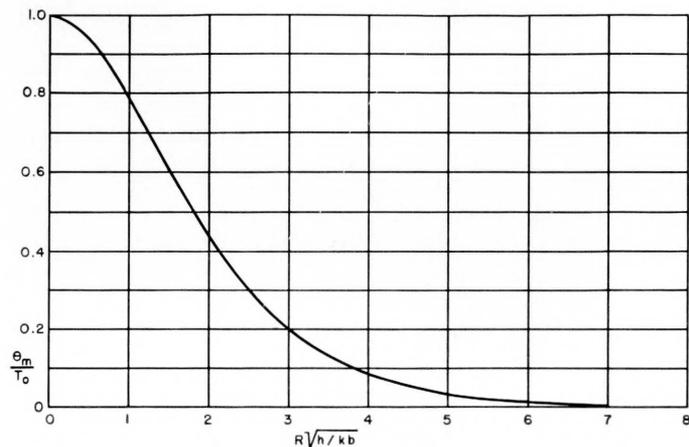


Figure 32—Ratio,  $\theta_m/T_o$ , of the temperature,  $\theta_m$ , above ambient at the center of a circular disk exchanging heat with the surrounding medium when no heat is added from an external source, to the temperature above ambient of the rim,  $T_o$ , as a function of  $R\sqrt{h/(kb)}$ .

Figure 32 gives the ratio  $\theta_m/T_o$ , of this deviation in temperature,  $\theta_m$ , at the center of the disk from the ambient temperature, to the temperature of the rim,  $T_o$ , above ambient, as a function of the parameter  $R\sqrt{h/(kb)}$ . For example, if the temperature of the rim is 10 degrees above ambient, and it is required that the error shall not exceed one degree at the center of the disk, then the curve shows that for a value of  $\theta_m/T_o = 0.1$ , the disk must be so designed that  $R\sqrt{h/(kb)}$  exceeds 3.8. To accomplish this, many kinds of metals and non-metals are available.

**32(c). The Case Where the Heat Exchanged With the Medium Is Negligible Relative to That Conducted to the Rim.**

Under this condition,  $h$  becomes zero and Equation (203) assumes the indeterminate form

$$\theta = \frac{0}{0} + T_o, \text{ since } I_0(z) \Big|_{(z=0)} = 1$$

To evaluate this, Equation (203) may be written

$$\theta = \frac{w}{h} \left[ \frac{I_0(R\sqrt{h/(kb)}) - I_0(x\sqrt{h/(kb)})}{I_0(R\sqrt{h/(kb)})} \right] + T_o \quad (208)$$

now when  $h$  becomes very small, approaching zero, we may write the Bessel functions as a series in which all values higher than the squares are negligible. Then from Equation (204), Equation (208) becomes



$$\left. \theta \right]_{h \rightarrow 0} = \frac{w}{h} \left[ \frac{1 + \frac{1R^2h}{4kb} - 1 - \frac{1}{4} \frac{x^2h}{kb}}{1} \right] + T_o$$

from which

$$\left. \theta \right]_{h=0} = \frac{w}{4kb} (R^2 - x^2) + T_o \quad (209)$$

This result checks with that readily derived by direct analysis for this assumed condition. It shows that the temperatures in the disk above the temperature of the rim are exactly half those in the uniform conductor above its terminals for corresponding positions relative to their centers, when half the strip length  $L/2 = R$ , and when they have the same thickness and thermal constants.

### 32(d). Case Where the Heat Conducted to the Rim Is Negligible Relative to That Exchanged With the Medium.

Under this condition, either the conductivity  $k$  or the thickness  $b$  or both may approach zero; in which case we have the Bessel functions with practically an infinite argument.

Now for very large arguments, Equation (205) shows that the Bessel function

$$I_o(z) \approx \frac{e^z}{\sqrt{2\pi z}} \quad (210)$$

Applying this value to Equation (203), we have

$$\left. \theta \right]_{k \text{ or } b \rightarrow 0} = \frac{w}{h} \left[ 1 - \frac{\epsilon^{x\sqrt{h/(kb)}}}{\epsilon^{R\sqrt{h/(kb)}}} \times \sqrt{\frac{2\pi R\sqrt{h/(kb)}}{2\pi x\sqrt{h/(kb)}}} \right] + 0$$

$$= \frac{w}{h} \left[ 1 - \epsilon^{-(R-x)\sqrt{h/(kb)}} \times \sqrt{\frac{R}{x}} \right]$$

Since the second member equals  $\epsilon^{-\infty} = 0$ , except where  $x = R$ , when  $k$  or  $b$  approaches zero, then everywhere except at the rim,

$$\theta = \frac{w}{h} \quad (211)$$

This result can also be deduced readily by simple physical reasoning.

### 32(e). Case Where the Two Sides of the Disk Are in Contact With Two Different Mediums at Different Temperatures.

This may occur, for example, when one side is in contact with air and the other side with water.

Let the two mediums have the actual temperatures  $T_1$  and  $T_2$ , that is, not relative to ambient, and convection constants  $h_1$  and  $h_2$  respectively, then, as will be shown later, both sides of the disk may be considered as being in contact with a single fictitious medium having an actual temperature

$$T_a = \frac{h_1 T_1 + h_2 T_2}{h_1 + h_2} \quad (212)$$

which, when used as the ambient temperature in the equations given for a single common medium, the re-

sulting equations will be applicable to the condition where two different mediums are used. The convection transfer of heat from both sides per unit area is

$$h = h_1 + h_2$$

Therefore if  $T$  is the actual temperature at any radius  $x$  of the disk and  $T_r$  is the actual temperature of the rim,  $T_r - T_a$  may be substituted for  $T_o$ , and  $T - T_a$  for  $\theta$  in the general Equation (203), and then the temperature at  $x$  is

$$T = \left( \frac{w}{h} + T_a \right) \left[ 1 - \frac{I_o(x\sqrt{h/(kb)})}{I_o(R\sqrt{h/(kb)})} \right] + T_r \left[ \frac{I_o(x\sqrt{h/(kb)})}{I_o(R\sqrt{h/(kb)})} \right] \quad (213)$$

where  $h = h_1 + h_2$ .

### 32(f). Example.

As an example of the use of these equations, consider a disk of stainless steel, 6 cm. in diameter and 0.025 cm. thick, connected at its rim to a heavy heat-conducting ring at a temperature above ambient of 10 Centigrade degrees, in still air. Let radiant energy incident upon the disk be absorbed at the rate of 0.1 watt per sq. cm., and let the convection loss from both sides combined be 0.0022 watt per sq. cm. per 1 degree C. Compute the temperature at the center of the disk above ambient.

Then in Equation (206),  $R = 3.0$  cm.,  $b = 0.025$  cm.,  $h = 0.0022$ ,  $w = 0.1$ , and the thermal conductivity of the stainless steel used is 0.15. Then the argument of the Bessel function,  $R\sqrt{h/(kb)} = 3\sqrt{0.0022/(0.15 \times 0.025)} = 2.295$ . From tables of modified Bessel functions of zero order and the first kind,  $I_o(2.295) = 2.819$ .

Then from Equation (206), the temperature at the center above ambient is

$$\theta_m = \frac{0.1}{0.0022} \left( 1 - \frac{1}{2.819} \right) + \frac{10}{2.819} = 29.3 + 3.55 = 32.85 \text{ deg.}$$

Cent.

### 33. COMPENSATED THERMOCOUPLE CIRCULAR DISK RADIATION METER.

If a circular disk is used as a device for measuring radiant power, it may be compensated for the effect of rim and ambient temperatures in a manner similar to the method used for a strip connected between terminals, as described in Section 30, Part IX. Instead of a uniform compensating strip as is used for the strip conductor, a circular sector is used for the disk. The rim of the compensating sector is in thermal contact with the heat-absorbing ring but on the opposite side from the disk. Thus, the front face of the disk is unobstructed, and the sector is shielded from the radiation.

The thermal parameter of the compensating sector,  $\sqrt{h/(k_1 b_1)}$ , must be the same as that of the disk,  $\sqrt{h/(kb)}$ . The disk and compensating sector need not be of the same material or thickness so long as the thermal constants referred to are equal.

The hot junction of the thermocouple is in thermal contact with the center of the disk and its cold junction in contact with the center of the compensating sector, which may be slightly enlarged at this point for good

contact. If desired for reasons of design, a corresponding sector may be extended in one piece to the opposite side of the ring, making the two sectors meet at the center at which an enlargement may be provided for uniting the sectors and for thermocouple contact.

The temperatures above ambient of the center of the disk and of the center of the sector are given by Equations (206) and (207) respectively. The thermocouple measures the difference between these two temperatures, that is

$$\theta = (\theta_m)_{\text{disk}} - (\theta_m)_{\text{sector}} = \frac{w}{h} \left[ 1 - \frac{1}{I_0(R\sqrt{h/(kb)})} \right]$$

which is proportional to the radiation power,  $w$ , alone, as it is independent of ambient and ring temperatures.

### 34. DERIVATION OF EQUATIONS (203) AND (212).

Figure 30 illustrates a circular disk connected at its rim to a heat-absorbing ring at a temperature  $T_o$  above ambient assumed constant. The diagram below the disk illustrates the temperature distribution.

To make the analysis more general, we will apply it to a sector of the disk having an angle  $\phi$ . Let  $x_1$  and  $x_2$  be two radii at a distance  $dx$  apart, and let the temperatures above ambient at these two radii be  $\theta_1$  and  $\theta_2$ , differing by  $d\theta$ . Then the rate at which heat passes the arc of the sector at radius  $x_1$  is

$$-kbx_1\phi \left( \frac{d\theta_1}{dx_1} \right) \quad (214)$$

and the rate at which heat passes the arc at  $x_2$  is

$$-kbx_2\phi \left( \frac{d\theta_2}{dx_2} \right) \quad (215)$$

Now the rate at which heat is applied to the circular strip  $dx$  wide subtended by the angle  $\phi$  is  $w x \phi dx$  and the rate at which heat is exchanged between the circular strip and the medium is  $h \theta x \phi dx$ . Therefore the net gain in heat per unit time in this strip is the difference between these two quantities or

$$(w - h\theta)x\phi dx \quad (216)$$

This must equal the difference between the two rates (214) and (215) or

$$kb\phi \left[ -x_2 \left( \frac{d\theta_2}{dx_2} \right) + x_1 \left( \frac{d\theta_1}{dx_1} \right) \right] \quad (217)$$

now

$$\frac{d\theta_2}{dx_2} = \frac{d\theta_1}{dx_1} + \frac{d}{dx_1} \left( \frac{d\theta_1}{dx_1} \right) dx_1$$

and

$$x_2 = x_1 + dx$$

Substituting these quantities in the quantity (217), reducing and dropping the sub numbers, and equating it to the quantity (216) we have

$$-kb \left[ \frac{d^2\theta}{dx^2} + \frac{1}{x} \frac{d\theta}{dx} \right] = w - h\theta \quad (218)$$

Since the angle  $\phi$  disappears from the equation, it

follows that the temperature distribution is independent of the angle of the sector and therefore is the same for the entire disk as for any sector of it. This is also evident by simple reasoning from the symmetry of the figure.

Equation (218) may be written

$$\frac{d^2\theta}{dx^2} + \frac{1}{x} \frac{d\theta}{dx} + \frac{w - h\theta}{kb} = 0 \quad (219)$$

To solve, let

$$\frac{w - h\theta}{kb} = y$$

then

$$\frac{d\theta}{dx} = -\frac{kb}{h} \frac{dy}{dx}; \quad \frac{d^2\theta}{dx^2} = -\frac{kb}{h} \frac{d^2y}{dx^2}$$

when these are inserted in Equation (219), we have

$$\frac{d^2y}{dx^2} + \frac{1}{x} \frac{dy}{dx} - \frac{h}{kb} y = 0 \quad (220)$$

for brevity let  $-h/(kb) = -\alpha^2 = j^2\alpha^2$  (221)

where

$$j = \sqrt{-1}$$

Then Equation (220) becomes

$$\frac{d^2y}{dx^2} + \frac{1}{x} \frac{dy}{dx} + j^2\alpha^2 y = 0 \quad (222)$$

Again let

$$j\alpha x = z, \quad (223)$$

then

$$\frac{dy}{dx} = j\alpha \frac{dy}{dz}, \quad \frac{d^2y}{dx^2} = (j\alpha)^2 \frac{d^2y}{dz^2}$$

Substituting these in Equation (222), we have

$$\frac{d^2y}{dz^2} + \frac{1}{z} \frac{dy}{dz} + y = 0 \quad (224)$$

This is a Bessel equation, the solution of which is

$$y = A_1 J_0(z) + B_1 Y_0(z) \quad (225)$$

in which  $J_0(z)$  and  $Y_0(z)$  are Bessel functions of the first and second kinds, respectively, of zero order, and  $A_1$  and  $B_1$  are constants of integration.

Inserting the value of  $z$  from Equation (223), we have

$$y = A_1 J_0(j\alpha x) + B_1 Y_0(j\alpha x) \quad (226)$$

which may be written as

$$y = A_1 I_0(\alpha x) + B_1 K_0(\alpha x) \quad (227)$$

where  $I_0(\alpha x) = J_0(j\alpha x)$  and  $K_0(\alpha x) = Y_0(j\alpha x)$  are modified Bessel functions of the first and second kinds, respectively, of zero order.

To determine constants  $A_1$  and  $B_1$ ; when  $x=0$ ,  $K_0(\alpha x) = -\infty$ . Since  $\theta$  must be finite,  $B_1$  must be zero, and Equation (227) becomes

$$y = A_1 I_0(\alpha x) \quad (228)$$

Substituting the value of  $y$  in Equation (228), we obtain

$$\frac{w - h\theta}{kb} = A_1 I_0(\alpha x) \quad (229)$$

Now when  $x=R$ ,  $\theta=T_o$ , which when substituted in Equation (229) and solving for  $A_1$  give



$$A_1 = \frac{w - hT_o}{kbI_o(\alpha R)}$$

Substituting this in Equation (229); inserting the value for  $\alpha$ ; and solving for  $\theta$ , we have

$$\theta = \frac{w}{h} \left[ 1 - \frac{I_o(x\sqrt{h/(kb)})}{I_o(R\sqrt{h/(kb)})} \right] + T_o \left[ \frac{I_o(x\sqrt{h/(kb)})}{I_o(R\sqrt{h/(kb)})} \right],$$

which is Equation (203).

**34(a). When the Two Sides of the Disk Are in Contact With Two Different Mediums at Different Temperatures.**

Let the actual temperatures of the two mediums be  $T_1$  and  $T_2$ , and their convection constants  $h_1$  and  $h_2$  respectively; and  $T$  the actual temperature of the disk at any position  $x$ ; and  $T_r$  that of the ring.

Then the total rate of heat exchange between both sides and their respective mediums per sq. cm. of the disk is

$$h_1(T - T_1) + h_2(T - T_2)$$

which by rearranging may be written

$$(h_1 + h_2) \left( T - \frac{h_1 T_1 + h_2 T_2}{h_1 + h_2} \right) \tag{230}$$

in which  $h_1 + h_2$  is the combined convection constant

for the two sides, and  $\frac{h_1 T_1 + h_2 T_2}{h_1 + h_2}$  designated  $T_a$  for

brevity is the temperature of a single medium having a convection constant  $h = h_1 + h_2$  common to both sides, which would be equivalent in effect to that of the two mediums.

Then in the equations for  $\theta$  previously given, if  $(T - T_a)$  is substituted for  $\theta$ ,  $h_1 + h_2$  for  $h$ , and  $(T_r - T_a)$  for  $T_o$ , we can obtain an equation for the disk temperature, as for example was done in Equation (213).

*Reference:*

<sup>1</sup> "Table of Functions," *Janke and Emde*, a book.

E. N.—No. 85 Cont.

—W. N. Goodwin, Jr.

## MINIATURE RECORDER USED IN RED CROSS BLOOD PROGRAM

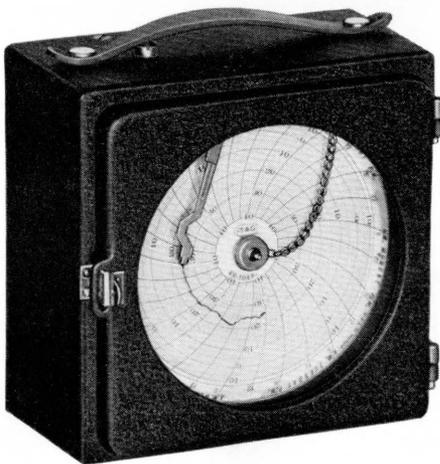
WHOLE blood stored in refrigerated "blood banks" is very susceptible to damage caused by small changes in its temperature. The National Institute of Health, in its requirement for the storage of citrated whole blood, specifies that the blood should be maintained between the temperatures of 4 and

been adversely affected by temperature.

A National Blood Program Directive emphasizes the necessity for continuous accurate records of the temperature of whole blood in storage. "It is extremely important in the storage of whole blood that a definite record be kept showing the temperature at all times within

storage refrigerators. Such temperature recording devices should be in operation on all refrigerators in which blood is stored. Graphs or charts which record the accurate temperature should be identified with the proper refrigerator, dated, and filed for any future reference."

The unit which is adaptable to this task is the Weston Model 8475



Weston Model 8475 Miniature Recorder.

10 degrees C. (39.2 and 50 degrees F.) and preferably between 4 and 6 degrees C. (39.2 and 42.8 degrees F.). The Weston Model 8475 Temperature Recorder has been used successfully to give assurance that the quality of the blood has not

*NOTE: This space has been left blank to prevent the destruction of copy if the reader chooses to correct his copy of Volume 8, Number 2, by clipping the diagram shown on the reverse side of this page.*

Miniature Recorder, a distant reading type equipped with five feet of connecting tubing. With this model it is possible to keep the bulb of the instrument within the refrigerator and the recorder on the outside for ease of rewinding the drive and

changing the chart. A standard chart having a range of  $-20$  to  $+20^{\circ}$  C. for twenty-four hour revolution is well suited to the blood refrigerator application.

E. N.—No. 100

—E. T. Oettinger.

If you plan to attend the 1954

## RADIO ENGINEERING SHOW

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#### CORRECTION NOTICE

We wish to call your attention to an error in the article "A Single Point Ammeter Using a Current Transformer," which appeared in Volume 8, Number 2, issue of WESTON ENGINEERING NOTES. The diagram shown in the middle of page 4 of that issue is incorrect. The correct diagram is printed below. Any reader desiring to correct his copy of Volume 8, Number 2, may clip the diagram below and paste it over the incorrect diagram.

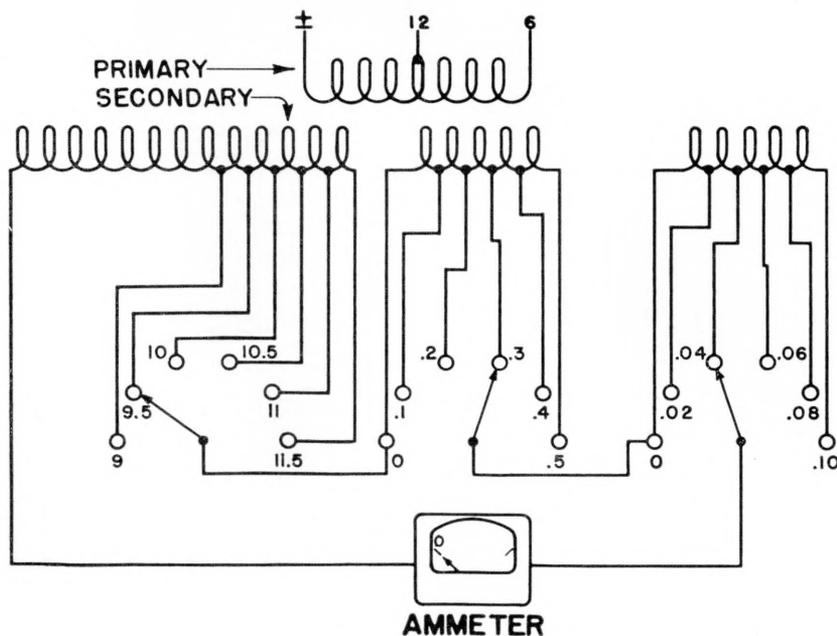


Figure 1.

### "Westonia"

E. F. Weston was listening to a lunch table discussion on fixed resistors. Finally he felt impelled to comment. "I wonder if the people who are bragging about the so-called pyrolytic-deposited carbon resistors know that it is really an old Weston development. The Doctor (he always refers to the original Weston in this manner) used this trick of depositing carbon from a hydrocarbon gas in his early work with incandescent carbon filament lamps. He picked up the trick, in turn, from his observations as a boy in the ovens of a gas plant in Wales. Look up Baekeland's Perkin Medal Address!"

So we did. And Baekeland said, in 1915, on the occasion of the award of the Perkin Medal to Dr. Weston, "Weston remembered that as a boy, when he went to visit the gas works to obtain some hard carbon for his Bunsen cell, this carbon was collected from those parts of the gas retort which had been the hottest, and where the hydrocarbon gas had undergone dissociation, leaving a dense deposit of coherent carbon.

"In this chemical phenomenon of dissociation at high temperature, he perceived a chemical means for 'self-curing' any weak spots in the filament of his lamp. The remedy was as ingenious as simple. In preparing his filament, he passed the current through it while the filament was placed in an atmosphere of hydrocarbon gas, so that in every spot where the temperature rose highest on account of greatest resistance, brought about by the irregular structure of the material, the gas was dissociated and carbon was deposited automatically until the defect was cured."

This is the process used in making pyrolytic carbon resistors except that the carbon is deposited on a porcelain or steatite tube heated in a furnace instead of on a carbon filament heated by current passing through it. And thus we seem to turn a full circle in some 70 years.